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Shifting of wrapped phase maps in the frequency domain using a rational number

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Abstract

The number of phase wraps in an image can be either reduced, or completely eliminated, by transforming the image into the frequency domain using a Fourier transform, and then shifting the spectrum towards the origin. After this, the spectrum is transformed back to the spatial domain using the inverse Fourier transform and finally the phase is extracted using the arctangent function. However, it is a common concern that the spectrum can be shifted only by an integer number, meaning that the phase wrap reduction is often not optimal. In this paper we propose an algorithm than enables the spectrum to be frequency shifted by a rational number. The principle of the proposed method is confirmed both by using an initial computer simulation and is subsequently validated experimentally on real fringe patterns. The technique may offer in some cases the prospects of removing the necessity for a phase unwrapping process altogether and/or speeding up the phase unwrapping process. This may be beneficial in terms of potential increases in signal recovery robustness and also for use in time-critical applications.

1. Introduction

Many signal recovery methods yield phase values that have 2π phase jumps within the recovered phase signal. This is called the wrapped phase, and the process of recovering a continuous form of the signal phase is called phase unwrapping [1].

In many real world applications that measure the phase of a signal, phase unwrapping is an essential task. Some examples are MRI [2], [3], synthetic aperture radar (SAR) [4], [5], and interferometry [6]. The difficulty of the phase unwrapping problem has resulted in a large number of attempts to reach acceptable solutions and hundreds of algorithms have been proposed and published. Most of these techniques can be classified into three categories: 1) path independent methods that set branch cuts to prevent the unwrapping path from crossing discontinuities, noisy areas and under-sampled regions [7], [8], [9]; 2) path dependent methods that use quality maps [10]; and 3) minimum norm methods [1], [11].

The phase unwrapping procedure can be avoided altogether in some cases. For example, the phase information in a fringe pattern can be extracted using the Fourier transform profilometry method. In this technique, the fringe pattern is Fourier transformed. Then the frequency spectrum is shifted to the origin, and the inverse Fourier transform is computed. Finally the phase is extracted using the arctangent function. The spectrum shift generally contributes to reducing (or fully eliminating) the phase wraps, making this method especially attractive to measure objects that have small height, but complex shape variations, e.g. printed circuit boards [12]. The phase shift as a phase wrap reduction method has recently been extended to phase stepping [13]. However, the use of the discrete Fourier transform limits the shift to integer values in all cases.

In this paper, we propose a method that allows non-integer shifts in the Fourier spectrum. This makes it possible to increase the resolution of the approach and eliminate phase wraps that in some cases cannot be removed by an integer shift. Also, this method can completely remove the tilt of an extracted phase map. The principle of the proposed method is first validated by using a computer simulation and then confirmed experimentally on real fringe patterns.

2. Computer simulation

A computer-generated 3D object was produced by using the *peaks* function in MATLAB (see Eq. 1). The resulting object is shown in Fig. 1(a) and 1(b) as a 3D plot and a 2D intensity image, respectively. It consists of 512 × 512 pixels and contains regions with both slow and rapid phase variations. For these reasons it has become a popular benchmark object in the literature for testing the performance of different fringe analysis algorithms [14].

$$\emptyset(x,y) = 3(1-x)^2 \exp(-x^2-(y+1)^2) - 10\left(\frac{x}{5}-x^3-y^5\right)\exp(-x^2-y^2) - \frac{1}{3}\exp(-(x+1)^2-y^2) \quad (1)$$

Where *x* and *y* are the sample indices for the x and y axes respectively.

From this object, four fringe patterns have been generated by using Eq. (2). These are simulated patterns that represent the result obtained by projecting shifted patterns upon the virtual 3D object described by Eq. 1. One of the four simulated phase-modulated fringe patterns is depicted in Fig. 1(c) as a grey scale range image.

$$g_0(x,y) = \cos(2\pi f_o x + \emptyset(x,y)) \quad (2a)$$

$$g_{90}(x,y) = \cos(2\pi f_o x + \emptyset(x,y) + \frac{\pi}{2}) \quad (2b)$$

$$g_{180}(x,y) = \cos(2\pi f_o x + \emptyset(x,y) + \pi) \quad (2c)$$

$$g_{270}(x,y) = \cos(2\pi f_o x + \emptyset(x,y) + \frac{3\pi}{2}) \quad (2d)$$

Where f_o is the spatial frequency of the carrier and here this is set to a value of 1/32 fringes per pixel (*i.e.*, there are exactly 32 pixels in each fringe). The number of fringes in the fringe pattern image is 512/32=16 exactly. For simplicity, it is considered here that there is no carrier frequency on the y-axis (*i.e.*, the projected fringes lie exactly parallel to the y-axis). The phase information in the fringe pattern can be extracted using the four-frame phase stepping algorithm described by Eq. (3) [15].

$$\varphi_w(x,y) = arctan2[g_0(x,y) - g_{180}(x,y), g_{270(x,y)} - g_{90}(x,y)] \quad (3)$$

Where $arctan2[.]$ is the four quadrant arctangent function which is named the *atan2* function in MATLAB. The extracted phase $\varphi(x, y)$ is shown in Fig. 1(d) and it contains 2π steps that should be removed by employing a phase unwrapping algorithm [1].

The phase unwrapping step might be avoided by using the Fourier transform method, proposed in [13], as follows. Initially, a complex array is constructed;

$$\varphi_{wc}(x, y) = e^{j\varphi_w(x,y)} \tag{4}$$

Where $j = \sqrt{-1}$. The 2D Fourier transform of $\varphi_{wc}(x, y)$ is then calculated as shown in Eq. (5).

$$\Phi(u, v) = \mathbb{F}[\varphi_{wc}(x, y)] \tag{5}$$

Where $\mathbb{F}[.]$ is the 2D Fourier transform operator, and the terms u and v are the horizontal and vertical frequencies, respectively. The 2D Fourier transform of the wrapped phase map is calculated using Eq.'s (4) and (5). The magnitude of the Fourier transform is shown in Fig. 1(e). The peak in the frequency domain is located at the frequencies $\Delta u = \frac{512}{32} = 16$ and $\Delta v = 0$.

The location and number of phase wraps can be changed using the Fourier transform as follows. First, the 2D Fourier transform of the wrapped phase map needs to be shifted in the frequency domain towards the origin by Δu and Δv as shown in Fig. 1(e). Both Δu and Δv values can be chosen arbitrarily, here they are set here to 16 and 0 respectively. Then the inverse 2D Fourier transform $\mathbb{F}^{-1}[.]$ is calculated, as shown in Eq. (6).

$$\varphi_{wcs}(x, y) = \mathbb{F}^{-1}[\Phi(u - \Delta u, v - \Delta v)] \tag{6}$$

After this, a new phase map can be generated by using Eq. (7).

$$a = \Re[\varphi_{wcs}(x, y)] \tag{7a}$$

$$b = \mathfrak{I}[\varphi_{wcs}(x, y)] \tag{7b}$$

$$\varphi_w(x, y) = arctan2(a, b) \tag{7c}$$

Where $\mathfrak{I}[.]$represents the imaginary part, and $\mathfrak{R}[.]$ represents the real part of the complex array $\varphi_{wcs}(x, y)$. The new phase map does not contain 2π phase jumps and it is shown in Fig. 1(f) and 1(g).

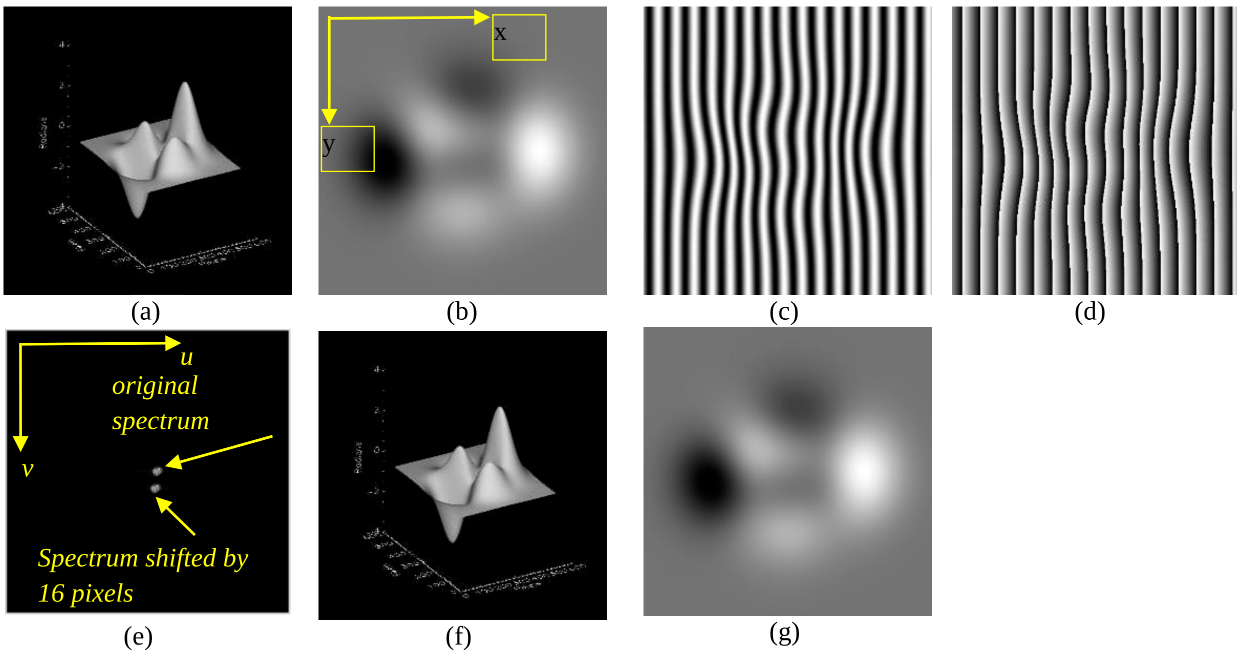

Fig. 1. (a) The 3D plot of the simulated object. (b) The 2D phase map of the object. (c) A fringe pattern. (d) The wrapped phase map. (e) The spectrum of the wrapped phase map and the new spectrum where the peak is shifted to the center. (f) The 3D plot of the unwrapped phase map. (g) The 2D phase map of the reconstructed object.

The success in the previous example contributes to the integer number of fringes in the image. In most of the real-world applications, the number of fringes in the image is arbitrary where phase unwrapping by the same method could bring the trouble.

The computer simulation described above was subsequently repeated, but with the spatial carrier frequency set to 1/33 fringes per pixel. The number of fringes in this image is 512/33=15.5152. The wrapped phase map is generated using Eq. (3) and it is shown in Fig. 2(a). The Fourier transform of this wrapped phase map is calculated using Eq.'s (4) and (5). The frequency spectrum should be shifted towards the origin by a distance $\Delta u = 15.51$ and $\Delta v = 0$ in order to remove the phase jumps. As only integer shifts are normally possible, the spectrum is shifted here using the values $\Delta u = 16$ and $\Delta v = 0$. The inverse Fourier transform and the phase map are then calculated using Eq.'s (6) and (7) respectively. The resultant wrapped phase map is shown in Fig. 2(b), and it can be noticed that the 2π phase jumps have not been removed completely.

In a second attempt to remove the phase wraps, the spectrum is shifted using the values $\Delta u = 15$ and $\Delta v = 0$. The resultant phase map is shown in Fig. 2(c). This figure reveals that the 2π phase jumps also have not been removed completely.

The authors suggest increasing the resolution of the Fourier transform in order to remove the phase wraps. This can be achieved by padding the wrapped phase map with zeros. Suppose that we would like to shift the spectrum towards the origin by a frequency equivalent of the original sample step of 16. Then the image size should be set to $16\left(\frac{512}{15.51}\right) = 528.175 \cong 528$. The wrapped phase image is then padded with zeros for both the horizontal and vertical directions in order to extend its size to 528×528 pixels. The padded image is shown in Fig. 2(d).

The padded image is converted to a complex array using Eq. (4), which is then Fourier transformed using Eq. (5). The spectrum is shown in Fig. 2(e). The spectrum is shifted using the values $\Delta u = 16$ and $\Delta v = 0$ as shown in Fig. 2(e). The inverse Fourier transform is calculated using Eq. (6), and the phase map is calculated using Eq. (7), which is shown in Fig. 2(f). This image has the size of 528×528 pixels and it is then cropped to the size of 512×512 pixels in order to extract the phase map. The cropped image is shown in Fig. 2(g). This figure reveals that the 2π phase jumps have been removed completely.

The mathematical difference between the object shown in Fig. 1(a) and the unwrapped phase map shown in Fig. 2(g) is calculated and it is shown in Fig. 2(h). The root mean square of the sum of this difference is calculated and it has a value of 3.1×10^{-6} radians.

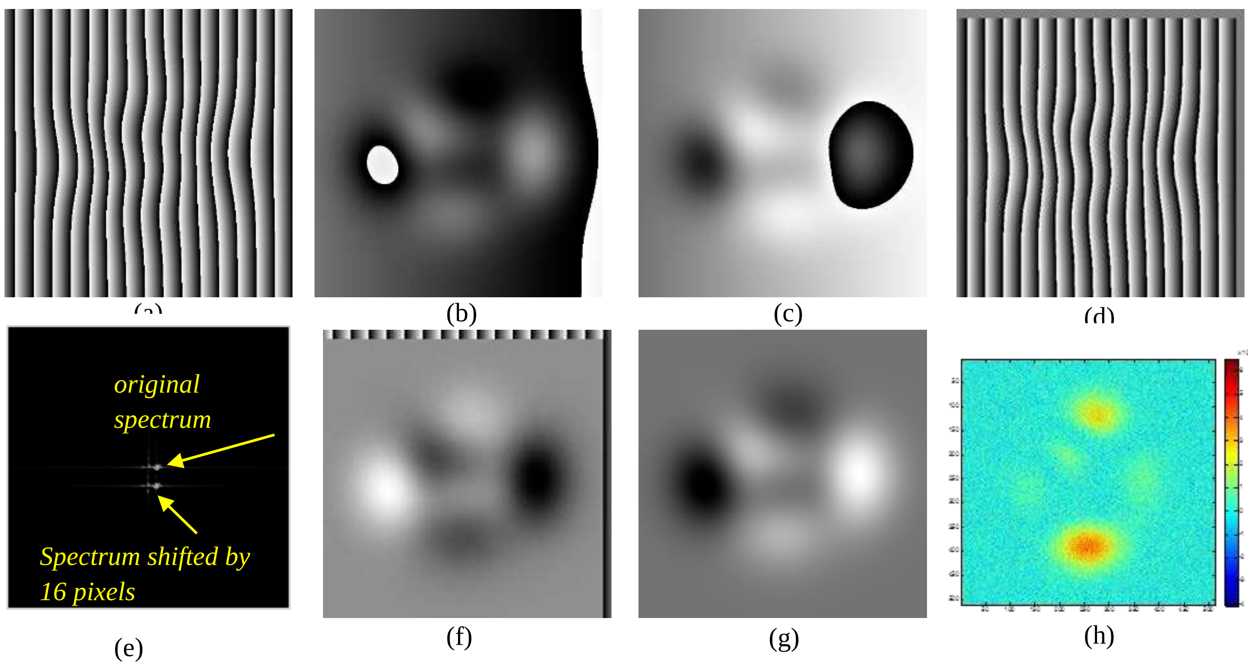

Fig. 2 (a) A simulated wrapped phase map with the size of 512×512 pixels and with a spatial carrier frequency f_o of 1/33fringes per pixel. (b) A 2D intensity map for a wrapped phase map that is calculated by shifting the spectrum in the frequency domain with 16 steps towards the origin. (c) A 2D intensity map for a wrapped phase map that is calculated by shifting the spectrum in the frequency domain with 15 steps towards the origin. (d) Zero-padding the wrapped phase map image in (a). The size of the padded image is 528×528 pixels. (e) The spectrum of the padded image and the spectrum is shifted towards the center of the image by 16 steps. (f) The phase of the shifted spectrum is shown as a 2D intensity map. The phase has the size of 528×528 pixels. (g) The phase image is cropped to the size of 512×512 pixels and it is shown as a 2D intensity map respectively. (h) The mathematical difference between the object shown in Fig. 1(a) and the unwrapped phase shown in Fig. 2(g).

3. Experimental results

In real-life applications, the number of fringes in a fringe pattern is, normally, not an integer number. Conventional methods shift the spectrum in the frequency domain using an integer number in order to unwrap the wrapped phase that is extracted from the fringe pattern. But this may not be able to remove the phase wraps even though the unwrapped phase does not exceed the 2π range [13].

This paper suggests a new method to shift spectrum in the frequency domain using a rational number. This technique is able to unwrap an image completely if the corerect unwrapped phase does not exceed the 2π range. This is achieved by zero-padding the wrapped phase map as explained below.

A computer keyboard is shown in Fig. 3(a) and this was measured using a fringe projection system. Four fringe patterns, with a phase shift of $\pi/2$ between each two consecutive fringe patterns, were projected sequentially onto the keyboard. Then four phase-modulated fringe patterns were captured using a camera. One of these fringe patterns is shown in Fig. 3(b) as grey scale range image. The wrapped phase map calculated according to Eq. (3) is shown in Fig. 3(c). All these images have the size of 512×512 pixels.

The wrapped phase map shown in Fig. 3(c) was Fourier transformed using Eq.'s (4) and (5), and the result of this is shown in Fig. 3(d). The maximum magnitude value in the spectrum is located at $\Delta u = 5$ and $\Delta v = 0$. The spectrum is shifted towards the origin using these values, and the shifted spectrum is shown in Fig. 3(d). The resultant phase map was then calculated using Eq.'s (6) and (7) and is shown in Fig. 3(e). This figure reveals that the 2π phase jumps have not been removed completely.

In a second attempt to remove the 2π phase jumps, the spectrum was shifted towards the origin using the values $\Delta u = 6$ and $\Delta v = 0$. The resultant phase map is shown in Fig. 3(f) and. This figure reveals that the 2π phase jumps have also not been completely removed.

The number of fringes in Fig. 3(b) is five and a half approximately, which is a non-integer number. The spectrum should be shifted to the center using the values $\Delta u = 5.5$ and $\Delta v = 0$ approximately. This cannot be carried out without increasing the resolution of the Fourier transform by zero-padding the wrapped phase image.

Suppose that we would like to shift the spectrum towards the origin by a frequency equivalent of the original sample step of 5.5. Then the image size should be set to $6\left(\frac{512}{5.5}\right) = 558.545 \cong 558$. The wrapped phase image is then padded with zeros for both the horizontal and vertical directions in order to extend its size to 558×558 pixels and it is shown in Fig. 3(g).

This padded image is Fourier transformed using Eq.'s (4) and (5). The spectrum is then frequency shifted towards the origin, using shift values $\Delta u = 6$ and $\Delta v = 0$. The inverse Fourier transform is then calculated using Eq. (6). The phase is extracted using Eq. (7) and it is shown in Fig. 3(h) and it has the size of 558×558 pixels. This image is cropped to the size of 512×512 pixels in order to extract the required phase map and this is shown in Fig. 3(i). This figure reveals that the 2π phase jumps have been completely removed.

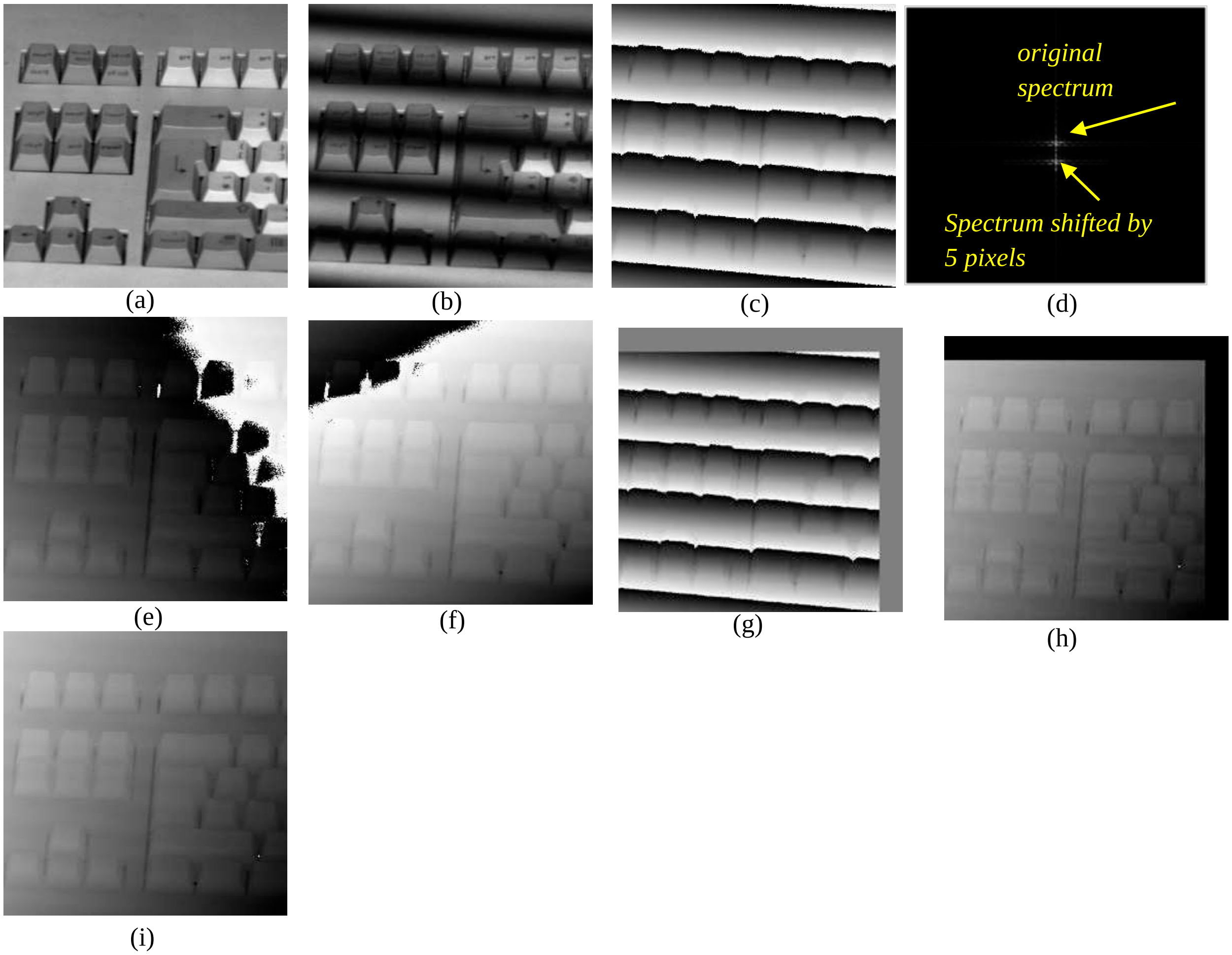

Fig. 3 (a) A keyboard object with the size of 512×512 pixels. (b) A fringe pattern. (c) The wrapped phase map calculated using the four frame phase-stepping algorithm. (d) The spectrum computed for the wrapped phase map and the spectrum is shifted towards the center of the image by $\Delta u = 5$ and $\Delta v = 0$. (e) The wrapped phase of the shifted spectrum object is shown as a as a 2D intensity image. (f) The wrapped phase of the shifted spectrum by $\Delta u = 6$ and $\Delta v = 0$ shown as 2D intensity map. (g) The wrapped phase image is zero padded to have the size of 558×558 pixels. (h) The spectrum of the padded image is shifted to the center of the image by $\Delta u = 6$ and $\Delta v = 0$ and then the phase is calculated. (i) The image in (h) is cropped to the size of 512×512 and the cropped image is shown as a 2D intensity image.

4. Conclusions

The number of fringes in a real fringe pattern is normally not integer. In this case, conventional methods that use the Fourier transform in order to avoid the unwrapping step may not be able to remove all the phase steps. This is because these methods are able to shift the spectrum using integer numbers only.

The authors propose a method to shift the spectrum using a rational number. The suggested method is able to remove all the phase wraps if the correct unwrapped phase is within the 2π range. This paper suggests padding the wrapped phase image with zeros and then calculating its Fourier transform. This has the effect of 'increasing' the resolution of the Fourier transform. After that, the spectrum is shifted to the origin using an integer number. But this has the effect of shifting the image by a rational number.

References

1. D. Ghiglia, M. Pritt, Two-Dimensional Phase Unwrapping Theory, Algorithms and Applications (John Wiley & Sons, 1998).
2. A. Fan, S. Govindarajan, R. Kinkel, N. Madigan, A. Nielsen, T. Benner, E. Tinelli, B. Rosen, E. Adalsteinsson, C. Mainero, "Quantitative oxygen extraction fraction from 7-Tesla MRI phase: reproducibility and application in multiple sclerosis," Journal of Cerebral Blood Flow & Metabolism 35, 131–139 (2014).
3. T. Kaaouana, T. Samaille, N. Thierry, C. Dufouil, Delmaire, D. Dormont, M. Chupin, "Phase contrast MRI for discriminating brain microbleed in a multicentre clinical study," in 1st International Conference on Advanced Technologies for Signal and Image Processing (ATSIP), 265 – 270 (2014).
4. X. Xie, P. Huang, Q. Liu, "Phase unwrapping algorithm based on extended particle filter for SAR interferometry, IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences E97-A, 405-408 (2015).
5. H. Zhong, J. Tang, S. Zhang, X. Zhang, "A quality-guided and local minimum discontinuity based phase unwrapping algorithm for InSAR/InSAS interferograms," IEEE Trans. Geoscience and Remote Sensing Letters 11, 215 – 219 (2014).
6. K. Falaggis, D. Towers, C. Towers, "Generalized phase unwrapping for multi-wavelength interferometry," Fringe13 Conference, Germany, 653-656 (2013).
7. R. Goldstein, H. Zebker, C. Werner, "Satellite radar interferometry: two-dimensional phase unwrapping," Radio Science 23, pp. 713-720 (1998).
8. H. Zebker, Y. Lu, "Phase unwrapping algorithms for radar interferometry: residue-cut, least-squares, and synthesis algorithms," J. Opt. Soc. Am. A.15, 586-598 (1988).
9. J. M. Huntley, J. R. Buckland, "Characterization of sources of 2π phase discontinuity in speckle interferograms," J. Opt. Soc. Am. A. 12, 1990-1996 (1995).
10. G. Liu, W. R., Y. Deng, R. Chen, Y. Shao, Z. Yuan, "A new quality map for 2-D phase unwrapping based on gray level co-occurrence matrix," IEEE Trans. Geoscience and Remote Sensing Letters 11, 444 – 448 (2014).
11. M. D. Pritt, "Phase unwrapping by means of multigrid techniques for interferometric SAR," IEEE Transactions on Geoscience and Remote Sensing 34, 728 – 738 (1996).

12. David Burton, Anthony Gooddall, John Atkinson, Michael Lalor, “The use carrier frequency shifting for the elimination of phase discontinuities in Fourier transform profilometry,” Optics and Lasers in Engineering 23, 245-257 (1995).
13. M. Qudeisat, M. Gdeisat, D. Burton, F. Lilley, “A simple method for phase wraps elimination or reduction in spatial fringe patterns,” Optics Communications 284, 5105–5109 (2011).
14. Chen W, Sun J, Su X, Bian X. Discuss the structure condition and sampling condition of wavelet transform profilometry. Journal of Modern Optics 2007; 54:2747-62.
15. Creath K., (1993) "Temporal phase measurement methods," chapter 2 of "*Interferogram analysis: digital fringe pattern measurement techniques*," Edited by Robinson WR. and Reid GT., Institute of Physics Publishing, Bristol and Philadelphia, pp. 94-140.