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Collaborative Optimization of Container Liner Slot Allocation and Empty Container Repositioning Based on Online Booking Platform

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Abstract: The shipping market is unpredictable and volatile due to some uncontrollable factors such as epidemic, conflicts and natural disasters. There is always an imperfect match between the supply capacity of liner companies and the actual demand of the market, which leads to a waste of slot resources and/or unsatisfied customer demand. Furthermore, the trade off between empty container transportation and laden container transportation is the crucial problem of strategic importance for liner companies. To deal with the above problem, this paper aims to develop a new solution to the collaborative optimization problem of container slot allocation and empty container repositioning by exploring the resource allocation, storage, and repositioning methods collaboratively. An online booking platform is introduced in this paper, and no-shows and customer preferences are considered in the analysis. An innovative integer programming model is established based on an online booking mode and a delivery-postponed strategy. A new branch-and-cut algorithm is then proposed to solve the problem. Finally, numerical experiments are conducted to verify the effectiveness of the proposed model and algorithm. The experimental results show that collaborative optimization can remarkably enhance the revenue of liner companies along with increasing the utilization of slot resources.

Keywords: collaborative optimization; slot allocation; empty container repositioning; online booking platform; delivery-postponed strategy; no-show

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1. Introduction

Ocean transportation is the cornerstone of global trade and supply chains [1]. In 2020, ocean transport carried more than 81% of the international trade volume, with container traffic reaching 815.6 million Twenty-feet Equivalent Units (TEUs) [2]. The recent growing demand of container transportation has heightened the unpredictability and volatility of the container shipping market. A report suggests that from 2021 to the first quarter of 2022, a significant increase in demand for container transport has caused an insufficient supply (capacity) of container vessels, increasing freight rates [3]. Thus, a new round of shipbuilding booms was sparked. Unfortunately, since the second half of 2022, the demand for container transportation has declined rapidly. With the mass of new container ships entering the market, container transportation freight has fallen sharply. According to Drewry's WCI World Container Index, from March 2022 to March 2023, the 40 ft container freight rate fell by more than 80%. The competition situation has been more intense with the rapid changes in the shipping market. This has motivated liner shipping lines to seek new solutions to sustaining revenue and enhancing competitiveness. In the case of limited capacity resources, container shipping lines need to allocate the slot reasonably and scientifically to maximize transportation efficiency and revenue under the premise of satisfying different customer preferences. Unfortunately, it is not easy, if at all possible, to increase freight rates in a fiercely competitive shipping market. Therefore, effective management of container slot allocation becomes an alternative method for lines to increase revenue [4].

The core part for the slot allocation problem of containers (SAPC) is that liner companies can dynamically receive and reject booking requirements to maximize revenue. The features of the SAPC include (1) liner's operating routes, schedules, and capacity, which are fixed in the short term; (2) varying freight rates for cargoes based on market segmentation; (3) transportation demand for laden containers that exhibits randomness and volatility within a certain range; and (4) the sale and reservation of seats occurring before the ship's arrival. Given that the market became more volatile in recent years more than ever, classical methods for container slot allocation need to be revisited. This would stimulate new research needs on the solutions to the SAPC. SAPC is a key topic that container lines have been focusing on [5] from different perspectives, but relatively less studies have been focused on from the perspective of concerning empty container repositioning [6,7].

Empty container repositioning is an inevitable challenge addressed within the shipping industry [8,9] due to imbalanced trades between import and export countries. Empty container repositioning is an internal operation for the liner company to redeploy empty containers from surplus ports to deficient ports, aiming to minimize the cost and maximize benefits. This is a multi-period dynamic decision-making and complex problem. The issue has recently attracted growing concerns [10] and has faced challenges from the aforementioned volatile market in recent years. It is becoming a critical operational problem for container lines. The world trade pattern has the following characteristics. (1) Spatial dimension differences between import and export sources. Take the Trans-Pacific containerized trade for instance, where container volume from East Asia to North America was 26.1 million TEUs in 2022, and 6.6 million TEUs in the reverse direction [3]. Apparently, East Asia is one of the major export regions in the world. On the contrary, North America is the part of main import area. (2) Time dimension differences between demand and supply of empty containers. According to [3], the ships made more port calls in the first half than the second half of the year from 2018 to 2021. Evidently, the world trade in the first half of the year is more vigorous than that in the second half. (3) Imbalances in the volume of imported and exported containers. These characteristics result in the uneven distribution of empty containers worldwide. In order to satisfy the shipper's requirement of empty containers, liner shipping companies should implement inter-regional empty container repositioning, which is inevitable to transport empty containers from surplus ports to deficient ports [8,11].

As a result, empty container repositioning occupies a substantial ratio of transportation volume and generates additional transportation costs. Since 1993, the volume of empty container repositioning has occupied around 20% of the volume of sea container transportation [12], and the cost of that accounts for 20% of the general operating cost around the worldwide shipping companies [2]. In terms of long-term benefits, empty container repositioning can enable container resource recycling while reducing container leasing costs for liner companies. Hence, to maximize revenue, container lines should effectively address both the shipper's container transportation demand and their empty container transport demand simultaneously. Therefore, it is necessary and beneficial for container lines to make an intelligent trade off between the slot allocation and the empty container repositioning of different voyages.

To develop a new solution to the collaborative slot allocation and empty container repositioning problem (CSAECRP), liners can apply the revenue management (RM) method, a practical approach to improving revenue, reducing costs, and maximizing anticipatory profit [5]. The core of the CSAECRP is how to reasonably allocate and sell the limited slot capacity on container ships at the right time according to different types of demands. The key characteristics of CSAECRP have been analyzed, including the timeliness of product or service, the fixed capacity of the service, the segmented market against a product or service, the different demands of the segment markets, the advanced sale or booking before the service period, and the predictable market demand. These characteristics fit the concept of RM [1,13,14] well and hence allow its use in the context of this study to seek an optimization of a company's products or services by predicting customer behavior. The RM can help companies increase revenue growth and survive in a fiercely competitive environment. It effectively maximizes expected profits [5,15]. Therefore, this is the first papers that adopts a delayed delivery strategy [16,17] in the RM theory to study the CSAECRP, making new contributions from an applied research perspective.

Optimization of the online booking system of liner shipping is an effective way to improve its competitiveness. Shippers and liner companies need to bear the risk in the traditional homogeneous booking mode. For liner companies, bearing the risk of wasting slot resources caused by the shipper's no-show is highly appreciated. If a liner company oversells slots, shippers need to take the risk that the cargo cannot be transported on time. With the prompt evolution of Internet technology, "Internet + Shipping" has become an emerging research trend in the shipping industry every step of the way. Many shipping e-commerce online trading platforms have emerged in the shipping market, such as Maersk's 'Maersk Spot' and COSCO's 'SynconHub' [18]. The e-commerce online trading platform enriches the shipper's booking channels. Shippers can obtain timely information about liner shipping slots and make a reservation according to their demands. At the same time, the liner company can obtain slot reservation information and adjust the slot delivery and price momentarily. The online booking mode realizes a green, paperless office and aids in reducing the risk regarding shippers being unable to complete a transportation service on schedule, whilst improving a shipper's trust to the lines company [18]. Obviously, such advantages of using on-line booking systems can effectively improve the competitiveness of liner companies [19]. Especially under the continuous influence of various unstable factors (such as epidemic viruses), shipping via online trading platforms has become an inevitable tool rather than an optional choice for the shipping industry [20].

Although showing some attractiveness, online booking and empty container repositioning studies are still, at large, treated separately in the existing literature and in practice. Aiming at taking advantage of both solutions, this paper pioneers a new integer programming model to tackle the CSAECRP based on the delayed delivery strategy in the RM theory in order to incorporate the solution of online booking and empty container repositioning. Compared to the very limited studies in this context (e.g., ref. [17]), this paper makes new methodological contributions by conducting new analysis on (1) the trade off between laden container transportation and empty container repositioning collectively and (2) improvement of the utilization rate of slot resources and customer satisfaction simultaneously in the process of solving the CSAECRP. Therefore, the findings can provide a scientific basis to rationalize the relevant decision making of liner shipping companies.

This paper is organized as follows: A literature review is provided in Section 2. Section 3 describes the problem. Integrated model formulations are presented in Section 4. Section 5 developed a branch-and-cut algorithm. Numerical experiments and sensitivity analysis are presented in Section 6. The research findings are summarized in Section 7.

2. Literature Review

This research is on the allocation problem of container–liner slot resources devices from the perspective of RM. This field of research encompasses various dimensions such as market segmentation, slot-booking strategies, and channels. Originating in the 1970s, RM aids to maximize revenue and profit through tailored services to personalized demands. RM is applied to tackling different products or services within constrained capacities, exploiting prearranged information [21]. Within the context of its applications in transport slot allocation, RM emerged in air transportation slot allocation, and then [22,23] introduced it into the maritime transportation industry. Following the slot allocation problem with RM, the research scenario is gradually evolving from a single route and single voyage [24,25] to multiple routes and multiple voyages [17,26].

Market segmentation serves as a prominent strategy in addressing resource allocation challenges within the liner market. It refers to the classification process of categorizing the market into distinct segments based on variations in customers' consumption demands and purchasing behaviors. Any market segment comprises consumer groups with similar demands. The segmentation strategy within the liner shipping market was initially proposed by [27], followed by the presentation of various segmentation criteria. The current market segmentation criteria have been summarized, including the type of containers, the partnership between the shipper and the liner, and the urgency of the cargoes (requirements for transportation time). In addition, the liner container slot resource allocation market is subdivided into dry bulk containers, refrigerated containers, and open-top containers sized for 20′ and 40′ containers separately by [28–30]. Refs. [31,32] classified the market into contract customers, general customers, and emergency customers based on the type of shipper. Refs. [31,33] divided the market into contract and spot markets. Ref. [16] subdivide the market into emergency and non-emergency cargo markets based on service time considerations.

Due to intense market competition, the shipping market has evolved into a service-oriented market. Aiming at upgrading market competitiveness, liner companies provide diverse services for different customers to boost revenue. Therefore, a series of booking strategies are derived, each tailored to address specific challenges and opportunities within the industry. These strategies include bidding strategies, booking limit strategies, nested booking limit strategies, overbooking strategies, and delayed delivery strategies. The bidding strategy was first proposed by [34]. Subsequently, ref. [35] pioneered the application of bidding strategies to revenue management network problems. Building upon this foundation, the bid-price control strategy [36] and discriminatory bidding strategy [37] were derived. Ref. [38] further contributed to the field by proposing the top three slot reservation acceptance strategies mentioned above. Ref. [39] verified that the booking limit strategy can benefit liner companies compared to the traditional first-come, first-served strategy by simulation. In terms of delivery strategies, ref. [40] pioneered a delay-in-delivery strategy that allowed liner companies to delay shipments to contracted customers. Expanding on this, ref. [17] derived and compared the overbooking (OB) with delivery-postponed (DP) strategies for liner companies.

Research on issues related to slot allocation will gradually focus on the new slot reservation channel under demand orientation. There are many drawbacks to the traditional single-slot sales channel. Firstly, when market demand is vigorous, a liner company may struggle to fulfill the shipping requirements of contract shippers. Conversely, during periods of weak market demand, the contract shipper cannot provide sufficient supply, resulting in a waste of slot resources. Recognizing these limitations, scholars are increasingly emphasizing the importance of broadening sales channels. Ref. [18] highlighted the significant advantages of shipping e-commerce in the face of COVID-19. Ref. [19] analyzed the SAPC in the e-commerce environment to maximize the marginal cost of liner companies. The result indicated that the e-commerce sales channel is predictable and outperforms traditional sales channels in terms of revenue. Furthermore, refs. [41,42] devised a container-open booking platform. Building upon this platform, ref. [43] developed a multi-agent architecture to realize timely message interaction among autonomous agents, shippers, freight forwarders, and liner companies.

To continuously increase liner companies' revenue, scholars have jointly optimized the SAPC with other issues. Refs. [44,45] have researched the combination problem of route optimization and SAPC. Additionally, some scholars addressed the joint optimization problems of SAPC and empty container repositioning. Ref. [46] investigated SAPC with multi-type containers amid seasonal demand fluctuations, taking into account the empty container transportation. Their findings underscored the higher priority of refrigerated and 40′ dry bulk containers over ordinary containers. Ref. [47] employed RM to tackle SAPC alongside the empty container repositioning issue for Asian ocean carriers, devising estimation methods for the expected cost of empty container repositioning. Ref. [8] established a bi-level linear programming model to address this jointly optimization problem. The upper model focused on the SAPC to maximize routes' profit, while the lower model minimized transportation costs associated with empty container repositioning. Ref. [46] explored the SAPC, considering empty container repositioning with the ship schedule adjustment, revealing that several factors, such as cargo transfer cost, potential profit of cargo handling, and profit of empty container repositioning, have significant effects on ship capacity decision making and port selection. Furthermore, ref. [1] combined the SAPC with empty container inventory management, considering market segmentation and no-shows scenarios. The brief summary of the literature is shown in Table 1.

Table 1. A summary of the existing literature in the field of slot allocation.

Research Perspective	Category	Literature
Routes and voyages (periods) single/multi	Single	[24,25]
	Multi	[26]
Market segmentation criteria	Container types (dry/refrigerated/20'/40')	[28–30]
	Shipper types (contract/general/emergency)	[31,32]
	Contract/spot markets	[31,33]
	Emergency/non-emergency markets	[16]
Booking strategy	Bidding/bid-price control/discriminatory bidding strategy	[38]
	Booking limit strategy	[39]
	Delay-in-delivery strategy	[40]
	Overbooking and delivery-postponed strategies	[17]
Joint optimization problems of SAPC and empty container repositioning	Shipping e-commerce platform	[18,19,41–43]
Booking strategy	Multi-type containers and seasonal demand fluctuation estimation methods	[47]
	A bi-level linear programming model	[8]
	Ship schedule adjustment empty container inventory management	[1,46]

In the research on optimization methods, most of the literature relies on commercial software such as Cplex [1,19], Gurobi [4], and WinQSB 2.0 [8]. These types of optimization software achieve high accuracy for small-scale problems; however, they struggle to deliver satisfactory solutions within a short time when addressing large-scale challenges. To overcome this limitation, some researchers have focused on developing optimization algorithms tailored for large-scale problem solving, such as the NSGA-II algorithm [30] and a pattern-search-based solution algorithm [48]. Others have proposed approximate algorithms to handle cabin allocation in stochastic scenarios, including sampling-based average approximation methods [5,49] and approximate dynamic programming approaches [50–52].

In the existing literature on slot allocation, the research achievements of collaboratively optimizing empty container repositioning and slot allocation are scanty, requiring new models and empirical evidence. Much of the literature primarily focuses on the transportation demand for laden containers, neglecting the objective need for empty container repositioning. To the best of our knowledge, the current studies on empty container repositioning have overlooked the transformation between empty and laden containers. Many studies simplify the problem of empty container repositioning as a single-period problem [10,12,46,53–59], disregarding its multi-period nature. Consequently, the collaborative optimization problem of multi-cycle empty container repositioning and slot allocation still has theoretical implications that need to be further explored. Moreover, most papers on resource allocation predominantly employ the traditional "offline booking" model, which may not adequately address the dynamic nature of the container liner shipping market. Comparatively, an online booking system can better fit the volatile market, with the container liner shipping sector being exposed in recent years due to its advantages in dealing with market dynamics in real time. This paper brings novelties to our understanding of this area through the collaborative optimization of multi-cycle slot allocation and empty container repositioning under an online booking mode. A new mixed integer programming model is established based on market segmentation and delivery delay strategies. The commercial solver (Cplex) solves the small-scale numerical experiment, and a branch-and-cut algorithm solves large-scale examples.

The contributions of this paper are summarized as follows: Firstly, a novel CSAE-CRP with a DP strategy based on online booking is proposed. It contributes to the new collaborative optimization on resource allocation, storage, and repositioning problems. Secondly, an online booking platform is introduced, with which the corresponding mixed integer linear programming (MILP) model is newly developed and no-show and discrimination pricing are incorporated and addressed. Lastly, a new branch-and-cut algorithm is proposed to resolve the MILP model.

3. Problem Description

3.1. Market Segmentation and DP Strategy

When providing cargo transportation services to customers, container liner companies should guarantee that cargo is transported securely and completely between the departure and destination ports. Moreover, liner companies are supposed to provide bespoke services owing to the customer's different preferences, including but not limited to transportation rates, transportation time, and the supply and service capabilities of empty containers.

Some cargo has a short shelf life, and the requirements for transportation time are strict, such as fresh meat, seafood, eggs, vegetables, fruits, flowers, and dairy products. These products are called "time-sensitive" (TS) cargo. To ensure the quality of TS cargo, lines are expected to transport containers laden with TS cargo to the destination port as soon as possible. The shorter the transportation time, the better the quality; hence, the shipper can obtain higher revenue. Therefore, shippers are agreeable to remit higher rates for TS cargo to shorten the waiting time for transportation. This prompts liner companies to provide the fastest transportation services for TS cargo at higher rates. In order to enhance the satisfaction of shippers, liner companies ought to subdivide the market and provide specialized services for the segmentation market. The market is divided into the TS cargo market and the ordinary cargo market in this paper.

The DP strategy proposed by [16,17] is adopted, since the capacity of container liners is limited. It refers to dividing the cargo into different grades according to the freight paid by the shippers. When the ship capacity of the present voyage is insufficient, some low-grade shipments will be delayed until the next voyage. According to the sensitivity of cargo to time, container cargo is divided into TS cargo and ordinary cargo. The TS cargo arriving at the port on time will be serviced quickly and transported on time. For ordinary cargo, when the remaining capacity of the liner is sufficient, it will be transported on the current voyage. In contrast, if the remaining capacity of the liner is inadequate or the cargo is no-show (delayed arrival at the port, that is, for various reasons, failing to appear at the designated port before the scheduled time that agreed with the liner company), it will be postponed to the next voyage. Through this strategy, liner companies could formulate proper slot allocation schemes to augment capacity utilization of liners and increase their revenue. The essence of the DP strategy is that the liner company allocates the limited slots' resources as much as possible to customers who are willing to pay higher freight for reliable and fast transportation services, so as to achieve the goal of maximizing revenue and utilizing slot resources.

3.2. Online Booking Platform

Recently, many liner shipping companies, such as Maersk and COSCO Shipping, have applied online booking platforms to selling slots. Online booking platforms are an essential way to improve the competitiveness of liner companies because they have the advantages of real-time information interaction, paperless offices, simplified processes, and dynamic allocation. The procedure of CSAECRP contains a lot of participants in the shipping supply chain, consisting of liner company (that own the ships), shippers (or consignees), secondary freight forwarder (which collect cargo transportation demand from shippers and then book container slots directly toward the liner company at lower freight, or send the demand to the first-tier freight forwarder), and a first-tier freight forwarder (who integrates slot demands from shippers or secondary freight forwarders and books slots from liners at the best price).

The process of CSAECRP replies on an online booking platform, as shown in Figure 1.

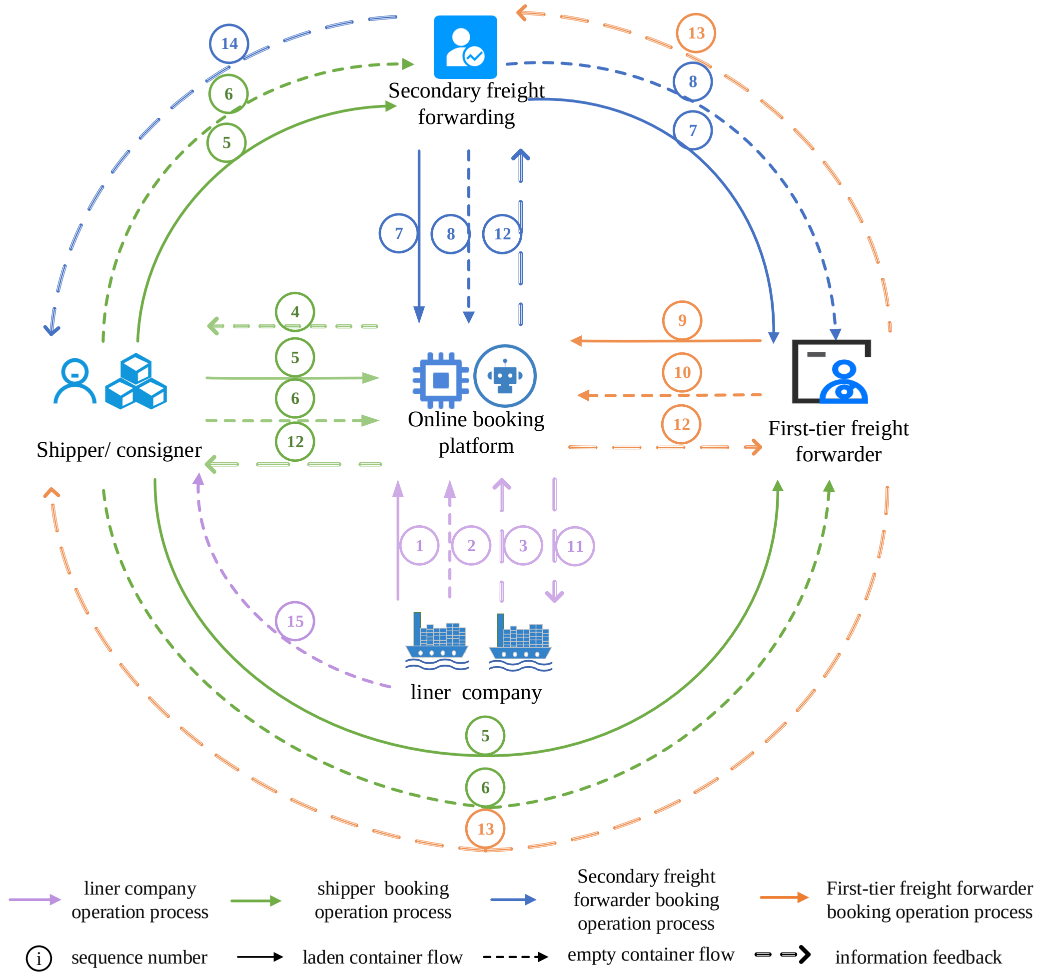

Figure 1. Online booking platform.

More specifically:

Step 1. Liner shipping company determines the market-allocated quantity of container slots.
Step 2. The liner shipping company confirms empty containers stockpiled in port.
Step 3. The liner shipping company determines the booking charge for different service levels.
Step 4. The shipper queries different service levels' slot information (including booking limit and price).
Step 5. The shipper submits the request for slot booking for the laden container.

Step 6. Shippers surrender empty containers on demand.
Step 7. Secondary freight forwarders commit a summary of laden container slot reservation requests.
Step 8. Secondary freight forwarders remit aggregated empty container demand.
Step 9. First-level freight forwarders commit to the collection of the laden container slot booking request.
Step 10. First-level freight forwarders submit the total empty container requirements.
Step 11. According to the decision support system, the platform feeds back information (including the slot allocation situation, empty container repositioning situation, and container leasing condition) to liner shipping companies.
Step 12. Information feedback of liner company, including the order acceptance or rejection.
Step 13. Information feedback from first-level freight forwards to secondary freight forwarders or shippers, including the order acceptance or rejection.
Step 14. Information feedback from secondary freight forwarders to shippers, including the order acceptance or rejection.
Step 15. The liner company provides empty containers for shippers.

Normally, the specific process of CSAECRP through the online booking platform is divided into four categories, including class one to class four. Class one: the shipper makes a reservation directly on the online booking platform. The specific process includes Step 1 to Step 6, Step 11 to Step 12, and Step15. Class two: the shippers entrust the secondary freight forwarder to book the slots on the online booking platform belonging to the liner company. The procedure contains Step 1 to Step 8, Step 11 to Step 12, and Step 14 to Step 15. Class three: the shippers delegate the first-level freight forwarder to book the slots through the online booking platform since different discounts on the booking price of slots are available to different customers. The proceeding is consisted of Step 1 to Step 6, and Step 9 to Step 15. Class four: the shippers commit the secondary freight forwarding to make a reservation of slots; the secondary freight forwarder will request the first-level freight forwarding to book the slots on the online booking platform after the booking requirements are summarized. The workflow is from Step 1 to Step 15.

3.3. Problem Description of CSAECRP

In this problem, without loss of generality, empty container repositioning is closely related to slot allocation. The former focuses on arranging the movement of empty containers in the shipping network in order to better meet the demand of customers for empty containers (carriers). The latter concerns on how to allocate slot resources for laden containers (i.e., converted from empty containers filled with goods) to provide satisfactory transportation services for shippers. To a large extent, they are complementary. Therefore, the CSAECRP of this paper is derived.

Since this paper stands in the perspective of liner companies, the decision-making problem is the acceptance or rejection of slot booking, the allocation and scheduling decision of slot resources and empty container resources, which does not involve the optimization of slot pricing. Thus, in order to simplify the problem, whether it is the owner, the second-level freight forwarder or the first-level freight forwarder, there is no difference for the liner company in the decision-making process. The model optimization is the problem that needs to be solved in the decision support system of the online booking platform. According to the market segmentation rule, DP strategy and the demand of customers, it determines the predetermined number of receptions (including TS goods and ordinary goods), the actual transportation volume of laden containers (similarly, involving TS cargoes and ordinary cargoes), the number of containers of ordinary goods delayed, the practical transportation volume of empty containers and the leasing number of empty containers between origin and destination ports at any voyage in each route.

4. Mathematical Models

4.1. Model Assumptions

To simplify the CSAECRP statement, the following assumptions are introduced:

1. All routes operated by liner companies can meet the weekly frequency.
2. Empty container repositioning is only transported by sea, regardless of other modes of transportation.
3. Empty container demand is satisfied through repositioning between ports and leasing.
4. The demand distribution of loaded and empty containers can be predicted according to historical data.
5. The loaded containers discharged into the port on the previous voyage will be converted entirely into empty containers before the ship's arrival on this voyage and will be returned to the port for storage.
6. The ordinary cargo can only be delayed once; the delayed freight will be preferentially arranged for transportation on the next voyage.
7. The capacity of the container leasing company is unlimited.
8. The identity or role of the booking person was not considered.
9. The storage space in the port is large enough.

4.2. Formulations of CSAECRP

Firstly, we introduce x_i^{vs}, which represents the container amount remained on a liner ship while the liner ship anchored in the ith port on voyage v along route s to complete the unloading operation but yet not start the loading operation. Here, we discuss two scenarios. Scenario 1 is shown in Figure 2: on the vth voyage along route s, the liner ship calls at port a, b, c, and d in sequence and finally returns to the port a. After the ship finishes the unloading task in port c on voyage v along route s, the source of containers still carried on the ship is divided into three parts. They are loaded and empty containers transported from port a to port d, from port b to d and b to a (the last port of this voyage), respectively. Hence, x_c^{vs} can be expressed as $x_c^{vs} = XC_{ad}^{vs} + XE_{ad}^{vs} + XC_{bd}^{vs} + XE_{bd}^{vs} + XC_{ba}^{vs} + XE_{ba}^{vs}$.

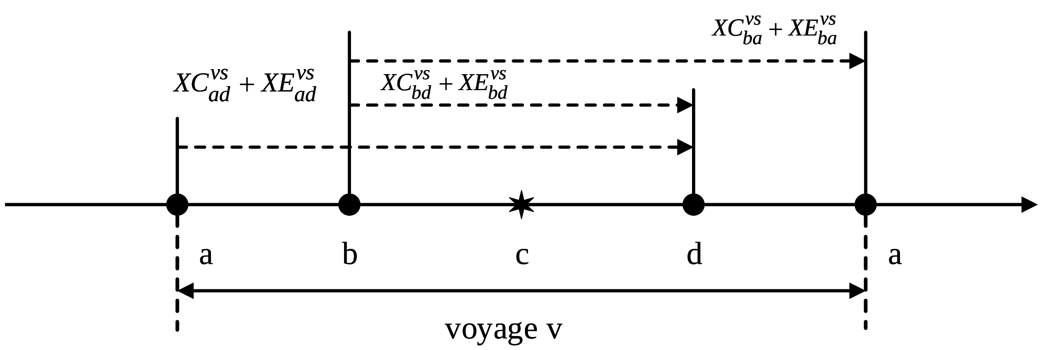

Figure 2. Diagram of the remained amount of containers after the liner ship has completed the unloading operation.

Based on the same route, we discuss scenario 2. As shown in Figure 3, after the liner ship finishes the unloading task at the first port (port a) on voyage v along route s, the excess containers come from two voyages. They can also be divided into three parts. They include the loaded and empty containers transported from port c on the prior voyage to port b, port d on the previous voyage to port b, and port d on the initial voyage to port c. Accordingly, x_a^{vs} can be expressed as $x_a^{vs}=XC_{cb}^{(v-1)s}+XE_{cb}^{(v-1)s}+XC_{db}^{(v-1)s}+XE_{db}^{(v-1)s}+XC_{dc}^{vs}+XE_{dc}^{vs}$. The above two situations can be summarized as Formulations (19) and (20).

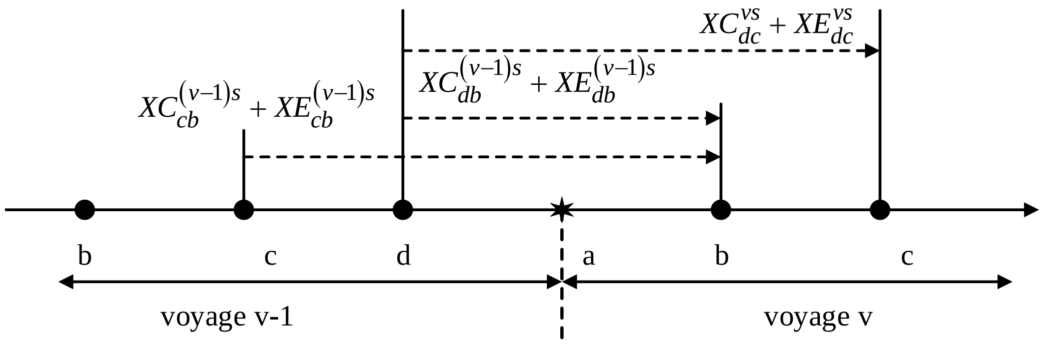

Figure 3. Diagram of the aboard number of containers after unloading the liner ship between various voyages.

The objective function is to maximize the profit of multiple routes, voyages, ports, and cargo, which consist of seven parts. Z refers to the total revenue of the liner company. The first two parts are the revenue gained by the liner company, in which the first part is revenue from container slot reservations for TS cargo and the second part is revenue for ordinary cargo. The rest are the costs of the liner company, which consist of the transportation cost of loaded containers and empty containers, the leasing cost of empty containers, the storage cost of delayed loaded containers and empty containers, and the fixed operating cost of the ship. The choice of what prices are set for items is justified by [1,7,17].

Constraints (1) and (2) represent the slot reservation quantity limit of the TS and ordinary cargo, respectively. Constraint (3) ensures that the transportation number of TS cargo is equal to their amount arriving at a port on time. Constraint (4) reveals that part of the ordinary cargo arriving at port on schedule can be transported on the current voyage; the rest will be delayed until the next voyage when the liner ship has space. Constraint (5) shows that the origin of ordinary cargo waiting to be transported on the present voyage can be divided into two parts. The first part is the ordinary cargo that is delayed to be transported from the previous voyage; the next part encompasses it arriving at the port on time for a set voyage. Constraint (6) stipulates that the number of ordinary cargoes allowed to be delayed until the next voyage cannot exceed the restriction. Constraint (7) requires TS cargo to be preferentially transported. Constraint (8) guarantees that the priority for a shipment belongs to TS cargo, with ordinary cargo being delayed. The, ships that have available storage can transport the ordinary cargo arriving on this voyage.

$$\max Z = \sum_{s\in S}\sum_{v\in V}\sum_{j\in P}\sum_{i\in P}\left(es_{ij}^{vs}\bullet XS_{ij}^{vs} + ef_{ij}^{vs}\bullet XF_{ij}^{vs} - cc_{ij}^{vs}\bullet XC_{ij}^{vs} - ce_{ij}^{vs}\bullet XE_{ij}^{vs}\right)$$
$$-\sum_{s\in S}\sum_{v\in V}\sum_{i\in P} cl_{ij}^{vs}\bullet XL_i^{vs} - \sum_{s\in S}\sum_{v\in V}\sum_{i\in P}\left(cs_{ij}^{vs}\bullet\left(\sum_{j\in P} XLF2_{ij}^{vs} + CE_i^{vs}\right)\right) - \sum_{s\in S}\sum_{v\in V} co^{vs}$$

s.t.

$$XS_{ij}^{vs} \le DS_{ij}^{vs} \qquad \forall i,j\in P,\forall v\in V,\forall s\in S \tag{1}$$

$$XF_{ij}^{vs} \le DF_{ij}^{vs} \qquad \forall i,j\in P,\forall v\in V,\forall s\in S \tag{2}$$

$$XLS_{ij}^{vs} = \left\lfloor \lambda s_{ij}^{vs}\bullet XS_{ij}^{vs}\right\rfloor \qquad \forall i,j\in P,\forall v\in V,\forall s\in S \tag{3}$$

$$XLF1_{ij}^{vs} + XLF2_{ij}^{vs} = \left\lfloor \lambda f_{ij}^{vs}\bullet XF_{ij}^{vs}\right\rfloor \qquad \forall i\in P,\forall s\in S, v=1 \tag{4}$$

$$XLF1_{ij}^{vs} + XLF2_{ij}^{vs} = XLF2_{ij}^{(v-1)s} + \left\lfloor \lambda f_{ij}^{vs}\bullet XF_{ij}^{vs}\right\rfloor \qquad \forall i,j\in P,\forall s\in S,\forall v\in V\cap v\ge 2 \tag{5}$$

$$XLF2_{ij}^{vs} \le \left\lfloor \gamma f_{ij}^{vs}\bullet XF_{ij}^{vs}\right\rfloor \qquad \forall i,j\in P,\forall v\in V,\forall s\in S \tag{6}$$

$$\sum_{j\in P} XLF1_{ij}^{vs} \le cap^{vs} - X_i^{vs} - \sum_{j\in P} XLS_{ij}^{vs} \qquad \forall i\in P,\forall s\in S, v=1 \tag{7}$$

$$XLF1_{ij}^{vs} \leq cap^{vs} - X_i^{vs} - \sum_{j \in P} (XLS_{ij}^{vs} + XLF2_{ij}^{(v-1)s}) \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{8}$$

$$XC_{ij}^{vs} = XLS_{ij}^{vs} + XLF1_{ij}^{vs} \quad \forall i, j \in P, \forall s \in S, v = 1 \tag{9}$$

$$XC_{ij}^{vs} = XLS_{ij}^{vs} + XLF2_{ij}^{(v-1)s} + XLF1_{ij}^{vs} \quad \forall i, j \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{10}$$

$$CE_i^{vs} = ICE_i^s + \sum_{\substack{j \in P \\ j \neq i}} XE_{ji}^{vs} - \sum_{\substack{j \in P \\ j \neq i}} XC_{ij}^{vs} - \sum_{\substack{j \in P \\ j \neq i}} XE_{ij}^{vs} + XL_i^{vs} \quad \forall i \in P, \forall s \in S, v = 1 \tag{11}$$

$$CE_i^{vs} = CE_i^{(v-1)s} + \sum_{\substack{j \in P \\ j \neq i}} XC_{ji}^{(v-1)s} + XL_i^{vs} + \sum_{\substack{j \in P \\ j \neq i}} XE_{ji}^{vs} - \sum_{\substack{j \in P \\ j \neq i}} XC_{ij}^{vs} - \sum_{\substack{j \in P \\ j \neq i}} XLF2_{ij}^{vs} - \sum_{\substack{j \in P \\ j \neq i}} XE_{ij}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{12}$$

$$XE_{ij}^{vs} \leq DE_{ij}^{vs} \quad \forall i, j \in P, \forall v \in V, \forall s \in S \tag{13}$$

$$\sum_{j \in P} XE_{ji}^{vs} + XL_i^{vs} + IEC_i^s \geq \sum_{j \in P} DE_{ij}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v = 1 \tag{14}$$

$$\sum_{j \in P} XE_{ji}^{vs} + XL_i^{vs} + CE_i^{(v-1)s} \geq \sum_{j \in P} DE_{ij}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{15}$$

$$\sum_{j \in P} (XC_{ij}^{vs} + XE_{ij}^{vs}) \leq cap^{vs} - X_i^{vs} \quad \forall i \in P, \forall v \in V, \forall s \in S \tag{16}$$

$$\sum_{j \in P} XE_{ij}^{vs} \leq cap^{vs} - X_i^{vs} - \sum_{j \in P} (XLS_{ij}^{vs} + XLF1_{ij}^{vs}) \quad \forall i \in P, \forall s \in S, v = 1 \tag{17}$$

$$\sum_{j \in P} XE_{ij}^{vs} \leq cap^{vs} - X_i^{vs} - \sum_{j \in P} (XLS_{ij}^{vs} + XLF2_{ij}^{(v-1)s} + XLF1_{ij}^{vs}) \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{18}$$

$$X_i^{vs} = \sum_{\substack{k<i<j \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{k<i<j \\ i,j,k \in P}} XE_{kj}^{vs} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XE_{kj}^{vs} \quad \forall i \in P, \forall s \in S, v = 1 \tag{19}$$

$$X_i^{vs} = \sum_{\substack{k<i<j \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{k<i<j \\ i,j,k \in P}} XE_{kj}^{vs} + \sum_{\substack{i<j<k \\ i,j,k \in P}} XC_{kj}^{(v-1)s} + \sum_{\substack{i<j<k \\ i,j,k \in P}} XE_{kj}^{(v-1)s} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XE_{kj}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{20}$$

$$XLS_{ij}^{vs}, XLF1_{ij}^{vs}, XLF2_{ij}^{vs}, XL_{ij}^{vs}, X_i^{vs}, XF_{ij}^{vs}, XC_{ij}^{vs}, XE_{ij}^{vs}, CE_{ij}^{vs} \in \mathbb{Z}^+ \bigcup \{0\} \quad \forall i, j \in P, \forall v \in V, \forall s \in S \tag{21}$$

Constraints (9) and (10) define that the loaded container transported on this voyage should contain the TS cargo transported on the current voyage, the ordinary cargo delayed to transport from the previous voyage, and the ordinary cargo transported on the current voyage. Constraints (11) and (12) show the state transition equation of the empty containers stored in a port after a ship's departure. Constraint (13) indicates the limitation of the empty container repositioning number. Constraints (14) and (15) ensure that the order for empty containers is met by repositioning, leasing, and storage numbers. Constraint (16) defines that the transportation quantity of loaded and empty containers should be less than the remaining capacity. Constraints (17) and (18) stipulate that the transportation priority of loaded containers is higher than that of empty containers. Constraints (19) and (20) show the recurrence formula of how many containers are remaining on the ship. Constraint (21) defines the value range of variables.

4.3. Basic Model

In order to validate the collaborative approach proposed in this study, a basic model is proposed as follows. It is a optimization model for the SAPC based the DP strategy. The objective function of basic model is maximize the revenue of liner company.

$$\max Z_2 = \sum_{s \in S} \sum_{v \in V} \sum_{j \in P} \sum_{i \in P} \left(es_{ij}^{vs} \bullet XS_{ij}^{vs} + ef_{ij}^{vs} \bullet XF_{ij}^{vs} - cc_{ij}^{vs} \bullet XC_{ij}^{vs} \right) - \sum_{s \in S} \sum_{v \in V} \sum_{i \in P} cl_i^{vs} \bullet XL_i^{vs} - \sum_{s \in S} \sum_{v \in V} co^{sv}$$

s.t.

$$XL_i^{vs} \geq \sum_{j \in P} DE_{ij}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \tag{22}$$

$$X_i^{vs} = \sum_{\substack{k<i<j \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XC_{kj}^{vs} \quad \forall i \in P, \forall s \in S, v = 1 \tag{23}$$

$$X_i^{vs} = \sum_{\substack{k<i<j \\ i,j,k \in P}} XC_{kj}^{vs} + \sum_{\substack{i<j<k \\ i,j,k \in P}} XC_{kj}^{(v-1)s} + \sum_{\substack{j<k<i \\ i,j,k \in P}} XC_{kj}^{vs} \quad \forall i \in P, \forall s \in S, \forall v \in V \cap v \geq 2 \tag{24}$$

$$XLS_{ij}^{vs}, XLF1_{ij}^{vs}, XLF2_{ij}^{vs}, XL_{ij}^{vs}, X_i^{vs}, XF_{ij}^{vs}, XC_{ij}^{vs} \in \mathbb{Z}^+ \bigcup \{0\} \; \forall i, j \in P, \forall v \in V, \forall s \in S \tag{25}$$

and Constraints (1)–(10).

Constraint (22) reveals that the demand for empty containers under the basic model is completely satisfied by the single channel of leasing empty containers. Similar to Constraints (19) and (20), Constraints (23) and (24) illustrate the calculation formula of the number of containers remaining on a liner ship while anchoring, without considering empty container repositioning. Constraint (21) is similar, with the valid range of variables being shown in Constraint (25).

5. Solution Algorithm

The branch-and-cut algorithm is used in this section to solve the CSAECRP. The important elements for this algorithm are introduced as follows:

5.1. Valid Inequalities

The aforementioned equations (i.e., Equations (1)–(21)) are able to be further strengthened by applying the following valid inequalities. Capacity constraint valid inequality:

$$CE_i^{vs} \geq \sum_{j \in P} XE_{ij}^{vs} \quad \forall i \in P, \forall v \in V, \forall s \in S \tag{26}$$

Constraint (26) refers to the capacity constraint of empty container repositioning. It defines that the repositioning number of empty containers cannot exceed their storage number at any port. Mutual exclusion valid inequality:

$$(1 - z_i^{vs}) \sum_{j \in P} XE_{ij}^{vs} \leq \sum_{j \in P} DE_{ij}^{vs} \quad \forall i \in P, \forall v \in V, \forall s \in S \tag{27}$$

An extra binary variable z_i^{vs} is introduced here. z_i^{vs} is equal to 1 if and only if the ith port on the voyage along the route of a ship is a deficient port. Constraint (27) is known as the logical inequality, which stipulates that empty container repositioning will not be carried out when the port is deficient (that is, the remaining empty containers in the previous stage of the port are not able to satisfy the requirement for containers at this stage.) General valid inequality:

$$\lfloor \mu_1 \rfloor \bullet XS_{ij}^{vs} + \lfloor \mu_2 \rfloor \bullet XF_{ij}^{vs} + \lfloor \mu_3 \rfloor \bullet XE_{ij}^{vs} \leq \mu_1 DS_{ij}^{vs} + \mu_2 DF_{ij}^{vs} + \mu_3 DE_{ij}^{vs} \quad \mu \in R^+ \tag{28}$$

Constraint (28) is a general valid inequality, which is inspired from the Chvátal–Gomory procedure [60,61]. (The proof of the validity of cuts is introduced in Appendix A).

5.2. Branch-and-Cut Algorithm

The application of the cut plane composed of the above valid inequalities will be illustrated in this section. The cuts and bound improvement suitable for solving the problem in this paper are introduced in this section. The branch-and-cut algorithm is outlined as follows:

Step 1. Initialize: Set $T = 0$, $UB = M$, $LB = 0$.

Step 2. Solve the LP relaxation: Obtain $X^* = \{x_1^0, x_2^0, x_3^0, \ldots\}$. This will detect an optimal solution, an updated upper bound solution or that the problem is infeasible.

Step 3. Branching: According to the most fractional branching strategy to create two new nodes.

Step 4. Fathoming rules: If the node relaxation is infeasible or $XLS_{ij}^{vs} + XLF1_{ij}^{vs} + XLF2_{ij}^{vs} + X_i^{vs} \prec cap^{vs}$ and $f(x) \prec UB * \tau, \tau \in (0.5, 1)$, fathom the node and return to Step 3. Otherwise, continue to Step 5.
Step 5. Insert all valid inequalities into the program.
Step 6. Solve the LP relaxation of new nodes, respectively.
Step 7. Update the upper and lower bounds again.
Step 8. If the termination condition is satisfied, end the procedure and input the X^{best}.

The pseudocode are presented in Algorithm 1.

Algorithm 1 Branch-and-cut algorithm.

Step1 Initialize: $T = 0, UB = M, LB = 0$.
Step2 Solve the LP relaxation, and then obtain $X^* = \{x_1^0, x_2^0, x_3^0, \ldots\}$
while $(X^* = \emptyset)$ **do**
  Break
  if $(X^* \in Z^+)$ **then**
    STOP, $X^{best} = X^*$
    if $(X^* \notin Z^+)$ **then**
      Set: $UB = f(X^*)$
    end if
  end if
end while
Step3 Branching: According to the most fractional branching strategy.
  Set: Choose node $x_i^j \notin Z^+$, set P1: $x_{i'}^{j'} \leq \lfloor x_i^j \rfloor$; P2: $x_{i'}^{j'} \geq \lfloor x_i^j \rfloor + 1$; T=T+1.
Step4 Fathoming: Fathom nodes according to the fathoming rules.
Step5 Insert: All valid inequalities (26)–(28) into the program.
Step6 Solve: The LP relaxation of P1 and P2, respectively. Obtain X_1^* and X_2^*.
Step7 Repeat: Update the upper and lower bounds.
while $X_1^* = \emptyset$ and $X_2^* = \emptyset$ **do**
  Break
  if $X_1^* = \emptyset$ and $X_2^* \in Z^+$ **then**
    STOP,$X^{best} = X_2^*$
  end if
  if $X_2^* = \emptyset$ and $X_1^* \in Z^+$ **then**
    STOP,$X^{best} = X_1^*$
  end if
  if $X_1^*(X_2^*) \in Z^+, X_2^*(X_1^*) \notin Z^+$ **then**
    $LB = f(X_1^*)$ or $f(X_2^*)$, P2(P1): repeat Step 3–Step 5
  end if
  if $X_1^* \in Z^+, X_2^* \in Z^+$ **then**
    if $X_1^* \geq X_2^*$ **then**
      STOP,$X^{best} = X_1^*$
    else
      $X^{best} = X_2^*$
    end if
  end if
  if $X_1^* \notin Z^+, X_2^* \notin Z^+$ **then**
    if $f(X_2^*)$ or $(X_2^*) < LB$ **then**
      Cut P1(P2), for P2(P1): repeat Step 3–Step 5
    end if
  end if
end while
Step8 Termination condition:
if T=100, Or $\frac{|UB-LB|}{LB} < \varepsilon$ **then**
  STOP, $f\left(X^{best}\right) = LB$
end if

6. Numerical Experiments

6.1. Data Description

Three routes of a liner shipping company are applied to study the CSAECRP in an online booking mode. As shown in Figure 4, the fixed-capacity container ships, which belong to the liner company, serve three routes. The container ships provide transportation services to shippers according to a fixed schedule (weekly) and a regular call order. The call order of the three routes is as follows:

Route1: Shanghai (SH)-Ningbo (NB)-Xiamen (XM)-Yantian (YT)-Singapore (SP)-Felixstowe (FT)-Rotterdam (RD)-Gdansk (GD)-Wilhelmshaven (WS)-Felixstowe (FT)-Port Kelang (PK)-Yantian (YT)

Route2: Tianjin (TJ)-Dalian (DL)-Qingdao (QD)-Shanghai (SH)-Ningbo (NB)-Singapore (SP)-Piraeus (PR)-Rotterdam (RD)-Hamburg (HB)-Antwerp (AT)-Rotterdam (RD)-Shanghai (SH)

Route3: Qingdao (QD)-Shanghai (SH)-Ningbo (NB)-Kaohsiung (KS)-Hong Kong (HK)-Yantian (YT)-Singapore (SP)-Piraeus (PR)-Colombo (CL)-Singapore (SP)-Hong Kong (HK)-Shanghai (SH)

Figure 4. Diagram of complex shipping route network.

The range of parameters is shown in Table 2, and the source of parameters is adjusted according to [17] and the field investigation by the authors. The capacity of container ships on the three routes is 12,000 TEU, 10,000 TEU, and 10,000 TEU, respectively. The transportation cost of a container is positively correlated with transportation distance. It is well known that the container freight rate is strongly associated with transportation costs. Therefore, the unit transportation freight rate and unit transportation cost of the container in this paper are set according to the navigation distance. In addition, the service provided for TS cargo is more attentive than that for ordinary cargo, and TS cargo does not bear the risk of delayed delivery. Consequently, it is reasonable to set the unit freight rate of TS cargo higher than that of ordinary cargo. The value is set at 1.25, based on the average data from the field investigation, and it can be adjusted in other cases to better reflect associated scenarios. The unit leasing cost of empty containers obeys the uniform random distribution U (150, 300), and the unit storage cost of containers obeys the uniform random distribution U (650, 750).

Table 2. Parameter setting for numerical test.

Parameters	Route1	Route2	Route3
es_{ij}^{v}	l_{ij}^{1}*0.5	l_{ij}^{2}*0.5	l_{ij}^{3}*0.5
ef_{ij}^{v}	l_{ij}^{1}*0.4	l_{ij}^{2}*0.4	l_{ij}^{3}*0.4
cc_{ij}^{v}	ef_{ij}^{1}* rand (0.5, 0.6)	ef_{ij}^{2}* rand (0.5, 0.6)	ef_{ij}^{3}*rand (0.5, 0.6)
ce_{ij}^{v}	cc_{ij}^{1}*rand (0.5, 0.6)	cc_{ij}^{2}*rand (0.5, 0.6)	cc_{ij}^{3}* rand (0.5, 0.6)
cl_{i}^{v}	rand (150, 300)	rand (150, 300)	rand (150, 300)
cs_{i}^{v}	rand (650, 750)	rand (650, 750)	rand (650, 750)
co^{v}	3000	3000	3000
DS_{ij}^{v}	$N(200, 2^2)$	$N(200, 2^2)$	$N(200, 2^2)$
DF_{ij}^{v}	$N(200, 2^2)$	$N(200, 2^2)$	$N(200, 2^2)$
DE_{ij}^{v}	$N(40, 2^2)$	$N(40, 2^2)$	$N(40, 2^2)$
IEC_{i}	rand (200, 500)	rand (200, 500)	rand (200, 500)
cap^{v}	12,000	12,000	12,000
λs_{ij}^{v}	0.95 + rand (−0.03, 0.03)	0.95 + rand (−0.03, 0.03)	0.95 + rand (−0.03, 0.03)
λf_{ij}^{v}	0.95 + rand (−0.03, 0.03)	0.95 + rand (−0.03, 0.03)	0.95 + rand (−0.03, 0.03)
γf_{ij}^{v}	0.3 + rand (−0.2, 0.2)	0.3 + rand (−0.2, 0.2)	0.3 + rand (−0.2, 0.2)

On the basis of historical digital data, the mean and standard deviation for the demand and the on-time arrival rate are presumed in this paper. The demand for TS and ordinary cargo obeys the normal distribution N (200, 22), and the demand for empty containers submits to the normal distribution N (40, 22) [52]. The BC algorithm described in Section 5 was accomplished in C++ by applying CPLEX 12.6.3. The calculation result of the numerical experiment was obtained using a computer, operating with Microsoft Windows 11 and an Intel® CoreTM i9-12900F CPU at 2.40 GHz and 32.0 GB of RAM . Calculations are shown in Table 3, and the obtained data are the mean values of the three experiments.

Table 3. The solutions of different cases.

			CPLEX		BC		
Case	**Route**	**Voyage**	**Obj. (USD)**	**Runtimes (s)**	**Obj. (USD)**	**Runtimes (s)**	**Cuts**
1	1	4	300,047,427	1.57	300,047,427	0.06	11
2	2	4	290,353,066	2.49	290,353,066	1.17	43
3	3	4	188,742,495	12.03	188,742,495	1.09	39
4	1,2	4	590,396,489	5.31	590,396,489	2.12	48
5	2,3	4	479,094,216	70.57	479,094,216	9.91	57
6	1,3	4	488,785,936	31.32	488,785,936	8.26	52
7	1,2,3	4	780,128,984	374.88	780,128,984	11.53	143
8	1,2,3	8	–	>10,800	1,558,274,566	1934.05	1348
9	1,2,3,4	4	1,023,417,086	300.28	1,023,417,086	72.81	144
10	1,2,3,4,5,6	4	1,453,286,074	1829.75	1,453,286,074	167.39	769

6.2. Test Results

Three new indicators (i.e., Equations (29) and (30)) are established to better illustrate the effectiveness of the strategy and collaborative optimization model. To fully demonstrate the trade off between empty container traffic volume and loaded container transportation volume, the concept of an empty weight ratio is introduced and can be formulated as

$$\alpha = \text{ave}(\frac{XE_{ij}^{vs}}{XC_{ij}^{vs}} * 100\%) \tag{29}$$

There are two sources of empty containers: one is empty container repositioning, and the other is empty container leasing. To show the trade off between empty container repositioning and the leasing volume, the concept of an empty container repositioning satisfaction rate β is introduced. This can be formulated as

$$\beta = \frac{\sum\limits_{s \in S} \sum\limits_{v \in V} \sum\limits_{j \in P} \sum\limits_{i \in P} XE_{ij}^{vs}}{\sum\limits_{s \in S} \sum\limits_{v \in V} \sum\limits_{j \in P} \sum\limits_{i \in P} (XE_{ij}^{vs} + XL_{ij}^{vs})} * 100\% \tag{30}$$

To evaluate the performance of BC, two sets of tests are compared for different problem sizes. The first set uses the CPLEX MIP solver to solve the model under default settings directly. The other group is computed with the BC algorithm. Ten different cases are tested in this paper, as shown in Table 3. Cases 1, 2, and 3 only contain one shipping route and four voyages, while Cases 4, 5, and 6 involve two routes and four voyages. The routes consisting of Cases 7 and 8 are the same, whereas Case 8 includes eight voyages. Evidently, Cases 9 and 10 contain more shipping routes. As shown in Table 3, the exact solution could be obtained by CPLEX in most instances. However, the CPU runtime of CPLEX increases significantly as the scale of the problem grows. While the size of the CSAECRP is enormous, CPLEX cannot obtain the exact solution in an acceptable time frame (10,800 s). Compared with the CPLEX, our algorithm has apparent advantages regarding its calculation time. Furthermore, the calculation precision of the BC algorithm is verified to be exact, showcasing its ability to find an optimal solution. The total revenue convergence process of Case 6 by applying the branch-and-cut algorithm is shown in Figure 5.

Figure 5 illustrates the number of containers of various types across multiple voyages on different shipping routes. Specifically, Groups 1(1) to 1(4) represent the container counts for different types across the first to fourth voyages on the first route. Similarly, the final group, which is 3(4), corresponds to the container counts for the fourth voyage on the third route. Each group comprises four columns of data, representing, from left to right, the number of containers for TS cargo, ordinary cargo, postponed cargo, and empty containers.

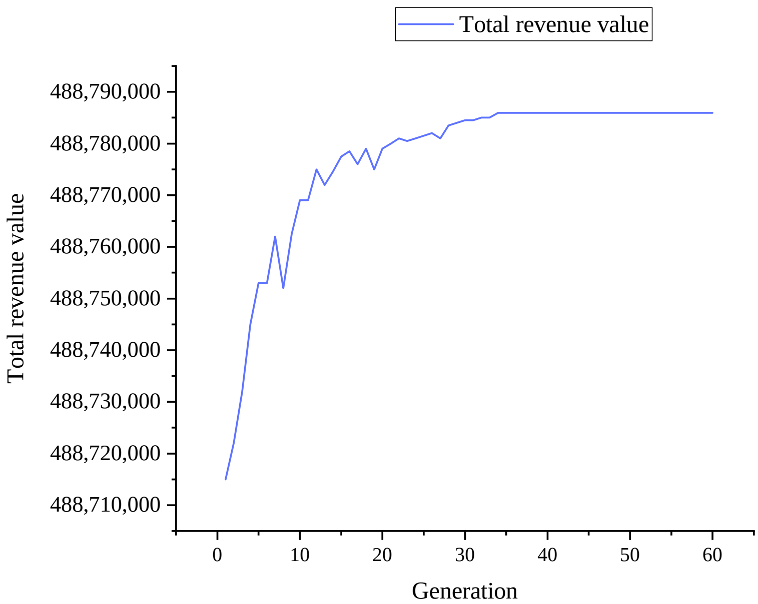

Figure 5. Solution process of the branch-and-cut algorithm.

It is important to note that the container numbers for TS and ordinary cargo are measured against the left vertical axis, while the numbers for postponed cargo and empty containers are aligned with the right vertical axis. As indicated in Figure 6, TS cargo has the largest volume of containers, followed by ordinary cargo. The quantity of TS cargo containers is approximately 1.2 to 1.4 times that of ordinary cargo containers. This is attributed to the fact that the freight rate for TS cargo is higher, leading liner companies to prioritize containers that offer higher revenue.

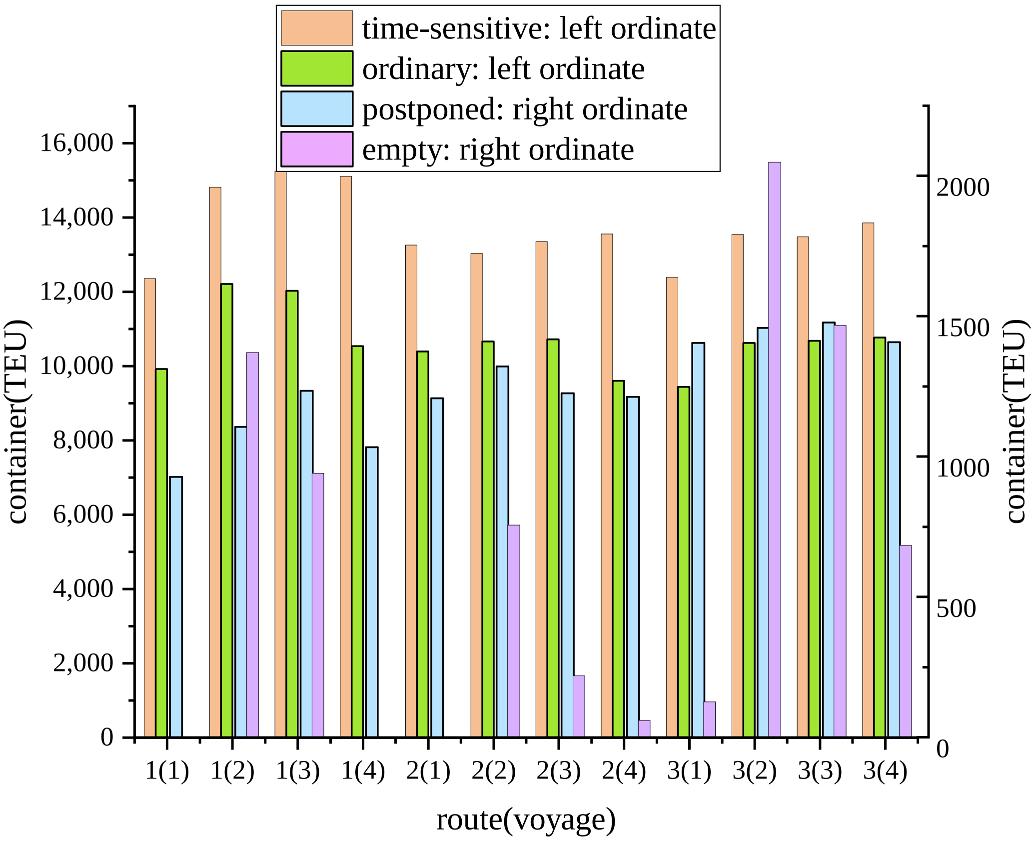

Figure 6. Diagram of number for four types of containers.

The results further reveal that the number of delayed containers tends to increase on voyages with higher volumes of TS cargo and empty containers, largely due to the limited capacity of ship resources. Additionally, it is evident that every route requires the repositioning of empty containers. Although repositioning empty containers does not generate direct revenue, it is necessary to meet the demand for empty containers from shippers. Given the high leasing costs for empty containers, repositioning them becomes a strategic choice aimed at balancing storage costs with leasing expenses. In some cases, the cost of repositioning empty containers between certain ports may exceed the leasing cost. Therefore, from a revenue management perspective, it is sometimes more cost-effective not to reposition empty containers between specific ports.

Table 4 presents the ratio of empty container transportation to laden container transportation for various voyages and routes. The values indicate that this ratio remains within 6%, underscoring the liner company's prioritization of laden containers with a higher revenue potential.

Table 4. The ratio of empty container transport number to loaded container transport number.

	Route1	**Route2**	**Route3**
Voyage 1	0	0	0
Voyage 2	2.358%	0.798%	4.354%
Voyage 3	2.444%	1.92%	3.324%
Voyage 4	0	0.207%	1.707%
Voyage 5	0.093%	0.253%	1.594%
Voyage 6	2.941%	2.017%	3.748%
Voyage 7	2.517%	1.627%	5.617%
Voyage 8	0.062%	0.014%	1.172%

The numerical variations in Table 4 reflect the periodic pattern of empty container repositioning. Specifically, the ratio is negligible (almost 0) for the first voyage of each route, primarily due to sufficient initial stocks of empty containers at the port, thus obviating the need for repositioning. In contrast, substantial increases in empty container repositioning occur during the second and third voyages, while volumes decrease for the fourth and fifth voyages. This cycle repeats with higher levels observed again during the sixth and seventh voyages.

During the initial voyage, the port's stockpile of empty containers meets the demand, eliminating the necessity for repositioning. However, to fulfill subsequent voyage requirements for empty container circulation, the liner company must reposition or lease additional containers starting from the second voyage. This strategic repositioning in the second and third voyages facilitates the necessary interchange and circulation between empty and laden containers within the route over the short term, thereby reducing empty container volumes during subsequent voyages.

Table 5 compares the results of two optimization models: the collaborative optimization model (denoted as "Co.") proposed in this paper under the enhanced delivery strategy and the basic optimization model (denoted as "Basic"), which uses a standard delay delivery strategy without considering empty container repositioning. The comparison focuses on three key metrics: profit, average space utilization, and average delivery delay rate.

Table 5. The solutions of different cases.

	Profit		**Average Space Utilization**		**Average Delivery-Delayed Rate**	
	Co.	**Basic**	**Co.**	**Basic**	**Co.**	**Basic**
	779,142,116	743,739,199	87.096%	76.944%	87.096%	76.944%
Enhancing rate	4.76%	-	10.151%	-	2.122%	-

As shown in Table 5, the profit of the collaborative optimization model is 4.76% higher than that of the basic model. Additionally, the collaborative model improves average space utilization by 10.151%, while the average delivery delay rate increases by 2.122%. These results demonstrate the effectiveness of the collaborative optimization model presented in this study.

The enhanced model, with its ability to improve slot resource utilization, offers significant advantages, helping liner shipping companies increase their revenue. The improvements in both profitability and space utilization underscore the model's potential to optimize operations and deliver better performance compared to traditional strategies.

6.3. Sensitivity Analysis

To validate the effectiveness and general applicability of the model and method proposed in this paper, a sensitivity analysis was conducted by considering several factors. Figure 6 illustrates the impact of changes in the unit freight rates for TS cargo and ordinary cargo on the total revenue of liner companies. The horizontal axis represents the percentage change in the unit freight rate, ranging from −0.9 (indicating a 90% reduction in the original unit rate) to 1 (indicating a 100% increase). The vertical axis represents the percentage change in total revenue, ranging from −57.91% (a 57.91% decline in profit compared to the original) to 120.50% (a 120.50% increase in revenue).

Three key insights can be drawn from Figure 7:

1. Total revenue increases with unit rate growth, and this upward trend becomes more pronounced as the unit rate rises.
2. When the unit freight rate decreases to 10–50% of the original, the change in total revenue stabilizes, indicating that revenue fluctuations remain relatively steady in this range.

3. The total revenue is more sensitive to changes in the unit rate of TS cargo compared to ordinary cargo, suggesting that variations in TS cargo rates have a more significant impact on overall profitability.

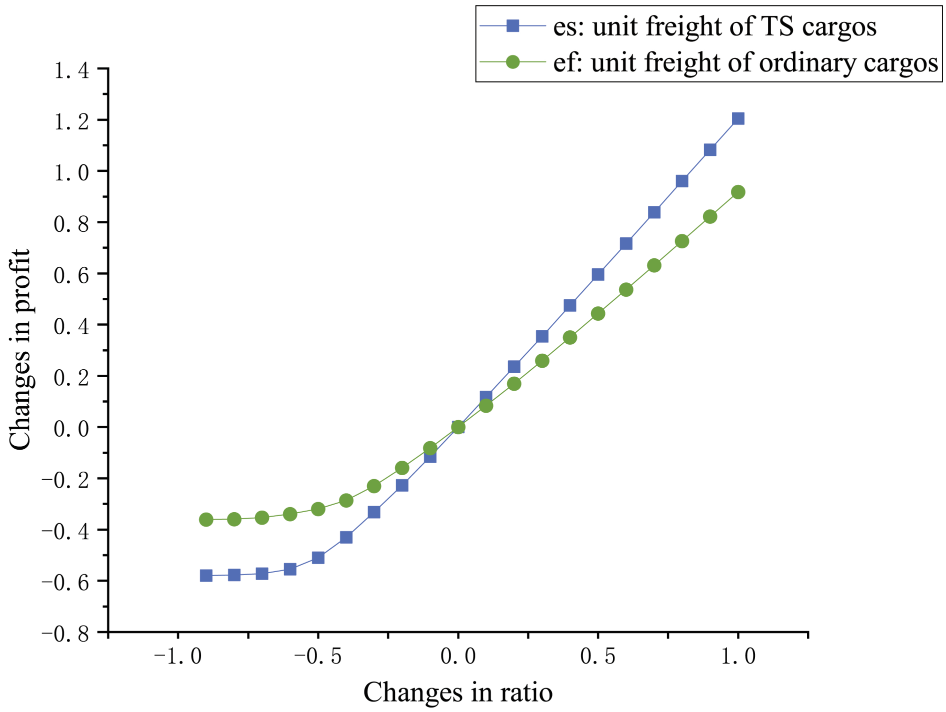

Figure 7. Sensitivity analysis of unit freight on the profit.

Additionally, this paper explores the impact of changes in unit costs on total revenue, including the unit transportation cost for laden containers, unit transportation cost for empty containers, unit leasing cost for empty containers, and unit stacking cost. Figure 8, consistent with Figure 7, shows the effect of these cost changes. The horizontal axis represents the percentage increase in unit costs, with the same variable range as in Figure 7.

It is important to note that the total profit change curve for the unit transportation cost of laden containers is based on the right vertical axis, with a range from −78.16% to 79.96%. The total revenue change curves for the other costs—empty container transportation, leasing, and stacking—are plotted on the left vertical axis. The ranges for these values are as follows: −0.26% to 1.28% for unit empty container transportation cost, −2.65% to 2.44% for unit empty container leasing cost, and −7.52% to 7.97% for unit stacking cost. Two key insights can be drawn from Figure 7:

1. Total profit decreases as unit costs increase, and the sensitivity of total profit to each cost varies significantly.
2. Among the different costs, the total revenue is most sensitive to the unit transportation cost of laden containers, while it is least sensitive to the cost of empty container repositioning. This is because the number of slots allocated to empty containers is relatively small compared to those allocated to laden containers.

This paper also examines the effect of changes in unit leasing costs on the sourcing of empty containers. Figure 9 illustrates the satisfaction ratio of empty container repositioning volume to the total volume of empty containers (i.e., the sum of repositioning volume and leasing volume). As depicted in Figure 9, with increasing unit leasing costs, the associated rental costs rise as well. Consequently, at the source of empty containers, the proportion of repositioned containers increases, while the proportion of leased containers decreases, which is a logical outcome. The sensitivity of the three routes to changes in empty container leasing costs is roughly similar. The variation in satisfaction ratios for these routes ranges from −10.71% to 2.02%, from −6.61% to 5.14%, and from −5.51% to 6.62%.

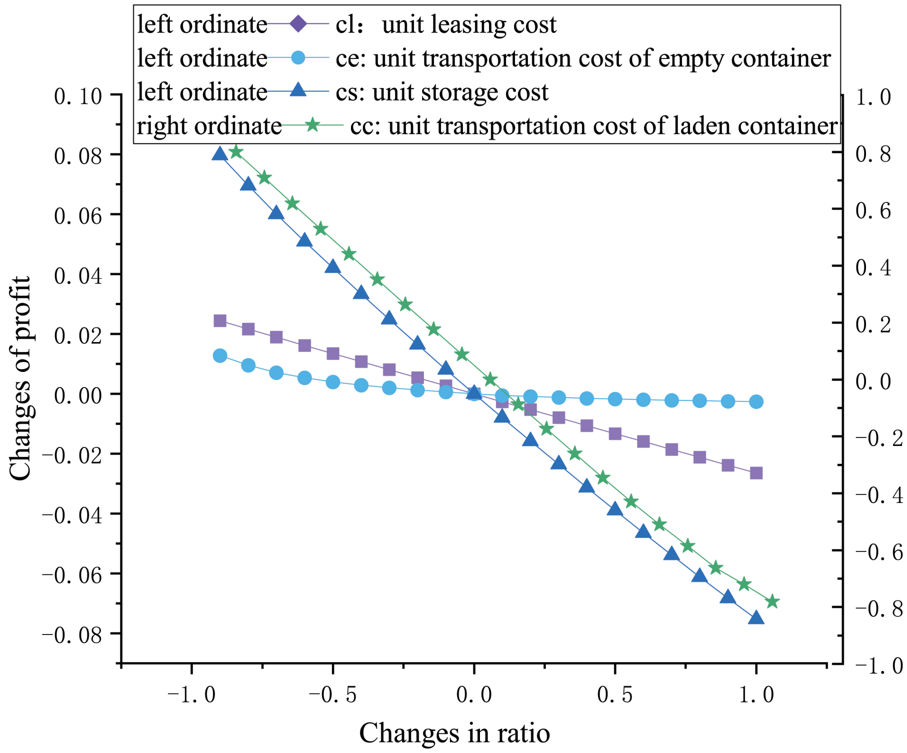

Figure 8. Sensitivity analysis of unit cost on the profit.

To explore the trade off between the allocation of laden and empty containers in the CSAECRP, this section analyzes the α indicator. Figure 10 investigates the impact of unit rates and unit costs on α. Figure 10a examines how changes in unit rates (including TS cargo and ordinary cargo rates) affect α, while Figure 10b illustrates the influence of changes in unit costs (covering the transportation costs for laden and empty containers, leasing costs, and stockpiling costs) on α.

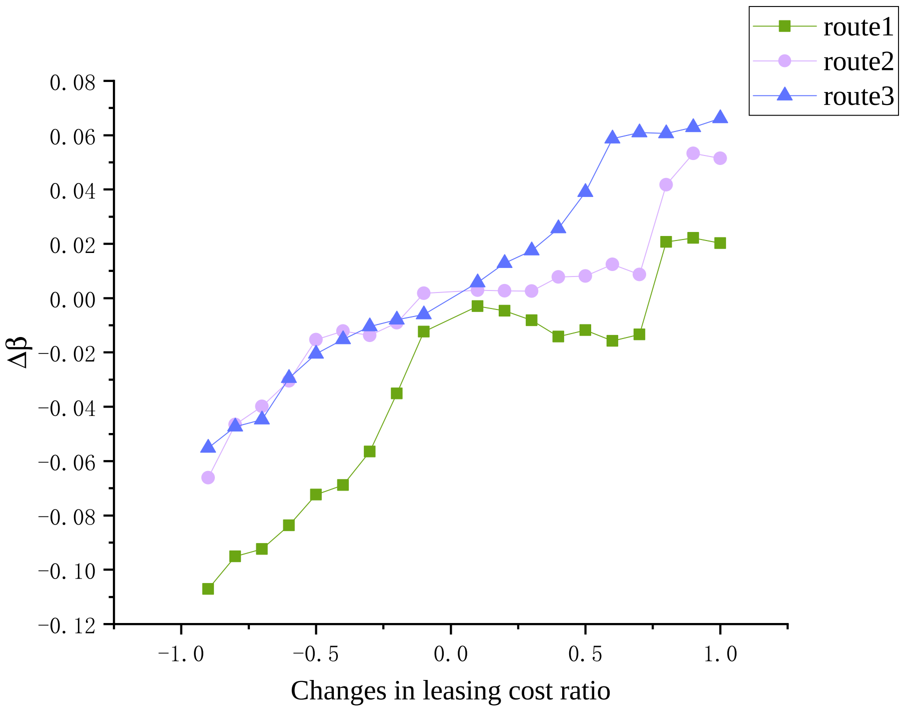

Figure 9. Sensitivity analysis of leasing cost on the empty container reposition satisfaction.

As shown in Figure 10a, as unit revenues increase, the value of α decreases rapidly at first and then stabilizes. When unit rates are low (indicating a depressed shipping market), liner shipping companies tend to allocate more slots to empty containers, resulting in a faster trade off between empty and laden containers. Notably, α is more sensitive to changes in the unit rates for TS cargoes. As the unit rate for TS cargo increases, α declines sharply from 13.55% to 2.63%. For ordinary cargo rates, α also decreases, though less dramatically, from 10.92% to 2.63%.

However, when unit rates reach a certain threshold (indicating a stable or flourishing shipping market), the number of slots allocated for empty containers decreases gradually, and the trade off between laden and empty containers becomes more stable. At this stage, the sensitivity to changes in unit rates for both TS cargo and ordinary cargo is similar, and it reduces slowly from 2.63% to 2%.

The impact of each unit cost on the value of varies, as shown in Figure 10b, leading to the following four conclusions:

Firstly, as the unit transportation and storage costs for laden containers rise, the value of α shows a noticeable upward trend, with greater sensitivity to transportation costs. Since all other factors remain constant, an increase in the unit transportation cost of laden containers reduces the revenue gained from transporting them. Consequently, the allocation of slots for empty containers increases, while slots for laden containers decrease, causing the value of α to rise sharply from 1.65% to 10.83%.

Secondly, as the unit storage cost rises, liner companies reduce the number of empty containers stored at the port and allocate more slots to empty containers to maximize revenue. As a result, the value of α increases from 0.49% to 4.99%.

Thirdly, with the rise in unit leasing costs, the value of α grows slowly from 2.03% to 3.01%. When leasing costs rise, liner companies prefer to reposition empty containers rather than lease them. However, since the transportation volume for empty containers is relatively small compared to laden containers, the change in α is modest.

Lastly, as the unit transportation cost for empty containers increases, the value of α declines rapidly from 7.93% to 1.54%. When the transportation cost for empty containers is low, liner companies are more inclined to reposition empty containers to save on leasing and storage costs, resulting in more slots being allocated for empty containers. However, as transportation costs for empty containers rise, companies shift to leasing, leading to a rapid reduction in the number of slots allocated for empty containers.

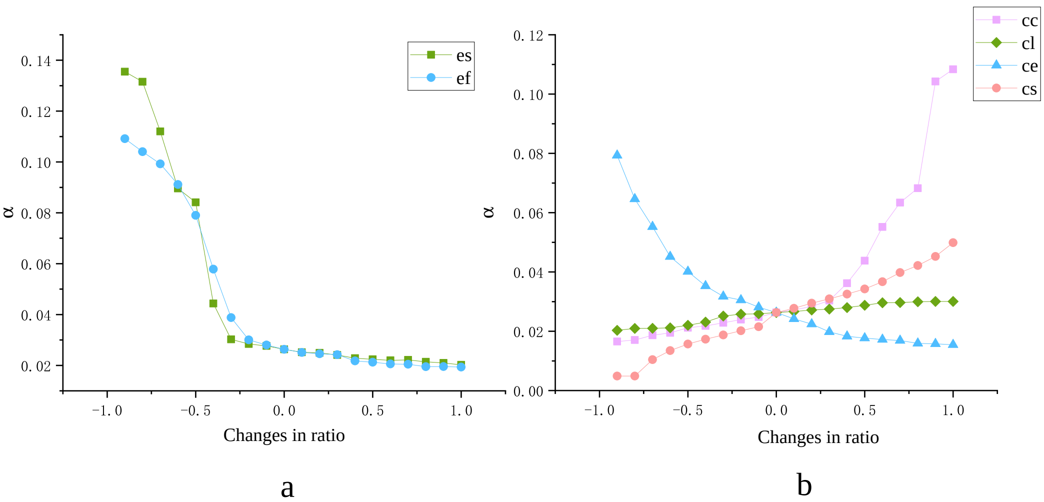

Figure 10. Sensitivity analysis on the empty-laden ratio.

6.4. Discussion

This paper compares the optimization results with the solution results determined by the basic slot allocation based on delayed delivery. The collaborative optimization results of this paper can increase the total revenue of a liner company by 4.76%; at the same time, the utilization rate of slot resources can be increased by 10.151%.

Compared to the findings in [1,17], this research highlights the pronounced cyclical characteristics of container transportation, particularly for empty container repositioning. Furthermore, the results identify key factors influencing the trade off between laden and empty container transportation, including the unit revenue of laden containers, transportation costs, and storage costs for empty containers. When the unit revenue of laden containers is low or port storage costs are high, it is a wise choice for liner companies to allocate more slots for empty container transportation.

7. Conclusions

Fierce competition in the liner shipping market impels liner companies to seek a scientific solution to promoting their revenue. The CSAECRP is explored in this paper. Different customer preferences were considered, such as cost, delivery time, and empty container demand based on DP strategies and an online booking platform. A novel integer programming model was developed to maximize total benefit for the liner company. Then, a new branch-and-cut algorithm was used to solve the problem. The effectiveness of the algorithm and model was validated by numerical experiments. The results indicate that collaborative optimization can increase revenue and improve the utilization rate of slots. Then, the sensitivity of total revenue, a trade off between empty and laden containers, and slot utilization to the initial data were analyzed. The results demonstrate that the three most important influential factors are the freight of TS cargoes, the freight of ordinary cargoes, and the transportation costs of laden containers.

Future studies should take into account container trans-shipment operations under complex shipping networks and slot leasing or exchange businesses between liner companies. Another limitation of this paper is that the cargo will only be delayed by one voyage, without concerning factors such as multiple voyage delays, which will trigger new research directions in the future. Moreover, we will further improve the convergence of the algorithm proposed in this paper and avoid invalid cutting points as much as possible. In addition, we will seek an optimization method that combines updated big data mining technology with artificial intelligence algorithms to better cope with the ever-changing shipping market demands and freight rates.

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Nomenclature

The following abbreviations are used in this manuscript:

Sets	
P	Set of ports, P={1, 2, ...i, j}
S	Set of routes, $S = \{1, 2, ...s\}$
V	Set of voyages, $V = \{1, 2, ...v\}$
Parameters	
γf_{ij}^{vs}	The maximum postponed proportion for ordinary cargoes from ith to jth port on the voyage v along route s
λf_{ij}^{vs}	The ratio of arriving port on schedule for ordinary cargoes from ith to jth port on the voyage v along route s
λs_{ij}^{vs}	The ratio of arriving port on schedule for TS cargoes from ith to jth port on the voyage v along route s
cap^{vs}	The capacity of the ship on the voyage v along route s
cc_{ij}^{vs}	Unit transportation cost of the loaded container from ith to jth port on the voyage v along route s (including handling charges)
ce_{ij}^{vs}	Unit transportation cost of the empty container from ith to jth port on the voyage the v along route s (including handling charges)
cl_{i}^{vs}	Unit leasing cost of empty container in ith port on the voyage v along route s
co^{vs}	Fixed operation cost of the ship on the voyage v along route s
cs_{i}^{vs}	Unit storage cost of container in ith port on the voyage v along route s
DE_{ij}^{vs}	Demand of empty container from ith to jth port on the voyage v along route s
DF_{ij}^{vs}	Demand of loaded container for ordinary cargoes from ith to jth port on the voyage v along route s
DS_{ij}^{vs}	Demand of loaded container for TS cargoes from ith to jth port on the voyage v along route s
ef_{ij}^{vs}	Unit freight of ordinary cargo from ith to jth port on the voyage v along route s
es_{ij}^{vs}	Unit freight of TS cargoes from ith to jth port on the voyage v along route s
IEC_{i}^{s}	The original volume of empty containers in ith port on the primary voyage along route s
Decision variables	
XE_{ij}^{vs}	The transportation number of empty containers from ith to jth port on voyage v along route s
XF_{ij}^{vs}	The received number via online booking platform for ordinary cargoes from ith to jth port on voyage v along route s
XL_{ij}^{vs}	The leasing number of empty containers in ith port on voyage v along route s
$\mathrm{XLF2}_{ij}^{vs}$	The postponed container number of ordinary cargoes from ith to jth port on voyage v along route s
XS_{ij}^{vs}	The received volume via online booking platform for TS cargoes from ith to j th port on voyage v along route s
Auxiliary decision variables	
CE_{i}^{vs}	The surplus number of empty containers after ship leaving the ith port on voyage v along route s
X_{i}^{vs}	The transportation number of loaded container from ith to jth port on voyage v along route s
XC_{ij}^{vs}	The transportation number of loaded container from ith to jth port on voyage v along route s
XLS_{ij}^{vs}	The transportation number of loaded container for TS cargoes from ith to jth port on voyage v along route s
$\mathrm{XLF1}_{ij}^{vs}$	The transportation number of loaded container for ordinary cargoes from ith to jth port on voyage v along route s

Appendix A

According to [62,63], an efficient cut plane must satisfy two key properties: 1. It must exclude globally infeasible solutions. 2. It must preserve all feasible solutions. These properties ensure not only the rapid convergence of the algorithm but also its ability to find the global optimal solution. Specifically, the cutting planes defined in Constraints (26) and (27) effectively remove globally infeasible solutions, thereby restricting the search to a limited solution space and guaranteeing convergence within a finite number of iterations. Simultaneously, these cutting planes maintain all feasible solutions, ensuring the algorithm's capacity to identify the global optimum.

Proof. For any iteration, the value of $\overset{t}{XE}{}_{ij}^{vs}$,$\overset{t}{CE}{}_{i}^{vs}$,$\overset{t}{DE}{}_{ij}^{vs}$ and $\overset{t}{z}{}_{i}^{vs}$ can be obtained. Assuming that $\sum\limits_{j\in P}\overset{t}{XE}{}_{ij}^{vs} > \overset{t}{CE}{}_{i}^{vs}$, the port $i$transfers a large number of empty containers, it can be determined as a surplus container port, that is, $\sum\limits_{j\in P}\overset{t}{DE}{}_{ij}^{vs} \prec \overset{t}{CE}{}_{i}^{vs}$. According to Constraint (13), $\overset{t}{XE}{}_{ij}^{vs} \leq \overset{t}{DE}{}_{ij}^{vs}$, $\sum\limits_{j\in P}\overset{t}{XE}{}_{ij}^{vs} \prec \overset{t}{CE}{}_{i}^{vs}$ can be inferred. However, this is contrary to the assumption, so the cutting plane (26) can effectively cut the infeasible solution.

If $\sum_{i \in P} \overset{t}{XE}{}_{ij}^{vs} \leq \overset{t}{CE}{}_{i}^{vs}$, when port i is a surplus container port, that is, $\overset{t}{z}{}_{i}^{vs} = 0$, it is deduced that $(1 - \overset{t}{z}{}_{i}^{vs})\overset{t}{XE}{}_{ij}^{vs} = \overset{t}{XE}{}_{ij}^{vs}$. On the basis of Constraint (13), $\overset{t}{XE}{}_{ij}^{vs} \leq \overset{t}{DE}{}_{ij}^{vs}$ is established forever. When a port is a deficient container port, that is, $\overset{t}{z}{}_{i}^{vs} = 1$, then $(1 - \overset{t}{z}{}_{i}^{vs})\overset{t}{XE}{}_{ij}^{vs} = 0$. Meanwhile, $\overset{t}{DE}{}_{ij}^{vs} \geq 0$ always exists. In other words, the cutting plane effectively avoids eliminating feasible solutions. Therefore, it is evident that the cutting planes (26) and (27) qualify as valid inequalities. Furthermore, the effectiveness of the cutting plane (28) has been validated in [60,61]. □

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