Biomechanical loading during running: can a two mass-spring-damper model be used to evaluate ground reaction forces for high-intensity tasks?

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Abstract: Running impact forces expose the body to biomechanical loads leading to beneficial adaptations, but also risk of injury. High-intensity running tasks especially, are deemed highly demanding for the musculoskeletal system, but loads experienced during these actions are not well understood. To eventually predict GRF and understand the biomechanical loads experienced during such activities in greater detail, this study aimed to 1) examine the feasibility of using a simple two mass-spring-damper model, based on eight model parameters, to reproduce ground reaction forces (GRFs) for high-intensity running tasks and 2) verify whether the required model parameters were physically meaningful. This model was used to reproduce GRFs for rapid accelerations and decelerations, constant speed running and maximal sprints. GRF profiles and impulses could be reproduced with low to very low errors across tasks, but subtler loading characteristics (impact peaks, loading rate) were modelled less accurately. Moreover, required model parameters varied strongly between trials and had minimal physical meaning. These results show that although a two mass-spring-damper model can be used to reproduce overall GRFs for high-intensity running tasks, the application of this simple model for predicting GRFs in the field and/or understanding the biomechanical demands of training in greater detail is likely limited.

Keywords: GRF modelling, Model parameter optimisation, Training load monitoring, Whole-body loading, Biomechanical demands

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Introduction

In running-based sports, the different structures of the body are repetitively exposed to biomechanical loads. These loads can lead to beneficial adaptations on the one hand (Couppe et al., 2008; Timmins, Shield, Williams, Lorenzen, & Opar, 2016), but also risk of injuries (Gabbett & Ullah, 2012). High-intensity running tasks especially (e.g. accelerating, decelerating, sprinting) (Akenhead, French, Thompson, & Hayes, 2014; Vigh-Larsen, Dalgas, & Andersen, 2018), are deemed highly demanding for the musculoskeletal system, but the biomechanical loads experienced during these actions are not well understood (Vanrenterghem, Nedergaard, Robinson, & Drust, 2017). Therefore, measuring and monitoring the ground reaction forces (GRFs) for these movements in non-laboratory settings would allow for a more detailed understanding of the biomechanical demands of training.

GRFs resulting from collisions with the ground during running are absorbed and returned by the body in a spring-like manner. Therefore, simple mass-spring models (single point mass attached to a spring) have been used to investigate various GRF characteristics (e.g. Blickhan, 1989; Dutto and Smith, 2002; Morin et al., 2005). The sinusoidal GRF profiles predicted by this model do however not accurately represent the typical double-peak GRF profiles of running (Alexander, Bennett, & Ker, 1986; Bullimore & Burn, 2007). These characteristic force peaks can substantially deviate between various tasks and are thus essential for examining the specific whole-body loads experienced during different running tasks. Based on the distinct contributions of the lower limb and upper body segments to the GRF during running (Bobbert, Schamhardt, & Nigg, 1991; Clark, Ryan, & Weyand, 2017), a two mass-spring-damper model can be used to describe the distinct impact and active peaks during simple elastic movements, i.e. steady running (Alexander et al., 1986; Derrick, Caldwell, & Hamill, 2000). However, the ability of this model (which is based on eight parameters that describe simple mechanical characteristics of the body) to reproduce GRF profiles for high-intensity running tasks is yet completely unknown.

If a simple two mass-spring-damper model can reproduce GRFs for non-elastic high-intensity tasks, while retaining physically meaningful model parameters, this might eventually be used to predict GRF in the field and understand the biomechanical demands of such activities in greater detail. Therefore, this study aimed to use a two mass-spring-damper model to reproduce GRF profiles for activities that are frequently performed during running-based sports. It was hypothesised that 1) this model could accurately replicate measured GRF and loading characteristics for high-intensity running tasks, and 2) that its model parameters could be used to evaluate the biomechanical demands of these activities.

Methods

Fifteen healthy and physically active team-sports athletes participated in this study. Participants provided informed consent according to Liverpool John Moores University ethics regulations. After a warm-up, participants performed rapid accelerations from standstill to sprinting, decelerations from sprinting to standstill, and running trials at constant speeds from 2 m/s to maximal sprinting speed (~6-9 m/s, individual specific), with 1 m/s stepwise increases. For each trial, GRF data were collected at 3000 Hz with a force platform (9287B, Kistler Holding AG, Winterthur, Switzerland), filtered using a 50 Hz second-order Butterworth low-pass filter and normalised to body mass. To evaluate the total magnitude of load experienced during the different running tasks, resultant GRFs (overall whole-body loading) were calculated from the three force components and used for this investigation.

A two mass-spring-damper model described by eight natural model parameters (Figure

1) was used to reproduce measured GRFs (Alexander et al., 1986; Derrick et al., 2000). The model consisted of a lower mass m_2 on a spring and damper, representing the support leg, with an upper mass m_1 on a spring on top, representing the rest of the body. The positions of the upper and lower mass without any external load was described by x_1 and x_2, while l_1 and l_2 were the natural lengths of the upper and lower springs respectively. The linear spring stiffness constants for the upper and lower spring were defined as k_1 and k_2, while c was the damper's damping coefficient. From these nine parameters the eight natural parameters were derived according to Equations 1-8 (Table 1), with BM being the total body mass. The model's motion was described by the accelerations of its upper and lower mass (Table 1, Equation 9 and 10), in which $a_{1,2}$, $v_{1,2}$ and $p_{1,2}$ were the upper and lower mass accelerations, velocities and positions respectively, λ the upper mass ratio relative to the lower mass, ω_1 and ω_2 the natural frequencies of the upper and lower spring, ζ the damper's damping ratio, and g the gravitational acceleration (-9.81 m/s^2). For each trial, a unique parameter set to fit modelled GRFs to measured GRFs was determined by solving Equations 9 and 10 (Table 1). The equations were solved with a purpose-written Python optimisation script, which included the L-BFGS-B numerical optimisation algorithm (Python, 2017; SciPy, 2017). Starting conditions for the optimisation were as described in Appendix A and parameters following from the optimisation process were used to calculate modelled GRFs (Table 1, Equation 11). Optimal model parameter combinations were determined by minimising the sum of the root mean square error (RMSE) of the GRF and its gradient, between modelled and measured GRF curves.

Modelled GRF accuracy was evaluated by RMSE and errors of relevant GRF loading characteristics impulse (area under the GRF curve), impact peak (force peak during the first 30% of stance) and loading rate (average GRF gradient from touch-down to impact peak). Error metrics were averaged across trials and participants for each task, i.e. accelerations, decelerations, and running at constant low (2-3 m/s), moderate (4-5 m/s) and high (>6 m/s) speeds. RMSE was rated very low (<1 N/kg), low (1-2 N/kg), moderate (2-3 N/kg), high (3-4 N/kg) or very high (>4 N/kg). GRF loading characteristic errors were rated very low (<5%), low (5-10%), moderate (10-15%), high (15-20%) or very high (>20%). Furthermore, correlation analyses were performed between modelled and measured impulses, impact peaks and loading rates, and rated as very weak (R^2<0.1), weak (R^2=0.1-0.3), moderate (R^2=0.3-0.5), strong (R^2=0.5-0.7), very strong (R^2=0.7-0.9) or extremely strong (R^2=0.9-1) (Hopkins, Marshall, Batterham, & Hanin, 2009).

Results

GRF profiles were reproduced with high accuracy across tasks (Figure 2; Table 2). RMSE was very low for accelerations, as well as low- and moderate-speed running, but increased for high-speed running and especially decelerations. Furthermore, impulses were modelled with very high accuracy (errors <1%). Consequently, the correlation between measured and modelled impulses was extremely strong ($p<0.001$) across tasks (Figure 3A) while errors were independent of task and magnitude (Figure 3B and C).

Since not all trials contained a distinct measured impact peak (e.g. accelerations (Figure 2A) or forefoot-strike sprints (Figure 2G)) and for several trials the impact peak could not be modelled (Figure 2B, F and H), only a select number of trials were included in the impact peak and loading rate analysis (Table 2). Impact peaks were modelled with low to moderate errors for constant speed running, but high to very high for accelerations and decelerations. Similarly, modelled loading rate errors were high to very high across tasks. Nevertheless, modelled and measured impact peaks and loading rates had an extremely strong correlation across tasks (Figure 3D and G). Absolute errors significantly ($p<0.001$) increased for higher impact peaks and loading rates (Figure 3E and H), but relative errors remained constant independent of task and magnitude (Figure 3F and I).

Despite the accurately reproduced GRF curves, all model parameters varied strongly between and within tasks (Figure 4; Table 3). Especially motion (p_1, p_2, v_1, v_2) and mass (λ) related parameters were highly variable for decelerations, while ω_1 and ω_2 strongly varied for all tasks. Although ζ varied less between tasks, within task variability was large.

Discussion and Implications

The purpose of this study was to investigate whether a simple two mass-spring-damper model can reproduce GRFs for high-intensity running tasks, while retaining physically meaningful parameters. Across tasks, GRF curves could be reproduced with low to moderate curve errors. The slightly higher errors observed in decelerations were likely due to the distinct GRF profiles. The model typically underestimated the high impact peaks and loading rates but overestimated the much lower second (active) peak (Figure 2C and D). Previous studies also reported increased modelled curve errors in tasks (Nedergaard, 2017) and individuals (Derrick et al., 2000) with considerably higher impact peaks. Nedergaard (2017) suggested higher curve errors to be due to lower spring stiffnesses, which reduces the magnitude of the impact peak (Derrick et al., 2000; Nedergaard, 2017). Moreover, Derrick et al. (2000) showed that to increase the impact peak, higher values are required for spring stiffnesses ω_1 and ω_2, upper mass velocity v_1 and mass ratio λ, together with a reduced damping ratio ζ. In this study, mean v_1 and λ

values were indeed substantially higher for decelerations compared to other tasks, but ω_1, ω_2 and ζ were in a similar range as other tasks (Figure 4; Table 3). For GRF profiles with high impact peaks, the model likely needs to adjust as many parameters as possible to reproduce this first peak, while maintaining an accurate representation of the rest of the curve characteristics (e.g. active peak, stance time).

Impulses were modelled with very high accuracy (≈0.01 Ns/kg) and had a perfect correlation (R^2=1) with measured impulses. These results are in accordance with errors (≈0.01 Ns/kg) and correlations (R^2=0.98-1) found by Nedergaard (2017), but much lower than Derrick et al. (2000) who reported impulse errors of 5.5-8.5 Ns (≈0.08-0.12 Ns/kg). Since the latter study only optimised ω_1, ω_2 and p_2, the better results in the present study are likely the result of including all model parameters in the optimisation process. Therefore, the two mass-spring-damper model can give very good estimates of overall loading across tasks.

In contrast to overall loads, subtle loading characteristics (impact peak and loading rate) were modelled less accurately. The initial force peak due to the lower limb colliding with the ground (Clark et al., 2017), is typically followed by a slight decrease in GRF before gradually increasing to the active peak caused by the upper body (Bobbert et al., 1991). For accelerations and steady running this force decrease is small and forms a minor part of the whole GRF profile. Since curve gradients and RMSEs were used as model parameter optimisation criteria, a continuously rising curve from touch-down to mid-stance (thus ignoring the impact peak) affected these criteria minimally. This explains that for 99% of the decelerations, in which the impact peak dominates the GRF profile, impact peaks were visible in the modelled curves, compared to only 34-48% for accelerations and steady running. Moreover, impact peaks (and loading rates) were typically underestimated with errors increasing for higher impact peaks. In general, differences between measured impact and active peaks increased for higher impact peaks (compare for example Figures 2C and D). Most model parameters affecting the impact peak influence the active peak simultaneously (Derrick et al., 2000). Therefore, the model likely underestimated the higher impact peaks more, to limit the overestimation of the second peak.

Despite the higher errors, correlations between measured and modelled impact peaks and loading rates were extremely strong (R^2=0.96-0.97) (Figure 3D and G). These correlations are stronger than Udofa et al. (2016), who used a two mass model to reproduce GRFs found correlations of R^2=0.82 between measured and modelled impact peaks, across different running speeds (3-6 m/s) and loading conditions. The strong linear relationships observed in this study (Figure 3A, D and G) might be used to adjust modelled impact peaks and loading rates to get more accurate estimates of these loading characteristics.

A limitation of the two mass-spring-damper model is the assumption of spring-like (elastic) behaviour, meaning a constant spring stiffness during stance. Moreover, the model's damper absorbs energy while energy producing elements are not included. The leg is however known to be stiffer during landing than take-off (Blickhan, 1989), while the muscle-tendon units produce more work during the push-off phase (Cavagna, 2006). Although the high-intensity tasks investigated in this study seriously violated these model assumptions, reproduced GRF profiles were fairly accurate. The model likely overcompensates for the absence of active elements by substantially increasing its stiffness (i.e. higher ω_1 and ω_2), in accordance with reduced energy requirements for higher leg stiffness (Dutto & Smith, 2002; McMahon & Cheng, 1990). This might explain why higher stiffness was observed for accelerations and high-speed running, where the muscles need to produce more energy, compared to decelerations, where energy is primarily absorbed (Figure 4; Table 3). Due to the strong variability within tasks however, parameters should be interpreted with caution.

Another limitation of this study is the complexity of model parameter combinations. As described above, different parameters represent multiple physical aspects (e.g. leg stiffness) and affect various GRF characteristics (e.g. impact peak, stance time) at the same time (Derrick et al., 2000). During the optimisation process, numerical solvers searched for optimal modelled GRF solutions in the highly complex eight-dimensional parameter space. Therefore, numerous similarly good solutions might be found for comparable GRF curves, leading to the high parameter variability and physically unrealistic parameter values observed across trials (Table 3). For example, many modelled GRF solutions were found to have λ values larger than 20, meaning that for those trials the lower mass (support leg) was negligible relative to the rest of the body. Model parameters found in this study therefore have little physical meaning, limiting the biomechanical interpretability of the model. Moreover, an exploration during which the parameter search spaces were restricted to physically meaningful values did not lead to more consistency in parameter values within or between tasks, while the accuracy of modelled GRF profiles was reduced (Appendix B).

A possible explanation for the limited model parameter interpretability described above, is the choice to reproduce a three-dimensional (resultant) GRF with a one-dimensional model. The authors chose to reproduce the total force magnitude to allow for investigating the overall whole-body load experienced during the different running tasks. Consequently, horizontal segmental movements leading to the horizontal forces included in the resultant GRF, had to be accounted for by the vertical motion in the model. Since vertical motion was described by the eight model parameters, this might have contributed to the inconsistent parameter values observed and the lack of physical meaning. Horizontal movements and forces are, however, relatively small compared to the vertical components, and are thus unlikely to have considerably affected the results in this study. Moreover, exploratory work revealed that using the vertical component of GRF only, did not noticeably improve the reproduced GRF profiles or enhance the interpretability of the model parameters.

In this study, GRFs were reproduced by adjusting model parameters to fit measured GRFs. However, in applied sport settings (e.g. football pitch, running track, etc.), measured GRF is not available and other methods are required to estimate model parameters and predict GRF. Since the two mass-spring-damper model's motion is described by the acceleration of its masses, currently popular body-worn accelerometers (Akenhead & Nassis, 2016; Cardinale & Varley, 2017) might be used to estimate the parameters and predict GRFs in the field. However, the large variability and minimal physical meaning of the model parameter values likely limit the usefulness of this approach.

Conclusion

This study aimed to use a two mass-spring-damper model to reproduce GRF profiles for activities that are frequently performed during running-based sports. As hypothesised, the model could be used to reproduce overall GRF profiles for high-intensity running tasks. However, the required model parameters varied strongly between trials and had minimal physical meaning, rejecting our second hypothesis. Therefore, the application of this specific two mass-spring-damper model for predicting GRFs in the field and/or understanding the mechanical aspects of the running tasks investigated in greater detail is likely limited.

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Appendix A

The model parameter optimisation process for accurately reproducing ground reaction forces (GRFs) described in this study, requires the definition of starting conditions for the different parameters. Therefore, a pilot analysis was performed with the model parameters as defined by Derrick et al. (2000), who used the two mass-spring-damper model for constant speed running at 3.83 m/s (± 5%) (Table A1). To verify if these parameters were appropriate as starting conditions for reproducing the GRF profiles for the high-intensity tasks investigated in this study, GRF data for these tasks were modelled for four randomly selected participants. The parameter values reported by Derrick et al. (2000) were used as initial starting conditions for the parameters. After this optimisation process, the resulting median model parameters (Table A1) from this analysis were then used as starting conditions for the whole data set.

Appendix B

The two mass-spring-damper model parameters found in this study varied strongly within and between tasks and had little physical meaning, limiting the model's interpretability. However, due to the highly complex eight-dimensional parameter space, several parameter combinations might result in similarly accurate modelled ground reaction force (GRF) solutions. If the model can accurately replicate GRF profiles across tasks within a range of values that are more physically meaningful, this may improve the interpretability of the model parameters. Therefore, GRF profiles were reproduced with the two mass-spring-damper model within a predefined range of model parameter values. The model's mass ratio λ was fixed at a value of 3 au (i.e. lower mass ~25% of the total body mass), which was estimated from previously described segmental properties of the foot, shank, thigh and pelvis (Dempster, 1955). In addition, the remaining parameter search windows were limited to a range of values that was deemed theoretically reasonable and physically meaningful (note: p_2 was calculated from v_2).

- p_1 = -0.4 – 0.1 m
- v_1 = -3 – 1 m/s
- v_2 = -0.5 – 2 m/s
- ω_1 = 0 – 50 N/m/kg
- ω_2 = 0 – 174 N/m/kg
- λ = 3 au
- ζ = 0.1 – 1.5 au

Root mean square errors (RMSE) of the reproduced GRF profiles from a limited range of parameter values increased for accelerations (+106%), decelerations (+6%) and running at constant low (+29%), moderate (+10%) and high (+20%) speeds, compared to using free parameters search windows. Moreover, the model parameters required to reproduce the measured GRF profiles strongly varied within the defined parameter boundaries (Figure B1). There was no consistency of parameters values within or between any of the parameters or tasks. Moreover, many trials required parameter values equal to the set upper or lower limit of different parameters, indicating the need for higher or lower values than physically reasonable. Therefore, it was concluded that the two mass-spring-damper model cannot be used to replicate GRF profiles with high accuracy across a range of running tasks, using physically meaningful model parameters.

Tables

Table 1 Equations describing the eight natural parameters of the two mass-spring-damper model

Initial position of the upper mass	$p_1 = x_1 - l_1 - l_2$	Equation 1
Initial position of the lower mass	$p_2 = x_2 - l_2$	Equation 2
Initial velocity of the upper mass	$v_1 = \dot{p_1}$	Equation 3
Initial velocity of the lower mass	$v_2 = \dot{p_2}$	Equation 4
Mass ratio	$\lambda = \frac{m_1}{m_2}$	Equation 5
Natural frequency of the upper spring	$\omega_1 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{(1+\lambda) \cdot k_1}{\lambda \cdot BM}}$	Equation 6
Natural frequency of the lower spring	$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \sqrt{\frac{(1+\lambda) \cdot k_2}{BM}}$	Equation 7
Damping ratio of the damper	$\zeta = \frac{c}{2 \cdot \sqrt{k_2 \cdot m_2}}$	Equation 8
Acceleration of the upper mass	$a_1 = -{\omega_1}^2 \cdot (p_1 - p_2) + g$	Equation 9
Acceleration of the lower mass	$a_2 = -{\omega_2}^2 \cdot p_2 + {\omega_1}^2 \cdot \lambda \cdot (p_1 - p_2) - 2 \cdot \zeta \cdot \omega_2 \cdot v_2 + g$	Equation 10
Ground reaction force	$GRF = -\frac{BM \cdot \omega_2}{1+\lambda} \cdot (\omega_2 \cdot p_2 + 2 \cdot \zeta \cdot v_2)$	Equation 11

Table 2 Modelled ground reaction force curve and loading characteristics errors

	RMSE		**Impulse error**		**Impact peak error**		**Loading rate error**	
	N/kg	%	Ns/kg	%	N/kg	%	N/kg/s	%
Accelerations (n=189)	0.69 ±0.47	9.9 ±6.4	0.01 ±0.01	0.6 ±0.5	2.43 ±1.49	18.9 ±11.7	487 ±342	31.3 ±19.9
Decelerations (n=240)	2.48 ±1.17	33.9 ±28.3	0.01 ±0.01	0.7 ±0.5	7.43 ±4	20.6 ±13.7	431 ±276	18.7 ±9.4
Constant speed running								
Low (2-3 m/s; n=126)	0.48 ±0.22	7.6 ±5.8	0.01 ±0	0.4 ±0.3	1.53 ±1.25	10.2 ±8.5	200 ±116	19.1 ±9.8
Moderate (4-5 m/s; n=126)	0.78 ±0.25	9.4 ±3.9	0.01 ±0	0.3 ±0.2	1.54 ±0.86	7.5 ±4.2	254 ±101	20.8 ±6.9
High (>6 m/s; n=176)	1.21 ±0.56	13.6 ±7.1	0.01 ±0	0.3 ±0.2	2.99 ±1.74	12 ±8.1	287 ±156	18.4 ±9.7
All tasks (n=857)	1.28 ±1.06	17 ±19.1	0.01 ±0.01	0.5 ±0.4	5.74 ±3.85	17.4 ±12.2	385 ±247	20.3 ±10.7

Mean ± standard deviations for root mean square errors (RMSE), impulse, impact peak and loading rate errors of the modelled GRF profiles for different tasks. Values are either absolute or relative errors compared to the measured GRF. Impact peak and loading rate (grey shaded) was modelled for 34%, 99% and 48% of the acceleration, deceleration and constant speed running trials respectively.

Table 3 Mean ± standard deviation values for the eight model parameters for the different tasks

	p_1 (m)	**p_2 (m)**	**v_1 (m/s)**	**v_2 (m/s)**	**ω_1 (N/m/kg)**	**ω_2 (N/m/kg)**	**λ (au)**	**ζ (au)**
Accelerations	0.09 ±8.19	-0.7 ±5.47	16.5 ±146.03	0.37 ±5.03	32 ±27	102 ±155	0.4 ±2.29	0.9 ±3.9
Decelerations	-12.97 ±26.35	-0.33 ±1.18	80.98 ±184.71	45.87 ±132.34	24 ±32	114 ±91	161.4 ±474.73	0.4 ±0.5
Constant speed running								
Low (2-3 m/s)	0.63 ±3.14	0.07 ±1.22	-2.89 ±56.17	-0.12 ±1.22	31 ±28	72 ±78	5.87 ±5.9	0.9 ±2.4
Moderate (4-5 m/s)	0.91 ±5.2	0.09 ±0.8	12.67 ±137	-0.2 ±1.13	37 ±35	101 ±106	4.16 ±6.34	0.6 ±1.1
High (>6 m/s)	-2.21 ±13.37	-1.74 ±10.31	-1.83 ±115	0.98 ±12.62	34 ±35	134 ±148	1.93 ±4.99	1.9 ±7
All tasks	-4 ±17.12	-0.57 ±5.32	28.49 ±146.94	13.71 ±74.86	31 ±32	109 ±129	49.38 ±267.09	0.9 ±3.7

Table A1 Initial conditions for the model's eight parameter values for reproducing GRF

	p_1 (m)	p_2 (m)	v_1 (m/s)	v_2 (m/s)	ω_1 (N/m/kg)	ω_2 (N/m/kg)	λ (au)	ζ (au)
Derrick et al. (2000)	0.015*	0.0074	-0.73	-0.66	207**	626**	2	0.35
Optimised	-0.01	0.00	-1.29	-0.19	18.33	58.32	2.81	0.31

The starting parameter values for the model optimisation process as described by Derrick et al. (2000) and those following from a pilot analysis using data for high-intensity running tasks. New (optimised) starting parameters are median values.

* *The upper mass position p_1 was not reported and its value was estimated to be double that of the position p_2 of the lower mass.*

** *The natural spring frequency values were estimated from the reported spring stiffness values k_1 and k_2.*

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Figure captions

Figure 1 The two mass-spring-damper model consisted of a lower mass (m_2) representing the support leg and an upper mass (m_1) representing the rest of the body. Both masses were given an initial position (p_1, p_2) and velocity (v_1, v_2), and the mass ratio λ was defined as the upper mass relative to the lower mass (m_1/m_2). The stiffnesses of the upper and lower spring were defined by their natural frequencies (ω_1, ω_2) and the model was damped by a damping coefficient ζ. The model's motion was described by the acceleration of its two masses (a_1, a_2) based on the eight natural model parameters, from which the modelled GRF was calculated.

Figure 2 Typical examples of measured (black continuous line) and modelled (red dotted line) ground reaction force (GRF) profiles including the root mean squared error (RMSE) between both curves.

Figure 3 Errors for relevant ground reaction force (GRF) loading characteristics impulse, impact peak and loading rate for accelerations (blue circles), decelerations (red triangles), and running at a constant low (light grey crosses), moderate (dark grey crosses) and high (black crosses) speed. Negative errors are an underestimation of the measured value and positive errors and overestimation.

Figure 4 Model parameter values for accelerated, decelerated, and low-, moderate- and high-speed running. Means (black dotted line) and standard deviations (grey dashed line) were taken across tasks.

Figure B1 Model parameter values for accelerated, decelerated, and low-, moderate- and high-speed running. Mass ratio λ was fixed at 3 au, while the other parameters were bound to a range of values deemed physically reasonable.