Levi, E, Jones, M and Zoric, I

Phase Voltage Harmonic Imbalance in Asymmetrical Multiphase Machines with Single Neutral Point

http://researchonline.ljmu.ac.uk/4733/

Citation (please note it is advisable to refer to the publisher's version if you intend to cite from this work)


LJMU has developed LJMU Research Online for users to access the research output of the University more effectively. Copyright © and Moral Rights for the papers on this site are retained by the individual authors and/or other copyright owners. Users may download and/or print one copy of any article(s) in LJMU Research Online to facilitate their private study or for non-commercial research. You may not engage in further distribution of the material or use it for any profit-making activities or any commercial gain.

The version presented here may differ from the published version or from the version of the record. Please see the repository URL above for details on accessing the published version and note that access may require a subscription.

For more information please contact researchonline@ljmu.ac.uk
Phase Voltage Harmonic Imbalance in Asymmetrical Multiphase Machines with Single Neutral Point

I. Zoric, M. Jones, E. Levi
Faculty of Engineering and Technology
Liverpool John Moores University
Liverpool, U.K

Abstract—Multiphase (n-phase) machines are often designed with l sub-windings on stator, each having k phases, and the machine is typically operated with l isolated neutral points. However, such a machine can also operate with a single neutral point, which improves the fault-tolerant properties. When a machine is inverter supplied, low order harmonics may be present due to the low switching frequency and nonlinear inverter properties. Moreover, low order zero-sequence harmonics can be deliberately injected to increase dc bus voltage utilisation. This paper investigates a phenomenon that has not been reported so far in relation to asymmetrical multiphase machines with a single neutral point, namely that the presence of balanced low order harmonics in leg voltages produces unbalanced phase voltage harmonics and consequently unbalanced phase current harmonics. By analysing the neutral point (common mode) voltage harmonics, imbalance in the phase voltage harmonics is explained. Analytical expressions for neutral point voltage harmonics and phase voltage harmonics are provided for asymmetrical machine configurations with a single neutral point having arbitrary numbers of sub-windings and sub-winding phases. Theoretical considerations are verified using simulations and experiments with asymmetrical twelve- and nine-phase loads with a single neutral point, respectively.

Keywords—Multiphase machines, phase voltage harmonics, harmonic imbalance, low-order harmonics

I. INTRODUCTION

With variable speed drives reaching multi-MW power rating, multiphase machines offer numerous advantages over three-phase ones. Lower current and power per phase allow use of reduced rating switches, torque ripple is at the higher frequencies, and, above all, fault-tolerant operation is highly desirable in safety critical application [1, 2]. Some examples where advantages of the multiphase machines are of particular importance include fault-tolerant remote off-shore wind power generation systems [3, 4], increase of the dc link voltage by series connection of the voltage source inverters (VSIs) [5], multi-motor applications [6, 7], and electric ship propulsion [8] and generation [9].

Good fault tolerance is one of the most important benefits of the multiphase machines. In many cases use of a multiphase machine with a multiple three-phase winding topology has been considered as the most viable solution due to the readily available three-phase VSIs. Furthermore, the fault tolerant strategy can be very simple in the cases with multiple neutral points, since a whole three-phase winding set can be taken out of service in the case of a fault [10]. Off course, available power is reduced. On the other hand, research shows that switching off an entire sub-winding is not an optimal solution and that configurations with the single neutral point provide superior performance in post-fault operation. By utilising flux/torque non-producing planes or by changing the transformation matrix, a smaller drop in available power under open-phase fault can be achieved while optimising operation for torque ripple or stator resistive loss minimisation [11].

If asymmetrical or split-phase machines are used, some additional benefits result as related to an easier manufacturing of the stator winding, since stator core can contain the same number of slots as for a three-phase machine [12], as long as the complete winding is formed as a set of three-phase sub-windings. Extensive research has been carried out on modelling and finding appropriate decoupling matrices for split-phase machines with both multiple and single neutral points [13-17]. Analysis of the torque ripple harmonics has been also carried out [12, 18]. Nevertheless, the properties of low order harmonics in this particular type of multiphase machines have not been explored in the past and this is the subject this paper deals with. As shown in the paper, harmonic imbalance in the phase voltages does not appear in asymmetrical six-phase machines, which have been by far the most frequent object of study in the multiphase drive area. Indeed, such an unbalance does not take place as long as the complete winding consists of only two sub-windings.

It is shown in this paper that balanced low-order leg voltage harmonics, usually present in high power multiphase drives with low PWM switching frequency, can produce unbalanced phase voltage harmonics in the case of the asymmetrical machines with a single neutral point. Unequal phase voltage harmonics may lead to unequal thermal losses among phases and uneven stress on the switching devices, which should be taken into consideration during the drive design stage. Starting from the analysis of the neutral point voltage harmonics, analytical expressions for phase voltage harmonics are provided and verified by simulations and experiments. The developed theory is general and it covers all cases of the asymmetrical machines with a single neutral point. The appearance of phase voltage harmonic imbalance requires existence of at least three sub-windings in the stator winding, as shown in the paper.

II. ASYMMETRICAL MACHINE DESCRIPTION AND LEG VOLTAGE HARMONICS

A definition of an asymmetrical machine and phase displacement angles, in the case of an arbitrary number of phases, is provided first. Stator windings of an n-phase asym-
metrical machine, considered here, consist of \( k \)-phase sub-windings, where \( k \) is a prime number. Number of the sub-windings is equal to \( l \), so that the total phase number \( n \) equals:

\[
n = l \cdot k
\]

(1)

Spatial angular shift within each sub-winding is equal to \( \frac{2\pi}{n} \). In an asymmetrical machine the phase shift between first phases of sub-windings is \( \pi n \). A schematic of the spatial phase shifts for the asymmetrical \( n \)-phase machine, composed of \( l \) sub-windings with \( k \) phases, is shown in Fig. 1. Symbol \( i \) represents the phase number within the sub-winding; hence it varies from 1 to \( k \). Symbol \( j \) is introduced to denote the sub-winding number, and it varies from 1 to \( l \); then the spatial angular position of the \( i \)th phase in the \( j \)th sub-winding can be defined as:

\[
\theta_{j,i} = \frac{\pi}{n} (2l(i-1)+j-1), \quad j = 1, 2, ..., l, \quad i = 1, 2, ..., k
\]

(2)

It should be noted that for all further equations indices \( j \) and \( i \) are varied as in (2), if not explicitly specified otherwise. It is assumed that the machine is inverter supplied and that the leg voltages may contain harmonics. All leg voltage harmonics of the given harmonic order are considered balanced; hence they have the same amplitude in each leg. Moreover, all harmonics of the leg 1 are taken as being in balanced; hence they have the same amplitude in each leg.

The fundamental is of course the 1st harmonic. Without loss of generalisation, harmonic order \( h \) is a positive integer. Firstly, the neutral point voltage is analysed, and afterwards the expression for phase voltages is developed. In what follows it is assumed that the machine phases are balanced, so the impedance of every phase is the same.

III. Neutral Point Voltage Analysis

The single neutral point voltage is found by firstly establishing the neutral point voltage of each sub-winding.

From (3), the neutral point voltage for the harmonic order \( h \) of the winding set \( j \) can be found as a sum of all leg voltages of that sub-winding, divided with the number of phases in the sub-winding:

\[
v_{j,h}^{NP}(t) = \frac{1}{k} \sum_{i=1}^{k} A_h \cos \left( \frac{h \pi}{n} (2l(i-1)+j-1) \right)
\]

(4)

(4) and applying trigonometric identities, a simplified expression is obtained:

\[
v_{j,h}^{NP}(t) = \frac{A_h}{k} \cos \left( \frac{h \pi}{n} (j-1) \right) \sum_{i=1}^{k} \cos \left( (i-1) \frac{h \pi}{2k} \right)
\]

(5)

The summands in summation of (5) yield zero when the harmonic order is not an integer multiple of the number of phases in the sub-winding, \( k \). Otherwise the summation is equal to \( k \). Therefore, neutral point voltage for the harmonic order \( h \) of the sub-winding \( j \) is defined as:

\[
v_{j,h}^{NP}(t) = \begin{cases} A_h \cos \left( \frac{h \pi}{n} (j-1) \right), & h = h_n k, \quad h_n = 1, 2, ..., k \end{cases}
\]

(6)

Here the newly introduced variable \( h_n = h/k \) represents the normalised harmonic order. This is introduced to simplify the equations that follow and provide a tool for derivation of the phase voltage harmonics in the general case.

Finding the value of the single neutral point voltage for the harmonic order \( h \) is achieved by summing the values of individual neutral point voltages of each sub-winding (6) and dividing the result with the number of sub-windings \( l \). Since harmonics of the order \( k \neq h_n k \) cannot exist in any neutral point voltage of the sub-windings, they also do not exist in the single neutral point voltage. Keeping in mind that in this analysis the phase voltage is defined as a difference between leg and single neutral point voltage, it follows that leg and phase voltage harmonics are identical for the cases where \( h \neq h_n k \). Consequently, there is no unbalance and these cases are omitted from future analysis.

Hence the single neutral point voltage harmonics are given with:

\[
v_{h}^{NP}(t) = \frac{1}{l} \sum_{j=1}^{l} A_h \cos \left( kh_n \left( \frac{\pi}{n} (j-1) \right) \right)
\]

(7)

When the sum of cosine functions in (7) is calculated the neutral point voltage for the harmonic order \( h \) is defined as follows:

\[
v_{h}^{NP}(t) = \frac{A_h}{l} \sin \left( \frac{h \pi}{2n} \right) \cos \left( kh_n \left( \frac{\pi}{n} (l-1) \right) \right)
\]

(8)

When the phase angle in the expression (8) is divided by the harmonic order and compared with phase disposition angles, given in Fig. 1, it can be seen that it is always equal to the half of the phase disposition angle of the first phase in the last sub-winding \((l-1)\pi n\). This fact will be used later to show the source and nature of the unbalance in the phase voltage.
harmonics.

Expression (8) also shows that the amplitudes of the neutral point voltage normalised harmonics are not dependent on the number of phases in the sub-winding. This means that every normalised neutral point voltage harmonic \( h_n = h/k \) has the same amplitude for any asymmetrical machine with a single neutral point that has the same number of sub-windings. Visualisation of the neutral point voltage normalised harmonics (\( h_n \)) amplitudes in the case of the machines with 2, 3, 4, and 5 sub-windings is shown in Fig. 2 (all values are per-unit).

For the sake of clarity, an example can be made by comparing the nine-phase \((k=3, l=3)\) and the 15-phase \((k=5, l=3)\) machines, the second row in Fig. 2. For example, cases of the 1st and the 3rd normalised harmonics \((h_n = h/k)\) are the 3rd and the 9th harmonics of the 9-phase machine, while in the 15-phase case they are the 5th and the 15th harmonics. Since both machines have the same number of sub-windings, their normalised harmonics are the same. Therefore, the 3rd harmonic of the 9-phase machine has the same amplitude as the 5th harmonic of the 15-phase machine. The same applies for the 9th and the 15th harmonics of the 9-phase and 15-phase machines, respectively.

Even-order normalised harmonics are either zero or one, but since even harmonics cannot exist in symmetrical supply, they are not analysed here. It should be pointed out that (8) will become limit equation when \( h_n \) becomes integer multiple of 2\( l \). One way of solving this is to use L’Hôpital’s rule, so instead of \( \sin(\pm \pi) \), \( \cos(\pm \pi) \) is obtained.

### IV. Phase Voltage Analysis

Phase voltage is calculated as a difference between leg voltage (3) and neutral point voltage (8) as follows:

\[
\begin{align*}
\Phi_{j,i,h} & = \mathcal{A}_h \cos \left( \frac{h}{k} \left( \frac{\pi}{n} + \frac{\pi}{2} (j - i) \right) \right) - \\
& - \mathcal{A}_h \sin \left( \frac{h}{l} \frac{\pi}{2} \right) \cos \left( \frac{h}{k} \left( \frac{\pi}{2} - (j - i) \right) \right)
\end{align*}
\]

(9)

When (9) is simplified and \( h_n \) is replaced by \( h/k \), the expressions for amplitude and phase of the phase voltage harmonics become as follows:

\[
\Phi_{j,i,h} = \mathcal{A}_h \left( \frac{\sin \left( \frac{h}{2k} \right)}{\sin \left( \frac{h}{2n} \right)} \right)^2 - \frac{1}{2} \left( \frac{\sin \left( \frac{h}{2k} \right)}{\sin \left( \frac{h}{2n} \right)} \right) \sin \left( \frac{h}{k} \left( \frac{\pi}{n} + \frac{\pi}{2} (j - i) \right) \right)
\]

(10)

where, again, \( j \) and \( i \) designate the sub-winding and the phase within that sub-winding, respectively. When the expression for phase voltage amplitude (10) is analysed, it can be seen that all phases within one sub-winding have the same amplitude of the given voltage harmonic (because \( i \) is multiplied by 2\( l \), which is the period of the cosine function). On the other hand, voltage harmonic amplitudes in the phases in different sub-windings are different.

For the sake of clarity, this is demonstrated using the example of the nine-phase machine. Per-unit amplitudes of phase voltage 3rd harmonic, normalised with the amplitude of the leg voltage 3rd harmonic, are shown in Fig. 3. As expected, amplitudes of the 3rd harmonic of the phase voltages within each sub-winding are the same. On the other hand, there is a difference between phases of different sub-windings. Amplitudes of the 3rd harmonic of the phase voltages of the second sub-winding (2nd, 5th, and 8th phase) are different from the ones of the first sub-winding (1st, 4th, and 7th phase). As previously mentioned, this behaviour is easily explained when the phase angle of the neutral point voltage harmonics (8) is taken into consideration.

In the neutral point harmonic analysis, it has been shown that when the phase angle of the neutral point voltage is divided by the harmonic order, it is always equal to half of the phase shift angle of the first phase in the last sub-winding. Hence, the neutral point voltage is always positioned in the middle of the first group of the leg voltages, as shown schematically in Fig. 4. In this figure phase angles of the leg voltages are divided by the harmonic order; thus this analysis holds true for all harmonic orders.
The phase voltages are calculated as a difference between leg voltages and neutral point voltage. It can be seen that leg voltages are located at an equal angular distance from the neutral point voltage. Therefore, the corresponding phase voltages are equally influenced by the neutral point voltage and so have the same amplitude. In the case of the even number of sub-windings, Fig. 4 (left), each two pairs of the phase voltages have the same amplitude. On the other hand, in the case of the odd number of sub-windings, Fig. 4 (right), neutral point voltage coincides with one of the leg voltages, i.e. with \((l-1)/2\).

As stated before, phase voltage harmonics of the same order in one sub-winding have the same amplitude. Hence, the number of phases per sub-winding is irrelevant for the analysis. Consequently, only the first phases of all the sub-windings will be analysed, while the 2\(^{nd}\), 3\(^{rd}\), or any other phase in the sub-winding has the same amplitude of the considered phase voltage harmonic.

Amplitudes of the first five odd normalised phase voltage harmonics in the case of the machines with 2, 3, 4, and 5 sub-windings are given in Fig. 5. In the ideal case with a symmetrical supply, even harmonics do not exist. If even harmonics are present they are balanced and their amplitude is equal to zero or to the amplitude of the corresponding leg voltage harmonics. Hence, they are not of interest in the analysis and are omitted from Fig. 5.

Looking at Fig. 5, it can be seen that for the same number of sub-windings \(l\), phase voltage harmonic unbalance is symmetrical around the imaginary line \(l/2\), bold black line. If the line coincides with a phase number, \(l\) is odd and phase voltage harmonic amplitude of that phase has a unique value, while phase voltage harmonic amplitudes of the other phases are the same if they are at the same distance from \(l/2\). Alternatively, if the line \(l/2\) is between two phases, \(l\) is even and then phases equidistant from the line \(l/2\) have the same value of the phase voltage harmonic amplitudes.

Examining the harmonic order in Fig. 5, it can be seen that pattern of unbalance in the phase voltage harmonics is repeated after each 2\(l\) normalised harmonics. For example, if the machine has three sub-windings, the 1\(^{st}\), 7\(^{th}\), 13\(^{th}\), ... normalised harmonics of the phase voltages will have the same unbalance. If the given machine has three phases per sub-winding, for example a nine-phase machine, harmonics with the same unbalance pattern will be the 3\(^{rd}\), 21\(^{st}\), 39\(^{th}\), ..., effectively, the 3\(^{rd}\), (2\(n+3\))\(^{th}\), (4\(n+3\))\(^{th}\) ... harmonic. It should be noted that in the case when there are two sub-windings, i.e.

![Fig. 3. The 3\(^{rd}\) harmonic phase voltage amplitudes of the asymmetrical nine-phase machine with a single neutral point.](image3.png)

![Fig. 4. Influence of the neutral point voltage on the phase voltages in the case of the even (left) and odd (right) number of winding sets.](image4.png)

![Fig. 5. Per-unit amplitudes of the 1\(^{st}\), 3\(^{rd}\), 5\(^{th}\), 7\(^{th}\) and 9\(^{th}\) phase voltage normalised harmonics. The 1\(^{st}\) phases of all sub-windings, for the machines with 2, 3, 4, and 5 winding sets, are shown.](image5.png)

V. SIMULATION AND EXPERIMENTAL RESULTS

To verify the analysis, simulations have been carried out for a 12-phase asymmetrical induction machine with a single isolated neutral point. The machine is considered ideal and balanced and is supplied by 12-phase two-level inverter in 180° conduction mode. As a consequence, the supply generates all odd-order harmonics. Dc link voltage is set to 600V and the fundamental frequency is 50Hz. Leg, phase and neutral point voltages, with the corresponding spectra, are shown in Fig. 6. Only the first phase is shown in time domain (first three plots), but all phases are included in the FFT analysis (bottom three plots). The first 21 harmonics are illustrated.

Even harmonics are omitted from the plot, since they are equal to zero, along with the dc component. It can be seen that
and 21st harmonics. The amplitude of the fundamental is set to 60V, while all harmonics have the same amplitude of 20V. Fundamental frequency is 20Hz; hence harmonics are at 60Hz, 180Hz, 300Hz, and 420Hz, respectively. Switching frequency is 5kHz, which is in this case high enough so that the PWM switching process does not influence the harmonics of interest. Waveform and the spectrum of one of the references used to supply the R-L load are shown in Fig. 8. It should be noted that the reference settings are such as to ensure that the CB PWM stays in the linear modulation region.

Phase currents of the first and the second phase are shown in Fig. 9. It can be clearly seen that these two currents are different. Furthermore, in order to obtain more detailed insight into phase currents unbalance, spectra of all leg, phase, and neutral point voltages and phase currents for the 3rd, 9th, 15th, and 21st harmonics are shown in Fig. 10. Each vertical bar represents one phase. Dashed lines in the second row of Fig. 10 represent calculated harmonic amplitudes based on the measured amplitudes of the leg voltage harmonics as follows:

- blue dashed line – 9th harmonic of the 2nd, 5th, and 8th phases.
- black dashed line – 9th harmonic of 1st, 3rd, 4th, 6th, 7th, and 9th phases.
- red dashed line – 3rd, 15th, and 21st harmonics of 2nd, 5th, and 8th phases.
- green dashed line – 3rd, 15th, and 21st harmonics of 1st, 3rd, 4th, 6th, 7th, and 9th phases.

Similarly, red and blue dashed lines in the bottom plot of Fig. 10 represent calculated harmonic amplitudes based on the neutral point voltage. Red line represents 3rd, 15th, and 21st harmonics, while blue line represents 9th harmonic amplitude. It can be seen that unbalance in the phase voltage harmonics exists and measured harmonic amplitudes are in a good agreement with the predicted values. Small differences between the predicted and the measured values are predominantly due to the tolerances of the R-L load values.

Experimental verification has been performed using a nine-phase R-L load supplied from a bespoke nine-phase two-level voltage source inverter. The dc supply for the inverter is provided by a Sorensen SGI 600/25 dc voltage source. The inverter has hardware implemented dead time of 5µs. A simple carrier-based (CB) PWM is used as a modulation strategy. Inverter control is performed by dSPACE rapid prototyping platform. Measurements have been taken with Tektronix MSO2014 scope. Leg, phase and neutral point voltages are measured using a Tektronix P5205A active voltage differential probe, while phase currents have been measured using Tektronix TCP0030 active current probe. Resistance and inductance values of the used R-L load are 43Ω and 250mH, respectively. The load is connected to form a single neutral point. Dc link voltage has been set to 300V.

To verify the proposed theory, references provided to the modulator consist of a component at fundamental frequency and the 3rd, 9th, 15th, and 21st harmonics. The amplitude of the fundamental is set to 60V, while all harmonics have the same amplitude of 20V. Fundamental frequency is 20Hz; hence harmonics are at 60Hz, 180Hz, 300Hz, and 420Hz, respectively. Switching frequency is 5kHz, which is in this case high enough so that the PWM switching process does not influence the harmonics of interest. Waveform and the spectrum of one of the references used to supply the R-L load are shown in Fig. 8. It should be noted that the reference settings are such as to ensure that the CB PWM stays in the linear modulation region.

Phase currents of the first and the second phase are shown in Fig. 9. It can be clearly seen that these two currents are different. Furthermore, in order to obtain more detailed insight into phase currents unbalance, spectra of all leg, phase, and neutral point voltages and phase currents for the 3rd, 9th, 15th, and 21st harmonics are shown in Fig. 10. Each vertical bar represents one phase. Dashed lines in the second row of Fig. 10 represent calculated harmonic amplitudes based on the measured amplitudes of the leg voltage harmonics as follows:

- blue dashed line – 9th harmonic of the 2nd, 5th, and 8th phases.
- black dashed line – 9th harmonic of 1st, 3rd, 4th, 6th, 7th, and 9th phases.
- red dashed line – 3rd, 15th, and 21st harmonics of 2nd, 5th, and 8th phases.
- green dashed line – 3rd, 15th, and 21st harmonics of 1st, 3rd, 4th, 6th, 7th, and 9th phases.

Similarly, red and blue dashed lines in the bottom plot of Fig. 10 represent calculated harmonic amplitudes based on the neutral point voltage. Red line represents 3rd, 15th, and 21st harmonics, while blue line represents 9th harmonic amplitude. It can be seen that unbalance in the phase voltage harmonics exists and measured harmonic amplitudes are in a good agreement with the predicted values. Small differences between the predicted and the measured values are predominantly due to the tolerances of the R-L load values.
results are found to be in very good agreement with the proposed theory.

REFERENCES


