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Expectancy of Success, Attainment Value, Engagement, and Achievement:

A Moderated Mediation Analysis

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Abstract

The aim of this study was to examine how expectancy of success, attainment value, and their interaction predict behavioural engagement, and how behavioural engagement, in turn, predict achievement. Data were collected from 586 English students aged 10-11 years in their final year of primary school. Expectancy of success was positively related to subsequent achievement directly and indirectly, mediated by behavioural engagement over and above the variance accounted for by prior achievement and behavioural engagement. Indirect relations from expectancy of success to achievement were moderated by attainment value. Higher attainment value protected performance from low expectancy of success by increasing behavioural engagement. The compensatory role of high attainment value diminished at higher levels of expectancy of success.

Keywords: Expectancy of success, attainment value, behavioural engagement, academic achievement, expectancy-value theory
1.0 Introduction

The key proposition of expectancy-value theories (EVTs; Eccles, 2005; Pekrun, 1993; Wigfield & Eccles, 2000; Wigfield, Tonks, & Klauda, 2016) is that expectancy of success (henceforth referred to as expectancy) and subjective value interact to influence achievement-related behaviour, choice, and performance. Early studies examined expectancy and value additively rather than interactively. It is only recently that sophisticated latent variable modelling techniques have become available to examine expectancy × value interaction using naturalistic data (e.g., Nagengast et al., 2011). Furthermore, while EVT's propose that expectancy and value predict achievement, it is likely that achievement-related behaviours mediate the relations between expectancy, value, and achievement (e.g., Cole, Bergin, & Whittaker, 2008; Liem, Lau, & Nie, 2008). In the present study we use a moderated mediational model to examine this theoretical proposition in a sample of final year primary school students. This model proposes that the expectancy-value interaction is indirectly related to subsequent achievement, mediated by behavioural engagement.

1.1 Expectancy-Value Theories

Expectancy-value theories (EVTs; e.g., Ajzen & Fishbein, 1980; Eccles, 2005; Pekrun, 1988, 1993; Vroom, 1964; Wigfield & Eccles, 2000; Wigfield et al., 2016) provide a motivational account of the factors that influence achievement-related choices, engagement, and performance. The two central components of EVT's are expectancy of success and subjective value. Expectancy is defined as the belief held by a person about the probability of success in a forthcoming task or activity. Subjective value refers to the significance or meaning ascribed to the task. These can be the inherent interest resulting from a given activity (intrinsic value), the importance of achievement for one’s sense of self of identity (attainment value), and the instrumental usefulness of a task for one’s current life or for reaching future goals (utility value). Expectancies and values are formed from social
influences that include the expectations of key socialisers (such as parents and teachers), socialisers’ beliefs and behaviours, a person’s goals, beliefs, and self-concepts of ability, and memory of previous achievement-related events. Expectancies and values are thought to influence behaviour (e.g., effort and engagement), future choices (e.g., whether to continue to study a particular subject or not), and achievement.

Studies collecting naturalistic data using samples of secondary school children have shown that expectancy, often measured using competence beliefs (e.g., academic self-concept) as a proxy, and task value are positively related to achievement (e.g., Heyder, Kessels, & Steinmayr, 2017; Kosovich, Hullemen, Barron, & and Getty, 2014; Meece et al., 1990), effort (e.g., Chouinard, Karsenti, & Roy, 2007; Federici & Skaalvick, 2014; Pekrun, 1993), engagement (e.g., Fan, 2011; Fan & Williams, 2010; Wang & Eccles, 2013), lower school dropout (Fan & Wolters, 2014), and the intention to continue studying at school or beyond (e.g., Fan & Wolters, 2014; Taskinen, Schütte, & Prenzel, 2013; Xiang, McBride, Guan, & Solmon, 2003). In the few studies to collect data longitudinally, the positive relations of expectancy and value with achievement have been shown to remain when controlling for prior achievement (Pekrun, 1993; Pinxten, Marsh, De Fraîne, Noortgate, & Dame, 2014; Steinmayr & Spinath, 2009). A limitation of these and other studies, however, is that they do not consider the key proposition of EVTs; that expectancy and value interact.

1.2 Expectancy × Value Interactions

Foundational EVTs (e.g., Atkinson, 1957; Feather, 1959) emphasised that expectancy and value exert interactive influences on achievement-related choices, behaviours, and performance. That is, both expectancy and value are thought to be required to motivate achievement-related behaviour. The absence of either expectancy or value cannot be offset by correspondingly larger levels in the other (as would be the case if expectancy and value exerted additive influences). However, as highlighted by Nagengast et al. (2011), the
theorised expectancy × value interaction has rarely been investigated using naturalistic data due to limitations of available analytic tools. More recent studies have addressed this issue by applying latent interaction analysis to control for measurement error, which can be amplified for interaction terms in traditional non-latent regression-based approaches. In the present study, we also utilise latent variable modeling approaches to study how expectancy and value interact to predict engagement.

In samples of secondary school students, academic self-concept (as a proxy for expectancy) has been shown to interact with value to predict engagement with career aspirations (Guo, Parker, Marsh, & Morin, 2015; Guo, Marsh, Parker, Morin, & Yeung, 2015; Nagengast et al. 2011), academic achievement (Guo et al., 2016; Guo, Marsh, et al., 2015; Guo, Parker et al., 2015; Trautwein et al. 2012), and participation in extracurricular activities (Nagengast et al. 2011). With one exception (Guo, Marsh et al., 2015), the pattern of interactions was as predicted by EVT; relations between academic self-concept and educational outcomes (achievement, aspirations, and extracurricular activities) were amplified by high value. In Guo, Marsh et al. (2015), the relations between academic self-concept and achievement/educational aspirations were stronger for low utility value. High utility value protected achievement and educational aspirations at low academic self-concept. When academic self-concept was high, utility value did not play a protective role.

1.3 Indirect Relations between Expectancy, Value, and Achievement

In EVTs, expectancy and value are theorised to influence performance. Several studies have shown that the relations between expectancy-value and achievement in samples of secondary school students are indirect and mediated through variables such as engagement and learning style (Liem et al., 2008), achievement goals (Plante, O’Keefe, & Théorêt, 2013), task-specific goals (Greene, DeBacker, Ravindran, & Krows, 1999), and effort (Pekrun, 1993). In a study with undergraduate students, Cole et al. (2008) showed that higher value
was indirectly related to test scores, mediated through effort. In fact, it is theoretically plausible that expectancy and value could influence various cognitive, emotional, or behavioural mechanisms that affect achievement (e.g., Finn & Zimmer, 2014; Lauerman, Eccles, & Pekrun, 2016; Schunk & DiBenedetto, 2016). However, among different variables that could plausibly mediate relations between expectancy, value, and academic achievement, behavioural engagement likely functions as the most proximal antecedent of achievement. As such, we chose to focus on behavioural engagement in the present study.

Behavioural engagement was defined as effort, persistence, and exertion in classroom activities, accompanied by markers of mental effort, such as attention (Appleton, Christenson, Kim, & Reschly, 2006; Fredricks et al., 2011). The decision to include behavioural engagement was also influenced by the conceptual fit of behavioural engagement with EVT; achievement behaviour has always been considered a salient educational outcome of expectancy and value (Wang & Eccles, 2012). Furthermore, behavioural engagement is a useful omnibus construct that is able to incorporate a number of indicators (e.g., effort and persistence) representing a single underlying process. Importantly, higher behavioural engagement is associated with greater academic achievement (e.g., Dotterer, & Lowe, 2011; Hughes & Kwok, 2007; Patrick, Ryan, & Kaplan, 2007; Reyes, Brackett, Rivers, White, & Salovey, 2012; Wang, & Holcombe, 2010).

1.4 Aim and Hypotheses

Recent studies (Geo et al., 2016; Guo, Marsh, et al., 2015; Guo, Parker, et al., 2015; Nagengast et al., 2011; Trautwein et al., 2012) have made substantial advances in the empirical examination of EVT by examining how expectancy and value interact to predict educational behaviours, choices, and achievement. Only two of these studies, however, employed longitudinal designs and controlled for autoregressive relations with prior achievement (Guo, Parker, et al., 2015; Trautwein et al., 2012). Furthermore, none of these
studies have examined the possibility that expectancy and value might be indirectly related to achievement, or examined expectancy and value interactions in younger students.

The present study aimed to examine how expectancy and value interact to indirectly predict achievement via behavioural engagement. We proposed a moderated mediational model (see Figure 1) whereby expectancy, value, and their interaction, are indirectly related to achievement through behavioural engagement. Data were collected from students in their final year of primary school (aged 10-11 years), a younger sample of students than has hitherto been studied. Self-report data for expectancy, value, and behavioural engagement, were collected over three waves during a single school year and subsequent achievement near the end of the school year from standardised National Curriculum Tests in mathematics. Prior achievement was taken from teacher-estimated grades at the end of the previous academic year. Thus, the study comprised five points of measurement.

As the substantive constructs we examine (expectancy, value, behavioural engagement, and achievement) are subject-specific, in common with many earlier EVT studies (e.g., Meece et al., 1990) the study focussed on a single subject, that of mathematics. Like several previous studies (e.g., Guo, Marsh, et al., 2015; Guo, Parker, et al., 2015; Nagengast et al. 2011), we based our measure of expectancy on academic self-concept; items were adapted to focus on the probability of success in mathematics lessons and tests. Based on Wise and DeMars’s (2005, p.2) conceptualisation of test-taking motivation where the goal is to “...accurately represent what one knows and can do in the content area covered by the test,” we reasoned that attainment value was the most salient of the three values to test performance. High intrinsic or utility value is no guarantee that a student’s goal will be to perform well on tests such as the National Curriculum Tests. Accordingly, we focused specifically on attainment value in the present study. For comparative purposes, however, we test models with intrinsic and utility value in accompanying Supplementary Materials.
Furthermore, we controlled for autoregressive relations with prior achievement and behavioural engagement to offer a robust test of the relations from expectancy and value to subsequent achievement and engagement. Gender was included as a covariate as mathematics has been traditionally considered a gendered subject and is especially relevant when investigating constructs so closely related to ability/academic self-concept (Barkatsas, Kasimatis, & Gialamas, 2009; Watt, 2006). Succinctly stated, our focal hypotheses were the following:

\[ H1: \] Expectancy and attainment value will positively interact to predict subsequent behavioural engagement.

\[ H2: \] Behavioural engagement will be positively related to subsequent attainment.

\[ H3: \] The expectancy and attainment value interaction will be indirectly and positively related to subsequent achievement mediated through behavioural engagement.

By virtue of our design we are also able to model relations from prior achievement to subsequent behavioural engagement and from behavioural engagement to subsequent expectancy and attainment value. Although we do not offer specific hypotheses, as these relations were not the focus of our study and primarily included as control variables, we anticipate all relations would be positive.

2.0 Method

2.1 Participants

The participants at the first self-reported assessment (T2) were 586 students (male \( n = 275 \), female \( n = 277 \), missing \( n = 34 \); mean age = 10.2 years, \( SD = .38 \)) from 22 English primary schools. All participants were in their final year of primary school (Year 6) and nested in 36 different classes (\( M = 16.3 \) students per class). The majority of schools had only one Year 6 class; 12 had two classes. Students were not differentiated by ability in any of the schools. The ethnic heritage of students was White \( n = 431 \), Asian \( n = 17 \), Black \( n = 26 \), other
n = 26, and mixed heritage n = 37 (missing n = 49). At T3 and T4, sample size was n = 444 (male n = 221, female n = 223) and n = 433 students (male n = 207, female n = 226), respectively. These socio-demographic characteristics are broadly representative of other English primary schools (51% male, 75% White ethnic heritage; Department for Education, 2016). Participant attrition was caused by students being absent from school due to illness, having moved school, or students (or their parents/carers) exercising their right not to participate, but was unrelated to T1 and T3 achievement, attainment value, T2 and T4 engagement, gender, age, and ethnicity. Missing data was handled using full-information maximum likelihood (FIML) estimation which is less likely to result in biased parameter estimates than listwise or pairwise deletion (Graham, 2012).

2.2 Measures

Participants responded to self-report items on a five-point scale (1 = strongly disagree, 2 = disagree, 3 = neither, 4 = agree, 5 = strongly agree). All items are reproduced in the Supplementary Materials.

2.2.1 Expectancy. Expectancy was measured using four academic self-concept items that were adapted from the Self-description Questionnaire II (SDQ II; Marsh, 1990) to reflect an expectation of good marks in mathematics lessons and tests (e.g. ‘I get good marks in maths lessons’). The SDQ has excellent psychometric properties, including internal consistency and construct validity, and has been used in numerous studies to examine relations with achievement (Marsh, 2006; Marsh & Martin, 2011). The internal consistency in the present study was good (see Table 1).

2.2.2 Values. Values were measured using four items each for utility, attainment, and intrinsic value, adapted from the Michigan Study of Adolescent Life Transitions scales (Eccles, O’Neill, & Wigfield, 2005). Items were adapted to refer to mathematics lessons and tests (e.g., ‘Getting good marks on maths tests is important to me’ for attainment value). The
instrument from which these items were drawn has been used in various expectancy-value studies and has shown good internal consistency and construct validity (e.g., Archambault, Eccles, & Vida, 2010; Eccles, Vida, & Barber, 2004). In the present study, internal consistency for attainment value was acceptable (see Table 1). Only the attainment value subscale was used for analyses presented in this manuscript (see Supplementary Materials for models using the intrinsic value and utility value subscales).

2.2.3 Behavioural engagement. Behavioural engagement was measured using the five-item behavioural engagement scale from the *Engagement vs. Dissatisfaction with Learning Questionnaire* (Skinner, Kindermann, & Furrer, 2009). Items were adapted to refer to mathematics (e.g., ‘I participate in the activities and tasks in my maths lessons’). The internal consistency and construct validity of this scale has been documented by Skinner and Chi (2012), Skinner, Furrer, Marchand, and Kinderman (2008), and Skinner et al. (2009). The internal consistency in the present study was good (see Table 1).

2.2.4 Mathematics achievement. Mathematics achievement at the end of Year 5 was measured using teacher-estimated grades, retrieved from official school documentation, based on work in lessons and class tests. Estimated grades were based on work in lessons and class tests using standardized, criterion-referenced levels of progress used in the English National Curriculum (Department of Education, 2014a) on a 15-point scale. Accountability systems used in the English education system heavily incentivise robust and exact assessment of student work (James, 2012; Perryman, 2006). Furthermore, primary school teacher assessment of National Curriculum Levels of Progress has been shown to correlate strongly (*r* = .77 – .92) with scores on standardized mathematics tests (Harlen, 2004). Mathematics achievement near to the end of Year 6 was measured using three 30-minute National Curriculum Tests, one arithmetic test and two reasoning tests resulting in a combined score of 0 – 120. These national tests are taken under strict administrative conditions, set and
marked by an external awarding body approved by the Department of Education and monitored by the Office of Qualifications and Examinations Regulation. A review of National Curriculum marking reliability and accuracy by He, Hayes, and Wiliam (2010) reported high levels of internal consistency (Cronbach’s $\alpha = .96 – .97$).

### 2.3 Procedure

Data were collected at five time points, with self-report assessments spaced at three month intervals. The initial measurement of mathematics achievement was taken from the teacher-estimated grades at the end of Year 5 ($T_1$). Self-reported engagement was measured during the Autumn term of Year 6 ($T_2$), expectancy and attainment value were measured in the Spring term of Year 6 ($T_3$), and engagement was measured again in the summer term of Year 6 ($T_4$). The second measurement of mathematics achievement was taken near the end of Year 6 ($T_5$; see 2.2.4). Self-report data were collected using digital personal assistants and administered by the regular classroom teacher following a standardised script. Students used a unique identifier to link their responses from different waves of measurement together with their mathematics achievement. The project was approved by a Faculty Research Ethics panel. Written permission to collect data was provided by the head teacher at each participating school, and passive (opt-out consent by parents/ carers of participants) and verbal assent was sought from participants at each wave of self-report assessment.

### 2.4 Analytic Strategy

Data were analysed in two steps. Following a preliminary analysis of descriptive statistics, we first examined a measurement model of the study constructs using confirmatory factor analysis (CFA). This model was used to estimate latent bivariate correlations and check for temporal measurement invariance of engagement at $T_2$ and $T_4$. Second, we examined the hypothesised moderated mediational model (Figure 1) using structural equation modelling (SEM). All analyses were performed using Mplus 8 (Muthén & Muthén, 2017). To
account for deviations to the normal distribution of variables (see Table 1), all models were estimated using maximum likelihood estimation with robust standard errors (MLR; see Hox, Maas, & Brinkhuis, 2010). The Mplus ‘complex’ and ‘cluster’ commands were used to adjust standard errors for the clustering of participants within classrooms.

Model fit was assessed using the following indices: Root mean square error of approximation (RMSEA), standardized root mean square residual (SRMR), comparative fit index (CFI), and Tucker-Lewis index (TLI). A good fitting model is indicated by RMSEA and SRMR indices of <.08 and <.06, respectively, and CFI and TLI indices of >.95 (Hu & Bentler, 1999). These values, however, may be overly stringent for naturalistic data (e.g., Heene, Hilbert, Draxler, Ziegler, & Bühner, 2011; Lance, Butts, & Michels, 2006) and should not be treated as strict cut-off scores.

Latent interactions between expectancy and attainment value were specified using the unconstrained approach (Marsh, Wen & Hau, 2004). The four indicators of expectancy and attainment value were mean centred and randomly paired to create four new multiplicative indicators for a latent interact term. To allow for model estimation, and following the recommendations by Marsh et al., (2004), the means of expectancy and attainment value were fixed to zero, and the means of the latent interaction variables were fixed to equal the covariance of the control and value variables. As compared with the latent moderated structural equation approach (Klein & Moosbrugger, 2000), the unconstrained approach has the advantage of estimating model fit indices and incorporating the cluster and complex commands. In simulation studies, the unconstrained approach performs as well as constrained (Jöreskog & Yang, 1996) and residual-centered approaches (Marsh et al., 2004; Steinmetz, Davidov & Schmidt, 2011).

3.0 Results

3.1 Descriptive Statistics
Descriptive statistics are shown in Table 1. Factor loadings taken from the measurement model described below were good (λ ≥ .54). The proportion of variance in scores attributable to the classroom level, established using the intraclass correlation coefficient (σI) was relatively small for T2 and T4 engagement, T3 expectancy, and T3 attainment value (σI ≤ .05) and somewhat larger for achievement (σI = .10 and .07 for T1, and T5, respectively).

3.2 Measurement Model

A measurement model was built specifying engagement (T2 and T4) and T3 expectancy and attainment value as latent constructs with four indicators each. Residuals for corresponding items of engagement at T2 and T4 were allowed to correlate. The achievement variables (T1 and T5) were specified as single-item latent variables. Based on empirical findings (e.g., Hoy, Tarter, & Hoy, 2006; Watkins, Lei, & Canivez, 2007), the T1 and T5 achievement factor loadings were modelled as λ = .9 (σe = .1) rather than assuming the single indicators to offer a perfect measurement of achievement (λ = 1). The measurement model showed an excellent fit to the data, χ²(91) = 162.80, p < .001, RMSEA = .033, SRMR = .044, CFI = .966, and TLI = .956. A series of further CFAs were examined to check for measurement invariance in engagement at T2 and T4 (see Table 2). Metric (factor loadings constrained to be equal), scalar (intercepts constrained to be equal), and residual invariance (residual variance constrained to be equal) models all showed ΔCFI/ ΔTLI < .01 and ΔRMSEA < .015, indicating strong invariance for engagement over time (Chen, 2007; Cheung & Rensvold, 2002).

3.3 Latent Bivariate Correlations

In order to estimate latent bivariate correlations, gender (0 = male, 1 = female) was added to the measurement model. This model also showed a good fit to the data: χ²(130) = 245.07, p < .001, RMSEA = .032, SRMR = .043, CFI = .959, and TLI = .944. The latent
bivariate correlations are reported in Table 3. T₃ expectancy and attainment value were positively related to T₄ engagement, and T₄ engagement positively related to T₅ achievement. Except for male students reporting lower T₂ engagement, there were no significant gender differences.

3.4 Structural Equation Modelling

The moderated mediational model depicted in Figure 1 was examined in a SEM with paths for gender (0 = male, 1 = female) linked to all other variables. This model showed an acceptable fit to the data: $\chi^2(203) = 493.91, p < .001$; RMSEA = .045; SRMR = .082; CFI = .933; and TLI = .917. The Homoscedastic Fit Index (HFI) was used to establish whether omitted interaction terms or non-normal distributions of variables may have contributed to a reduction in model fit (Gerhard, Büchner, Klein, & Schermelleh-Engel, 2017). A HFI value of .996 (values ≥ .95 are indicative of a good fit) indicated that model fit was not adversely influenced by omitted interaction terms or non-normal distributions of variables. Given the complexity of the model and the caveat regarding the applicability of cut-off values for fit indices with naturalistic data, we considered this model fit to be acceptable and proceeded to examine beta coefficients (see Table 4 and Figure 2).

3.4.1 Relations from T₃ expectancy, T₃ value, and their interaction, to T₄ engagement. T₄ engagement was predicted by T₃ expectancy ($\beta = .22, p = .001$) and the interaction between T₃ expectancy and T₃ attainment value ($\beta = -.28, p = .008$) over and above the variance accounted for by T₂ engagement ($\beta = .46, p < .001$) and T₁ achievement ($\beta = .21, p = .04$). T₃ value was not a significant predictor of T₄ engagement ($\beta = .03, p = .80$). Simple slopes were estimated to probe the interaction between expectancy and value. As EVT's propose symmetrical interactions, either expectancy or attainment value could be positioned as the moderator; accordingly, both possibilities were examined. Due to the negatively skewed, leptokurtic, distributions for expectancy and value (see Table 1), simple
slopes were estimated at ±.5SD as providing more substantively meaningful values that would extrapolate to other samples and contexts than ±1SD (see Dawson, 2013; Hayes, 2013). For comparative purposes, simple slopes at ±1SD are provided in the Supplementary Materials.

When attainment value was positioned as the moderator, a positive relation was shown between expectancy and engagement for mean value ($B = .14, p = .002$). At low value, this relation became stronger ($B = .21, p < .001$) and at high value, this relation became weaker and non-significant ($B = .07, p = .10$). When expectancy was positioned as the moderator, the relation between value and engagement was non-significant at low ($B = -.08, p = .65$), mean ($B = .04, p = .80$) and high value ($B = .15, p = .26$). Simple slopes with attainment value as the mediator are plotted in Figure 3.

### 3.4.2 Relations from T₄ engagement to T₅ achievement

T₅ achievement was predicted by T₄ engagement ($β = .59, p < .001$) over and above the variance accounted for by T₁ achievement ($β = .43, p < .001$) and T₂ engagement ($β = .36, p < .001$).

### 3.4.3 Relations from T₃ expectancy, T₃ value, and their interaction to T₅ achievement

T₅ achievement was directly predicted by T₃ expectancy ($β = .20, p = .018$), but not T₃ value ($β = -.01, p = .983$), or the interaction between T₃ expectancy and T₃ attainment value ($β = .12, p = .063$). However, of primary interest are the indirect effects of expectancy, value, and their interaction on achievement mediated by engagement. T₃ expectancy ($β = .13, SE = .06, 95\%\text{CIs} [.02, .23]$), and the interaction between T₃ expectancy and T₃ attainment value ($β = -.16, SE = .07, 95\% \text{CIs} [-.05, -.28]$), but not T₃ attainment value ($β = .02, SE = .07, 95\% \text{CIs} [-.09, .12]$), were indirectly related to T₅ achievement mediated by T₄ engagement. Conditional indirect simple slopes could be estimated from T₃ value to T₅ achievement at different levels of T₃ expectancy, or from T₃ expectancy to T₅ achievement at different levels of T₃ value. Since the simple slopes from T₃ value to T₄ engagement were
non-significant at all levels of T₃ expectancy (see section 3.4.1), we proceeded to only estimate conditional indirect simple slopes from T₃ expectancy to T₅ achievement at different levels of T₃ value. Indirect simple slopes for high and low attainment value were estimated at ±.5SD (see Table 5). At low attainment value, the indirect relationship between T₃ expectancy and T₅ achievement was substantial and significant. At high attainment value, this relationship was weaker and no longer statistically significant. These indirect relationships are plotted in Figure 4. For comparative purposes, simple slopes at ±1SD are provided in the Supplementary Materials.

3.4.4 Relations from T₁ achievement to T₂ engagement, and from T₂ engagement to T₃ expectancy and value. T₁ achievement predicted T₂ engagement (β = .16, p = .009). T₂ engagement predicted T₃ expectancy (β = .33, p < .001) and T₃ attainment value (β = .28, p < .001).

3.4.5 Relations with gender. Male students reported greater T₂ engagement (β = -.13, p = .02). All other relations with gender were not statistically significant (ps > .05).

4.0 Discussion

The aim of this study was to examine how expectancy, attainment value, and their interaction predicted subsequent engagement (controlling for the variance accounted for by prior achievement and engagement), and how engagement predicted subsequent achievement (controlling for the variance accounted for by prior achievement and engagement). In this model, engagement is proposed to mediate the relations between expectancy, attainment value, and their interaction, with subsequent achievement. To examine the interaction between expectancy and attainment value on achievement, indirect relations between expectancy and achievement, mediated by engagement, were examined at different levels of attainment value. Self-report data for expectancy, attainment value, and engagement, were collected from Year 6 primary school children over the course of one academic year. End-of-
year achievement was assessed using scores from National Curriculum Test scores in Mathematics and prior achievement assessed using teacher-estimated achievement from the end of Year 5.

The finding that expectancy, value, and their interaction predict engagement is consistent with previous studies examining EVT in samples of secondary school students (e.g., Fan & Williams, 2010; Wang & Eccles, 2013) and builds on work using latent variable modelling approaches to examine how expectancy value interactions predict salient educational outcomes (e.g., Guo et al., 2016; Trautwein et al. 2012). Rather than the expected positive interaction, however, expectancy was a stronger predictor of engagement at low expectancy. Classic EVT proposes that both expectancy and value are required to motivate behaviour (e.g., Atkinson, 1957; Feather, 1959). However, our results, mirroring those of Guo, Marsh, et al. (2015) found that high value could partially compensate for low expectancy. This suggests that possibility that expectancy and value could be both interactive and additive rather than being solely interactive. In our study expectancy remained a unique predictor of subsequent engagement as did with Guo, Marsh, et al., 2015, who showed for achievement and aspiration that high intrinsic value could partially compensate for low expectancy. Accordingly, we conclude that our results provide partial support of H1. It is also notable that the attainment value and expectancy interaction was not truly symmetrical; the relationship between expectancy and engagement was conditional on attainment value but not vice versa. This is a likely artefact of the greater shared variance between the interaction term and attainment value ($r = -.69$; see Figure 2) compared to that of expectancy ($r = -.32$; see Figure 2) reducing the power of attainment value as a predictor, but not as a moderator (see Landis & Dunlap, 2000).

The finding that behavioural engagement predicted subsequent academic achievement supports previous studies documenting this association in samples of elementary and
secondary school students (e.g., Dotterer, & Lowe, 2011; Hughes & Kwok, 2007; Patrick et al. 2007; Reyes et al., 2012; Wang, & Holcombe, 2010). The finding that expectancy, value, and their interaction indirectly predict achievement, mediated by behavioural engagement, supports earlier work using samples of secondary school and undergraduate students (Cole et al., 2008; Liem et al., 2008). These findings support $H2$.

The finding that the indirect relation between expectancy and subsequent engagement was moderated by attainment value makes a novel contribution to the literature. The conditional indirect slope was stronger at low levels of attainment value; that is, when expectancy was low, the achievement of students with low attainment value was not as good as for those with high attainment value. At higher levels of expectancy, the differential in achievement between high and low attainment value decreased. This pattern of results follows the expectancy-value interaction on engagement showing the how high value can partially compensate for low expectancy (also see Guo, Marsh, et al., 2015). Other studies of expectancy value interactions (Guo et al., 2016; Guo, Parker et al., 2015; Nagengast et al., 2011; Trautwein et al. 2012) have shown a more traditional pattern of expectancy-value interaction where achievement and achievement-related behaviours were improved when both expectancy and value were high. Accordingly, we conclude that our results provide partial support of $H3$.

4.1 Limitations and Directions for Future Research

Despite the novel contribution to the literature, using a robust design and a sophisticated analytic approach, there are two limitations that should be highlighted. First, although the longitudinal design we used allowed for the control of prior achievement and behavioural engagement, it did not allow for the control of concurrent relations between expectancy, value, and behavioural engagement. Designs that measure all variables at all waves of data collection are preferable. Unfortunately, limitations on data collection imposed
by participating schools meant that we were unable to do this. Future studies should strive where possible, to control for concurrent and prior relations between substantive variables. Second, our study focused on a single type of value, namely attainment, as being the most germane to the end-of-year achievement measured through a standardised test. It is possible that other values and cost may also have interacted with expectancy to influence behavioural engagement and achievement. Future studies may wish to consider including these.

4.2 Educational Implications

Educational practitioners would be advised that there are potential gains to be achieved for students who are low in their attainment value or expectancy. Specifically, high attainment value can buffer performance against low expectations of success and high expectations of success can buffer performance against low attainment value. Hence efforts directed towards raising attainment value may prove productive in students whose expectancy is low and resistant to change, especially when used to foster on-task engagement. Similarly, efforts to foster expectancy of success could prove useful for students whose attainment value is low and resistant to change. This could be achieved through combining task-value activities (e.g., those with an explicit link to achievement outcomes – such as standardised test performance) and task-value messages (e.g., teacher reinforcing the link between lesson activities and personal achievement outcomes; see Acee, Weinstein, Hoang, & Flaggs, 2018). Task-value activities and messages could also be directed towards helping students to imagine situations where achievement contributes to one’s esteem and self-worth (see Oyserman & James, 2009). However, this could be a risky strategy that ultimately could backfire if students employ avoidance strategies as a means of self-worth protection (see Martin, Marsh, & Debus, 2001). Such approaches should be used with caution.

4.4 Conclusion
In the present study we have demonstrated in a sample of 10-11 year old primary school children that expectancy, value, and their interaction, predict subsequent behavioural engagement, and achievement, beyond the variance accounted for by prior behavioural engagement and achievement. Expectancy was directly, and indirectly, related to subsequent achievement, mediated by behavioural engagement. Indirect relations were stronger when attainment value was low. Our results suggest expectancy and value can operate additively as well as interactively and that high attainment value has a protective role against low expectancy for student achievement, via greater engagement in lesson tasks and activities.
Endnotes

1 In UK parlance, mathematics is colloquially referred to as ‘maths’

2 From 2014 onwards the requirement for schools to track progress using National Curriculum Level descriptors was relaxed by the Department for Education (Department for Education, 2014b). Schools could exercise autonomy in choosing monitoring and assessment systems to track progress. Many schools, including those in our sample, initially chose to remain with National Curriculum Level descriptors due to familiarity.

3 Papers can be found at this URL: www.gov.uk/government/publications/key-stage-2-tests-2016-mathematics-test-materials
References


Table 1
*Descriptive Statistics for T₂ and T₄ Engagement, T₃ Expectancy, T₃ Attainment value, and T₃ and T₅ Achievement.*

<table>
<thead>
<tr>
<th></th>
<th>Range</th>
<th>Mean</th>
<th>SD</th>
<th>α</th>
<th>σ₁</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Factor Loadings</th>
</tr>
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<tbody>
<tr>
<td>T₂ Engagement</td>
<td>1–5</td>
<td>4.58</td>
<td>0.53</td>
<td>.76</td>
<td>.05</td>
<td>-2.08</td>
<td>7.12</td>
<td>.64 – .78</td>
</tr>
<tr>
<td>T₄ Engagement</td>
<td>1–5</td>
<td>4.51</td>
<td>0.57</td>
<td>.79</td>
<td>.02</td>
<td>-1.41</td>
<td>2.19</td>
<td>.60 – .83</td>
</tr>
<tr>
<td>T₃ Expectancy</td>
<td>1–5</td>
<td>4.08</td>
<td>0.78</td>
<td>.85</td>
<td>.04</td>
<td>-1.25</td>
<td>1.90</td>
<td>.54 – .86</td>
</tr>
<tr>
<td>T₃ Attainment Value</td>
<td>1–5</td>
<td>4.65</td>
<td>0.50</td>
<td>.73</td>
<td>.03</td>
<td>-2.18</td>
<td>7.29</td>
<td>.75 – .83</td>
</tr>
<tr>
<td>T₁ Achievement</td>
<td>1–15</td>
<td>6.92</td>
<td>2.36</td>
<td>—</td>
<td>.10</td>
<td>0.24</td>
<td>-0.30</td>
<td>—</td>
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<tr>
<td>T₅ Achievement</td>
<td>0–120</td>
<td>102.69</td>
<td>5.58</td>
<td>—</td>
<td>.07</td>
<td>-0.33</td>
<td>0.55</td>
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</tbody>
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*Note.* The intraclass correlation coefficient (σ₁) represents the proportion of variance attributable to the classroom level.
Table 2
*Tests of Measurement Invariance for Engagement at T\textsubscript{2} and T\textsubscript{4}.*

<table>
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<tr>
<th></th>
<th>$\chi^2$</th>
<th>RMSEA</th>
<th>SRMR</th>
<th>CFI</th>
<th>TLI</th>
<th>$\Delta$ RMSEA</th>
<th>$\Delta$ CFI</th>
<th>$\Delta$ TLI</th>
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<td>Configural</td>
<td>162.80(91)</td>
<td>.033</td>
<td>.044</td>
<td>.966</td>
<td>.956</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Metric Invariance</td>
<td>174.30(95)</td>
<td>.035</td>
<td>.047</td>
<td>.962</td>
<td>.952</td>
<td>+.002</td>
<td>-.004</td>
<td>-.004</td>
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<tr>
<td>Scalar Invariance</td>
<td>184.63(99)</td>
<td>.035</td>
<td>.060</td>
<td>.959</td>
<td>.950</td>
<td>&lt;.001</td>
<td>-.003</td>
<td>-.002</td>
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<tr>
<td>Residual Invariance</td>
<td>174.28(103)</td>
<td>.032</td>
<td>.072</td>
<td>.966</td>
<td>.960</td>
<td>-.002</td>
<td>+.007</td>
<td>+.010</td>
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</tbody>
</table>

*Note.* All models statistically significant at $p < .001$. 
Table 3

Latent Bivariate Correlations for $T_2$ and $T_4$ Engagement, $T_3$ Expectancy, $T_3$ Attainment Value, $T_3$ and $T_5$ Achievement, and Gender.

<table>
<thead>
<tr>
<th></th>
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<th>5.</th>
<th>6.</th>
<th>7.</th>
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<td>.45***</td>
<td>.60***</td>
<td>.16*</td>
<td>.13*</td>
<td>-.14**</td>
</tr>
<tr>
<td>2. $T_4$ Engagement</td>
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<td>.50***</td>
<td>.07</td>
<td>.40***</td>
<td>-.08</td>
</tr>
<tr>
<td>3. $T_3$ Expectancy</td>
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<td>.58***</td>
<td></td>
<td>.35***</td>
<td>.37***</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>4. $T_3$ Attainment Value</td>
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<td></td>
<td></td>
<td>.15*</td>
<td>.24***</td>
<td>-.04</td>
<td></td>
</tr>
<tr>
<td>5. $T_1$ Achievement</td>
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<td></td>
<td></td>
<td>.35***</td>
<td>-.04</td>
<td></td>
</tr>
<tr>
<td>6. $T_5$ Achievement</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>-.02</td>
<td></td>
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<tr>
<td>7. Gender</td>
<td>—</td>
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</tr>
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*p < .05. **p < .01. ***p < .001.
Table 4
*Standardised and Unstandardised Path Coefficients from the Moderated Mediation Model.*

<table>
<thead>
<tr>
<th></th>
<th>T₁ Achievement</th>
<th>T₂ Engagement</th>
<th>T₃ Expectancy</th>
<th>T₃ Achievement Value</th>
<th>T₄ Engagement</th>
<th>T₅ Achievement</th>
</tr>
</thead>
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<tr>
<td><strong>Standardised Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁ Achievement</td>
<td>0.16**</td>
<td></td>
<td></td>
<td></td>
<td>0.21*</td>
<td>0.43***</td>
</tr>
<tr>
<td>T₂ Engagement</td>
<td></td>
<td>0.33***</td>
<td></td>
<td></td>
<td>0.46***</td>
<td>0.36***</td>
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<tr>
<td>T₃ Expectancy (EX)</td>
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<td></td>
<td></td>
<td>0.22**</td>
<td>0.20*</td>
</tr>
<tr>
<td>T₃ Achievement Value (AV)</td>
<td>0.03</td>
<td>0.12</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₃ EX × AV</td>
<td>-0.28**</td>
<td></td>
<td></td>
<td></td>
<td>0.59***</td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-0.06</td>
<td>-0.13*</td>
<td>0.06</td>
<td>&lt;0.01</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td><strong>Unstandardised Coefficients</strong></td>
<td>0.03*</td>
<td>0.57***</td>
<td>0.20***</td>
<td>0.42***</td>
<td>0.14**</td>
<td>1.85*</td>
</tr>
<tr>
<td>T₁ Achievement</td>
<td>0.03*</td>
<td>0.57***</td>
<td>0.20***</td>
<td>0.42***</td>
<td>0.14**</td>
<td>1.85*</td>
</tr>
<tr>
<td>T₂ Engagement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.14**</td>
<td>1.85*</td>
</tr>
<tr>
<td>T₃ Expectancy (EX)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.14**</td>
<td>1.85*</td>
</tr>
<tr>
<td>T₃ Achievement Value (AV)</td>
<td>0.04</td>
<td>-0.03</td>
<td>-0.27**</td>
<td>1.74</td>
<td>0.09</td>
<td>0.09</td>
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<tr>
<td>T₃ EX × AV</td>
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<td></td>
<td></td>
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<td>8.64***</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.29</td>
<td>-0.12*</td>
<td>0.09</td>
<td>&lt;0.01</td>
<td>-0.02</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

*p < .05. **p < .01. ***p < .001.
Table 5
*Indirect and Total Effects of the Relations between $T_3$ Expectancy and $T_5$ Achievement at Differing Levels of $T_3$ Attainment Value, Mediated by $T_4$ Engagement.*

<table>
<thead>
<tr>
<th></th>
<th>Low Attainment Value (-.5SD)</th>
<th>Mean Attainment Value</th>
<th>High Attainment Value (+.5SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>$SE$</td>
<td>95%CIs</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>1.77</td>
<td>0.70</td>
<td>0.61, 2.93</td>
</tr>
<tr>
<td>Total Effect</td>
<td>3.62</td>
<td>0.82</td>
<td>2.26, 4.99</td>
</tr>
</tbody>
</table>

*Note.* Indirect and total effects represented by unstandardised coefficients.
Figure 1. The hypothesised model to examine whether $T_3$ expectancy, $T_3$ attainment value, and their interaction, are indirectly related to $T_5$ achievement through $T_4$ engagement, controlling for prior ($T_1$) achievement, and ($T_2$) engagement. Solid lines represent structural paths and dashed lines represent correlations. Although included in the model, paths to gender were omitted from for simplicity.
Figure 2. SEM to examine the moderated mediational model. Solid lines represent standardised coefficients for structural paths and dashed lines represent correlations (*p < .05, **p < .01, ***p < .001). Gender was related to T2 engagement but for simplicity was omitted from Figure 1.
Figure 3. The model implied simple slopes of interaction between T₃ expectancy and T₃ attainment value on T₃ engagement.
Figure 4. The model implied indirect simple slopes of the relations between $T_3$ expectancy and $T_3$ achievement at differing levels of $T_3$ attainment value.
Control-Value Appraisals, Enjoyment, and Boredom in Mathematics: A Latent Interaction Analysis

- Supplementary Materials -

This document contains materials designed to supplement the main text. The materials include the following:

Simple Slopes for the $T_3$ Expectancy and Attainment Value Interaction on $T_4$ Engagement at ±1SD

Conditional Indirect Simple Slopes for the $T_3$ Expectancy and Attainment Value Interaction on $T_5$ Achievement, mediated by $T_4$ Engagement, at ±1SD

The Moderated Mediational Model with Intrinsic Value as the Moderator

The Moderated Mediational Model with Utility Value as the Moderator

References

Tables S1-S7

Appendix containing all self-report items used in the study
Simple Slopes for the T₃ Expectancy and Attainment Value Interaction on T⁴ Engagement at ±1SD

In the main file we presented simple slopes for the T₃ expectancy value interaction on T₄ engagement at ±.5SD due to the negatively skewed, leptokurtic, distributions for expectancy and value. For comparative purposes we provide simple slopes estimated at ±1SD (values that are typically used in educational psychology research). At mean attainment value the relation between T₃ expectancy and T₄ engagement was positive and statistically significant ($B = .14, p = .002$). At low value, this relation became stronger ($B = .27, p < .001$) and at high value, this relation became weaker and non-significant ($B = .01, p = .92$). Simple slopes are plotted in Figure S1.

Conditional Indirect Simple Slopes for the T₃ Expectancy and Attainment Value Interaction on T₅ Achievement, mediated by T₄ Engagement, at ±1SD

In the main file, conditional indirect relations from T₃ expectancy to T₅ achievement, mediated by T₄ engagement, were estimated at ±.5SD values of attainment value. For comparative purposes we provide conditional slopes estimated at ±1SD (see Table S1). At low attainment value, the indirect relationship between T₃ expectancy and T₅ achievement was substantial and significant. At high attainment value, this relationship was weaker and no longer statistically significant. Conditional indirect slopes are plotted in Figure S2.

The Moderated Mediation Model with Intrinsic Value as the Moderator

In the main file the moderated meditational model used attainment value as the moderator in the expectancy-value interaction. We reasoned it was the most appropriate form of value with which to study relations with subsequent scores on a standardised test. For comparative purposes we present analyses with intrinsic value (see Table S2 for descriptive
statistics). All confirmatory factor analyses, and structural equation models, for intrinsic value (and utility value described below) used maximum likelihood estimation and ‘type = complex’. A measurement model with four intrinsic value indicators replacing the four attainment value indicators showed a good fit to the data: $\chi^2(130) = 236.31, p < .001$, RMSEA = .032, SRMR = .042, CFI = .962, and TLI = .950. Latent bivariate correlations are reported in Table S3. The moderated meditational model with the four intrinsic value indicators showed a good fit to the data: $\chi^2(203) = 371.63, p < .001$; RMSEA = .035; SRMR = .060; CFI = .940; and TLI = .926. Model coefficients are reported in Table S4. Critically, the interaction between $T_3$ expectancy and $T_3$ intrinsic value was not a statistically significant predictor of $T_4$ engagement ($\beta = -.19, p = .11$) or $T_5$ achievement ($\beta = -.07, p = .16$). Unlike attainment value, presented in the main manuscript, $T_3$ intrinsic value did not interact with $T_3$ expectancy to predict $T_4$ engagement.

The Moderated Mediation Model with Utility Value as the Moderator

For comparative purposes we also present the analyses with utility value as the moderator (see Table S2 for descriptive statistics). A measurement model with four utility value indicators replacing four attainment value indicators showed a good fit to the data: $\chi^2(130) = 206.02, p < .001$; RMSEA = .029; SRMR = .045; CFI = .961; and TLI = .949. Latent bivariate correlations are reported in Table S5. The moderated meditational model with the four utility value indicators showed a fit to the data that was borderline acceptable using traditional cut-off values (e.g., Hu & Bentler, 1999): $\chi^2(203) = 431.56, p < .001$; RMSEA = .041; SRMR = .067; CFI = .912; and TLI = .889. Model coefficients are reported in Table S6. Critically, a statistically significant interaction was reported for the $T_3$ expectancy and $T_3$ utility value on $T_4$ engagement ($\beta = -.36, p < .001$).
Simple slopes were estimated at ±.5SD and ±1SD. At mean utility value the relation between T₃ expectancy and T₄ engagement was positive but not statistically significant \((B = .09, p = .07)\). At high value, this relation became weaker and remained non-significant \((- .5 \text{ SD}: B = .03, p = .58; -1 \text{ SD}: B = -.03, p = .62)\). At low value, this relation became stronger and statistically significant \((- .5 \text{ SD}: B = .15, p = .01; -1 \text{ SD}: B = .21, p = .008)\). Simple slopes are plotted in Figure S3.

Conditional indirect slopes to estimate the relations from T₃ expectancy to T₅ achievement, mediated by T₄ engagement, were estimated at ±.5SD and ±1SD utility value (see Table S7). At low utility value, the indirect relationship between T₃ expectancy and T₅ achievement was substantial and significant. At mean and high utility value, this relationship was weaker and no longer statistically significant. Conditional indirect slopes are plotted in Figure S4.

Like attainment value (presented in the main manuscript), utility value interacted with T₃ expectancy to predict engagement and achievement. The patterns of results for utility value and attainment value were similar. Value protected engagement and achievement at low levels of expectancy and vice versa. Indirect relations between the expectancy value interaction and achievement were stronger for attainment than utility value; for attainment value they became non-significant at high value whereas for utility value they became non-significant at mean value. Based on Wise and DeMars’s (2005, p.2) conceptualisation of test-taking motivation we reasoned that attainment value would be the most germane subjective of the three types of value included in expectancy-value theory for performance on a standardised test. The finding that the indirect relation between the expectancy value interaction and achievement was stronger for attainment value than utility value (and that the expectancy intrinsic value interaction was not statistically significant) supports this reasoning.
References


Table S1
*Indirect and Total Effects of the Relations between* $T_3$ *Expectancy and* $T_5$ *Achievement at Differing Levels of* $T_3$ *Attainment Value, Mediated by* $T_4$ *Engagement*

<table>
<thead>
<tr>
<th></th>
<th>Low Attainment Value (-1SD)</th>
<th>Mean Attainment Value</th>
<th>High Attainment Value (+1SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>$SE$</td>
<td>95%CIs</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>2.34</td>
<td>0.88</td>
<td>0.90, 3.79</td>
</tr>
<tr>
<td>Total Effect</td>
<td>4.20</td>
<td>0.96</td>
<td>2.63, 5.67</td>
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*Note.* Indirect and total effects represented by unstandardised coefficients.
Table S2
Descriptive Statistics for $T_2$ and $T_4$ Engagement, $T_3$ Expectancy, $T_3$ values, and $T_3$ and $T_5$ Achievement

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<th>$\sigma_1$</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Factor Loadings</th>
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<tr>
<td>$T_2$ Engagement</td>
<td>1–5</td>
<td>4.58</td>
<td>0.53</td>
<td>.76</td>
<td>.05</td>
<td>-2.08</td>
<td>7.12</td>
<td>.64 – .78</td>
</tr>
<tr>
<td>$T_4$ Engagement</td>
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<td>4.51</td>
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<td>.60 – .83</td>
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<td>.04</td>
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<td>0.50</td>
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<td>-2.18</td>
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<td>.75 – .83</td>
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<td>.47 – .69</td>
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<td>2.36</td>
<td>—</td>
<td>.10</td>
<td>0.24</td>
<td>-0.30</td>
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<tr>
<td>$T_5$ Achievement</td>
<td>0–120</td>
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<td>5.58</td>
<td>—</td>
<td>.07</td>
<td>-0.33</td>
<td>0.55</td>
<td>—</td>
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</tbody>
</table>

*Note.* The intraclass correlation coefficient ($\sigma_1$) represents the proportion of variance attributable to the classroom level.
Table S3
*Latent Bivariate Correlations for T\textsubscript{2} and T\textsubscript{4} Engagement, T\textsubscript{3} Expectancy, T\textsubscript{3} Intrinsic Value, T\textsubscript{3} and T\textsubscript{5} Achievement, and Gender*

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<th>4.</th>
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<td>.46***</td>
<td>.51***</td>
<td>.16*</td>
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<td>-.13*</td>
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<td>.48***</td>
<td>.07</td>
<td>.40***</td>
<td>-.08</td>
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<td>—</td>
<td>.59***</td>
<td>.37***</td>
<td>.37***</td>
<td>.02</td>
</tr>
<tr>
<td>4. T\textsubscript{3} Intrinsic Value</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>.06</td>
<td>.22***</td>
<td>.03</td>
</tr>
<tr>
<td>5. T\textsubscript{1} Achievement</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>.35***</td>
<td>-.04</td>
</tr>
<tr>
<td>6. T\textsubscript{5} Achievement</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-.02</td>
</tr>
<tr>
<td>7. Gender</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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</tr>
</tbody>
</table>

*p < .05. **p < .01. ***p < .001.*
Table S4

*Standardised and Unstandardised Path Coefficients from the Moderated Mediation Model Including Intrinsic Value*

<table>
<thead>
<tr>
<th></th>
<th>T₁ Achievement</th>
<th>T₂ Engagement</th>
<th>T₃ Expectancy</th>
<th>T₃ Intrinsic Value</th>
<th>T₄ Engagement</th>
<th>T₅ Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standardised Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁ Achievement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₂ Engagement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₃ Expectancy (EX)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₃ Intrinsic Value (IV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₃ EX × IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-.05</td>
<td>-.13*</td>
<td>.08</td>
<td>.10*</td>
<td>-.03</td>
<td>-.02</td>
</tr>
</tbody>
</table>

| **Unstandardised Coefficients** |                |               |              |                    |               |               |
| T₁ Achievement           | .03*           |               |              |                    |               |               |
| T₂ Engagement            |                | .58***        | .85***       |                    |               |               |
| T₃ Expectancy (Ex)        |                |               |              |                    |               |               |
| T₃ Intrinsic Value (IV)   |                |               | .11*         | 2.02*              |               |               |
| T₃ EX × IV               |                |               | -.02         | -.46               |               |               |
| Gender                   | -.27           | -.12*         | .11          | .14*               | -.02          | -.25          |

* *p < .05. **p < .01. ***p < .001.
Table S5

*Latent Bivariate Correlations for $T_2$ and $T_4$ Engagement, $T_3$ Expectancy, $T_3$ Utility Value, $T_3$ and $T_5$ Achievement, and Gender*

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $T_2$ Engagement</td>
<td>—</td>
<td>.59***</td>
<td>.45***</td>
<td>.52***</td>
<td>.13*</td>
<td>.13*</td>
<td>-.13*</td>
</tr>
<tr>
<td>2. $T_4$ Engagement</td>
<td>—</td>
<td>—</td>
<td>.44***</td>
<td>.52***</td>
<td>.06</td>
<td>.40***</td>
<td>-.08</td>
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<tr>
<td>3. $T_3$ Expectancy</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>.58***</td>
<td>.37***</td>
<td>.37***</td>
<td>.02</td>
</tr>
<tr>
<td>4. $T_3$ Utility Value</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>.17</td>
<td>.17**</td>
<td>.08</td>
</tr>
<tr>
<td>5. $T_5$ Achievement</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>.35***</td>
<td>-.04</td>
</tr>
<tr>
<td>6. $T_5$ Achievement</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-.02</td>
</tr>
<tr>
<td>7. Gender</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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</table>

*p < .05. **p < .01. ***p < .001.*
### Table S6

**Standardised and Unstandardised Path Coefficients from the Moderated Mediation Model Including Utility Value**

<table>
<thead>
<tr>
<th></th>
<th>(T_1) Achievement</th>
<th>(T_2) Engagement</th>
<th>(T_3) Expectancy</th>
<th>(T_3) Utility Value</th>
<th>(T_4) Engagement</th>
<th>(T_5) Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standardised Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_1) Achievement</td>
<td>.20**</td>
<td></td>
<td></td>
<td>.22*</td>
<td>.46***</td>
<td></td>
</tr>
<tr>
<td>(T_2) Engagement</td>
<td></td>
<td>.37***</td>
<td>.40***</td>
<td>.46***</td>
<td>.38***</td>
<td></td>
</tr>
<tr>
<td>(T_3) Expectancy (EX)</td>
<td></td>
<td></td>
<td></td>
<td>.15</td>
<td></td>
<td>.26**</td>
</tr>
<tr>
<td>(T_3) Utility Value (UV)</td>
<td>.13</td>
<td></td>
<td></td>
<td>- .36***</td>
<td>- .08</td>
<td></td>
</tr>
<tr>
<td>(T_3) EX × UV</td>
<td></td>
<td></td>
<td></td>
<td>- .22**</td>
<td>- .35</td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-.05</td>
<td>-.13*</td>
<td>.07</td>
<td>.14*</td>
<td>-.03</td>
<td>-.01</td>
</tr>
<tr>
<td><strong>Unstandardised Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_1) Achievement</td>
<td>.04*</td>
<td></td>
<td></td>
<td>.04*</td>
<td></td>
<td>1.09***</td>
</tr>
<tr>
<td>(T_2) Engagement</td>
<td></td>
<td>.62***</td>
<td>.41***</td>
<td>.42***</td>
<td></td>
<td>8.23***</td>
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<tr>
<td>(T_3) Expectancy (EX)</td>
<td></td>
<td></td>
<td></td>
<td>.09</td>
<td>2.26**</td>
<td></td>
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<tr>
<td>(T_3) Utility Value (UV)</td>
<td>.11</td>
<td></td>
<td></td>
<td>-.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T_3) EX × UV</td>
<td></td>
<td>-.22**</td>
<td>- .35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-.26</td>
<td>-.12*</td>
<td>.10</td>
<td>.14*</td>
<td>-.03</td>
<td>-.26</td>
</tr>
</tbody>
</table>

*\(p < .05\). **\(p < .01\). ***\(p < .001\).
### Table S7
*Indirect and Total Effects of the Relations between $T_3$ Expectancy and $T_5$ Achievement at Differing Levels of $T_3$ Utility Value, Mediated by $T_4$ Engagement*

<table>
<thead>
<tr>
<th></th>
<th>Low Utility Value (-.5SD)</th>
<th>Mean Utility Value</th>
<th>High Utility Value (+.5SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>SE</td>
<td>95%CIs</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>1.54</td>
<td>0.84</td>
<td>0.16, 2.93</td>
</tr>
<tr>
<td>Total Effect</td>
<td>3.80</td>
<td>0.86</td>
<td>2.63, 5.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Low Utility Value (-1SD)</th>
<th>Mean Utility Value</th>
<th>High Utility Value (+1SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>SE</td>
<td>95%CIs</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>2.17</td>
<td>1.10</td>
<td>0.35, 3.99</td>
</tr>
<tr>
<td>Total Effect</td>
<td>4.43</td>
<td>1.08</td>
<td>2.65, 6.20</td>
</tr>
</tbody>
</table>

*Note.* Indirect and total effects represented by unstandardised coefficients.
**Figure S1.** The model implied simple slopes of interaction between T₃ expectancy and T₃ attainment value on T₃ engagement.
Figure S2. The model implied indirect simple slopes of the relations between T₃ expectancy and T₅ achievement at differing levels of T₃ attainment value.
Figure S3. The model implied indirect simple slopes of the relations between T₃ expectancy and T₄ engagement at differing levels of T₃ utility value (±.5 SD for upper Figure and ±1SD for the lower Figure).
Figure S4. The model implied indirect simple slopes of the relations between T₃ expectancy and T₃ achievement at differing levels of T₃ utility value (±.5 SD for upper Figure and ±1SD for the lower Figure).
Appendix

Expectancy of Success:
1. I get good marks for my maths work
2. I am good at maths tests
3. I get good marks in maths lessons
4. I think that I will do well in my next maths test

Intrinsic Value:
1. I find maths lessons interesting.
2. I find doing maths interesting.
3. I am interested in learning maths.
4. Maths is not interesting (Reverse scored).

Attainment Value:
1. Getting good marks on maths tests is important to me.
2. Getting a good mark on maths tests is important to me.
3. I want to get good marks in my maths tests.
4. I want to show how good I am at maths on tests.

Utility Value:
1. Maths can help with things in everyday life.
2. Maths will help me later in life.
3. Being good at maths will help me get in a good set in secondary school.
4. Maths is a good skill to have outside of school.

Behavioural Engagement:
1. I try to do well in my maths lessons.
2. In my maths lessons, I try as hard as I can.
3. I participate in the activities and tasks in my maths lessons.
4. I pay attention in my maths lessons.
5. When I’m in my maths lessons, I listen very carefully.