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Collaborative Optimization of Container Liner Slot Allocation and Empty Container Repositioning Within Port Clusters

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Abstract: The imbalance between supply and demand for slot resources and empty containers has led to resource waste and excessive operating costs for liner companies. Moreover, intense competition among ports has made both container ship slots and port equipment resource allocation inefficient. To address these challenges, this paper aims to solve the collaborative optimization problem of slot allocation and empty container repositioning within port clusters concerning inventory control. A cooperative possession strategy and a hybrid (T, s) inventory control policy are incorporated in this paper. A novel mixed-integer programming model is proposed, enabling us to simultaneously track slot allocation, empty container repositioning, empty container leasing, and slot renting. To solve the model, a new branch-and-bound algorithm based on Lagrangian relaxation and the ascendancy principle (BBLRAP) is developed. Numerical experiments are conducted to demonstrate the effectiveness of the proposed model and algorithm. The results show that the new collaborative optimization method, incorporating the cooperative possession strategy and (T, s) inventory policy, can increase liner company revenues by expanding market share, reducing costs, and improving the utilization of slot resources, ultimately achieving a win–win outcome for both liner companies and their partners. Compared to state-of-the-art studies, the following paper makes new contributions to proposing a cooperative possession strategy within port clusters for the first time. This paper ensures that liner companies and partners achieve a win–win situation in the cooperative game, expanding market shares and improving customer satisfaction.

Keywords: cooperative possession strategy; (T, s) inventory control policy; port cluster; container slot allocation; empty container repositioning

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1. Introduction

Maritime transportation is crucial for global trade, with over 80% of the world's trade volume relying on ocean shipping [1]. A key component of this industry is container liner shipping, which has recently suffered significant fluctuations in terms of freight rates. More specifically, liner shipping freight rates soared from 2021 to early 2022, prompting liner companies to expand their fleets. According to the statistics of the United Nations Conference on Trade and Development [2], the global ship capacity has since increased by 3.2% annually. However, since mid-2022, factors such as intensified trade policies, geopolitical tensions, and inflation have led to a 0.4% contraction in global seaborne trade [2]. As a result, the container transportation market has shifted from experiencing strong demand to being in a weakened state.

With the influx of new vessels, excess capacity has emerged amid declining demand, intensifying competition within the liner shipping market. In this environment, it is challenging to increase freight rates in the short term. Effective management of container slot allocation has emerged as a promising strategy for sustaining revenue [3]. Therefore, addressing the container slot allocation problem (CSAP) within the new aforementioned context, which was rarely seen in history, is critical for liner companies [4]. It is important to recognize that the development of regional port clusters is an inevitable trend in the future of port evolution. To prevent harmful and disorganized competition among ports in the same coastal region, integrating ports into regional clusters has become essential [5]. Consequently, research on liner slot allocation should be expanded from individual ports to port clusters.

In a market downturn, the key measure to sustain liner companies is to apply a cooperative possession strategy to expand market shares. The emerging concept of the "sharing economy" offers new collaborative opportunities among liner companies [6]. To achieve cooperation between different liner companies, the key is the sharing of slot resources for an intelligent fee between the same origin and destination ports within a port cluster. In the face of weak market demand, slot renting between partners is deemed a win–win measure. Slot sharing between partner companies operating on the same routes can maintain customer allegiance and increase market shares while also optimizing resource utilization and revenue.

Additionally, the repositioning of empty containers is vital for operational efficiency. Liner companies need to dispatch empty containers or lease empty containers from container leasing companies to meet the shipper's demand [7]. Effective inventory management and repositioning strategies can effectively reduce the associated costs and increase revenue, especially given imbalanced global trade patterns [8]. Although transporting laden containers is more visibly beneficial, empty container transport is essential for long-term profitability [9]. Utilizing vacant slots for empty containers mitigates resource waste and lowers leasing costs, contributing to a greener supply chain. Therefore, strategic decisions around transporting empty containers between surplus and deficit clusters become important factors that influence slot optimization.

To maximize revenue, liner companies need to coordinately consider their own empty container supply demand while meeting shippers' laden container transportation requirements as much as possible. The resource coordination between the laden and empty container slots in O-D port pairs within the port cluster is crucial to the overall business layout of the liner company. The decision-making of slots rent-in and rent-out between ports within port clusters will directly affect the overall revenue of the liner company [10,11]. The issues of optimizing the utilization rate of slot resources and boosting revenue are the focus of liner companies and even the broader shipping industry. Therefore, it is necessary and beneficial for container lines to achieve an intelligent trade-off between the slot allocation quantity, the slot rental amount, and the empty repositioning volume within ports under the same port cluster in different voyages and at different periods. In addition, the kernel of the empty container repositioning problem is to make a smart decision on the empty container dispatching volume, empty container inventory number, and empty container rental quantity during multiple periods.

Due to the interplay of multiple interdependent factors, the problem being investigated in this paper is complex and challenging. It puts the state-of-the-art of the field forward in a sense that it provides a decision-making tool for intelligent decision-making concerning all the influential factors such as the trade-off between slot allocation, empty container repositioning, empty container inventory, and slot leasing across multiple routes, voyages, and ports within port clusters. It investigates the collaborative optimization problem of container slot allocation and empty container repositioning (COPCSAECR) based on a cooperative possession strategy and (T, s) inventory control policy within the scope of port clusters for the first time. Based on the characteristics of COPCSAECR, a novel mixed-integer programming model and a new BBLRAP algorithm are proposed to address the problem. Careful consideration of numerous influencing factors provides liner companies with more accurate and scientific operational decision support. In addition, it ensures that liner companies and their partners achieve a win–win situation in the cooperative game, expanding market shares and improving customer satisfaction.

The rest of this paper is organized as follows: Section 2 contains a literature review. The problem description and model assumptions are presented in Section 3. Section 4 describes the model's construction, and the solution algorithm is given in Section 5. Section 6 introduces numerical experiments and result analysis, and the conclusion of this paper is in Section 7.

2. Literature Review

Revenue management theory was first introduced by [12,13] in the study of slot resource allocation in maritime transportation. Since then, substantial research has been conducted with a focus on container liner slot allocation based on revenue management [3,14–16]. The research is categorized into three main areas: revenue-oriented strategy, slot reservation channels, and slot resource sharing.

A series of booking strategies have evolved, including bidding [17,18], booking limits [19], overbooking [20], and delivery-delay strategies [20,21]. Given the strategy of market segmentation, the shipping market is commonly subdivided into contract markets and spot markets based on the attributes of shippers [22–24]. Due to the fierce market competition environment, liner companies have shifted towards providing diverse services, utilizing differentiated slot allocation strategies to boost revenue. Ref. [4] applied the loyalty strategy and expansion strategy to study the slot allocation problem of time-sensitive cargoes. The results show that the expansion strategy outperforms the loyalty strategy in expected total revenue. Ref. [25] further developed a service-oriented strategy to maintain customer loyalty, while [26] proposed a dynamic allocation strategy to decide whether to accept each dynamically arrived booking request. Ref. [1] revealed that the service diversity strategy is more profitable than the traditional first-come-first-served strategy.

Slot sales channels have expanded, with many scholars exploring dual-channel allocation involving traditional and online platforms. Ref. [27] found that e-commerce platforms, particularly during the epidemics, offer significant growth potential for liner companies. Ref. [28] showed that online environments increase overall revenue compared to traditional methods. On this basis, some scholars [29–31] are committed to developing and designing an online booking platform to achieve real-time interaction of slot information among shippers, freight forwarders, and liner companies.

Alliance cooperation among liner companies based on slot resource sharing has been another critical research focus [32,33]. Ref. [34] studied slot exchanges and purchases between liner companies, showing that it is optimal for a liner company to adopt the joint strategy of slot exchange and purchase. Ref. [10] examined joint route allocation and slot allocation under co-chartering strategies, while [35] found that slot sharing through exchange models significantly boosts both revenue and service levels. Fairness in cooperation has also drawn attention. Ref. [36] explored profit-sharing agreements within shipping alliances, developing a two-stage model to balance alliance profits and individual company benefits. Furthermore, [37] suggested that slot allocation research should expand from a single port pair focus to the broader port cluster level, yet little research has addressed this issue.

The issue of empty container repositioning is a popular research topic in the liner shipping industry. It is also presented as a key influential factor at the operational level of liner companies. Since the problem is unavoidable, empty container repositioning has been extensively studied, with much research focusing on cost optimization [38–40]. Among them, [41] believe that empty container repositioning can bring perceived value to liner companies. Furthermore, the joint study of empty container repositioning and route optimization [8,42,43] has been extended.

Meanwhile, [26,44] found that the joint optimization of empty container repositioning and inventory control can further reduce costs while achieving efficient container management. Inventory control strategies, such as the (D, U) and (s, S) models, have also been applied to optimize repositioning [8,37,44–46], demonstrating that co-optimization with inventory management improves empty container circulation efficiency. A few scholars have combined slot allocation and empty container repositioning to further reduce operating costs and increase revenue [47,48].

Several studies [1,47,49] have confirmed that the COPCSAECR can improve the profits of liner companies. However, the COPCSAECR's consideration of inventory control from the perspective of port clusters has not yet been seen in the existing literature. Moreover, the above research (as exhibited in Table 1), including the cases of using both a slot exchange strategy and a co-charting strategy, is based on a strong assumption of the sharing of equal slot resources. Obviously, it overlooks the varying value of slots across different routes and periods, which can challenge the principle of fairness in cooperation among liner companies. To address this, this paper will, for the first time, introduce a cooperative possession strategy to ensure fairness and achieve mutually beneficial outcomes. Given the unique characteristics of COPCSAECR within port clusters, the inventory control strategy differs from conventional empty container repositioning problems. Thus, a (T, s) inventory policy is newly applied in this work by combining periodic inventory control policy and quantitative inventory control policy masterly.

Table 1. A brief summary of the literature.

Literature	Research Scope	Slot Allocation Strategy	Empty Container Replenishment Source	Inventory Policy
[1]	Single route/multi(O-D) ports	Overbooking with different service classes	Empty container rental and repositioning	Optimal policy
[3,16,22–24]	Multiple routes/multiple (O-D) ports	Market segmentation	/	/
[4]	Single route/multiple (O-D) ports	Loyal strategy and expansive strategy	Empty container repositioning	/
[8,37]	Single route/multiple (O-D) ports	/	Empty container repositioning	(D,U) policy
[9]	Multiple routes/multiple ports–inland depots	/	Empty container rental and repositioning	/
[10]	Multiple routes/multiple (O-D) ports	Slot co-chartering	/	/
[14,15,19]	Single route/multiple (O-D) ports	Market segmentation/booking limitation strategy	Empty container repositioning	/
[20]	Multiple routes/multiple (O-D) ports	Overbooking and delivery-delay-allowed strategies	/	/
[21]	Single route/multiple (O-D) ports	Delivery-delay strategies	/	/

Table 1. *Cont.*

Literature	Research Scope	Slot Allocation Strategy	Empty Container Replenishment Source	Inventory Policy
[25]	Single route/multiple (O-D) ports	Service-oriented strategies	/	/
[26]	Single route/multiple (O-D) ports	/	Empty container repositioning	Queuing theory
[27–31]	/	Online booking slots sale channel	/	/
[32,34]	Single or multiple routes/multiple (O-D) ports	Slot exchange	Empty container repositioning	/
[33,35,36]	Multiple routes/multiple (O-D) ports	Slot exchange	/	/
[44]	Multiple routes/ multiple inland nodes	/	Empty container rental and repositioning	(s,S) policy
[45]	Multiple routes/multiple inland nodes	/	Empty container rental and repositioning	(R,Q) policy
[46]	Multiple routes/multiple ports–inland depots within port cluster	/	Empty container rental and repositioning	(D,U) and (T, s)policy
[50]	Multiple routes/multiple inland nodes	/	Empty container rental and repositioning	(R,T) policy
This paper	Single route/Port cluster	Cooperative possession strategy for empty container rental and repositioning	(T, s) policy	

3. Problem Description

3.1. Cooperative Possession Strategy

Liner companies strive to maximize profits by balancing the demand for empty containers with the need to transport laden containers efficiently. While pursuing these goals, liner companies must not focus solely on short-term profit maximization. Instead, they should prioritize customer satisfaction and market shares for sustainable, long-term profitability.

The cooperative possession strategy aims to enhance customer satisfaction, increase market share, and foster customer loyalty by promoting win–win cooperation on specific routes and within designated port clusters. It involves liner companies leasing and selling shipping slots to other liner companies at discounted rates based on a cooperation agreement. The policy is specifically designed to facilitate the leasing of container ship slots between ports of departure and destination within the same port cluster, effectively meeting transportation demands.

The cooperative possession strategy offers liner companies multiple benefits, including reduced operating costs, expanded market share, improved ship slot utilization, and increased revenue. When customer demand is strong, leasing slots from partner liner companies with cooperation agreements not only meets customer needs, enhances service levels, and boosts customer satisfaction but also saves on the costs associated with launching new routes and investing in new capacity. Conversely, when customer demand is weak, selling surplus slots to cooperating liner companies helps optimize slot utilization and increase revenue.

3.2. (T, s) Inventory Policy

For liner companies, owning and managing a certain scale of self-operated containers purchased and owned by the companies themselves) holds significant commercial value, as it aligns with the economies of scale in the shipping industry. According to statistics, containers owned by liner companies account for 50–60% of the world's total container volume [44]. The primary purpose of liner companies owning and managing containers is to meet shippers' demand for transport carriers. A crucial strategy for achieving this is by effectively managing the inventory of empty containers. Empty container inventory control includes empty container repositioning, empty container replenishment (leasing), and empty container storage. For self-operated empty containers, both repositioning and storage need to be carefully managed. Additionally, planned rental containers, which function similarly to self-owned containers, have also to be included in the scope of empty container storage and transportation. Given the limited storage space and the large scale of container operations, the development of a scientific and dynamic inventory management strategy is essential. In this paper, both empty container transfer and laden container transportation are taken into account simultaneously by liner shipping. As is well known, liner shipping is characterized by strong periodicity and multi-stage operations. Its fixed departure schedules and consistent operating time make empty container management well-suited for applying a periodic inventory control strategy. The empty container inventory control management discussed in this paper features fixed ordering times and phased ordering. Additionally, transportation and storage volumes are constrained by liner slot availability and the maximum storage capacity of the yard.

Given these constraints, a (T, s) inventory control policy extended from (T, S) [46] and (T, R) [50] is applied to optimize the scientific management and efficient handling of empty containers. This approach involves reviewing the port yard inventory every T period. If, during a review, the inventory position (calculated as the current stock at the port yard plus planned deliveries minus back-orders) falls below the reorder level s, a replenishment order is placed to bring the inventory position back up to or above s. The value of s is determined by D (the sum of advance order period and transportation time), μ (the average demand of empty container), α (confidence Level), and σ (the standard deviation of empty container demand). The specific calculation equation is exhibited in Formula (21). The replenishment order is fulfilled through two channels: empty container transfers and empty container leasing.

The (T, s) inventory control policy offers distinct advantages in balancing inventory costs and service levels, reducing the risk of excessive inventory. Regarding inventory review timing, compared to continuous review strategies, (T, s) requires lower technical capital investment and fewer human resources. Furthermore, the regular review intervals of (T, s) align well with the fixed shipping schedules characteristic of container liner operations. In terms of inventory order quantity, (T, s) is more cost-efficient than the (D, U) policy or (s, S) policy, as it requires maintaining the minimum inventory to meet empty container demand, thereby reducing storage costs. However, the (T, s) policy lacks the prompt ordering capability of continuous review strategies and does not ensure the sufficient inventory levels provided by (D, U) and (s, S). It remains the most suitable approach for managing empty container control in liner companies, particularly when minimizing inventory and transportation costs while pursuing the maximum revenue.

3.3. Description of COPCSAECR Under Port Clustering

Given the dual attributes of containers, liner companies face a multifaceted challenge in addressing container transportation needs. As a result, liner companies must effectively manage not only the transportation of laden containers but also the logistical requirements associated with empty containers, as shown in Figure 1. Initially, a liner company must supply empty containers to shippers by leasing empty containers (that is, emergency leasing) or repositioning empty containers (which involves the planned leasing process). A critical aspect of this step is to determine the optimal split ratio between the volume of leased containers and the volume of transferred [44]. Planned leasing essentially equates to utilizing the liner company's own containers, requiring active management that includes the payment of both storage and leasing fees. Conversely, emergency leasing avoids incurring storage fees for the liner company. Correspondingly, emergency rental costs are higher. When deciding the volume of empty container transfers or leasing, one shall rely on a (T, s) inventory control policy, which underpins effective management of empty containers.

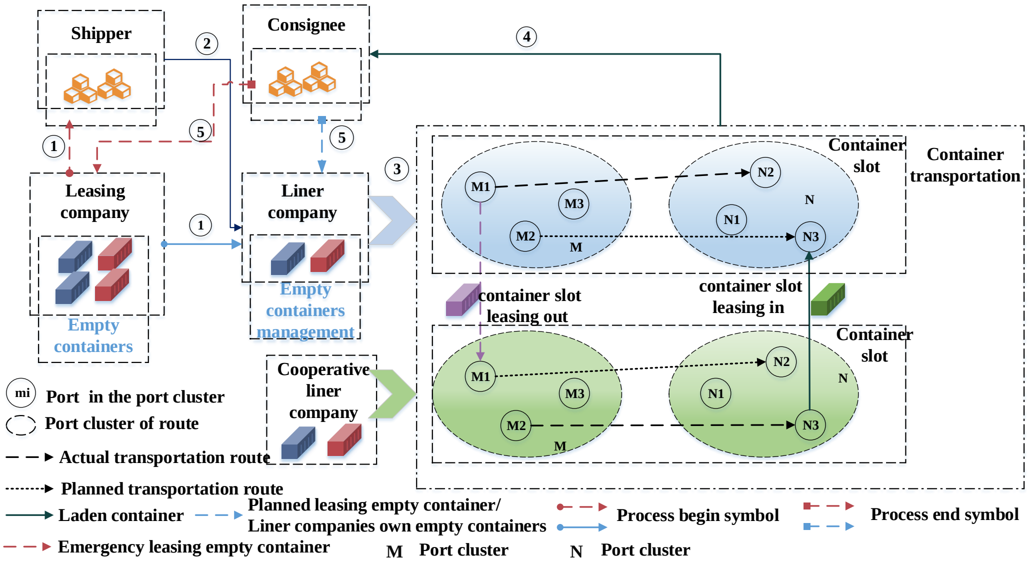

Figure 1. The process of COPCSAECR.

Subsequently, in process 2, the laden containers are delivered to the liner company after loading cargo into the empty containers by the shippers. Process 3 is the allocation of container liner slots, which presents a critical dilemma, as it must accommodate both laden and empty containers. Laden containers inherently hold a higher priority due to their capacity to generate immediate revenue for liner companies. The resources of a single liner company are limited and guided by the cooperative possession strategy; therefore, liner companies can benefit from leasing arrangements with partner firms within designated port clusters. Thus, the primary challenge for liner companies is to effectively determine the allocation of slots for laden containers, manage the requirements for empty container transportation, and assess the optimal volume of leased slots. This is vital for enhancing operational efficiency and ensuring long-term profitability. The laden containers are transported to the consignee in process 4. Finally, in process 5, the cargo is unloaded from the laden container, and the empty containers will be picked up. The emergency leased empty containers will be recovered by the empty container leasing company, while the remaining empty containers will be managed by the liner company.

Overall, the COPCSAECR under the same port cluster discussed in this paper aims to simultaneously address the following critical trade-off questions. Specifically, it examines the balance between empty container transfer volume and leased volume within the context of the (T, s) inventory control policy. Additionally, it assesses the relationship between emergency container rental and planned container rental volumes. Lastly, it investigates the trade-off between the allocation of laden versus empty container slots, as well as between the number of slots rented in and rented out.

4. Mathematical Models

The model assumption is introduced in Section 4.1. Then, the formulations of the model are given in Section 4.2.

4.1. Model Assumption

To simplify the COPCSAECR, 10 assumptions are described:

1. Whether the container is leased for a long term or a short term, the transportation cost from the container yard to the port is included in the leasing cost.
2. Empty container leasing companies have sufficient transportation capacity to ensure that leased containers arrive promptly.
3. Container leasing is divided into planned leasing and emergency leasing, where emergency leases are more expensive than scheduled leases. Containers leased on an emergency basis are returned to the leasing company immediately after the work is completed.
4. Any liner company within the port cluster can operate weekly routes.
5. The demand for laden and empty containers can be forecasted using historical big data.
6. All loaded containers that arrived at the port on the previous voyage will be converted into empty containers and returned to the port yard before the ship's arrival on the current voyage.
7. The empty containers required on this voyage will be delivered to the shipper during this voyage. All cargo will be loaded, and the laden container will be delivered to the designated yard at the port before the ship arrives.
8. Liner companies sign contracts with corporate partners, and the cost of leasing space within the same port group is fixed.
9. The maximum slot available for rental by other partner liner companies is predetermined.
10. Only 20-foot containers are considered, with a 40-foot container counted as two 20-foot containers.

4.2. Formulations

To track the COPCSAECR, a novel mixed-integer programming model is proposed here. The objective function aims to maximize the revenue of liner companies operating across multiple routes, voyages, and port pairs within the port cluster. It consists of 10 components. The first two parts are the revenue obtained by the liner company, namely the revenue from laden container transportation and the revenue from renting out slots. The remaining seven components represent the company's cost expenditures. The first cost is the expense of renting slots, followed by the cost of laden container transportation. The third cost pertains to the transportation of laden containers carried in slots rented out to partners. Additional costs include empty container repositioning, emergency rental of empty containers, storage, planned rental of empty containers, and fixed ship costs.

$$\begin{aligned}\max Z = {} & \sum_{l\in L}\sum_{v\in V}\sum_{n_j\in P}\sum_{m_i\in P}\left[gr^{lv}_{m_in_j}x\mathrm{g}^{lv}_{m_in_j} + fr^{lv}_{m_in_j}xro^{lv}_{m_in_j} - cc^{lv}_{m_in_j}xri^{lv}_{m_in_j} - ct^{lv}_{m_in_j}(xf^{lv}_{m_in_j} + xro^{lv}_{m_in_j})\right] \\ & - \sum_{l\in L}\sum_{v\in V}\sum_{n_j\in P}\sum_{m_i\in P}\left(cs^{lv}_{m_in_j}xe^{lv}_{m_in_j} + cr^{lv}_{m_in_j}xa^{lv}_{m_in_j}\right) - \sum_{l\in L}\sum_{v\in V}\sum_{m_i\in P}ch^{lv}_{m_i}SE^{lv}_{m_i} - \sum_{l\in L}\sum_{m_i\in P}ce^{l}_{m_i}xb^{l}_{m_i} \\ & - \sum_{l\in L}\sum_{v\in V}fc^{lv}\end{aligned} \tag{1}$$

s.t.

$$SE^{lv}_{m_i} = oe^{l}_{m_i} + xb^{l}_{m_i} + \sum_{n_k \in P} xe^{lv}_{n_k m_i} - \sum_{n_j \in P} xe^{lv}_{m_i n_j} - \sum_{n_j \in P} \left(xg^{lv}_{m_i n_j} - xa^{lv}_{m_i n_j} \right)_{v = 1, \forall l \in L, \forall m_i, n_j \in P} \quad , \tag{2}$$

$$SE^{lv}_{m_i} = SE^{l(v-1)}_{m_i} + \sum_{n_k \in P} \left(xg^{l(v-1)}_{n_k m_i} - xa^{l(v-1)}_{n_k m_i} \right) + \sum_{n_k \in P} xe^{lv}_{n_k m_i} - \sum_{n_j \in P} xe^{lv}_{m_i n_j} - \sum_{n_j \in P} \left(xg^{lv}_{m_i n_j} - xa^{lv}_{i m_i n_j} \right), \tag{3}$$

$$\forall v \in V \bigcap v \geq 2, \forall l \in L, \forall m_i \in P,$$

$$xri^{lv}_{m_i n_j} \leq MI^{lv}_{m_i n_j} \qquad \forall l \in L, \forall v \in V, \forall m_i, n_j \in P, \tag{4}$$

$$xri^{lv}_{m_i n_j} = xg^{lv}_{m_i n_j} - xf^{lv}_{m_i n_j} \qquad \forall l \in L, \forall v \in V, \forall m_i, n_j \in P, \tag{5}$$

$$xro^{lv}_{m_i n_j} \leq MO^{lv}_{m_i n_j} \qquad \forall l \in L, \forall v \in V, \forall m_i, n_j \in P. \tag{6}$$

$$XS^{lv}_{m_i} = \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xf^{lv}_{m_k n_j} + \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xe^{lv}_{m_k n_j} + \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xro^{lv}_{m_k n_j} + \sum_{\substack{, n_j < m_k < m_i \\ n_j, m_k \in P}} xf^{lv}_{m_k n_j} + \sum_{\substack{n_j < m_k < m_i \\ n_j, m_k \in P}} xe^{lv}_{m_k n_j} \tag{7}$$

$$+ \sum_{\substack{n_j < m_k < m_i \\ n_j, m_k \in P}} xro^{lv}_{m_k n_j} \qquad v = 1, \forall l \in L, \forall m_i \in P,$$

$$XS^{lv}_{m_i} = \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xf^{lv}_{m_k n_j} + \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xe^{lv}_{m_k n_j} + \sum_{\substack{m_k < m_i < n_j \\ n_j, m_k \in P}} xro^{lv}_{m_k n_j} + \sum_{\substack{m_i < n_j < m_k \\ n_j, m_k \in P}} xf^{l(v-1)}_{m_k n_j} + \sum_{\substack{m_i < n_j < m_k \\ n_j, m_k \in P}} xe^{l(v-1)}_{m_k n_j} \tag{8}$$

$$+ \sum_{\substack{m_i < n_j < m_k \\ n_j, m_k \in P}} xro^{l(v-1)}_{m_k n_j} + \sum_{\substack{n_j < m_k < m_i \\ n_j, m_k \in P}} xf^{lv}_{m_k n_j} + \sum_{\substack{n_j < m_k < m_i \\ n_j, m_k \in P}} xe^{lv}_{m_k n_j} + \sum_{\substack{n_j < m_k < m_i \\ n_j, m_k \in P}} xro^{lv}_{m_k n_j}$$

$$\forall v \in V \bigcap v \geq 2, \forall l \in L, \forall m_i \in P,$$

$$\sum_{n_j \in P} \left(xf^{lv}_{m_i n_j} + xro^{lv}_{m_i n_j} + xe^{lv}_{m_i n_j} \right) \leq SC^{lv} - XS^{lv}_{m_i} \quad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{9}$$

$$dg^{lv}_{m_i n_j} \beta \leq xg^{lv}_{m_i n_j} \leq dg^{lv}_{m_i n_j} \quad \forall l \in L, \forall v \in V, \forall m_i, n_j \in P, \tag{10}$$

$$dg^{lv}_{m_i n_j} \beta \leq xf^{lv}_{m_i n_j} \leq xg^{lv}_{m_i n_j} \quad \forall l \in L, \forall v \in V, \forall m_i, n_j \in P, \tag{11}$$

$$\sum_{n_j \in P} xf^{lv}_{m_i n_j} \leq SC^{lv} - XS^{lv}_{m_i} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{12}$$

$$\sum_{n_j \in P} xg^{lv}_{m_i n_j} \leq oe^{l}_{m_i} + xb^{l}_{m_i} + \sum_{n_j \in P} xa^{lv}_{m_i n_j} \qquad v = 1, \forall l \in L, \forall m_i \in P, \tag{13}$$

$$\sum_{n_j \in P} xg^{lv}_{m_i n_j} \leq SE^{l(v-1)}_{m_i} + \sum_{n_j \in P} xa^{lv}_{m_i n_j} \qquad \forall v \in V \bigcap v \geq 2, \forall l \in L, \forall m_i \in P, \tag{14}$$

$$\sum_{n_j \in P} xe^{lv}_{m_i n_j} \leq SC^{lv} - XS^{lv}_{m_i} - \sum_{n_j \in P} xf^{lv}_{m_i n_j} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{15}$$

$$\sum_{n_j \in P} xro^{lv}_{m_i n_j} \leq SC^{lv} - XS^{lv}_{m_i} - \sum_{n_j \in P} \left(xf^{lv}_{m_i n_j} + xe^{lv}_{m_i n_j} \right) \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{16}$$

$$SE^{lv}_{m_i} \leq N^{lv}_{m_i} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{17}$$

$$s^{lv}_{m_i} \leq N^{lv}_{m_i} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{18}$$

$$\sum_{n_k \in P} xe^{lv}_{n_k m_i} = s^{lv}_{m_i} - oe^{l}_{m_i} - xb^{l}_{m_i} \qquad v = 1, l \in L, \forall m_i \in P, \tag{19}$$

$$\sum_{n_k \in P} xe^{lv}_{n_k m_i} = s^{lv}_{m_i} - SE^{l(v-1)}_{m_i} - \sum_{n_k \in P} \left(xg^{l(v-1)}_{n_k m_i} - xa^{l(v-1)}_{n_k m_i} \right) \qquad \forall l \in L, \forall v \in V \bigcap v \geq 2, \forall m_i \in P, \tag{20}$$

$$s^{lv}_{m_i} = D \sum_{n_j \in P} \mu^{lv}_{m_i n_j} + \alpha \sqrt{\left(\sum_{n_j \in P} \sigma^{lv}_{m_i n_j} \right)^2 D + \left(\sum_{n_j \in P} \sigma^{lv}_{m_i n_j} \right)^2 \sum_{n_j \in P} \mu^{lv}_{m_i n_j}} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{21}$$

$$\sum_{n_j \in P} xe^{lv}_{m_i n_j} \left(SE^{lv}_{m_i} - s^{lv}_{m_i} \right) \geq \qquad \forall l \in L, \forall v \in V, \forall m_i \in P \tag{22}$$

$$\sum_{n_j \in P} xe^{lv}_{m_i n_j} \left(SE^{lv}_{m_i} - s^{lv}_{m_i} \right) \geq 0 \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \tag{23}$$

$$xa^{lv}_{m_i n_j} \leq F^{lv}_{m_i n_j} \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \forall n_j \in P, \tag{24}$$

$$\begin{aligned} & xf^{lv}_{m_i n_j} \geq 0;\ \ xe^{lv}_{m_i n_j} \geq 0;\ \ xro^{lv}_{m_i n_j} \geq 0;\ \ xa^{lv}_{m_i n_j} \geq 0;\ \ xb^{lv}_{m_i n_j} \geq 0;\ \ xg^{lv}_{m_i n_j} \geq 0; \\ & xri^{lv}_{m_i n_j} \geq 0 \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \forall n_j \in P, \end{aligned} \tag{25}$$

$$\begin{aligned} & xf^{lv}_{m_i n_j} \in Z;\ \ xe^{lv}_{m_i n_j} \in Z;\ \ xro^{lv}_{m_i n_j} \in Z;\ \ xa^{lv}_{m_i n_j} \in Z;\ \ xb^{lv}_{m_i n_j} \in Z;\ \ xg^{lv}_{m_i n_j} \in Z; \\ & xri^{lv}_{m_i n_j} \in Z \qquad \forall l \in L, \forall v \in V, \forall m_i \in P, \forall n_j \in P. \end{aligned} \tag{26}$$

Constraints (2) and (3) represent the remaining inventory of empty containers after the ship departs the port. Constraint (4) ensures that the slot leased between any port pair does not exceed the slot available from other partners. Constraint (5) guarantees that the transportation requirements of customer bookings are met through the liner company's own capacity and slot leasing. Constraint (6) states that the slots leased out by the liner company must remain within the maximum available slots. Constraints (7) and (8) specify the number of containers remaining on board after unloading but before loading begins. Constraint (9) reveals that the transport volume of laden containers, empty containers, and slot rental volume must not exceed the ship's remaining capacity. The acceptance of customer booking slots by liner companies is defined by formulation (10), while the constraints regarding slot allocation for the liner company are presented in formulations (11) and (12).

Constraints (13) and (14) succinctly illustrate that the volume of empty containers provided by the liner company must meet transportation demand. Constraint (15) ensures that, while fulfilling the transportation needs of laden containers, any remaining capacity can be allocated for empty container transport. Constraint (16) states that the liner company's remaining space can be leased to other companies after meeting the transportation demands for both laden and empty containers. Constraints (17) and (18) specify the limits on empty container storage capacity at ports.

Based on the (T, s) inventory control policy, Constraints (19) and (20) indicate that the empty container inventory must remain above the replenishment point during any voyage cycle. The empty container inventory for the current cycle consists of the volume of repositioned empty containers, the planned rental number of empty containers, the conversion volume from previous laden containers, and the prior inventory volume. Constraint (21) defines the replenishment point for each cycle. Formulations (22) and (23) outline the empty container repositioning constraints, specifying that if an empty container is transferred, the remaining inventory must exceed the replenishment point. Constraint (24) sets a limit on the quantity of emergency rental empty containers. Formulation (25) establishes a non-negativity constraint, while formulation (26) imposes an integer constraint.

5. Solution Algorithm

While existing commercial software like CPLEX and Gurobi can provide exact solutions, they often better fit small-scale problems. It is difficult for them to solve the large-scale COPCSAECR under a port cluster involving multiple shipping routes and multiple cycles since it is intractable to enumerate within the acceptable time. To address these limitations, this paper introduces a new BBLRAP.

5.1. Upper Bound-Based Lagrangian Relaxation Method

Firstly, the $xri^{lv}_{m_i n_j}$ in the objective function is replaced using Constraints (5), resulting in the following new model with fewer decision variables.

$$maxZ^{lv} = \sum_{n_j \in P}\sum_{m_i \in P}\left[\begin{array}{c} \left(gr_{m_i n_j} - cc_{m_i n_j}\right)xg_{m_i n_j} + \left(fr_{m_i n_j} - ct_{m_i n_j}\right)xro_{m_i n_j} - \left(cc_{m_i n_j} + ct_{m_i n_j}\right) \\ xf_{m_i n_j} - cs_{m_i n_j}xe_{m_i n_j} - cr_{m_i n_j}xa_{m_i n_j} - ch_{m_i n_j}SE_{m_i n_j} \end{array}\right] - \Re,$$

where Z^{lv} is the maximum revenue on voyage v of route l, and the vector $\Re = \sum_{m_i \in P} ce_{m_i}xb_{m_i} - fc$. In order to simplify the objective function, we introduce vector X and vector Γ. Vector X consists of variables, and vector Γ includes the freight and cost coefficients. Specifically, $X = (\ldots xg_{m_i n_j}\ldots xro_{m_i n_j}\ldots xf_{m_i n_j}\ldots xe_{m_i n_j}\ldots xa_{m_i n_j}\ldots SE_{m_i n_j}\ldots)$ and $\Gamma = \left(\ldots gr_{m_i n_j}\ldots cc_{m_i n_j}\ldots fr_{m_i n_j}\ldots ct_{m_i n_j}\ldots cs_{m_i n_j}\ldots cr_{m_i n_j}\ldots ch_{m_i n_j}\ldots ce_{m_i n_j}\right)$. Thus, the objective function of new model can be denoted by $\max Z^{lv'} = \sum_{n_j \in P}\sum_{m_i \in P}\Gamma^T X - \Re$.

Next, we utilize the Lagrangian relaxation method to derive the upper bound of the model. In our algorithm, we focus on relaxing constraint (16) to simplify the problem and decouple specific decision variables. A non-negative Lagrangian multiplier $\varphi^{lv}_{m_i n_j}$ is introduced to support the relaxation of the new model. The resulting relaxed model is as follows:

$$Z^{lv}(\varphi) = \max Z^{lv'} + \sum_{m_i \in P}\sum_{n_j \in P}\varphi^{lv}_{m_i n_j}(SC^{lv} - XS^{lv}_{m_i} - \sum_{n_j \in P} xf^{lv}_{m_i n_j} - \sum_{n_j \in P} xe^{lv}_{m_i n_j} - \sum_{n_j \in P} xro^{lv}_{m_i n_j}). \tag{27}$$

This is subject to (2), (3), (6)–(15), and (17)–(27), where the multiplier $\varphi^{lv}_{m_i n_j}$ is updated iteratively by the conventional sub-gradient method [51] to obtain the best possible upper bond value. During this iteration , the multiplier $\varphi^{lv}_{m_i n_j}$ is updated accordingly.

$$\varphi^{lv}_{m_i n_j}(t+1) = \max\left\{0, \varphi^{lv}_{m_i n_j}(t) + \theta_n\left(SC^{lv} - XS^{lv}_{m_i} - \sum_{n_j \in P} xf^{lv}_{m_i n_j} - \sum_{n_j \in P} xe^{lv}_{m_i n_j} - \sum_{n_j \in P} xro^{lv}_{m_i n_j}\right)\right\}, \tag{28}$$

$$\theta_{n+1} = \frac{\alpha\theta_n\left[Z^{lv}(\varphi) - Z_{LB}\right]}{\sum_{l \in L}\sum_{v \in V}\sum_{m_i \in P}\sum_{n_j \in P}(SC^{lv} - XS^{lv}_{m_i} - \sum_{n_j \in P} xf^{lv}_{m_i n_j} - \sum_{n_j \in P} xe^{lv}_{m_i n_j} - \sum_{n_j \in P} xro^{lv}_{m_i n_j})^2}. \tag{29}$$

The step size θ_n is defined by (29), and $0 < \alpha < 1$.

5.2. Pruning Strategy Based on Ascendancy Principle

To enhance the calculation efficiency and prevent multiple identical solutions across different nodes, we implement three ascendancy rules to eliminate unpromising nodes.

Ascendancy rule 1. δ' denotes the children node of δ, and node δ can be removed from the search sets if $Z\left(\delta'\right) = Z(\delta)$ and $xf(\delta) \geq xf\left(\delta'\right)$. If the child node in the search sets has the same revenue as its parent node, and the parent node has a larger amount of laden container slot allocated, it can be inferred that the parent node has a slimmer chance of obtaining the optimal value.

Ascendancy rule 2. Node δ can be pruned if $xf(\delta) < xe(\delta)$ or $xf(\delta) < xro(\delta)$ or$xf(\delta) + xe(\delta) + xro(\delta) + XS(\delta) > SC(\delta)$. Slot allocation is prioritized significantly. Following the cooperative possession strategy, when a liner company allocates slots, laden containers take precedence, followed by empty containers. Surplus capacity will be sold, while insufficient capacity will lead to leasing. If priority levels or slot resource constraints are violated, the node will be pruned.

Ascendancy rule 3. The node δ can be cut if $xf(\delta) + xe(\delta) + xro(\delta) + XS(\delta) \prec SC(\delta)$, and greater revenue can be obtained while satisfying resource constraints.

5.3. Framework of the Branch and Bound Algorithm Based on Lagrangian Relaxation and Ascendancy Rules

This framework outlines the new branch-and-bound algorithm. During the search process, feasible solutions with extended search values are stored in a designated set S. In S, nodes are arranged in descending order of relaxation target values. The best-first strategy is applied for branching. The algorithm initially selects the node with the highest relaxation solution and then verifies it based on ascendance criteria 2 and 3. If the node is not clipped, its relaxation solution becomes the initial upper bound, while the objective function value serves as the lower bound. According to the process shown in Algorithm 1, the feasible nodes are systematically explored and eliminated according to the ascendance rule to prevent redundant searches. This continues until the termination condition is met, which is either a set number of iterations or a sufficiently small gap between the upper and lower bounds.

Algorithm 1: BBLRAP

```
Input: $S$: feasible solutions set with extended search values, $t$: iterations, $T$: Maximum number of iterations;
Output: $UB(X^*)$: The upper bound of the optimal solution, $LB(X^*)$: The lower bound of the optimal solution, $X^*$: Optimal solution;
initialization: $t = 0$, LB = 0, S = $\Phi$;
Produce the primary node and add to the $S$;
while $S \neq \Phi$ and $t < T$ or $\frac{|UB-LB|}{LB} < \varepsilon$ do
    Select the node $\delta$ with the maximum upper bound from $S$;
    if node $\delta$ is not cut according to the ascendancy rule 2 and 3 then
        Calculate the upper bound $Z_{\delta}^{lv}(\varphi)$ for node $\delta$, and $UB = Z_{\delta}^{lv}(\varphi)$;
        if $UB(\delta) < UB$ then
            $LB = Z^{lv'}(\delta)$, Update $X(\delta)$ and $\varphi_{m_i n_j}^{lv}$ accordingly;
        end
        for suitable node from $S$ do
            Select node $\delta$ based on the best-first search strategy and generate child node $\gamma$ ;
            if $LB \leq UB(\gamma) < UB$ then
                if node $\gamma$ is not pruned based on the ascendancy rule 1 then
                    $LB = Z^{lv'}(\gamma), UB = Z_{\delta}^{lv}(\varphi)$, remove node $\delta$, add $\gamma$ to $S$, and Update $\varphi_{m_i n_j}^{lv}$ accordingly;
                end
            else
                $UB(\gamma) < LB$ Remove node $\gamma$ from $S$, reselect node from $S$;
            end
        end
    end
end
Return $UB(X^*)$, $LB(X^*)$ and $X^*$.
```

6. Numerical Experiments

6.1. Numerical Experiments Description

In this section, extensive computational experiments are conducted to evaluate the effectiveness of the collaborative optimization model and the newly proposed branch-and-bound algorithm within port clusters. A series of numerical experiments are performed, focusing on 4 container shipping routes and 23 ports across 7 port clusters operated by company A, which serves as the representative liner company in this study. The transportation demand and unit revenue for OD pairs within these port clusters on various routes are derived from real data provided by company A, with modifications made to ensure commercial confidentiality. The range of parameters is shown in Table 2.

Table 2. The value domain of the parameters.

Parameters	Range	Parameters	Range
$cs_{m_i n_j}^{lv}$	$\left\lfloor 0.03 * d_{m_i n_j}^{lv} \right\rfloor$ [41,52]	$ct_{m_i n_j}^{lv}$	$\left\lfloor 1.3 * cs_{m_i n_j}^{lv} \right\rfloor$ [53]
$cc_{m_i n_j}^{lv}$,	$\left\lfloor rand(0.8, 0.85) * ave\left\{gr_{mn}^{lv}\right\} \right\rfloor$	SC^v	18,000 TEUs, 18,000 TEUs, 15,000 TEUs,
$ce_{m_i n_j}^{lv}$			15,000 TEUs
D	1.29 [46]	α	3.5 [46]
β	90%	$MI_{m_i n_j}^{lv}$	$\left\lfloor dg_{m_i n_j}^{lv} * rand(0.15, 0.20) \right\rfloor$
$Mo_{m_i n_j}^{lv}$	$\left\lfloor dg_{m_i n_j}^{lv} * rand(0.15, 0.20) \right\rfloor$	fc^v	3000, 3000, 1500, 1500
ch^{lv}	$rand[250, 300]$, $rand[300, 350]$, $rand[300, 400]$, $rand[250, 300]$,		
	$rand[250, 300]$, $rand[200, 300]$, $rand[250, 300]$ (m = 1,2,..,7)		
oe^{lv}	$rand[400, 600]$,$rand[80, 200]$,$rand[100, 300]$,$rand[20, 100]$,		
	$rand[400, 600]$, $rand[50, 200]$, $rand[200, 300]$ (m = 1,2,..,7)		
μ^{lv}	$rand[0, 600]$, $rand[0, 300]$, $rand[0, 300]$, $rand[0, 100]$,		
	$rand[0, 300]$, $rand[0, 100]$, $rand[0, 200]$ (m = 1,2,..,7)		
σ^{lv}	$rand[10, 30]$, $rand[10, 20]$, $rand[10, 20]$, $rand[10, 20]$,		
	$rand[10, 20]$, $rand[10, 20]$, $rand[10, 20]$ (m = 1,2,..,7)		
N^{lv}	$rand[2500, 7500]$, $rand[2000, 3500]$, $rand[4000, 7000]$, $rand[1500, 2000]$,		
	$rand[5000, 6000]$, $rand[1500, 2000]$, $rand[1500, 2000]$ (m = 1,2,..,7)		
F^{lv}	$rand[300, 500]$, $rand[150, 250]$, $rand[150, 250]$, $rand[50, 100]$,		
	$rand[150, 250]$, $rand[50, 100]$, $rand[100, 200]$ (m = 1,2,..,7)		

The sailing distances between ports within different port clusters across various routes (denoted by d_{ij}^{lv}) are derived from website data (https://voc.myvessel.cn/position, accessed on 1 September 2024). The transportation cost of containers is proportional to the sailing distance. According to [23,54], it can be calculated that the maximum transportation default rate allowed by liner companies is in the range of 9.4% to 10.4%. Thus, the value of β in this paper is set as 90%. The BBLRAP algorithm described in Section 5 was accomplished by applying Python 3.6. The exact solution was obtained by the application CPLEX 12.6.3 software. The calculation result of the numerical experiment was obtained through a computer whose operating system was Microsoft Windows 11 with an IntelR CoreTM i9-12900F CPU at 2.40 GHz and 32.0 GB of RAM. The order of port calls within the seven port clusters on the four routes is shown in Figure 2.

Route 1: Xiamen (XM)—Nansha (NS)—Hong Kong (HK)—Yantian (YT)—Cai Mep (CM)—Singapore (SIN)—Piraeus (PR)—Hamburg (HB)—Rotterdam (RD)—Zeebrugge (ZB)—

Valencia (VC)—PR—Khalifa—(KL)—SIN—XM.
Route 2: Shanghai (SH)—Ningbo (NB)—XM—YT—SIN—Felixstowe (FT)—ZB—Gdansk (GDS)—Wilhelmshaven (WLS)—SIN—YT—SH.
Route 3: SH—NB—XM—YT—Manzanillo (MZ)—Houston (HT)—Tampa (TP)—Mobile (MB)
Route 4: NB—SH—Pusan (PS)—Long Beach (LB).

The seven port clusters are divided into groups, as shown in Table 3.

Table 3. Port cluster division.

Port Cluster	Port	Port Cluster	Port
1	SH/NB/XM/YT/HK/NS	2	CM/ SIN
3	PR/HB/RD/ZB/VC/GDS/WLS/FT	4	KL
5	PS	6	LB/MZ
7	HT/TP/MB		

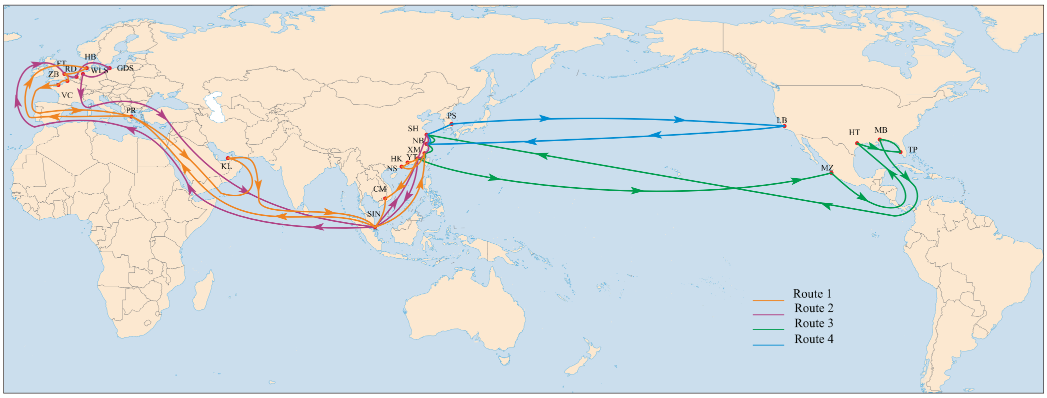

Figure 2. Navigation map.

6.2. Result

To verify the effectiveness of BBLRAP, this study established 10 example groups of varying sizes to compare results with CPLEX across two dimensions: computational effectiveness and execution time. As illustrated in Table 4, Tests 1, 3, 5, and 7 address a single route with four voyages, while Tests 2, 4, 6, and 8 tackle a single route with eight voyages. "UB" and "LB" denote the upper and lower bounds of the goals achieved through BBLRAP iterations, respectively. "UB Gap" and "LB Gap" signify the discrepancies between "UB" and "LB" compared to the exact objective function value. Table 4 indicates that both "UB Gap" and "LB Gap" values are sufficiently small, demonstrating tight convergence of upper and lower bounds, thus affirming the high accuracy of BBLRAP's solutions.

Table 4. Calculation result comparison of BBLRAP and Cplex.

Test	l	v	BBLRAP					Cplex	
			UB (USD)	LB (USD)	Time (s)	UB Gap	LB Gap	Obj. (USD)	Time (s)
1	1	4	526,450,341	525,134,931	61	0.17%	0.08%	52,555,375	517
2	1	8	102,822,033	102,534,324	84	0.17%	0.11%	102,647,236	723
3	2	4	53,345,097	53,137,247	59	0.23%	0.16%	53,222,403	547
4	2	8	106,968,519	106,551,742	78	0.22%	0.17%	106,733,188	719
5	3	4	83,955,690	83,771,086	63	0.13%	0.09%	83,846,548	579
6	3	8	170,428,714	170,105,049	81	0.11%	0.08%	170,241,242	791
7	4	4	11,354,619	11,337,593	35	0.09%	0.06%	11,344,400	114
8	4	8	22,694,350	22,644,450	51	0.12%	0.10%	22,667,117	275
9	1, 2, 3, 4	4	201,194,953	200,591,789	397	0.21%	0.19%	200,973,639	6159
10	1, 2, 3, 4	8	402,289,571	402,093,018	826	–	–	–	>10,800

Regarding algorithm execution time, BBLRAP exhibits significant advantages. As problem size increases, the solution time of CPLEX escalates rapidly. In particular, for large-scale problems (e.g., Test 10), CPLEX fails to complete within the preset 10,800 s, while BBLRAP consistently identifies the objective function value, maintaining compact upper and lower bounds. This highlights BBLRAP's superior performance in solving large-scale problems compared to CPLEX.

This subsection analyzes the numerical experimental results for Route 1 (representing the Asia–Europe route) and Route 3 (representing the Asia–North America route). Table 5 presents the number of laden containers, empty containers, rent-out slots, rent-in slots, and emergency leasing of empty containers between port clusters on Route 1 (note that all values are the average of the four voyages). From Table 5, it is evident that the volume of laden containers transported from Port Cluster 1 to Port Cluster 3 is approximately double that of the reverse route, illustrating the imbalanced nature of import and export trade between China and Europe.

Additionally, the allocation of slots for laden and empty containers within Port Cluster 1 is approximately equal. This indicated that when demand for laden container transportation is low, utilizing available slots for empty container repositioning can help liner companies reduce operating costs and enhance profits. Table 5 reveals that Port Clusters 2 and 3 are surplus areas for empty containers, while Port Cluster 1 faces a shortage. Consequently, empty container transfer operations are primarily focused on Route 1 from Port Cluster 2 to Port Cluster 1 and from Port Cluster 3 to Port Cluster 1.

Moreover, as indicated in Table 5, the current market demand for liner companies is relatively weak, leading to available slots for rent-out among various port clusters, which further boosts their revenue. The last column represents the ratio of emergency leasing of empty containers to the demand for empty containers. The data show that emergency leasing predominantly occurs within the same port cluster or between adjacent port clusters over short distances, as the cost of long-distance emergency leasing is significantly higher.

Table 5. The numerical experiment results of Route 1.

Origin	**Destination**	**Container Slot**				**Empty Container Emergency Leasing**	
Cluster	**Cluster**	**Laden**	**Empty**	**Rent out**	**Rent in**	**Number**	**Share**
1	1	617	646	122	0	1047	82.9%
1	2	2365	0	257	65	1764	74.6%
1	3	8394	0	733	273	0	0
2	1	303	919	55	0	0	0
2	2	297	0	4	0	81	27.2%
2	3	2186	0	229	113	0	0
3	1	4071	2816	731	0	0	0
3	2	1789	737	191	0	0	0
3	3	4082	156	866	0	739	17.4%
3	4	1437	0	359	0	0	0
4	1	76	0	62	0	0	0
4	2	52	0	71	0	0	0

Similar to Table 5, Table 6 presents the allocation of slots and the number of empty container emergency leases between port clusters on Route 3, with all values averaged over four voyages. As shown in Table 6, the number of laden containers transported from Port Cluster 1 to Port Clusters 6 and 7 is approximately three times greater than in the reverse direction. Empty container transportation primarily occurs from Port Cluster 7 to Port Cluster 1 and within Port Cluster 1 itself. Slot rent-in is concentrated in areas with high demand for laden container transportation, particularly on Route 3 from Port Cluster 1 to Port Clusters 6 and 7.

Emergency leasing of empty containers occurs exclusively within Port Cluster 1. Due to the significant distances between port clusters, the cost of emergency leasing is too high. Consequently, demand for empty containers is primarily met through repositioning and planned leasing. As illustrated in Table 5 and 6, when demand for laden container transportation within a port cluster is low, the allocation of laden and empty container slots is approximately equal, as seen in Port Cluster 1. In contrast, when a port cluster is in an empty container surplus area, the number of laden container slots is roughly twice that of empty container slots, as observed in the routes from Port Cluster 3 to Port Cluster 1 and from Port Cluster 7 to Port Cluster 1.

Table 6. The numerical experiment results of Route 3.

Origin Cluster	Destination Cluster	Container Slot				Empty Container Emergency Leasing	
		Laden	Empty	Rent out	Rent in	Number	Share
1	1	910	791	249	0	1543	90.7%
1	6	2310	0	208	135	0	0
1	7	6594	0	714	122	0	0
6	1	828	0	149	0	0	0
6	7	1037	0	134	0	0	0
7	1	2203	1343	563	0	0	0
7	7	983	388	147	0	0	0

To verify the effectiveness of the two strategies proposed in this paper, three scenarios are established for comparative analysis: Scenario 1 applies both the (T, s) inventory control strategy and the cooperative possession strategy; Scenario 2 utilizes only the cooperative possession strategy; Scenario 3 employs only the (T, s) policy. Figure 3 presents the results from the four voyages on Route 1. Notably, Figure 3 features dual axes, with total revenue represented on the left y-axis and the remaining costs on the right y-axis. From Figure 3, it is evident that the combined application of both strategies boosts the total revenue for the liner company. Additionally, Scenario 2 demonstrates that, in the absence of a reasonable and effective control strategy for empty container inventory, both empty container leasing costs and inventory costs rise sharply while transportation costs decrease. This leads to an ineffective accumulation of empty containers in certain areas while other regions experience shortages.

Figure 4 illustrates the fluctuations in emergency leasing volumes of empty containers across eight voyages on four routes. The figure reveals distinct cyclical characteristics in the emergency rental of empty containers. For Routes 1 and 2, there is a significant increase in emergency leasing during the 3rd and 7th voyages. Interestingly, Routes 3 and 4 display opposite cyclical patterns, with the lowest emergency leasing volumes occurring during the 2nd, 3rd, 6th, and 7th voyages. These cyclical trends suggest that both the demand for empty containers and the transportation demand for laden containers exhibit significant cyclical behavior. As the demand for laden container transportation rises, the need for empty containers correspondingly increases; however, the capacity for empty container transport is constrained by available slots. Consequently, as the repositioning of empty containers decreases, the number of emergency leases will inevitably rise.

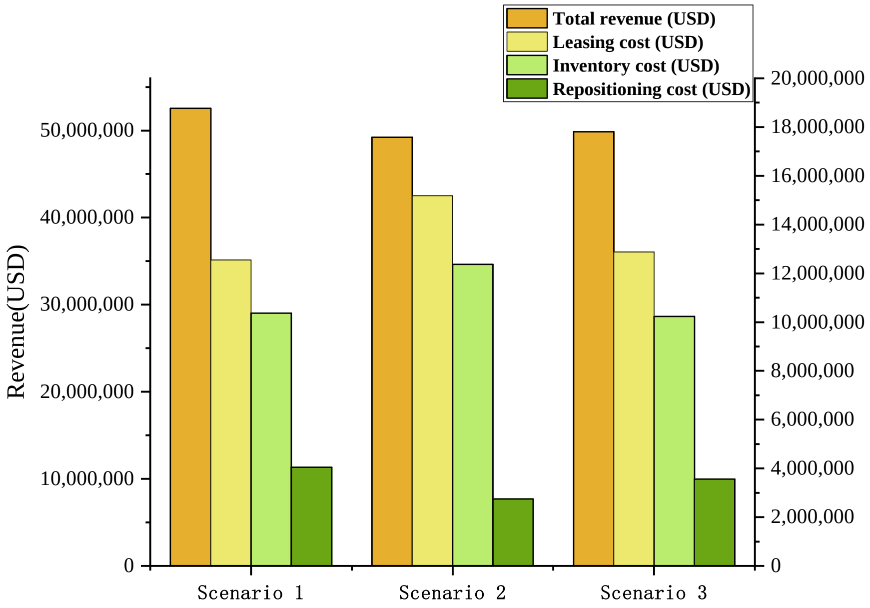

Figure 3. Comparison of results for different strategies.

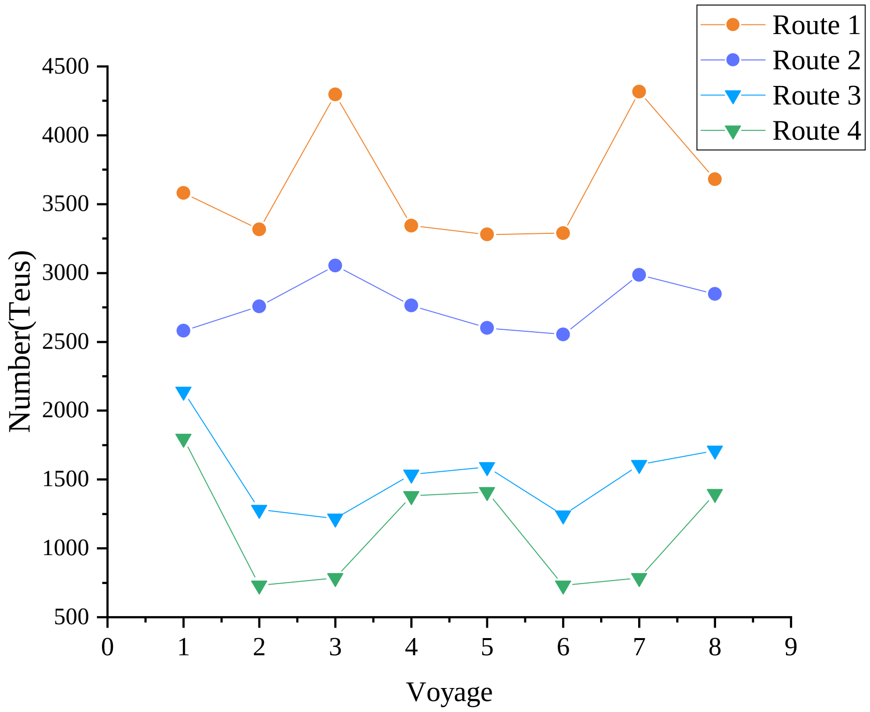

Figure 4. Emergency leasing number of empty containers on different routes and voyages.

6.3. Sensitivity Analysis

To explore the impact of various factors on liner companies' revenue, this paper analyzes the relationship between changes in two unit revenue factors and three unit cost factors with the resulting changes in overall revenue. Figure 5 illustrates the effects of fluctuations in unit container transport revenue and unit slot rent-out revenue on total revenue. The horizontal axis represents the range of changes in these influencing factors (where −0.4 indicates a 40% reduction in unit cost), while the vertical axis depicts the rate of change in revenue (i.e., the ratio of the change in revenue to the original revenue). The data in Figure 5 reveal a clear positive linear relationship between total revenue and changes in both factors. Notably, total revenue exhibits greater sensitivity to variations in unit container transport revenue; when unit transport revenue shifts from −40% to

40%, total revenue varies from −98.65% to 101.67%. In contrast, changes in total revenue in response to unit slot rent-out costs are comparatively modest, ranging from −15.11% to 15.75%.

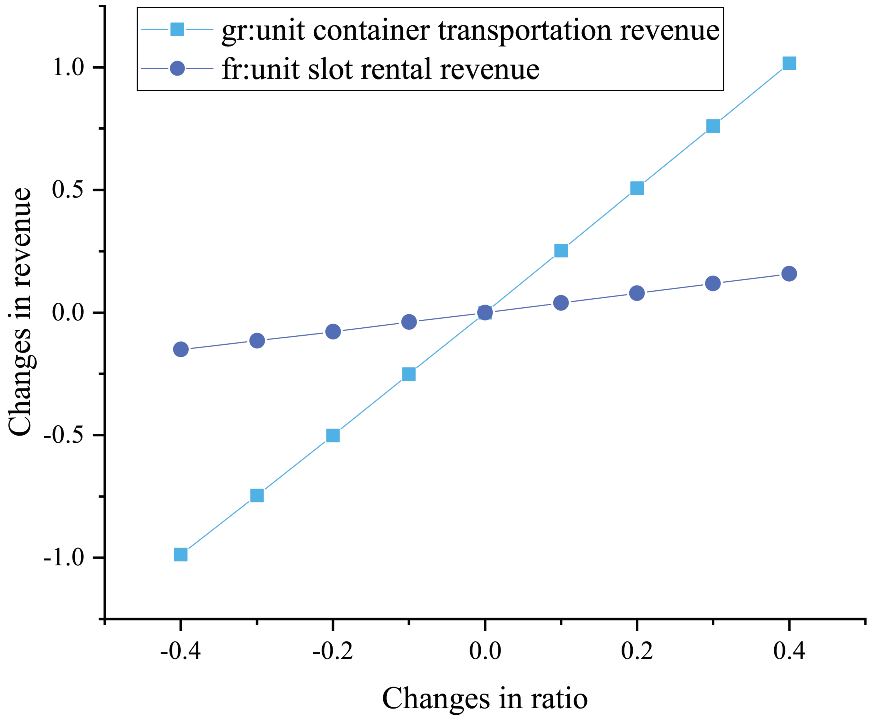

Figure 5. The influence of different parameters on the total revenue.

Figure 6 illustrates the effects of changes in unit slot rent-out costs, unit-laden container transportation costs, and empty unit container transportation costs on total revenue. In general, total revenue declines as each of these three unit costs increases. Among them, total revenue is most sensitive to changes in unit-laden container transportation costs and least sensitive to changes in unit slot rent-out costs. Specifically, the range of total revenue variations is from 52.06% to −51.97%, from 14.18% to −13.42%, and from 15.75% to −1.06%. The heightened sensitivity to laden container transportation costs is due to the large number of slots allocated for laden containers, while the current market demand remains relatively weak, resulting in a smaller number of slots rented for laden transport.

Figure 7 explores the effect of changes in unit slot rent-out revenue on the trade-offs among three key decision variables: liner slot bookings, empty container repositioning, and slot rent-out. On the horizontal axis, the value 0.6 represents a scenario where unit slot rent-out revenue has decreased to 0.6 times its original value, while the vertical axis represents the rate of change for each decision variable. Notably, the change rates for slot bookings and empty container repositioning are measured using the left vertical axis, whereas the changes for slot rent-out are indicated by the right vertical axis.

Both slot bookings and empty container repositioning decrease with an increase in unit slot rent-out revenue, eventually reaching a stable point. In contrast, the number of slots rented out increases as unit slot rent-out revenue rises until it also stabilizes. This is because as the unit slot rent-out revenue increases, more slots will be sold to partners to earn greater profits. Consequently, the slot bookings received by the liner company and the slot allocated to empty containers are reduced. However, once the number of slots rented out reaches its maximum capacity, further increases in unit slot rent-out revenue do not affect the number of slots rented out.

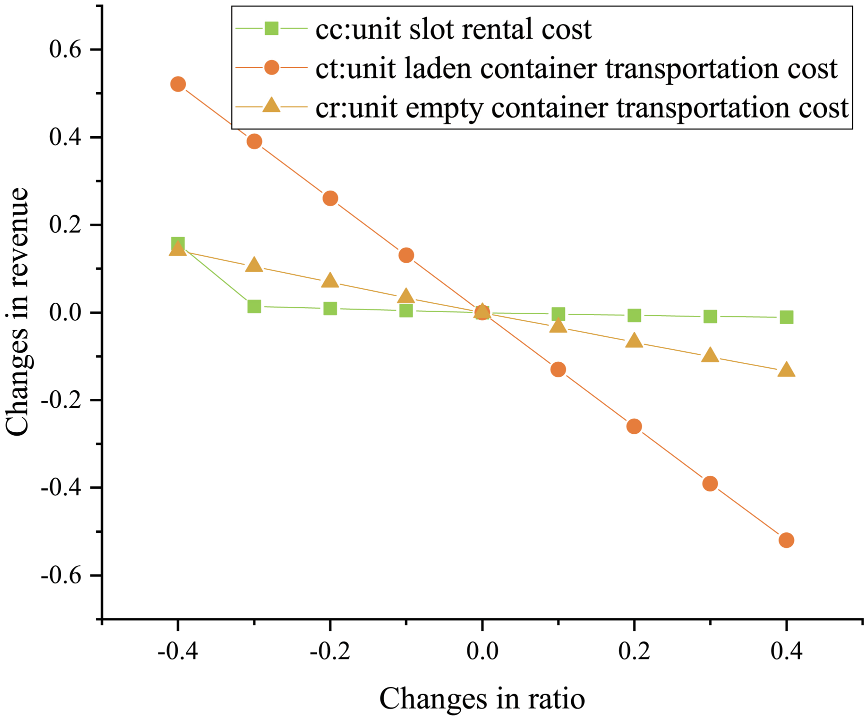

Figure 6. The influence of different costs on the total revenue.

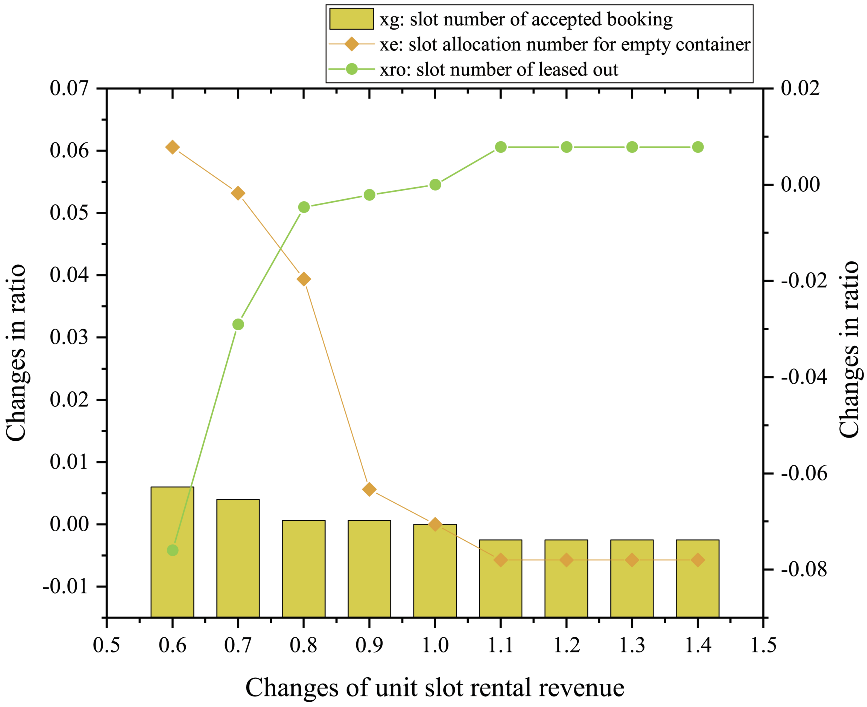

Figure 7. The impact of changes in unit slot rent-out revenue on decision variables.

The coordinate axis configuration of Figure 8 is similar to that of Figure 7. As depicted in Figure 8, the number of slots rent-in decreases as the unit slot renting cost increases, with the rate of reduction becoming more pronounced. Interestingly, when the unit slot renting cost is lower than the existing cost, both the empty container repositioning volume and the slot booking acceptance number remain largely unchanged. However, when the unit slot renting cost exceeds the current cost, these two decision variables begin to decline. This trend arises because when the slot renting cost decreases, the number of slots that can be rent-in is restricted by a maximum rentable threshold. Conversely, when the slot renting cost becomes too high, the liner company chooses to avoid renting slots or rents as few as possible. Under these conditions, the liner company may reject transportation requests with lower profit margins and minimize empty container repositioning to control operational costs.

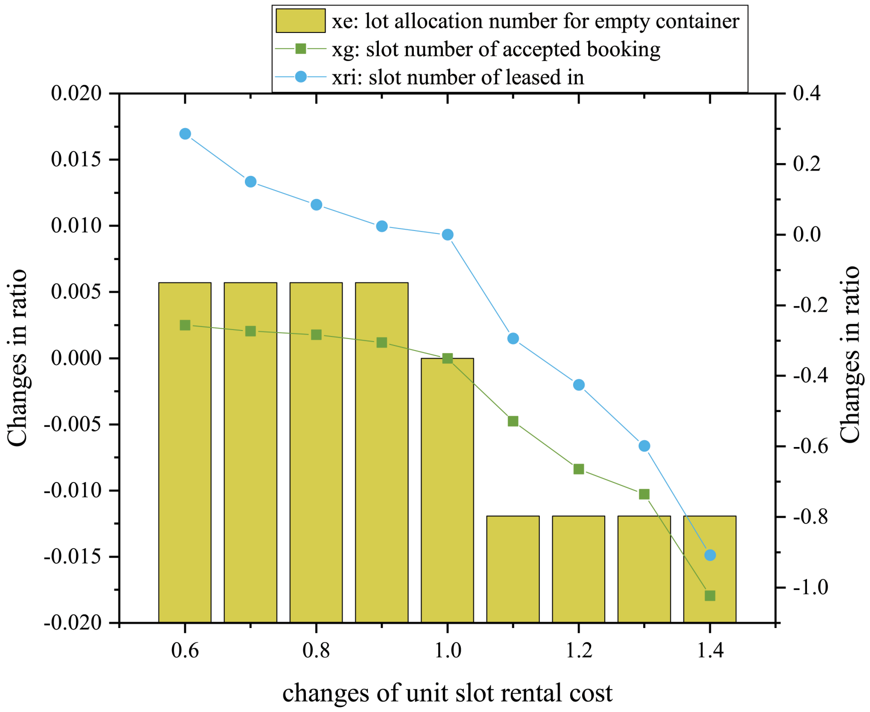

Figure 8. The impact of changes in unit slot rent-in revenue on decision variables.

6.4. Discussion and Implication

To further validate the effectiveness of the proposed model, this paper utilizes the dataset from the literature [1] and incorporates the data in this research to evaluate the impact of different optimization methods on the revenue of liner companies. Based on the application data from literature [1], including demand, unit transportation revenue, unit transportation, and storage costs, ports were simply divided into two port clusters according to the geographical area. Slot rental costs, rental revenue, and constraints were derived from the data in this study, and the total revenue was taken as the average of 10 experiments. As illustrated in Table 7, the optimization method proposed in this paper increased total revenue by 138.44% compared to the FCFS method. Notably, in FCFS, all empty containers are sourced from leasing without empty container repositioning or inventory management optimization. Additionally, the overall revenue improved by 7.45% relative to the optimization method described in [1], highlighting the effectiveness of the collaborative optimization approach proposed in this paper.

Table 7. The performance comparison among different methods in the case of the literature.

	(T, s) Policy and Cooperative Possession Strategy	**Optimal Policy and Overbooking with Different Service Classes Strategy**	**First Come First Serve (FCFS)**
Revenue	235,414,516	219,092,152	98,730,148
Enhancing percentage	138.44%	121.91%	–

The results of the numerical experiments demonstrate the following key findings:

1. The combined application of the (T, s) inventory control policy and the cooperative possession strategy significantly reduces both leasing and inventory costs for empty containers. This dual approach enhances the circulation and repositioning of empty containers by leveraging available slots, fosters win–win partnerships by expanding market share, optimizes slot utilization, and ultimately boosts the total revenue of liner companies.
2. Container transportation demonstrates notable cyclical characteristics, with distinct patterns emerging across different routes. To address these fluctuations, liner companies should strengthen collaboration with their partners to promote the cooperative possession strategy. This would help manage cyclical demand variations, improve slot utilization, and maximize revenue.
3. The findings underscore the importance of establishing a quick-response platform for slot leasing in partnership with collaborators. Such a platform would facilitate the seamless execution of the cooperative possession strategy. Moreover, the agreed-upon cooperative leasing price for slots should range from 72% to 76.5% of the slot selling price between port clusters.
4. In regions where port clusters face a shortage of empty containers and demand for laden container transportation is low, the allocation of slots between laden and empty containers tends to reach equilibrium. Increasing empty container transportation within such port clusters can further reduce inventory costs, balance the distribution of empty container resources, and mitigate vicious competition among ports.
5. Compared with empty container supply costs, the empty container leasing cost is the largest among all types of empty container supply costs. Liner companies should increase the long-term leasing of empty containers within the Asian port cluster and the empty container transportation volume to the Asian port cluster.
6. This paper leverages regional port clusters in China, Southeast Asia, Europe, Western North America, and Eastern North America to effectively manage and reposition empty containers. It provides a theoretical foundation for the coordinated optimization of slot allocation and empty container repositioning within the port cluster, offering significant practical value for liner companies aiming to reduce operating costs and enhance profitability.

7. Conclusions

This paper develops a new COPCSAECR under port cluster concerning the sources and channels of empty container supply and explores the effect of combining the (T, s) policy with the cooperative possession strategy. A new mixed integer joint optimization model was developed, and BBLRAP was developed to address the problem. Numerical experiments were conducted to verify the effectiveness and accuracy of the proposed new model and the BBLRAP solution algorithm.

The numerical experiments illustrate that establishing appropriate pricing has become critical to achieving higher profits under a cooperative possession model. As unit revenues for laden containers increase, competition in the liner market is expected to intensify. Market share will be a key determinant of a liner company's success. With the cooperative possession strategy, liner companies have a greater opportunity to emerge as "winners" in this increasingly competitive landscape. The sensitivity analysis revealed that the pricing of slot rent-in and rent-out directly influences the decision on whether a liner company accepts a shipper's transportation request. After analysis, it is found that the leasing price for slots ranging from 72% to 76.5% of the slot selling price between port clusters can bring the greatest benefits to liner companies. The optimization method proposed in this paper can effectively integrate the resources within the port cluster and carry out scientific allocation and transportation to prevent disorderly competition.

Although showing some attractiveness, the above work could benefit further studies from two perspectives, including (1) incorporating multimodal transportation and (2) considering cargo transhipment and liner connections. Currently, the research scope of empty container transportation is constrained to sea transportation and, in the future, could be extended to multimodal transportation. This paper has yet to take into account the transhipment of cargo and the connection of cargo between liners, which will be investigated in future work. Moreover, we plan to apply big data mining technology to make more precise demand predictions and explore optimization algorithms with improved iterative performance.

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Nomenclature

Sets

G port cluster sets on the route, $G = \{1, 2, ..., m, n\}$.

L route sets, $L = \{1, 2, ..., l\}$.

P port sets within port cluster, $P = \{1, 2, ..., m_i, n_j\}$.

V voyage sets, $V = \{1, 2, ..., v\}$.

Parameters

β Minimum satisfaction rate of laden container transportation demand.

$\mu_{m_i n_j}^{lv}$ The average empty container demand of unit decision period between port i in port cluster m and port j in port cluster n on voyage v of route l.

$\sigma_{m_i n_j}^{lv}$ The standard deviation of empty container demand per unit decision period between port i in port cluster m and port j in port cluster n on voyage v of route l.

$cc_{m_i n_j}^{lv}$ Unit slot rental cost from port i in port cluster m to port j in port cluster n on voyage v of route l.

$ce_{m_i}^{l}$ The unit empty container planned leasing cost at port i in port cluster m of route l.

$ch_{m_i}^{lv}$ The unit empty container storage cost at port i in port cluster m on voyage v of route l.

$cr_{m_in_j}^{lv}$ The unit emergency rental cost of empty container from port i in port cluster m to port j in port cluster n on voyage v of route l.

$cs_{m_in_j}^{lv}$ The unit empty container transportation cost (including loading and unloading costs) from port i in port cluster m to port j in port cluster n on voyage v of route l.

$ct_{m_in_j}^{lv}$ The unit laden container transportation cost (including loading and unloading costs) from port i in port cluster m to port j in port cluster n on voyage v of route l.

D The total duration comprises the advance decision period and the transportation interval for empty container repositioning. The decision period refers to the time required for the shipper to submit a request for empty containers to the liner company in advance. In this paper, the transportation interval is defined as one week.

$dg_{m_in_j}^{lv}$ The laden container transportation demand from port i in port cluster m to port j in port cluster n on voyage v of route l.

$F_{m_in_j}^{lv}$ The upper limit of emergency rental container from port i in port cluster m to port j in port cluster n on voyage v of route l.

fc^{lv} The fixed cost of ship for voyage v on the route l (including fuel cost, ship maintenance cost, etc.).

$fr_{m_in_j}^{lv}$ Unit slot rental revenue from port i in port cluster m to port j in port cluster n on voyage v of route l.

$gr_{m_in_j}^{lv}$ Unit container transportation revenue from port i in port cluster m to port j in port cluster n on voyage v of route l.

$MI_{m_in_j}^{lv}$ The maximum slots amount that a liner company can rent from the partner between port i in port cluster m and port j in port cluster n on voyage v of route l.

$MO_{m_in_j}^{lv}$ The maximum slots amount that a liner company can rent out to partner between port i in port cluster m and port j in port cluster n on voyage v of route l.

$N_{m_i}^{lv}$ The empty container storage capacity at port i in port cluster m on voyage v of route l.

$oe_{m_i}^{l}$ Initial empty container volume of port i in port cluster m on the initial voyage of route l. (empty container volume owned by the liner company).

$s_{m_i}^{lv}$ Empty container inventory replenishment point at port i in port cluster m on voyage v of route l.

SC^{lv} The ship capacity on the voyage v of route l.

Decision variables

$xa_{m_in_j}^{lv}$ The emergency leasing number of empty container from port i in port cluster m and returned to port j in port cluster n on voyage v of route l.

$xb_{m_i}^{l}$ The planned rental volume of empty container at the port i in port cluster m on the route l.

$xe_{m_in_j}^{lv}$ The slot allocation number for empty container from port i in port cluster m to port j in port cluster n on voyage v of route l.

$xf_{m_in_j}^{lv}$ The slot allocation number for laden container from port i in port cluster m to port j in port cluster n on voyage v of route l.

$xg^{lv}_{m_i n_j}$ The slot number booked by customers via online booking platform from port i in port cluster m and returned to port j in port cluster n on voyage v of route l.

$xro^{lv}_{m_i n_j}$ The slot number of leased out by the liner company from port i in port cluster m to port j in port cluster n on voyage v of route l.

Auxiliary decision variables

$SE^{lv}_{m_i}$ The remaining number of empty container after the ship leaves the port (i.e., inventory) at the port i in port cluster m on voyage v of route l.

$xri^{lv}_{m_i n_j}$ The slot number of leased in from port i in port cluster m to port j in port cluster n on voyage v of route l.

$XS^{lv}_{m_i}$ The container number on board when the ship arrives at the port i in port cluster m and completes unloading but does not start loading on voyage v of route l.

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