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Luminosity functions and cosmic star formation rate out to $z = 1.6$

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Evolution of star formation in the UKIDSS Ultra Deep Survey field – I. Luminosity functions and cosmic star formation rate out to $z = 1.6$

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1 INTRODUCTION

A key goal and surmountable challenge for observational cosmology today is to assemble a reliable picture of the formation and evolution of galaxies, and the buildup of stellar mass in the Universe. The secular evolution of galaxies is a result of the formation of new stars, and so understanding the processes triggering and truncating star formation in addition to placing constraints on the evolution of the star formation rate (SFR), in turn allows us to constrain theoretical models of galaxy formation and evolution. A multitude of star formation indicators have been utilized in evaluating the SFR, however each of these is subject to its own biases and caveats, in addition to the limitations technology places on the redshift range where each method is applicable. Ultraviolet (UV) flux provides the most direct measurement of SFR by tracing hot young massive stars (e.g. Bunker et al. 2004; Ouchi et al. 2004; Arnouts et al. 2005; Baldry et al. 2005; Bouwens

ABSTRACT

We present new results on the cosmic star formation history in the Subaru/XMM–Newton Deep Survey (SXDS)–Ultra Deep Survey (UDS) field out to $z = 1.6$. We compile narrow-band data from the Subaru Telescope and the Visible and Infrared Survey Telescope for Astronomy (VISTA) in conjunction with broad-band data from the SXDS and UDS, to make a selection of 5725 emission-line galaxies in 12 redshift slices, spanning 10 Gyr of cosmic time. We determine photometric redshifts for the sample using 11-band photometry, and use a spectroscopically confirmed subset to fine tune the resultant redshift distribution. We use the maximum-likelihood technique to determine luminosity functions in each redshift slice and model the selection effects inherent in any narrow-band selection statistically, to obviate the retrospective corrections ordinarily required. The deep narrow-band data are sensitive to very low star formation rates (SFRs), and allow an accurate evaluation of the faint end slope of the Schechter function, $\alpha$. We find that $\alpha$ is particularly sensitive to the assumed faintest broad-band magnitude of a galaxy capable of hosting an emission line, and propose that this limit should be empirically motivated. For this analysis, we base our threshold on the limiting observed equivalent widths of emission lines in the local Universe. We compute the characteristic SFR of galaxies in each redshift slice, and the integrated SFR density, $\rho_{\text{SFR}}$. We find our results to be in good agreement with the literature and parametrize the evolution of the SFR density as $\rho_{\text{SFR}} \propto (1 + z)^{4.58}$ confirming a steep decline in star formation activity since $z \sim 1.6$.

Key words: surveys – galaxies: evolution – galaxies: formation – galaxies: high-redshift – galaxies: star formation – cosmology: observations.
et al. 2010); however, UV measurements are subject to large dust corrections and so carry a large uncertainty in addition to the danger of missing heavily obscured regions of star formation. The dust enshrouding young stars can itself be used to trace star formation, as it re-radiates the incident UV radiation in the infrared (IR) (e.g. Le Floc’h et al. 2005; Pérez-González et al. 2005). However, this method in turn struggles to detect some unobscured star formation.

Less commonly used methods include using radio emission to trace supernova remnants associated with short-lived massive stars (e.g. Mauch & Sadler 2007) and tracing X-ray emission produced in X-ray binaries (of which one component is a massive O or B star). Alternatively, emission lines generated in Hα regions can be used to trace either the recombination of atoms or the forbidden lines that arise in this low-density environment. Emission lines in the optical and UV are of course subject to dust corrections much as many other tracers; however, Hα luminosity is particularly well calibrated for extinction effects.

The evolution of the SFR density, \( \rho_{\text{SFR}} \), is well studied, and a strong upward evolution is found with redshift out to \( z \sim 1 \) indicating that the peak of star formation activity occurs at \( z > 1 \). The compilation of Hopkins & Beacom (2006) provides a broad overview of this relationship out to \( z \sim 6 \) but demonstrates that, even within the relatively well-constrained epoch, estimates of SFR evolution still vary according to indicator used. For instance the evolution of \( \rho_{\text{SFR}} \) is often parametrized as \( \rho_{\text{SFR}} \propto (1 + z)^{\alpha} \), and in one of the earliest studies (using a rest frame UV colour to measure SFR) Lilly et al. (1996) found \( \alpha = 4 \). However, their sample was hampered by a small survey area, and the larger UV-selected samples of e.g. Cowie, Songaila & Berger (1999), Wilson et al. (2002) and Prescott, Baldry & James (2009) found much shallower slopes of \( \alpha = 1.5, 1.7 \pm 1.0 \) and \( 2.5 \pm 0.3 \), respectively. Meanwhile, the far-infrared (FIR) luminosity found in Le Floc’h et al. (2005) translates to a \( \alpha = 3.3^{+0.7}_{-0.2} \).

Studies making use of continuum emission as a measure of the star formation activity of galaxies, whether it be UV emission, or its re-radiation in the IR, allow us to quantify star formation in bright galaxies even down to low SFRs. The samples however are generally small, as sources not only require spectroscopy but must also be apparent in broad-band filters in order to be detected – the imposed broad-band flux limit means that samples in the optical become effectively stellar-mass limited, and therefore do not detect the lower mass population of galaxies responsible for an important contribution to the overall \( \rho_{\text{SFR}} \).

The use of narrow-band (NB) filters to detect emission-line galaxies provides a complementary technique to broad-band-selected samples. A NB survey selects objects based on the strength of line emission, and so while the equivalent width (EW) of emission lines comes into play, samples are close to SFR-limited, allowing the analysis of SFR over a much broader range of stellar masses. Recently, many authors have utilized this technique to compile large samples of star-forming galaxies and examine the evolution of the SFR as traced by either Hα luminosity (which scales directly with SFR; Kennicutt 1992) or alternatively the [O iii] and [O ii] forbidden lines which give a more loosely calibrated indication of the rate of star formation.

Hα surveys have been hugely successful, from the pioneering work of Bunker et al. (1995) detecting only a handful of emitters, to the recent advances from the High-Redshift Emission Line Survey (HZEELS; Geach et al. 2008; Sobral et al. 2009) that detected thousands of emission-line sources. Many authors have taken this approach, and the Hα luminosity function (LF) is well studied in a variety of environments (e.g. Gallego et al. 1995; Yan et al. 1999; Tresse et al. 2002; Fujita et al. 2003; Kodama et al. 2004; Umeda et al. 2004; Shioya et al. 2008 and Villar et al. 2008 to name but a few). Since IR spectroscopy is required to trace the Hα line beyond \( z = 0.4 \), strong emission lines blueward of Hα are required to study the higher redshift population using existing optical data sets. The [O iii] doublet for example has been traced to \( z = 1.6 \) by Hogg et al. (1998), Hicks et al. (2002), Teplitz et al. (2003), Ajiki et al. (2003), Drozdovsky et al. (2005), Gilbank et al. (2010) and Ly et al. (2012).

Ly et al. (2012) incorporated the [O iii] tracer as well and derived LFs in 11 redshift slices in the Subaru Deep Field (SDF) based on Hα, [O iii] and [O ii] luminosities. Fitting a Schechter function to their data, they found strong evolution in the faint-end slope, \( \alpha \), and the characteristic number density \( \phi^\ast \), but little evolution in the characteristic luminosity of galaxies, \( L^\ast \). They interpreted these results as a comparatively stronger evolution of the low-luminosity-galaxy population than those with higher intrinsic luminosities. They found a steep evolution in \( \rho_{\text{SFR}} \), in good agreement with previous results, a trend confirmed by the work of Sobral et al. (2013) who compiled the first view of star formation from \( z = 0.4 \) out to \( z = 2.2 \) using only Hα-selected galaxies. Sobral et al. (2013) parametrized the evolution in \( \rho_{\text{SFR}} \) as \( \log \rho_{\text{SFR}} = -2.1/(z + 1) \) suggesting a \( \alpha \sim 3 \) across the redshift range \( 0 < z < 1.6 \). Conversely however, they found no evolution of \( \alpha \), and set the parameter to \( -1.6 \pm 0.08 \) for the last 11 Gyr, concluding that previous claims of evolution stemmed from heterogeneous samples. This value of \( \alpha \) was in good agreement with Hayes, Schaerer & Östlin (2010) who found a slope of \( \alpha = -1.72 \pm 0.20 \) at \( z = 2.2 \) for an Hα-selected sample. Even consistently selected samples however (i.e. those using the same emission line to derive each LF) still show discrepancies in the value of \( \alpha \) and its evolution with redshift (or lack of). The Hα emitters at \( z = 2.2 \) found in Tadaki et al. (2011) for example result in a slope of \( \alpha = -1.37 \), much shallower than the studies of Hayes et al. (2010) and Sobral et al. (2013) and suggesting little evolution from high redshift to the present day. Hayes et al. (2010) however, while in agreement with a steep value of \( \alpha \) argued that their results did indeed confirm an evolution in \( \alpha \) compared to the shallower slope of \( \alpha = -1.35 \) at \( z = 0 \) determined by Gallego et al. (1995). Evidently, values of \( \alpha \) across redshift remain unclear, and tentative claims of the parameter’s evolution divide opinion further still. It is important therefore to better understand the factors which most affect the derivation of \( \alpha \), in order to place more meaningful constraints on the faint end slope.

This paper demonstrates how the maximum-likelihood method can be used to determine LFs, and the benefits of this approach over existing methods. NB selection is inherently sensitive to only a certain region of parameter space, and retrospective corrections are ordinarily applied in order to account for the undetected objects. Here, we use a maximum-likelihood analysis to model the selection effects statistically and robustly under assumptions about the underlying galaxy population. We explain in detail how we implement this method, and confirm the technique by replicating the results of Sobral et al. (2012) and Sobral et al. (2013).

Our LFs are derived from NB-selected emission-line galaxies in the Subaru/XMM–Newton Deep Field (SXDF), coincident with the Ultra Deep Survey (UDS) field of the UK Infrared Telescope (UKIRT)’s Infrared Deep Sky Survey (UKIDSS). We use the Hα, [O iii] and [O ii] emission lines to determine field LFs in 12 redshift slices, spanning 10 Gyr of cosmic time, out to \( z \sim 1.6 \). We place constraints on the characteristic SFR in each redshift slice, and make estimates of \( \rho_{\text{SFR}} \) for comparison to values in the literature. The outline of this paper is as follows. In Section 2, we summarize the data used in this analysis, in Section 3, we discuss the source
extraction and selection of NB emitters, and present the colour-magnitude plots of NB-selected objects. Section 4 discusses our spectroscopically confirmed subset, the star–galaxy separation technique applied in order to remove late-type stellar contaminants, our photometric redshift analysis and the resultant photometric redshift distributions, and finally our method of redshift slice assignment for emitters selected in each filter. Section 5 details our application of the maximum-likelihood technique for determining LFs, and discusses in detail the factors affecting the NB selection technique. We also describe how this analysis carefully models the selection effects ordinarily corrected for retrospectively. In Section 6, we present LFs and derive the characteristic SFR and $\rho_{\text{PSF}}$ in each redshift slice for comparison to literature results. Section 7 discusses the factors affecting our results and the effect of varying assumptions about the underlying galaxy population on the resultant values of $\alpha$. We summarize our conclusions in Section 8.

An $H_0 = 70 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, $\Omega_M = 0.3$ and $\Omega_{\Lambda} = 0.7$ cosmology is assumed throughout, and all magnitudes are in the AB system.

2 DATA

The data used (summarized in Table 1) are taken from the SXDF-UDS field in order to make use of the deep optical data from the Subaru/XMM–Newton Deep Survey (SXDS), in addition to deep IR imaging from UKIDSS UDS.

The SXDS (Furusawa et al. 2008) covers 1.3 square degrees and delivers deep optical imaging in the $B$, $V$, $R$, $i$ and $z$ bands in five overlapping pointings of Suprime-Cam.

UKIDSS, comprised of five sub surveys of various areal coverages and depths (Lawrence et al. 2007) targeted the SXDF with the deepest of these surveys, the UDS (Foucaud et al. 2007, Almaini et al., in preparation) covering 0.77 square degrees. Here we utilize the Eighth Data Release (DR8) from this survey (details in Table 1). Additionally, we incorporate $u$-band data across the entire field from Megacam on the Canada France Hawaii Telescope (CFHT), and Spitzer IRAC imaging from the Spitzer-UDS (SpUDS) survey in channels 1 and 2.

Narrow-band imaging was obtained from Suprime-Cam in four NB filters, and smoothed using a Gaussian kernel to recover the same point spread function (PSF) as the broad-band optical data from Subaru (0.82 arcsec). Details of these data can be found in Ouchi et al. (2008) and Ouchi et al. (2009).

In addition, we make use of two NB filters on the Visible and Infrared Survey Telescope for Astronomy (VISTA). VISTA’s visible-infrared camera (VIRCAM; Dalton et al. 2006), is a wide-field near-IR camera consisting of sixteen 2048 × 2048 Raytheon VIRGO HgCdTe arrays distributed over the focal plane of VISTA. The paw-print of the 16 detectors provides a non-contiguous instantaneous field of view of 0.6 deg$^2$. Half of detector 16 is affected by a time-varying quantum efficiency which makes flat-fielding that detector extremely challenging (see Jarvis et al. 2013 for further details). The NB980 and NB990 filters (Orr, Wallace & Dalton 2008) share a slot on VIRCAM – each covering eight detectors, and so to obtain a full paw-print from either of these filters the survey area has to be observed twice, with the instrument rotated 180°. Consequently, the two VISTA NB images suffer from mismatched seeing in the top and bottom halves of the images, and so we smooth the data to the PSF of the worst-seeing parts of the VISTA paw-print (1.42 arcsec).

In order to select a reliable sample with homogeneousy determined photometric redshifts, we use only the region of sky covered by both SXDS and UDS imaging so as to provide 11 bands of photometry for every detection. The usable overlap of these deep broad-band surveys and the Subaru and VISTA NB imaging data amounts to 0.63 square degrees and 0.38 square degrees, respectively. The six NB filters used for selection are shown together with Suprime-Cam broad-band filters across the corresponding wavelength ranges in Fig. 1.

3 SAMPLE CONSTRUCTION

We use the Source extraction software SExtractor (Bertin & Arnouts 1996) to detect objects in each of the NB images. SExtractor works by constructing a background map and then looking for groups of connected pixels above a threshold calculated from the root mean square (RMS) noise in the image. We require a group of 10 connected pixels at a significance of 1.5 times the background RMS to confirm a detection. We then estimate the NB image depths by placing 10 000 randomly positioned 2 arcsec apertures on each image and iteratively fitting a Gaussian to the sigma-clipped histogram of resultant fluxes. The catalogue is then cut to demand

<table>
<thead>
<tr>
<th>Telescope: Instrument</th>
<th>Filter</th>
<th>Filter type</th>
<th>Central wavelength</th>
<th>FWHM</th>
<th>$\sigma$ limit (2′′ aperture)</th>
<th>Citation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subaru: Suprime-Cam</td>
<td>NB</td>
<td>BB</td>
<td>5029 Å</td>
<td>73 Å</td>
<td>24.91</td>
<td>SXDS; Ouchi et al. (2008)</td>
</tr>
<tr>
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<td>NB</td>
<td>BB</td>
<td>5703 Å</td>
<td>68 Å</td>
<td>24.52</td>
<td>SXDS; Ouchi et al. (2008)</td>
</tr>
<tr>
<td>Subaru: Suprime-Cam</td>
<td>BB</td>
<td>BB</td>
<td>8150 Å</td>
<td>119 Å</td>
<td>25.65</td>
<td>SXDS; Ouchi et al. (2008)</td>
</tr>
<tr>
<td>Subaru: Suprime-Cam</td>
<td>NB</td>
<td>BB</td>
<td>9183 Å</td>
<td>131 Å</td>
<td>25.38</td>
<td>SXDS; Ouchi et al. (2008)</td>
</tr>
<tr>
<td>VISTA: VIRCAM</td>
<td>NB</td>
<td>BB</td>
<td>9781 Å</td>
<td>105 Å</td>
<td>23.17</td>
<td>Jarvis et al. (2013)</td>
</tr>
<tr>
<td>VISTA: VIRCAM</td>
<td>NB</td>
<td>BB</td>
<td>9909 Å</td>
<td>105 Å</td>
<td>23.54</td>
<td>Jarvis et al. (2013)</td>
</tr>
<tr>
<td>CFHT: Megacam</td>
<td>$u$</td>
<td>BB</td>
<td>3740 Å</td>
<td>500 Å</td>
<td>26.72</td>
<td>Foucaud et al., (in preparation)</td>
</tr>
<tr>
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<td>BB</td>
<td>BB</td>
<td>4473 Å</td>
<td>1079 Å</td>
<td>27.39</td>
<td>Furusawa et al. (2008)</td>
</tr>
<tr>
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<td>BB</td>
<td>BB</td>
<td>5482 Å</td>
<td>984 Å</td>
<td>27.10</td>
<td>Furusawa et al. (2008)</td>
</tr>
<tr>
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<td>BB</td>
<td>BB</td>
<td>6531 Å</td>
<td>1160 Å</td>
<td>26.85</td>
<td>Furusawa et al. (2008)</td>
</tr>
<tr>
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<td>BB</td>
<td>BB</td>
<td>7695 Å</td>
<td>1543 Å</td>
<td>26.66</td>
<td>Furusawa et al. (2008)</td>
</tr>
<tr>
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<td>$z$</td>
<td>BB</td>
<td>9149 Å</td>
<td>1384 Å</td>
<td>25.95</td>
<td>Furusawa et al. (2008)</td>
</tr>
<tr>
<td>UKIRT: WFCAM</td>
<td>J</td>
<td>BB</td>
<td>12 500 Å</td>
<td>1570 Å</td>
<td>24.98</td>
<td>UKIDSS UDS; Almaini et al., (in preparation)</td>
</tr>
<tr>
<td>UKIRT: WFCAM</td>
<td>H</td>
<td>BB</td>
<td>16 500 Å</td>
<td>2910 Å</td>
<td>24.27</td>
<td>UKIDSS UDS; Almaini et al., (in preparation)</td>
</tr>
<tr>
<td>UKIRT: WFCAM</td>
<td>K</td>
<td>BB</td>
<td>22 000 Å</td>
<td>3530 Å</td>
<td>24.59</td>
<td>UKIDSS UDS; Almaini et al., (in preparation)</td>
</tr>
<tr>
<td>Spitzer: IRAC</td>
<td>ch1</td>
<td>BB</td>
<td>35 500 Å</td>
<td>7411 Å</td>
<td>23.55</td>
<td>Spitzer SpUDS; PI Dunlop</td>
</tr>
<tr>
<td>Spitzer: IRAC</td>
<td>ch2</td>
<td>BB</td>
<td>44 900 Å</td>
<td>10 072 Å</td>
<td>22.88</td>
<td>Spitzer SpUDS; PI Dunlop</td>
</tr>
</tbody>
</table>
Figure 1. Wavelength coverage and transmission functions for all selection filters. Four NB filters on Subaru, plus two on VISTA (bold black lines) together with broad-band filters at corresponding wavelengths also from the Subaru telescope (dashed lines) are shown.

an NB-detection above 5σ relative to our estimate of the sky noise in order for a source to be selected as a possible line emitter.

We make a very conservative area cut, discarding first the official SXDS masked areas, and then visually inspecting the NB images (with object detections overlaid) to discard additional areas of noisy coverage towards the edges of the CCD and then artefacts, areas around bright stars and CCD bleeds in the broad-band images. The areas removed from the study as a result of the visual inspection varied slightly from filter to filter.

### 3.1 Subaru selection

For objects selected in one of the four Subaru NB filters, the sample was culled to the 5σ limiting magnitude of the shallowest SXDS field. We next consider the interpolated broad-band colour at the wavelength of the selection filter as a measure of the continuum level for each object. We calculate this continuum (IntNB$_{cont}$) by interpolating between the two broad-bands adjacent to the NB assuming that continuum flux follows a power law:

$$\text{IntNB}_{cont} = BB1 - (BB1 - BB2) \frac{\log(\lambda_{NB}/\lambda_{BB1})}{\log(\lambda_{BB2}/\lambda_{BB1})}.$$

As filter NB921 lies centrally in the $z$ band, we calculate colour in this filter relative to the $z$ band only. Broad-band magnitudes are obtained by running SExtractor in dual image mode, and colours are measured in 2 arcsec apertures. Objects are expected to scatter around zero in (IntNB$_{cont}$ – NB), with emission-line sources showing a positive colour. In practice, the median colour of NB-selected sources is slightly offset from zero, due to the mismatch of interpolated broad-band magnitude and the true continuum magnitude at the wavelength of the NB. Furthermore, as we select objects across a broad range of NB magnitudes, we are essentially sampling different redshifts (in addition to different parts of the galaxy population) and so with changing NB magnitude, we introduce a systematic error that gives rise to a gradient in the median colour of objects (typically of the order of ~0.015 mag). We remove the colour gradient by calculating the sigma-clipped median colour of sources as a linear function of NB magnitude, and subtracting this so that the resultant locus of colours lies along zero.

Potential emission-line objects must display an NB excess of at least 3σ relative to the sigma-clipped median colour and 3σ relative to the scatter from photometric uncertainty (in the relevant SXDS field) to be selected as an emitter. Photometric scatter is determined from the image depth estimates described above. Fig. 2 shows colour–magnitude diagrams for each of the NB selections, with green lines showing the lines of 1, 2 and 3σ colour excess, and the red lines depicting the 1, 2 and 3σ photometric colour error trumpets.

We assume our observations are background limited, as even at the bright magnitudes where photon shot noise comes into play, colour scatter in (IntNB$_{cont}$–NB) is determined empirically and so the requirement of a 3σ excess takes this scatter into account.

### 3.2 VISTA selection

For selection in one of the two VISTA filters (seen alongside the Subaru filters in Fig. 1), detections are made and NB excesses evaluated following broadly the same method developed with the Subaru data. The only variation in the selection process is the manner in which we evaluate the continuum contribution to the NB measurement. The two VISTA NBs are close in wavelength, and so they sample objects at (effectively) the same point in their emission spectra; measured fluxes should therefore only differ significantly when an emission line is present in one of the two filters. This allows us to detect objects in one NB filter, and use photometry from the other as a measure of continuum flux for each object.

The double-NB selection technique circumvents the error introduced through inaccurate determination of interpolated broad-band colour (the main source of scatter introduced into the NB-broad-band colour) and so we anticipate the locus of VISTA-selected objects to be much tighter, allowing for a much less conservative 3σ cut in colour scatter, i.e. a smaller colour excess is now robust at the 3σ level. Colour–magnitude diagrams for these two NBs are also shown in Fig. 2 demonstrating the anticipated narrow locus, however the far shallower depth of the VISTA imaging results in a shallower sampling of the star-forming population.

### 4 REDSHIFTS

Our NB selection, spanning six NB filters from 5029 Å to 9909 Å is sensitive to all the major diagnostic emission lines in the region from [O ii] to Hα + [N ii], and samples redshift slices from $z = 0.14$ ([O ii] in the NB570 filter) to $z = 1.6$ where [O ii] falls in to the VISTA filters. With more than 7000 objects meeting the selection criteria, it is essential that we are able to reliably discriminate between the different lines falling in each filter while employing a technique suitable for dealing with such a volume of data. The combination of the deep SXDS–UDS imaging, and complementary $u$ band and IRAC data, provides 11 broad-bands of photometry available for every detection, allowing reliable determinations of objects’ spectral energy distributions (SEDs) and consequently accurate photometric redshifts. We have obtained spectra for a small subset of objects (discussed in Section 4.3) and use the redshifts derived as a training set for the photometric redshift code ‘EAZY’ (Brammer, Van Dokkum & Coppi 2008).

### 4.1 Photometry

Photometry for the full selection of NB-excess-selected objects is computed in all 11 broad-bands using the IRAF task phot in 2 arcsec apertures. Astrometric corrections between Suprime-Cam data and the other images were performed using the method of Simpson et al. (2012). All Suprime-Cam imaging was reduced to provide a uniform PSF of 0.82 arcsec, and the $J$, $H$, and $K$-band images from Wide Field Camera (WFPCAM) are consistent with this to within 0.05 arcsec, and so no corrections are applied to the
Figure 2. The colour–magnitude diagrams for the four Subaru NB filters: NB503, NB570, NB816 and NB921, and the two VISTA NB filters: NB980 and NB990. Detections are restricted to the 5σ limiting NB magnitude in the shallowest SXDS field. The solid green line depicts the median colour of sources (corrected for gradient and offset from zero – see the text for details), dashed green lines show the 1, 2 and 3σ limits on intrinsic colour scatter. Solid red lines depict the 1, 2 and 3σ photometric errors, respectively for the shallowest region of the five SXDS fields. The solid blue circles represent the entire sample of NB-excess-selected objects.

Measured photometry, $B$- and $K$-band images were smoothed using a Gaussian kernel to recover the same PSF as the $u$ and IRAC images, respectively, and fluxes from the $u$-band image and the two IRAC images were then scaled according to the ratio of fluxes in the unsmoothed and smoothed images. All broad-band photometry is corrected for Galactic extinction of $A_V = 0.070$ mag (Schlegel, Finkbeiner & Davis 1998) using the Cardelli, Clayton & Mathis (1989) reddening law.
4.2 Star–galaxy separation

Our NB-excess technique is inherently susceptible to the spurious selection of any object with a spectrum not well described by a power law. This leads to contamination of the sample by late-type stars which are known to exhibit very red, complex SEDs due to the many absorption features present in their spectra. We employ the two-colour $BzK$ technique of Daddi et al. (2004) to identify and eliminate these stars from the red filters susceptible to this contamination. For the Subaru NB filters this applies to NB816 and NB921. Daddi et al. (2004) derive equation (2), and class all objects below this line as stars.

\[
(z - K) < 0.3 (B - z) - 0.5
\]  

As we are contaminated only by late-type, red, stars, we employ the additional criterion that objects must have $B - z > 2.5$ to be classed as stars and eliminated from the selection.

The $BzK$ colours of our sample are shown in Fig. 3. The population of blue objects falling below the Daddi et al. (2004) $BzK$ solid black line arises from scatter as a consequence of very faint $K$ magnitudes ($\sim 26$). We eliminate 256 stars in total, amounting to 2 per cent of the NB-excess sample.

Despite the red wavelength of the two VISTA filters, only NB990 is in danger of the spurious selection of stars due to the point in a late-type star spectrum it samples. Running the NB990 selection through our $BzK$ criteria however eliminates nothing from the small sample. Table 2 summarizes the number of detections in each filter plus numbers remaining in the sample after quantifying NB-excess and removing stars from the sample.

### Table 2

<table>
<thead>
<tr>
<th>Filter</th>
<th>Detections</th>
<th>Excess objects</th>
<th>Galaxies</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB503</td>
<td>43 781</td>
<td>4368</td>
<td>4368</td>
</tr>
<tr>
<td>NB570</td>
<td>37 846</td>
<td>1749</td>
<td>1749</td>
</tr>
<tr>
<td>NB816</td>
<td>97 382</td>
<td>3816</td>
<td>3595</td>
</tr>
<tr>
<td>NB921</td>
<td>109 126</td>
<td>5395</td>
<td>5560</td>
</tr>
<tr>
<td>NB980</td>
<td>16 193</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>NB990</td>
<td>17 755</td>
<td>191</td>
<td>191</td>
</tr>
</tbody>
</table>

The spectra were reduced using the Cosmic Evolution Survey (COSMOS) reduction pipeline before extracting redshift information using the runz redshifting code (Saunders, Cannon & Sutherland 2004). We determine reliable redshifts for 129 of the sample, and assign 108 of these the highest quality flag. The remainder of the subset is made up of low signal-to-noise detections (where emission-line flux does not in fact meet our final criteria), spectra displaying a single unidentifiable line, spectra affected by cross-talk from other slits and a small number of stars targeted to confirm our star–galaxy separation technique.

The modest but reliable subset of spectroscopic redshifts confirms that we are indeed selecting emission-line galaxies at the redshifts anticipated, and serves as a reference to determine the success of our photometric redshift technique applied to the rest of the sample.

4.4 Photometric redshift analysis

The default parameters of EAZY provide photometric redshifts in good agreement with the distribution expected for emission-line objects selected in these filters; however, to optimize the SED fitting, we modify this default parameter set so as to minimize $\sigma_{\text{NMAD}}$ for the spectroscopic subset (equation 3; Brammer et al. 2008), where

\[
\sigma_{\text{NMAD}} = 1.48 \times \text{median} \left( \frac{\Delta z - \text{median}(\Delta z)}{1 + z_{\text{spec}}} \right).
\]  

The default parameters of EAZY provide photometric redshifts in good agreement with the distribution expected for emission-line objects selected in these filters; however, to optimize the SED fitting, we modify this default parameter set so as to minimize $\sigma_{\text{NMAD}}$ for the spectroscopic subset (equation 3; Brammer et al. 2008), where

\[
\sigma_{\text{NMAD}} = 1.48 \times \text{median} \left( \frac{\Delta z - \text{median}(\Delta z)}{1 + z_{\text{spec}}} \right).
\]
The number of catastrophic outliers (>5σ_{NMAD}) for definite spectroscopic redshifts (of the highest quality flag) equates to 5 per cent of the spectroscopic subset, with a value of σ_{NMAD} = 0.026. Including objects with less certain spectroscopic redshifts raises this to 9 per cent. While photometric redshifts are commonplace for large survey data sets, particularly for K-selected (i.e. stellar-mass-limited) samples, photometry is frequently restricted to broad-bands only. Here we possess additional information that all objects remaining after the selection process must display a NB excess at the wavelength of the filter.

While naively, we might interpret the presence of an NB excess to mean that all sources must possess a major emission line redshifted to the wavelength of the selection filter, in practice the problem proves to be more complex. We design a pure emission-line template to investigate the effect of including NB data in addition to broad-band photometry, before fine tuning the setup parameters. Our emission line template consists of the major emission lines Lyα, [O ii], Hβ, [O iii], Hα and [N ii] with negligible continuum flux. However, including this template now means that the complexities of the emission-line ratios come into play, along with the nature of the NB excess. While the objects do indeed lie in distinct redshift slices, NB excess can arise from a vast number of emission lines. We find that the inclusion of NB data, along with an emission-line template, biases redshift determination towards the redshift slices associated with the emission lines in our template at the expense of a good fit to the broad-band photometry, thus adding redshift interlopers to our selection. The 11 bands of photometry are deep, and as EAZY is well equipped to determine redshifts based on this information alone we discard the NB plus emission-line template setup at this stage.

We find the best agreement between z_{phot} and z_{spec} for the default template set, using broad-band data only (see Fig. 4). This way redshifts are assigned independently of their detection method, allowing the correct identification of spectral breaks and additional better fitting emission lines than those included in the emission-line template. For the comparison in Fig. 4, we also incorporate matches to the full set of spectra available in the UDS field. The larger SXDS–UDS spectroscopic sample (Simpson et al. 2012, and papers cited therein) includes many spectra that are the result of targeted follow-up of X-ray or radio sources, and is therefore heavily biased towards active galactic nuclei (AGN). Objects with spectra from the larger sample (depicted by black dots in Fig. 4) often appear at redshifts other than those typical for our selection, as they sample many objects with emission lines usually weak in star-forming galaxies.

4.5 AGN contamination

We identify eight (of the 129) objects as AGN according to the line-ratio diagrams of Lamareille (2010) or based on the presence of [Ne ii] emission. As we can use only our own spectroscopic follow-up from the Magellan telescopes to quantify AGN contamination in an unbiased manner, we infer a contamination fraction of 6+8−5% per cent. This correction is small and highly uncertain, and because AGN activity is undoubtedly associated with some star formation, removing these sources then leaves us susceptible to a possible overcorrection and consequently an underestimate of star formation activity. We therefore choose not to correct our results for AGN contamination.

4.6 Resultant photometric redshift distribution

EAZY provides an output probability density distribution P(z) for each object it evaluates. We use z_{m2}, the redshift marginalized over the posterior redshift probability distribution (also taking into account the K-band apparent magnitude), to determine the redshift distribution of our objects and discriminate between the emission lines giving rise to NB excess. The resultant photometric redshift distribution can be seen in Fig. 5. The advantage of the posterior redshift probability is two-fold: we combine the best estimate of redshift (using all SED templates) with a priori constraints on the likelihood of an object being at a particular redshift (given our knowledge of its colour and K magnitude prior), while taking into account the probability that the object lies at an alternative, less well-fit redshift. z_{m2} effectively provides a weighted mean, however the resultant distribution of redshifts and that for the best-fitting redshift (z_a) are in fact very similar (traced by the two histograms in Fig. 5), which demonstrates that we are not altering the best-fitting redshifts dramatically through marginalization and use of the prior. We note that the small number of catastrophic outliers discussed in Section 4.4 produce doubly peaked P(z) distributions, with z_a often selecting a redshift of relatively small probability compared to the probability at the value of z_{m2}. Similarly, the small peaks of objects lying at z_a = 0 are the result of objects missing a red band of photometry and producing a small peak at z_a = 0 in their P(z) distributions. This lends further support to the advantages of z_{m2} which in these cases avoids the small z_a = 0 spike via the weighted mean. Fig. 5 shows that we are selecting emission-line galaxies at the anticipated redshifts and highlights the peaks in the distribution associated with Hα, [O iii] and [O ii], in addition to Lyα in the two bluest filters (Ouchi et al. 2008; not included in this analysis).

The lowest panel of Fig. 5 shows the combined data sets of NB980 and NB990. As these two filters are very close in wavelength (Δλ ~ 120 Å), we choose to combine the two data sets to combat the small number of detections (with a 5σ detection cutoff in the shallowest filter; NB980). The NB980 and NB990 data sets are
evaluated separately, and then combined before assigning objects to a redshift slice. The combined data set (hereafter NB985) is treated as a single selection from here on.

4.7 Redshift slice assignment

Before assigning objects to a particular redshift slice we consider the stack of $P(z)$ values for each NB selection of objects (seen in Fig. 6). We bin objects into the redshift slices associated with the peaks in the data, and assign the upper and lower limits of the bins according to which objects were transposed to the appropriate redshift. The number of objects per slice is the number of real objects assigned to a redshift slice from our catalogue, that were used to derive an LF for this redshift range.

Table 3. Number of objects assigned to each redshift slice. The redshift range quoted refers to the limits in $z_{\text{min}}$; according to which objects were assigned to a slice for a particular filter/line combination.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Redshift</th>
<th>Line</th>
<th>Objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB503</td>
<td>0.10 $&lt;$ 0.35 $&lt;$ 0.50</td>
<td>[O I]</td>
<td>142</td>
</tr>
<tr>
<td>NB570</td>
<td>0.00 $&lt;$ 0.14 $&lt;$ 0.30</td>
<td>[O I]</td>
<td>42</td>
</tr>
<tr>
<td>NB570</td>
<td>0.30 $&lt;$ 0.53 $&lt;$ 0.70</td>
<td>[O I]</td>
<td>96</td>
</tr>
<tr>
<td>NB816</td>
<td>0.00 $&lt;$ 0.25 $&lt;$ 0.35</td>
<td>H$\beta$</td>
<td>152</td>
</tr>
<tr>
<td>NB816</td>
<td>0.35 $&lt;$ 0.63 $&lt;$ 0.80</td>
<td>[O I]</td>
<td>985</td>
</tr>
<tr>
<td>NB816</td>
<td>0.80 $&lt;$ 1.19 $&lt;$ 1.50</td>
<td>[O I]</td>
<td>1013</td>
</tr>
<tr>
<td>NB921</td>
<td>0.00 $&lt;$ 0.40 $&lt;$ 0.50</td>
<td>H$\alpha$</td>
<td>279</td>
</tr>
<tr>
<td>NB921</td>
<td>0.50 $&lt;$ 0.83 $&lt;$ 1.10</td>
<td>[O I]</td>
<td>930</td>
</tr>
<tr>
<td>NB921</td>
<td>1.10 $&lt;$ 1.46 $&lt;$ 1.90</td>
<td>[O I]</td>
<td>2204</td>
</tr>
<tr>
<td>NB985</td>
<td>0.00 $&lt;$ 0.50 $&lt;$ 0.60</td>
<td>H$\alpha$</td>
<td>153</td>
</tr>
<tr>
<td>NB985</td>
<td>0.60 $&lt;$ 0.99 $&lt;$ 1.29</td>
<td>[O I]</td>
<td>33</td>
</tr>
<tr>
<td>NB985</td>
<td>1.29 $&lt;$ 1.64 $&lt;$ 2.08</td>
<td>[O I]</td>
<td>23</td>
</tr>
</tbody>
</table>

where $\text{flux}_{\text{NB}}$ is the flux in the NB, and $\text{flux}_{\text{cont}}$ is the continuum contribution to that NB flux.

4.8 Line fluxes

NB selections are by necessity made in an observable parameter space, here based on line flux. Apparent aperture line fluxes (hereafter: line fluxes) for objects in each redshift slice are derived from the initial $\text{SExtractor}$ detection measurements. The entirety of the NB excess is assumed to arise from an emission line and so line flux ($\mu$Jy) is computed as equation (4), and converted to W m$^{-2}$.

$$\text{Line flux} = \text{flux}_{\text{NB}} - \text{flux}_{\text{cont}},$$

4.9 Maximum likelihood luminosity functions

We employ the maximum-likelihood method of Marshall et al. (1983) in order to fit LFs to the data. We assume that the distribution of line luminosities in a given redshift slice will follow a Schechter function, given in log form by equation (5):

$$\phi(L) \, d\log L = \ln 10 \phi^* \left( \frac{L}{L^*} \right)^{\alpha+1} e^{-\left(\frac{L}{L^*}\right)} \, d\log L,$$

Figure 6. Stacked probability density distributions for each NB-selection of objects, shown over the redshift range relevant for this study. Red (H$\alpha$), green ([O I]) and blue ([O II]) shaded regions represent the redshift ranges assigned to a slice for a particular filter/line combination.
where $\phi^*$, $L^*$ and $\alpha$ are the characteristic number density, characteristic luminosity, and the gradient of the faint-end slope, respectively (Schechter 1976). We assume negligible cosmic evolution across the small redshift interval per slice. Next, we consider how the distribution of true line luminosities maps on to our observed distribution of line fluxes.

Splitting the line flux range where we are sensitive to lines into bins small enough to expect no more than one object per bin, we can write the likelihood of finding an object in bins $F_i$ and no objects in bins $F_j$, as equation (6) for a given LF:

$$
\Lambda = \prod_{F_i} \Psi(F_i) \mathrm{d} \log F \prod_{F_j} e^{-\Psi(F_j) \mathrm{d} \log F},
$$

where $\Psi(F_i)$ is the probability of detecting an object with line flux between $F_i$ and $10^{\log F}$. This simplifies to equation (7), where $F_k$ is the product over all bins:

$$
\Lambda = \prod_{F_i} \Psi(F_i) \mathrm{d} \log F \prod_{F_k} e^{-\Psi(F_k) \mathrm{d} \log F}.
$$

We then define the likelihood function as $S = -2 \ln \Lambda$ and minimize this (equation 8) in order to determine the parameters of the model ($\phi^*, L^*, \alpha$) that best reproduce the observed data

$$
S = -2 \sum \ln \Psi(F_i) + 2 \int \Psi(F) \mathrm{d} \log F.
$$

To obtain $\Psi(F_i)$ (the observed line LF) from $\phi(L)$ (the true line LF) we must consider the selection effects imposed in observing the data and statistically model these perturbations to $\phi(L)$ before minimizing the likelihood function.

### 5.1 Treatment of the filter profile

To produce an LF, we must first assess the NB filter response for the emission line of interest as a function of redshift. A given line flux may arise from a line near the centre of a detection filter, or an intrinsically brighter line lying closer to the edge of the filter where transmission is lower.

We define $T(z)$, the relative transmission with redshift, to determine the redshift range to which each filter/line combination is sensitive, accounting for filter transmission in addition to contributions from other lines. $T(z)$ is calculated by propagating a synthetic spectrum through the filter transmission curve (an example of the resultant line transmission is seen in Fig. 7), using standard line ratios ([N II]/H$\alpha = 3.3$, [O III]/H$\beta = 3$, [O I] in the low density limit; Osterbrock & Ferland 2006) normalized to the line of interest. $F$, our line flux, now becomes

$$
F = \left( \frac{L}{4 \pi d^2(z)} \right) T(z),
$$

where $L/4 \pi d^2(z)$ is the true flux, convolved with filter transmission $T(z)$. $\Psi(F)$, the probability of seeing a line of flux $F$, now becomes

$$
\Psi(F) \mathrm{d} \log F = \int \phi \left( \frac{4 \pi d^2(z)}{T(z)} \right) \left( \frac{dV}{dz} \right) \mathrm{d}z \mathrm{d} \log L,
$$

where $dV/dz$ is the comoving volume interval in our field per unit redshift, and the integral is performed over the redshift range to which the filter is sensitive.

### 5.2 Line detectability

The detection of an emission line in our study requires it to reside in an observable part of parameter space in the NB magnitude – colour-excess plane, i.e. the region within the three dotted lines in Fig. 8. The detection fraction as a function of line flux, is then given by equation (11):

$$
f_{\text{det}}(F) = \frac{\int_{0}^{m_{\text{faint}}} \phi_{\text{BB}}(m_{\text{BB}}) \mathrm{d}m_{\text{BB}}}{\int_{-\infty}^{m_{\text{faint}}} \phi_{\text{BB}}(m_{\text{BB}}) \mathrm{d}m_{\text{BB}}},
$$

where $\phi_{\text{BB}}(m_{\text{BB}})$ is a Schechter fit to the interpolated broadband magnitudes of all galaxies within a particular filter’s redshift range, and is derived from the photometric redshift catalogue of Grützbauch et al. (2011). The numerator of $f_{\text{det}}$ denotes the sample of galaxies our technique is capable of detecting (thick red lines in Fig. 8), and the denominator denotes the entire population of galaxies capable of hosting the line down to some integration limit, $m_{\text{limit}}$. $m_{\text{faint}}$ is the assumed faintest magnitude of a galaxy capable of hosting a particular emission line luminosity. Different authors set this limit differently and somewhat arbitrarily. Here, we adopt a limit based on the rest frame EW of the line of interest seen in the local Universe, and set $\text{EW}_{\text{limit}} = 100$ Å (Kennicutt 1992). The resultant Schechter fit to our observed line fluxes is sensitive to the
Figure 8. Lines of constant line flux in the NB magnitude (colour-excess) plane. The dotted lines represent the limiting EW for an emission line to meet the selection criteria, and the faintest broad-band magnitude that when combined with a given line flux will meet the 5σ NB threshold. A and B represent points on a line of constant line flux that correspond to the bright and faint broad-band limiting magnitudes, respectively. Thick red lines highlight the portion of each line of constant line flux that is detectable in our sample.

The faint limit for the majority of lines is set by \( m_{\text{limit}} \), i.e. the true limiting magnitude of the population of galaxies capable of hosting that line, as detailed above.

For exceptionally faint lines however, the detection limit of our catalogue comes into play. For an object to be detected its broad-band magnitude must be bright enough that when added to its line flux its NB magnitude is above the 5σ detection threshold, and so for very faint lines this constraint sets the faint limit on broad-band magnitude. Point B1 represents this limit for a particular line flux in Fig. 8.

Equation (11) then becomes equation (12):

\[
f_{\text{det}}(F) = \frac{\int_{m_{\text{BB}} - \infty}^{m_{\text{BB}}(F)}}{\int_{m_{\text{BB}}(F)}^{m_{\text{BB}}(\text{limit})}} \phi_{\text{BB}}(m_{\text{BB}}) \, dm_{\text{BB}}
\]

and \( \Psi(F) \) now takes its final form:

\[
\Psi(F) \, d \log F = f_{\text{det}}(F) \int_0^{\Delta m_{\text{NB}}} \phi \left( \frac{4\pi d_l^2 F}{T(z)} \right) \, dV \, d \log z.
\]

We note that the presence of a strong correlation between SFR and stellar mass (e.g. Noeske 2008, however see Sobral et al. 2011 for an alternative view) would mean that low–luminosity lines would be less likely to be found in more massive galaxies. We will address this issue and its effect on line detectability in Paper 2, where we will study LFs as a function of stellar mass.

5.3 Detection completeness

Finally, we assess the detection completeness of the NB selection catalogue retrieved via SExtractor. The completeness of detections is dependent on the depth of the detection image and on the size of the aperture used. As our analysis is carried out in 2 arcsec apertures and a retrospective aperture correction applied (see below), we assess completeness in the same manner down to the 5σ limiting magnitude. We construct a series of images inserting randomly positioned fake point sources in each image, allowing us to determine recovery fraction as a function of magnitude. We estimate incompleteness per \( \Delta m_{\text{NB}} = 0.05 \) bin, and create an appropriate number of sources with NB excesses distributed following the observed distribution of detected objects, and with NB magnitudes uniformly distributed within the bin. The effect is small as we only populate the faintest bins, typically adding fewer than 10 objects to an LF. This number rises to 21 objects (a 2 per cent increase) for the second most densely populated LF (\([\text{O} \text{II}]\) in NB816) and 91 objects (a 4 per cent increase) for \([\text{O} \text{II}]\) in NB921 where the number of detections is more than a factor of 2 larger again. Corrections are particularly low due to the stringent 5σ NB-detection requirement.

5.4 Galactic extinction and aperture correction

Each LF is corrected for Galactic extinction of \( A_V = 0.070 \) mag using the Cardelli et al. (1989) reddening law (following the same method as in Section 4.1). As the analysis is carried out using aperture photometry, we must assess the fraction of flux that is...
lost by only considering the light within our 2 arcsec aperture. We calculate the ratio of total NB flux to aperture NB flux using the output SExtractor values (FLUX_BEST and FLUX_APER, respectively) for each redshift slice, and scale each output LF according to the median ratio in that slice. The correction is typically small (a factor of ~1.2) even at the lowest redshifts, however for the smoothed VISTA NB data we find a slightly larger correction (~1.8) due to a much broader PSF.

6 RESULTS
We derive LFs to examine the evolution of the Hα, [O III] and [O II] LFs. The resultant Schechter fits provide LFs in 12 redshift slices between 0.14 and 1.64. Using the parametrizations of these LFs, we can derive SFRs and compute ρSFR to examine the evolution of star formation with redshift.

6.1 Luminosity functions
The derived LFs are presented in Table 4 and Fig. 9. Volumes quoted are representative of the volume covered where filter transmission is greater than 0.5. Observed (aperture-corrected) values are presented, followed by the extinction-corrected value of L*, assuming 1 mag of extinction at Hα (Kennicutt 1992, 1998) and as applied in Fujita et al. (2003), Pasqual et al. (2005), Sobral et al. (2012) and Sobral et al. (2013). Fig. 9 presents the best-fitting LFs for the data at each redshift with the maximum-likelihood fit shown in bold coloured lines. Symbols represent the binned LF in bins of ~0.1 dex.

Alongside each panel of LFs, we include a graphic representation of the error ellipses of the 1, 2 and 3σ contours in L* − φ* space representing the boundaries of the volume in 2D (φ*, L*) space. We anchor α to the median value across the redshift slices for each line as shown in each figure, while L* and φ* are allowed to vary (i.e. α is constant per panel to allow comparison of LFs).

6.1.1 Treatment of shallow data
As discussed in Section 5, the maximum-likelihood analysis minimizes equation (8) allowing φ*, L* and α to vary. However, this analysis requires a deep and plentiful data set in order to produce a well-constrained fit, particularly for α. NB570 is the shallowest of the Subaru NB images, and due to its blue wavelength samples [O III] emitters at a very low redshift (in the smallest cosmic volume of the analysis); consequently the LFs produced from the NB570 filter selection ([O III] at z = 0.14 and [O III] at 0.53) are subject to low number statistics, with only 42 and 83 objects, respectively making up each LF.

### Table 4. LFs in 12 redshift slices for Hα, [O III] or [O II] emission. Numbers of objects quoted refer to the final number of objects per LF (after culling to the broadest 3σ colour-scan limit of the SXDS fields, and taking into account the completeness correction).

<table>
<thead>
<tr>
<th>Redshift</th>
<th>Objects</th>
<th>Volume (10^4 Mpc^3)</th>
<th>log φ*_{Hα} (Mpc^-3)</th>
<th>log L*_{Hα} (W)</th>
<th>α_{Hα}</th>
<th>log L*_{[OIII]} (W)</th>
<th>Extinction-corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>142</td>
<td>1.22</td>
<td>−2.43^{+0.17}_{−0.21}</td>
<td>33.83^{+0.19}_{−0.16}</td>
<td>−1.03^{+0.17}_{−0.15}</td>
<td>34.23^{+0.19}_{−0.17}</td>
<td>1.33^{+0.78}_{−0.41}</td>
</tr>
<tr>
<td>0.4</td>
<td>271</td>
<td>2.95</td>
<td>−2.44^{+0.14}_{−0.17}</td>
<td>34.16^{+0.13}_{−0.11}</td>
<td>−1.14^{+0.14}_{−0.13}</td>
<td>34.55^{+0.13}_{−0.11}</td>
<td>2.85^{+0.64}_{−0.64}</td>
</tr>
<tr>
<td>0.5</td>
<td>151</td>
<td>4.76</td>
<td>−3.27^{+0.55}_{−0.94}</td>
<td>34.69^{+0.46}_{−0.27}</td>
<td>−2.16^{+0.33}_{−0.31}</td>
<td>−0.13^{−0.13}_{−0.13}</td>
<td>−0.13^{−0.13}_{−0.13}</td>
</tr>
<tr>
<td>0.5^{a}</td>
<td>151</td>
<td>4.76</td>
<td>−2.23^{+0.12}_{−0.12}</td>
<td>34.34^{−0.09}_{−0.07}</td>
<td>−1.23^{+0.12}_{−0.13}</td>
<td>34.74^{−0.09}_{−0.07}</td>
<td>4.33^{−0.73}_{−0.73}</td>
</tr>
<tr>
<td>Redshift</td>
<td>Objects</td>
<td>Volume (10^4 Mpc^3)</td>
<td>log φ*_{[OIII]} (Mpc^-3)</td>
<td>log L*_{[OIII]} (W)</td>
<td>α_{[OIII]}</td>
<td>log L*_{[OII]} (W)</td>
<td>SFR* (M⊙ yr^{-1})</td>
</tr>
<tr>
<td>0.14</td>
<td>42</td>
<td>0.36</td>
<td>−3.67^{+1.14}_{−1.11}</td>
<td>34.60^{−0.96}_{−0.96}</td>
<td>−1.63^{+0.60}_{−0.24}</td>
<td>35.14^{−0.96}_{−0.96}</td>
<td>10.17^{−0.95}_{−0.95}</td>
</tr>
<tr>
<td>0.63</td>
<td>943</td>
<td>8.09</td>
<td>−2.57^{+0.11}_{−0.13}</td>
<td>34.44^{−0.08}_{−0.08}</td>
<td>−1.27^{+0.11}_{−0.11}</td>
<td>34.99^{−0.08}_{−0.08}</td>
<td>7.16^{−1.20}_{−1.20}</td>
</tr>
<tr>
<td>0.83</td>
<td>910</td>
<td>12.35</td>
<td>−2.25^{+0.07}_{−0.09}</td>
<td>34.28^{−0.07}_{−0.09}</td>
<td>−0.76^{+0.22}_{−0.19}</td>
<td>34.83^{−0.07}_{−0.08}</td>
<td>4.95^{−0.83}_{−0.83}</td>
</tr>
<tr>
<td>0.99</td>
<td>32</td>
<td>14.69</td>
<td>−14.45^{+∞}_{−∞}</td>
<td>38.65^{−0.36}_{−0.40}</td>
<td>−3.78^{−0.36}_{−0.40}</td>
<td>−0.23^{−0.23}_{−0.23}</td>
<td>−0.23^{−0.23}_{−0.23}</td>
</tr>
<tr>
<td>0.99^{a}</td>
<td>32</td>
<td>14.69</td>
<td>−3.00^{+0.22}_{−0.22}</td>
<td>34.70^{−0.12}_{−0.12}</td>
<td>−0.78^{−0.21}_{−0.21}</td>
<td>35.25^{−0.12}_{−0.12}</td>
<td>12.97^{−3.67}_{−3.67}</td>
</tr>
<tr>
<td>Redshift</td>
<td>Objects</td>
<td>Volume (10^4 Mpc^3)</td>
<td>log φ*_{[OII]} (Mpc^-3)</td>
<td>log L*_{[OII]} (W)</td>
<td>α_{[OII]}</td>
<td>log L*_{[HII]} (W)</td>
<td>SFR* (M⊙ yr^{-1})</td>
</tr>
<tr>
<td>0.35</td>
<td>112</td>
<td>2.22</td>
<td>−2.31^{+0.19}_{−0.29}</td>
<td>33.90^{+0.20}_{−0.16}</td>
<td>−1.06^{+0.38}_{−0.34}</td>
<td>34.65^{+0.20}_{−0.16}</td>
<td>1.11^{−0.34}_{−0.34}</td>
</tr>
<tr>
<td>0.53</td>
<td>83</td>
<td>3.95</td>
<td>−5.55^{+1.75}_{−1.75}</td>
<td>35.38^{−0.84}_{−0.84}</td>
<td>−2.70^{−0.17}_{−0.17}</td>
<td>−0.16^{−0.16}_{−0.16}</td>
<td>−0.16^{−0.16}_{−0.16}</td>
</tr>
<tr>
<td>0.53^{a}</td>
<td>83</td>
<td>3.95</td>
<td>−2.85^{+0.28}_{−0.41}</td>
<td>34.13^{+0.22}_{−0.17}</td>
<td>−1.68^{+0.33}_{−0.31}</td>
<td>34.98^{+0.22}_{−0.17}</td>
<td>2.34^{+1.54}_{−0.76}</td>
</tr>
<tr>
<td>1.19</td>
<td>981</td>
<td>19.06</td>
<td>−2.41^{+0.09}_{−0.07}</td>
<td>34.61^{+0.07}_{−0.06}</td>
<td>−0.99^{+0.14}_{−0.13}</td>
<td>35.37^{−0.12}_{−0.13}</td>
<td>5.76^{+1.01}_{−0.74}</td>
</tr>
<tr>
<td>1.46</td>
<td>2217</td>
<td>23.09</td>
<td>−2.03^{−0.06}_{+0.06}</td>
<td>34.76^{−0.04}_{+0.04}</td>
<td>−0.91^{+0.11}_{+0.11}</td>
<td>35.52^{−0.04}_{+0.11}</td>
<td>8.09^{−0.71}_{−0.71}</td>
</tr>
<tr>
<td>1.64</td>
<td>27</td>
<td>28.31</td>
<td>−2.98^{+0.79}_{−0.79}</td>
<td>35.27^{+1.15}_{−1.15}</td>
<td>−3.10^{+1.15}_{−1.15}</td>
<td>−0.10^{−0.10}_{−0.10}</td>
<td>−0.10^{−0.10}_{−0.10}</td>
</tr>
<tr>
<td>1.64^{a}</td>
<td>27</td>
<td>28.31</td>
<td>−1.68^{+0.50}_{−0.43}</td>
<td>34.73^{+0.11}_{+0.11}</td>
<td>−0.91^{+0.11}_{+0.11}</td>
<td>35.49^{+0.10}_{+0.10}</td>
<td>7.47^{+1.93}_{−1.67}</td>
</tr>
</tbody>
</table>

*aA Gaussian prior on α is applied to these luminosity functions.
Figure 9. Graphic representation of our best-fitting LFs (Gaussian prior on $\alpha$ for VISTA LFs and NB570 at $z = 0.53$) and their 1, 2 and 3$\sigma$ error ellipses ($\alpha$ fixed to the median value per line for comparison) in $L^* - \phi^*$ space. The smaller of the two error bars on each point in the left-hand panels are Poissonian, larger error bars have the 1$\sigma$ variation as estimated by our jackknifing technique added in quadrature. $H\alpha$ (top panels) $[O\text{ III}]$ (central panels) and $[O\text{ II}]$ (lower panels). The upper SFR scale on these plots gives an indication of the raw SFR values for these LFs, these numbers have not been corrected for dust extinction.
Furthermore, VISTA data are ~1 mag shallower than NB570, and so our data struggles to probe \( L^* \) in the \( z = 0.5, 1.0 \) and 1.6 redshift slices, in addition to very low numbers of emitters being detected.

The shallower depth of this data means that \( \alpha \) is not as well constrained as for the deeper more detection-rich LFs; however, the NB985 combined data set is selected close in wavelength to the NB921 filter, and consequently probes redshift slices only marginally more distant than the NB921 selection. To combat the shallow data we fit LFs for the three VISTA redshift slices with \( \alpha \) constrained by a Gaussian prior based on the value for the NB921 selection for the same emission line. This is a valid method of evaluation where we do indeed have some prior knowledge of the value of \( \alpha \). We take a similar approach with the NB570 [O\textsc{ii}] LF at \( z = 0.53 \), setting the prior according to the [O\textsc{ii}] LF at \( z = 0.35 \).

### 6.1.2 The effect of cosmic variance

In order to study the effect of our limited number of detections, we undertake a jack-knife analysis in every redshift slice. For each slice, we use random sampling with replacement to construct 1000 synthetic catalogues of NB emitters such that each sample is of the same size as our true catalogues. We fit LFs to each of these synthetic catalogues and measure the standard deviation in each parameter. These 1σ deviations are comparable to but no bigger than the uncertainties in the true data set. Errors quoted in Table 4 refer only to the 1σ uncertainty in the real data. In Fig. 9, we plot two error bars for each point. The smaller of the two error bars represents the Poissonian error, and the larger includes the standard deviation on the number of objects in each luminosity bin from the jack-knifed samples added in quadrature.

### 6.2 Star Formation Rates

We use the best-fitting Schechter function in each of the 12 redshift slices to calculate the characteristic SFR and \( \rho_{\text{SFR}} \) across 10 Gyr of cosmic time. SFRs are calculated according to the relations of Kennicutt (1998) for H\( \alpha \) (equation 14), [O\textsc{ii}] (equation 15) and using the standard line ratios: \( H\alpha/\beta = 2.78 \) and \([\text{O\textsc{iii}}]/\beta = 3 \) (Osterbrock & Ferland 2006) to derive equation (15) for [O\textsc{iii}].

\[
\rho_{\text{SFR}}(M_\odot\text{yr}^{-1}) = 7.9 \times 10^{-35} L_{\odot} E(H\alpha) \tag{14}
\]

\[
\rho_{\text{SFR}}(M_\odot\text{yr}^{-1}) = 7.35 \times 10^{-35} L_{\odot} E([\text{O\textsc{iii}}]) \tag{15}
\]

\[
\rho_{\text{SFR}}(M_\odot\text{yr}^{-1}) = 1.39 \times 10^{-34} L_{\odot} E(H\alpha) \tag{16}
\]

Fig. 10(a) presents the fully corrected characteristic star formation rate (SFR\(^*\); aperture and extinction corrections described in Sections 6.1 and 5.4) for each redshift slice in this analysis, red points showing H\( \alpha \) derived estimates, green, [O\textsc{iii}] and blue, [O\textsc{ii}]. Error bars represent SFRs for the corresponding 1σ upper and lower bounds on the characteristic SFR. Fig. 10(b) presents the integrated \( \rho_{\text{SFR}} \) for each LF (full integration, extinction-corrected) together with the fits of Sobral et al. (2013) and Hopkins & Beacom (2006) using the data of Baldry & Glazebrook (2003).

### 7 DISCUSSION

#### 7.1 Evolution of the Star Formation Rate

We find characteristic SFRs broadly consistent between all three indicators out to \( z \sim 1.6 \). The characteristic SFR increases by a factor of \( \sim 10 \) between \( z \sim 0.14 \) and \( z \sim 1.6 \), in good agreement with literature results. The \( H\alpha \) and [O\textsc{iii}] derived SFRs show a distinct increase with redshift consistent with other studies, but we note that the [O\textsc{iii}] estimates (while broadly consistent with the other indicators due to their uncertainty) would produce a more convincing increase with redshift if for instance the [O\textsc{iii}]/\beta ratio were to decrease with redshift.

The parametrization of \( \rho_{\text{SFR}} \) according to \( \rho_{\text{SFR}} \propto (1 + z)^\gamma \) gives \( \gamma = 4.58 \) when handling errors as the maximum and minimum values of \( \rho_{\text{SFR}} \) produced from the 1σ confidence limits of each measurement of \( \phi^*, L^* \) and \( \alpha \). We note that applying the same...
least-squares fit to the data of Ly et al. (2007) gives a slope of $\gamma = 4.20$. Applying a blanket 10 per cent error on $\rho_{\text{SFR}}$ to our data results in $\gamma = 3.02$, similar to the parametrization of Sobral et al. (2013). The discrepancy between these results and shallower slopes in the literature is attributed to the small volumes probed in our low-redshift slices. For instance, using the figures of Somerville et al. (2004) we expect uncertainty in $\rho_{\text{SFR}}$ in the lowest redshift slices to be comparable to the measurements themselves due to cosmic variance. Larger areas of sky are required for this technique to be of use in determining the $\rho_{\text{SFR}}$ at low redshift as the redshift slices sampled in NB surveys are especially susceptible to cosmic variance. We return to this point in Section 7.4. We note also that the [O\text{II}] LF of Gilbank et al. (2010) at $z \sim 1$ gives a $\rho_{\text{SFR}}$ in good agreement with the progression of [O\text{II}] values seen in panel 10(b).

This technique is however sensitive to particularly low SFRs, probing well-below SFR* ($\leq 1 \text{ M}_\odot \text{ yr}^{-1}$) in the deepest data sets allowing a more accurate evaluation of $\alpha$. While the role of these faint galaxies’ contributions to the overall $\rho_{\text{SFR}}$ is unclear, if we wish to understand the nature of this faint population it is important to first constrain $\alpha$.

7.2 The faint-end slope

The gradient of the faint-end slope, $\alpha$, is the subject of much debate. The recent work of Sobral et al. (2013) made a significant advance on previous studies by examining estimates of $\alpha$ across a broad redshift range using the first sample of consistently selected emission line objects out to $z = 2.23$. An important part of determining the most likely LF at any redshift however, is to determine the significance of the galaxies we are capable of detecting in the analysis within the general population at redshift $z$.

As discussed in Section 5.2, we compute the ratio of detectable galaxies to the total number of galaxies capable of hosting the line. Investigating this ratio, we find that the best-fitting $\alpha$ of each LF is in fact very sensitive to the assumed faintest broad-band magnitude of a galaxy that can host an emission line (i.e. the denominator of this fraction). The analysis of Sobral et al. (2013) assesses emission-line detection fraction down to the limit of their NB data, and so we investigate the effect of assessing our data in the same way.

Evaluating our data set with $m_{\text{limit}}$ set to the limiting NB magnitude ($m_{\text{limit}}(\text{NB})$) we find that the resultant values of $\alpha$ are consistently steeper than for our $m_{\text{limit}}$(EW) analysis, the medians of the two samples being offset by 0.3. Furthermore, the range in values of $\alpha$ is reduced (from 0.90 to 0.66) through use of the ($m_{\text{limit}}(\text{NB})$) limit across the same redshift range.

Taking the well-constrained NB921 H\alpha and NB921 [O\text{II}] LFs, we vary the input limiting EW value for $m_{\text{limit}}$. The sensitivity of $\alpha$ to the input value is displayed in Fig. 11. As $EW_{\text{limit}}$ is increased, a larger fraction of the population of galaxies can host emission lines, and so the detection fraction decreases and the resultant $\alpha$ steeps. Conversely, however, as the input $EW_{\text{limit}}$ is lowered below 100 Å the numerator of equation (12) (the red portion of a line in Fig. 8) begins to decrease when $m_{\text{limit}}$ crosses the 5$\sigma$ detection limit. As the detection limit moves to brighter magnitudes, the galaxies undetected due to their position beneath the horizontal observed EW limit in Fig. 8 become a larger fraction of the overall population, and so the detection fraction again decreases, and $\alpha$ steeps. The shallowest $\alpha$ therefore, should correspond to the point where the NB detection limit, $m_{\text{limit}}$, is equal to $m_{\text{limit}}$.

7.3 The effect of $\alpha$ on $\rho_{\text{SFR}}$

The effect of $\alpha$ on $\rho_{\text{SFR}}$ is dependent on the value of $\alpha$ itself. For shallow $\alpha$, the effect is small, however, for steeper slopes the $\rho_{\text{SFR}}$ estimate is altered more dramatically for a small change in $\alpha$. For instance, taking the well-constrained NB921 [O\text{II}] LF at $z = 1.46$ and altering $\alpha = -0.91$ by $\Delta \alpha = 0.1$ in either direction (holding $\phi^\prime$, $L^\prime$ constant) causes the $\rho_{\text{SFR}}$ to rise by $\sim$5 per cent (steeper (more negative) $\alpha$) but only decrease by $\sim$3 per cent for the corresponding $\Delta \alpha = 0.1$ towards shallower (less negative) $\alpha$.

Taking the same approach with the NB921 H\alpha LF at $z = 0.4$, $\alpha = -1.14$, and the equivalent $\Delta \alpha = 0.1$ changes in $\alpha$ result in a 11 per cent increase in $\rho_{\text{SFR}}$ for steeper $\alpha$ and an 8 per cent decrease for a shallower slope. This behaviour is reflected in the lower panel of Fig. 11; however, it is also apparent that $\rho_{\text{SFR}}$ increases more rapidly towards smaller limiting EW values for $m_{\text{limit}}$. We interpret this as the result of an unrealistic limit to $m_{\text{limit}}$ causing the detection fraction of galaxies to become very small and produce an unrealistic correction inconsistent with the data.

7.4 Comparison to other studies

The NB921-selected [O\text{II}] LF and $\rho_{\text{SFR}}$ at $z = 1.46$ is in excellent agreement with the results of Sobral et al. (2012)’s UDS+COSMOS selection. Conversely though, comparison of our NB921-selected sample of H\alpha emitters to the selection of Sobral et al. (2013) at $z = 0.4$ reveals our $\rho_{\text{SFR}}$ is much lower than the UDS+COSMOS value. However, Sobral et al. (2013) found a difference in $\rho_{\text{SFR}}$ between their UDS and COSMOS-selected samples, with the UDS value being a factor of 3 lower than that for COSMOS. Their best-fitting LF using UDS emitters alone produces $\rho_{\text{SFR}} = 0.014 \text{ M}_\odot \text{ yr}^{-1}$ (D. Sobral, private communication) a value slightly higher than our own. We are able to fully account for this difference if we use the same method of redshift slice assignment as in Sobral et al.
(2013) (where objects with $K$ mag $> 23.0$ were assigned according to $B-R$, $i-K$ colour). Our work uses a later UDS data release and so deeper near-IR imaging results in more accurate photometric redshifts for faint objects – these are in good agreement with Grützbauch et al. (2011). We also confirm the reliability of our aperture corrections through the agreement of $\rho_{SFR}$ at $z = 0.4$ between Sobral et al. (2013)’s UDS-only sample and our H$\alpha$ emitters according to $B-R$, $i-K$ colour ($\rho_{SFR} = 0.0138 \, M_\odot \, yr^{-1}$). The recovery of $\rho_{SFR}$ despite the differing aperture sizes used for detection (Sobral et al. 2013 use a 3 arcsec aperture at low redshift) lends support to the success of our global corrections for aperture flux loss.

As discussed in Section 6.1.2, the uncertainty in our results due to the finite number of objects detected per redshift slice is investigated via a jack-knife analysis. However, it is not possible to tell if the values of $\rho_{SFR}$ we compute are representative of the overall $\rho_{SFR}$ at each redshift. This depends on the large-scale structure in the UDS at the redshifts we sample. It is not possible to tell whether these regions are particularly under or overdense without a comparison to many more similar surveys. For example, in the lowest redshift slice that we sample, [O III] at $z = 0.14$, we would require a contiguous volume ~140 times larger in order for the 1σ uncertainty in number counts of galaxies to reduce to 20 per cent according to the calculations of Somerville et al. (2004). The issue can be more efficiently addressed by observing widely separated regions of sky and comparing the variations seen to between such fields to predictions. For example the divergence of $\rho_{SFR}$ by a factor of ~3 between this work and Sobral et al. (2012)’s UDS+COSMOS measurement at low redshift (H$\alpha$ at $z = 0.40$) is quantitatively consistent with the predictions of Somerville et al. (2004). Finally, we note that our value of $\rho_{SFR}$ at $z = 1.46$ is in excellent agreement with both the $\rho_{SFR}$ of Sobral et al. (2012)’s UDS+COSMOS measurement, and Ly et al. (2007)’s SDF measurement. Based on these comparisons, we note that our estimates of $\rho_{SFR}$ in the lower redshift slices may differ from the true values by up to a factor of ~8 in the lowest redshift slice, $z = 0.14$. The resultant steep value of $\gamma$ should therefore not be a cause for concern when the uncertainty due to cosmic variance is so large.

8 CONCLUSIONS

We have developed a robust method for evaluating LFs which correctly handles the selection criteria of NB samples, negating the need for retrospective corrections to the data set. As a result, the method is suitable for highly irregular filters, and is applicable to a broad range of data sets. This is the first maximum-likelihood analysis of NB emitters using one of the largest NB-selected data sets to date. Our results can be broadly summarized as follows.

(i) We have presented maximum-likelihood LFs in 12 redshift slices out to $z = 1.64$, and demonstrated that this technique produces results in concordance with literature LFs using similar data sets over similar redshift ranges. In particular, we confirm the results of Sobral et al. (2013) and Sobral et al. (2012) in the $z = 0.4$ and $z = 1.46$ redshift slices for the H$\alpha$ and [O III] NB921 samples, respectively.

(ii) We find the evolution of the $\rho_{SFR}$ to rise according to $\rho_{SFR} \propto (1 + z)^{1.58}$ but conclude that this is strongly affected by cosmic variance in the small volumes probed at low redshift through this technique.

(iii) While the areal coverage of this survey is not large enough to place constraints on $\rho_{SFR}$ at low redshift, the sensitivity to very low SFRs that deep NB observations provide makes this technique ideal for constraining the faint end slope.

(iv) We discover that the detection fraction of emission lines plays an important role in the determination of the faint end slope. We propose that the limit of integration for the total number of galaxies capable of hosting an emission line should be empirically motivated, and here we base this on the line of interest’s maximum observed EW in the local Universe.

(v) The deep NB data will still provide useful information on the population of objects making up the faint-end slope, and allow an evaluation of SFR with stellar mass. We will present this analysis in a future paper.

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