A risk-based game model for rational inspections in port state control

Abstract

This paper analyses the game relationship between port authorities and ship owners under the new inspection regime (NIR). Based on 49328 inspection reports from Paris Memorandum of Understanding (MoU) (2015-2017), we present a Bayesian Network (BN) model to determine vessel detention rates after adding company performance as a new indicator in PSC inspection. A strategic game model is formulated by incorporating the BN model outcomes. The optimal inspection rate from the game model can help improve port authority performance in PSC. An empirical study is conducted to illustrate the insights of the results and provide suggestions for port authorities.

Key words: Port State Control, Game theory, Bayesian network, Nash solution, NIR, maritime safety, maritime risk

1. Introduction

Traditional flag state control has its limits in terms of ensuring the implementation of maritime safety regulations by ship owners, particularly those choosing open registration. Therefore, Port State Control (PSC), which renders port authorities the ability to inspect foreign vessels in their own ports, is set up in order to avoid the entries of sub-standard ships into their waters and the occurrence of maritime accidents. Since established in 1982, PSC is gradually viewed as the last safety line of defending sub-standard vessels and improving maritime safety because it effectively reduces the appearance of the vessels not fully following the relevant maritime safety regulations. Nevertheless, it is not perfect, leaving the gaps to be addressed and new solutions to be found. According to the PSC inspection records, every year there are still a large number of vessels that do not comply the regulations reckoned by port authorities and fail to pass their inspections, indicating the deficiencies of the PSC system in terms of motivating ship owners to improve vessel quality. Because of the high maintenance cost, some ship owners do not tackle the safety loopholes of their vessels in time. Although facing possible high punishment when his vessel is detained, a ship owner still gambles and takes the risk, as it is impossible for port authorities to inspect all the vessels entering their ports. From the perspective of port authorities, on one hand, excessive PSC inspections may harm the competitiveness of the ports and increase the burden of ship owners, leading ship owners to turn to other destinations that may have a more relaxed inspection policy (Li et al., 2014). On the other hand, a loose inspection policy is not helpful to stimulate ship owners to implement high intensive maintenance effort, which in return attracts sub-standard ships and possibly causes the occurrence of accidents and consequently economic loss and damage to reputation.

Therefore, striking a PSC inspection balance between port authorities and ship owners requires a scientific decision for rational policymaking. While the port authorities aim at motivating ship owners to maintain their vessels at a high safety level to mitigate maritime accidents, ship owners care more about minimization of the associated costs. Such conflict of interests thereupon forms the game relationship between the two stakeholders.

To improve the PSC inspection system, the much-anticipated New Inspection Regime (NIR) was launched in 2011. According to Paris MoU Annual Report (2011), it is viewed as the most significant change that transforms and modernizes the PSC system in recent years. Under the new inspection system, the vessel visiting a port will be attributed a ship risk profile through an associated information system, which determines the priority of ship inspections, the intervals between the inspections of a ship and the scope of the inspections. Based on the feedback, the port authority will decide the details of the inspections, (inspection types, detention results, and detention periods). The Paris MoU hoped that the implementation of NIR could efficiently improve the performance of PSC inspection system.

It is noteworthy that an important element that helps to categorize the ship risk profiles in NIR is the performance of International Shipping Management (ISM) companies. Before the implementation of NIR, ISM companies are just third-party managers who, for a negotiated fee and with no shareholding ties with their clients, undertake the responsibility of managing vessels in which they have no financial stake (Mitroussi, 2003). They accepted ships from and managed them on behalf of ship owners without much concern on their technical soundness given that they had no responsibility on vessels' failures of passing PSC inspections. However, this practice has been changed since the NIR was introduced in 2009 and implemented in 2011 on Paris MoU. Paris MoU establishes a shipping company (including ISM) performance formula that takes into account detention and deficiency records of the vessels under the company's management over a period of 36 months. Based on the deficiency and detention rates, the performance of ISM companies is classified into groups of four grades: high, medium, low and very low. A list of 'ISM managers' of poor performance has been developed, consisting of the ISM companies who have shown an unwillingness or inability to comply with the international conventions on maritime safety and/or on the protection of marine environment. Once a vessel is detained, the reputation of its associated ISM will be affected, leading to an increase frequency of inspections in future.

To ensure their profits and maintain their reputation, ISM companies are putting much effort to make them adaptive to the NIR and improving their management level. Considering the vessel quality, ISM companies raise their vessel acceptance criteria to ensure the successful inspection results that the ships under their management can receive. The involvement of ISM companies obviously influences the game between port authorities and ship owners in today's PSC practice.

For port authorities, when regulating their policies under NIR, it is of vital importance to take the company performance indicator into account. However, in this research, as we only focus on the period in which the vessel is already at the port, ISM companies are considered as a factor influencing the decision-making of port authorities, because the selection and determination of ISM companies happen before the occurrence of the inspections. Therefore, quantifying the influence of company performance on inspection results becomes the major issue when analysing the PSC inspection game under NIR in this research. Further research may consider ISM companies as a player in the inspection game if the time range of the game is widened.

This study aims at developing a risk-based game model based on Bayesian network (BN) to determine the optimal inspection strategy of a port authority under different circumstances after the implementation of NIR. Based on 49328 primary historical inspection reports obtained from the Paris MoU database in 2015-2017, those related to bulk carriers (i.e. 10000 records) are selected to build a BN risk model. The BN risk model provides a novel way to obtain the detention rates relating to different company performance levels and vessel quality. They can be used as important input in the subsequent game model construction. Through calculating every payoff during an inspection, a payoff matrix is utilized to present the new BN risk-based PSC game model.

The main contributions of this paper include: 1) to the authors' best knowledge, since NIR went into effect in 2011, company performance is, for the first time, viewed as an important factor influencing the decisions of port authorities in PSC inspection practice and scientific research; 2) BN and game theory are innovatively incorporated to exploit a rational way to precisely quantify the relationship between the port authority and ship owner during an inspection process; 3) it is the first attempt and presentation of a non-cooperative strategic game between port authorities and ship owners after the implementation of NIR. An optimal inspection policy for port authorities is derived from a Nash equilibrium solution; 4) New managerial insights about the optimal inspection rate (policy) are obtained. For instance, with the increase of punishment severity, the optimal inspection rates present a decreasing trend regardless the vessel condition. The declining speed of the optimal inspection rates slows down with the increase of punishment severity; 5) the proposed optimal inspection policy is able to provide real-time PSC decisions for port authorities in dynamic situations accordingly, where the risks constantly change; 6) suggestions are proposed to help port authorities of different economic constrains to make rational decisions. For instance, when a port authority has limited economic constrains, it should choose the optimal inspection rate as suggested by the game model; otherwise it can increase the punishment to an appropriate level as suggested by the model, to tackle the sub-standard effort and illegal actions of ship owners.

The remainder of this paper is organized as follows. Section 2 reviews the current literature focusing on the risk analysis relating to PSC inspections and presents the state of the art of game applications in maritime safety research. Section 3 describes the process of developing a theoretical framework for an optimal inspection policy based on the combination of BN and the game theory. It is followed by an empirical study, result analysis and implications in section 4. Finally, Section 5 concludes this study with reference to its scientific and practical contributions, limitations and future research directions.

2. Literature review

2.1 Risk analysis on PSC inspections

Since PSC inspections play an increasingly important role in maritime safety, more and more researchers have conducted related studies from both qualitative to quantitative perspectives. Various risk assessment approaches have been developed and applied in the past decades, demonstrating the diversity of this research field.

Knowing that intense maritime traffic may cause significant navigational challenges in Istanbul Strait, Kara (2016) applied a weighted point method to assess the risk level of each vessel experiencing the PSC inspections under the Black Sea MoU. However, the weighting and scoring method adopted in this study was at large based on subjective expert judgements, which potentially caused bias on the results.

Avoiding subjectivity in weighting has been extensively studied. Xu et al. (2007) presented a risk assessment system based on support vector machine to estimate the risk of candidate vessels according to historical data before conducting on-board inspections. Evaluations showed that the proposed system could improve the accuracy of risk assessment. Furthermore, Gao et al. (2008) combined support vector machine and K-nearest neighbour approaches to develop a new risk assessment model capable of coping with noisy data. Consequently, this method significantly improved the accuracy of the results. Although showing attractiveness, such methods still reveal problems in their practical applications in tackling dynamic risk prediction (e.g. ship detention probability) in different environments. This problem hinders the practical contribution of risk assessment approaches in PSC inspections. To solve this issue, Yang et al. (2018) utilized the BN to develop a detention rate prediction tool for port authorities. The advantages of BN over other risk assessment approaches in dynamic prediction provides important insights for us to seek the optimal inspection policies under different environments in NIR. However, Yang et al. (2018) only addressed risk analysis and did not conduct further studies on how the dynamic risk results can realise the optimization of inspection policy making of port authorities in PSC.

Based on 183,819 PSC inspection records, Knapp & Franses (2007) applied binary logistic regression to measure the effect of inspections on the probability of casualties, especially for the accidents involving very serious consequences. Meanwhile, the model determined the magnitude of improvable areas for sub-standard vessels. Later in the same year, they did a further econometric analysis about the influence of different risk factors on the detention probability, and the results indicated that only vessel types and PSC regimes were influential elements of great significance.

Compared to other risk assessment approaches, BN is widely used to evaluate maritime and port risks because of its advantages in forward prediction analysis and backward risk diagnosis (e.g. Ren et al., 2008; Eleye-Datubo et al., 2008; Zhang et al., 2013; Goerlandt & Montewka, 2015; Banda, O.A.V. et al., 2016; Pristrom et al., 2016)). However, few researchers investigated its effectiveness and potential in analysing the risks relating to PSC inspections. Hänninen & Kujala (2014) explored the dependencies of PSC inspection findings and vessel's involvement in accidents and incidents by using two learning algorithms to train BNs. The results showed that vessel type, inspection type and the number of structural conditions related deficiencies were among the most important factors influencing accident involvement. In addition, Yang et al. (2018) proposed a data-driven BN model involving multiple risk factors, to analyse their individual and combined effect on PSC inspections, and to develop a real-time prediction tool for port authorities to rationalize their inspections.

However, such studies focused on the PSC inspection system before the implementation of NIR, meaning the influence of company performance on inspection results is overlooked. As an important factor in new PSC inspection system, company performance is introduced when building BN for PSC inspections in this study. Furthermore, none of them had ever undertaken further studies to look at how the dynamic risk analysis result can assist port authorities in the development of rational inspection policies in their PSC practice.

2.2 Game theory applications in transportation

Game theory has been widely applied to stimulate policy making in transportation. Among different transport modes, road transportation shows a dominating position in terms of the use of game theory (e.g. Alberto et al, 1995; Hideyuki Kita, 1999; Chidambaram et al, 2014) and sea transport has taken a role of backseat in this aspect. Most of the researchers in road transportation focus on transportation network issues. Bell (2000) proposed a two-player non-cooperative game to minimise the expected trip costs, as well as measure the performance reliability of the transportation network through analysing the mixed strategy by Nash equilibrium. Levinson (2005) developed the congestion theory and pricing theory through two-player and three-player games. Based on this study, it proposed an improved model that corrected the calculated tolls and Nash equilibria predicted for the three-player game model in transportation network. Sasaki (2014) also did similar research on the optimal choices of a fare collection system considering the game-theoretical interactions between the transit agency and passengers. Other topics related to transportation network include the network design (Lin & Lee, 2010; Laporte et al., 2010), the cost allocation (Rosenthal, 2017), traffic-response signal control (Ruth et al., 2015), the role of privately owned road system in road network (Sofia, 2012) and green transportation (Bae et al., 2011).

In the maritime transportation field, inspection games are mainly presented from a quantitative orientation. In this game, port authorities tried to constrain the illegal actions of ship owners through inspection policies, regulations and punishments, while ship owners pursued the minimum costs to pass the inspections (Avenhaus et al, 1996; Baston and Bostock, 1991; Canty et al., 2001; Von Stengel, 1991; Rothenstein and Zamir, 2002).

In order to analyse the policies of PSC inspections, Li and Tapiero (2010) outlined a random payoff game-theoretical framework for vessel inspections at ports considering two kinds of error prone decisions (e.g. detaining a standard vessel or releasing a sub-standard vessel). The authors presented some particular Stackelberg solutions given different scenarios to highlight the effects and the implication of inspection costs and their derivatives. They paid enough attention on the inspections of potentially non-complying ship operators to regulations and sub-standard performance. Based on this research, Li et al. (2015) further developed a game model to decide on the optimal inspection level and the target of the inspection. A bi-matrix game between port authorities and ship owners was built based on the same two types of error prone decisions discussed in 2010. Different from the previous studies, this time the authors generated a Nash equilibrium solution representing the optimal inspection rate for port authorities. A numerical study was conducted to illustrate the optimal inspection strategy, which yielded significant savings for port authorities, as well as prevented potential violations of ship owners. Although showing significant insights for port authorities, there are still several deficiencies existing in both studies , i.e. 1) both studies were conducted before the implementation of NIR, not taking into account company performance as an important factor influencing the decision-making of port authorities in today's PSC practice; 2) when carrying out the numerical studies in the two works, the authors assumed that the work of the authorities was perfect and had no inspection risk exists, which was obviously idealized and thus had limited practical contributions. Hence, when establishing the new game model in this paper, both the contribution of company performance and the influence of inspection risk on the decisions of port authorities are investigated and considered, highlighting the main differences with and improvements from the two most related papers in the existing literature.

Environmental control is another form of the inspection game in transport studies. Bird and Kortanek (1974) explored various theoretical cooperative n-person games in order to aid the formulation of regulations concerning sources of pollutants in the atmosphere subject to the given least cost solutions. Russell (1990) introduced a specific type of stochastic model by allowing for errors of inference on the part of the agency due to imperfect monitoring instruments. Gueth & Pethig (1990) analysed a signalling game between a polluting firm that could save costs by illegal waste emission and a monitoring agency whose responsibility was to prevent such pollution.

In maritime safety area, terrorist threat draws attention. Reilly et al. (2012) used the game theory to model the interactions between a government agency, a carrier and a terrorist. A heuristic solution procedure was constructed to identify effective prohibitions and validated by a real case study in the continental US. The model was also suitable for rail networks. Sandler & Arce (2003), Sandler & Enders (2003) utilized the game theory to model terrorism as well. In the other maritime transport research areas, among the game studies were port competition (Ishii et al., 2013; Song et al., 2016), ship overload (Chen & Hu, 2014) and safety supervision (Yuan, 2008).

In general, PSC inspections present a strategic problem, and the inspection policies demand to be settled properly and optimally. Game theory, as a mathematical tool to study the conflicts and cooperation between decision-makers, is selected to solve this issue. Meanwhile, due to the implementation of NIR, company performance becomes a key influencing variable and indeed needs to be considered as a risk factor in decision-making process of PSC inspections, revealing a new research gap to be fulfilled.

3. Theoretical game between port authorities and ship owners

The process of developing a game model consists of three essential steps: 1) confirming the participated players, 2) figuring out the strategy of each player, and 3) determining the payoff of each strategy. When making decisions, both port authorities and ship owners will make their choices based on the payoffs of the strategies under different situations. As one of the important factors in game model, the inspection risk plays a key role in determining the payoffs. Hence, in order to quantify the inspection risk, BN is combined with the game model for the first time to precisely reflect the actual conditions in PSC after the implementation of NIR. Meanwhile, the BN model proposed in this paper takes into account company performance as an important risk factor influencing the inspection results, and the final model is able to reveal the detention rates under various conditions involving different company performance levels. In the subsequent game model construction, the detention rate can be used as an indicator of the company performance, presenting a game model between port authorities and ship owners considering the effect of company performance for the first time since PSC inspection regime changed.

3.1 BN for PSC inspection after the NIR[1]

When a vessel accepts an inspection at port, it may face two types of risks (Li et al., 2015). One is that the vessel is found non-conforming to the inspection requirements and detained when in fact it is a standard vessel. The other is the inspection shows the vessel conforms to the regulations when in fact it is a sub-standard vessel. The existence of the two types of inspection risks largely affects the estimate of the detention rate, which is an important factor influencing the inspection policy of port authorities and thus the construction of the game model. In order to make the model best reflect the reality, the inspection risks need to be

[1] The analysis of PSC inspection before the NIR has been conducted in Yang et al., (2018).

considered in our model.

Taking advantage of causal inference, BN is utilized in this paper to obtain the detention rate under different situations. As a powerful risk assessment approach, it has the ability to calculate the detention rate in consideration of the inspection risks detected from historical inspection records. Whenever the information about a specific inspection is collected, ship owners or the port authorities can use the BN to calculate the detention rate of the vessel. Compared to other risk assessment model, it combines the visualization with mathematical knowledge, enabling the analysis of the relationships between different risk factors. The process of developing BN is presented as follows[1].

3.1.1 Data acquisition

The dataset in this study consists of 49328 inspection records from 2015-2017, which are derived from the Paris MoU online inspection database (www.parismou.org/inspection-search/inspection-search). Each inspection record presents the details of the inspection and information of the inspected vessel.

A careful analysis of the inspection records indicates that bulk carriers play a dominating role, as the number of inspection records of bulk carriers counts 20% of the total. Therefore, bulk carriers are selected as the research target in this paper.

3.1.2 Variable identification

The variables in the BN are identified from the inspection records, including vessel flag, vessel age, company performance, type of inspection, port of inspection, date of inspection, number of deficiencies, and detention. It is noteworthy that company performance is included as one of the major risk factors in the Paris MoU online inspection database. Since the implementation of NIR, most ISM companies have raised their adoption policies to maintain their reputation in spite of facing possible toll losses. As a result, company performance is currently one of the relevant indexes reflecting vessel safety conditions and inspection results.

These variables are explained with the particular reference to their state definitions as follows.

Table 1. Identified variables in PSC inspections

VARIABLE	STATE
Vessel flag	White, Grey, Black, Black (high risk) (The performance of each state decreases successively.)

[1] The detailed information of the construction of BN for PSC is found in Yang et al., (2018). Here a brief introduction of each involved step is provided to keep the integrity of the whole risk-based game framework in this paper. In addition, it is also necessary given the facts that 1) the data used in this work is new, reflecting the changed practical situation after the introduction of the NIR in 2009; and 2) the BN model is updated by the addition of ISM performance as a new factor/node influencing ship detention rates.

Vessel age	0 to 5 years, 5 to10 years, 10 to15 years, 15 to 20 years, over 20 years
Company performance	High, Medium, Low, Very low
Type of inspection	Initial inspection, More detailed inspection, Expanded inspection
Port of inspection	Belgium, Canada, France, Germany, Greece, Italy, Netherlands, Spain, UK
Data of inspection	2015, 2016, 2017
Number of deficiencies	0, 1 to 3, 4 to 9, more than 10 (The number of inspected deficiencies are integer, e.g. '0' means 0 deficiency in inspection, and '1 to 3' means the number of deficiencies are 1, 2 or 3).
Inspection group	High detention Risk, Low detention Risk
Vessel group	High detention Risk, Low detention Risk
Detention	Yes, No

** The justification of the selection of the variables and their grades refers to Yang et al., (2018).*

It is noteworthy that the two intermediate level risk variables are introduced based on the principle of divorcing approach (Jensen, 2001; Yang et al., 2018) to avoid that the size of conditional probability tables (CPTs) are too large to effectively control. Additionally, the two nodes are comprehensive factors representing the overall level of vessel-related detention risk and inspection-related detention one, which act as the indicators reflecting the safety level of vessels in the game model later.

3.1.3 BN construction

The structure of BN in this study is learned via a data-driven approach, called TAN learning (Friedman et al., 1997; Carvalho et al., 2007). Through the Netica software, the result is presented in Figure 1. Although showing similarity, the new BN model in Figure 1 is different with the one in Yang et al. (2018) due to the addition of the node "Company performance" and the totally new data representing the situation after the NIR. It is because of this difference that the comparison of the detention results with and without the NIR makes significant practical insights on the importance of the NIR to improve maritime safety.

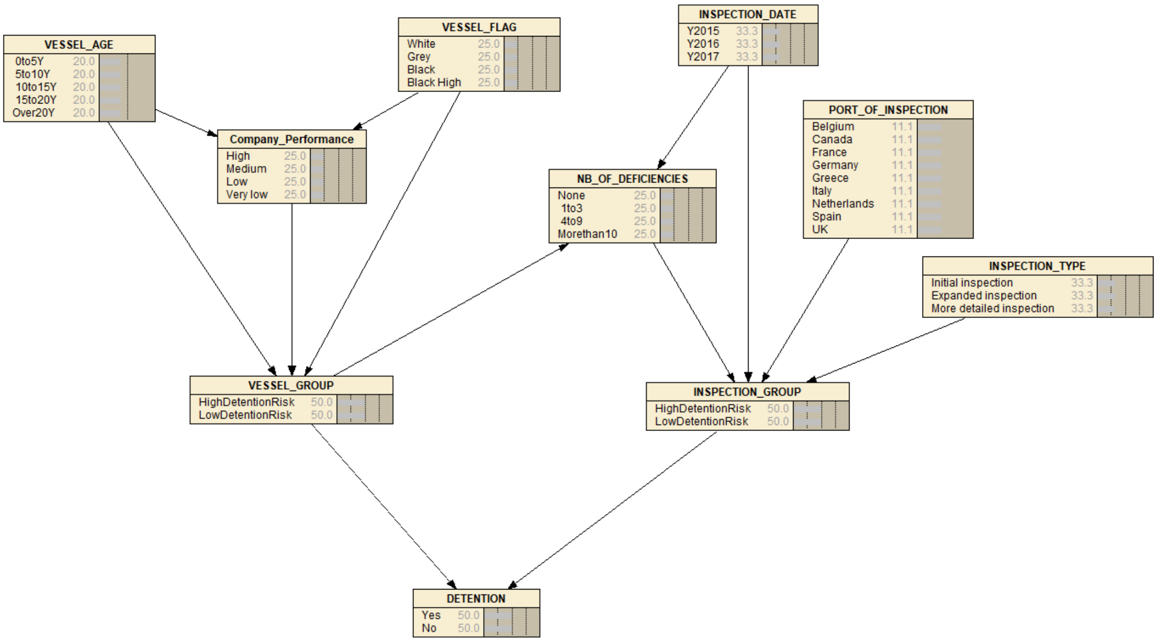

Figure 1. The structure of BN

3.1.4 Conditional probability distribution and risk prediction

After confirming the structure of the BN, the conditional probabilities of the nodes are required to model the uncertainties of risk variables. In this study, the CPTs are formulated by a gradient descent approach (Jensen, 1999; Bottou, 2010).

Once the BN structure and CPTs are properly constructed, the unobservable situations associated with the PSC inspections can be predicted through the generated posterior probabilities when observable evidence (e.g. root nodes such as vessel age and vessel flag) is provided. Therefore, BN is served as the prediction tool to provide a precise foreseen detention rate (validated by real data, see Yang et al. 2008) under different situations in the inspection game.

3.2 Game model construction

The inspection games between port authorities and ship owners are more like 'supervise-being supervised' activities. In this type of game, the main objective of port authorities is to optimize the social welfare (Florens & Foucher, 1999). Therefore, port authorities take measures to ensure maritime safety, such as maritime safety regulations and conventions and the punishment on illegal ship owners. Although these measures cannot completely eradicate potential maritime hazards, they can certainly stimulate ship owners to improve the quality of their vessels. Simultaneously, ship owners aim at maximizing their benefits, resulting in the search of a balance between the costs and detention. The conflict of objectives forms the game relationship between the port authorities and ship owners.

3.2.1 Assumptions

Before constructing the game model, several assumptions are proposed to conform to the definition of the strategic game. According to the definition and interpretations from Osborne & Rubinstein (1994), the following assumptions are made in this study:

(1) Vessels investigated in this study are bulk carriers, as explained in Section 3.1.1. (i.e. taking up 20% of the overall reports).

(2) The purposes of different stakeholders are as follows.

Ship owners: for maximizing personal interest

Port authorities: for minimizing social welfare losses

(3) The game is a strategic game, and each player holds the correct expectation about the other players' behaviour and acts rationally based on the information about the way that the game was played in the past.

(4) The players make decisions independently and simultaneously, and each player is unaware of the choices being made by the other players.

(5) There are two types of bulk carriers, standard vessels (i.e. M) and sub-standard vessels (i.e. m).

(6) The accident losses caused by standard or sub-standard vessels are the same.

3.2.2 Parameter identification

From the definition of the strategic game, it consists of three elements: 1) a finite set of players, 2) a nonempty set of strategies for each player and 3) a preference relation on the set of strategies. Under a wide range of circumstances, the preference relation of a player in a strategic game can be represented by a payoff function (also called a utility function). The value of the function is referred as payoff (utility). Therefore, when building the inspection game in this study, the parameters need to be identified from these three aspects.

Players

It is obvious that the inspection game involves two players: port authorities and ship owners.

Strategies

(1) Strategy of the port authorities

There are two strategies for the port authorities to treat the vessels arriving at their ports

- Inspect the vessel (with probability X)

- Not inspect the vessel (with the probability $1 - X$)

(2) Strategy of ship owners

When confronted with the inspections, ship owners can pay either a high intensive effort to ensure the vessel to be standard or a low effort to leave the vessel sub-standard. The strategies are expressed as follow.

- High intensive effort: Standard vessels (with probability Y)
- Low effort: Sub-standard vessels (with probability 1-Y)

Payoffs

In any game, payoffs are numbers that represent the motivations of the players. Depending on different games, payoffs may represent profit, quantity, continuous measures (cardinal payoffs), and/or the rank of desirable outcomes (ordinal payoffs). According to the objectives of the players in PSC inspections, the payoffs in the inspection games is defined as profit.

Based on the literatures (e.g. Li et al., 2015) and the inspection record reports, the profit of ship owners consists of the following components: expected detention cost, expected accident loss, inspection cost, maintenance cost and the port charges. Accordingly, the profit of port authorities includes social welfare increase due to detention, the social welfare loss due to accidents, inspection cost, and the port charges.

These parameters influencing payoffs are explained with a particular reference to their state definitions as follows.

(1) Expected detention cost

Related to the choice of port authorities, the expected detention cost is the risk that ship owners face when accepting inspections. Only when the port authorities decide to inspect their vessels, it incurs. Meanwhile, because the inspection results are subject to errors, there exist detention rates (likelihood) for both standard vessels and sub-standard vessels.

Detention rate: D

Detention rate is the probability that a vessel fails to pass the inspection. Meanwhile, it acts like a bond linking ship owners, port authorities and ISM companies. Its value can be obtained through the BN model in Section 3.

Detention-related cost of ship owners: C_{D1}

In general, a ship is not released from detention before all necessary repairs are made, and it even needs to sail to another shipyard for repair if it is not possible to repair these deficiencies at the places of the inspections. Such detention-related cost during the detention period is summarized as the consequence of detention.

According to the definition of risk (i.e. *Risk = Likelihood * Consequence*), the expected detention cost is the product of detention rate and detention-related cost.

$$Expected\ detention\ cost = detention\ rate\ (D) * detention\ related\ cost\ (C_{D1})$$

(2) Expected social welfare increase due to detention

Other than the expected detention cost to ship owners, detention also brings social welfare increase to port authorities. The punishment to shipowners makes their vessels safer and better, as well as generates additional earnings for the ports. This part is set as C_{D2}.

However, the detention-related cost of ship owners does not equal to the increase of social welfare because some cost types of the former are not included in the latter, e.g. operating cost, and fuel cost (if a ship needs to sail to another place for repair).

Similar to the expected detention cost, the expected social welfare increase is the product of detention rate and the punishment.

$$Expected\ social\ welfare\ increase = detention\ rate\ (D) * punishment\ (C_{D2})$$

(3) Expected accident loss

Expected accident loss is the risk of the vessel being caught in maritime accidents. It is composed of accident rate and accident loss.

Accident rate: P

Maritime transportation is risky and hazardous. When sailing at sea, every vessel will face the dangers of maritime accidents. On this occasion, shipowners' effort really matters. A standard and compliant vessel is less likely than a sub-standard one to be caught in an accident.

- P_M: accident probability of standard vessel
- P_m: accident probability of sub-standard vessel

Accident loss: C_A

Accident loss is the consequential cost related to ship owners when an accident happens. Different effort of ship owners can influence the severity of loss, and standard vessel is more likely to better deal with emergencies and cause less loss. Because of limited data availability, the value of vessel is chosen to represent the accident loss in this paper.

- C_{A1}: accident loss of standard vessels
- C_{A2}: accident loss of sub-standard vessels

As a result, the expected accident loss is calculated via the following equation,

$$Expected\ Accident\ loss = accident\ rate\ (P) * accident\ loss\ (C_A)$$

(4) Social welfare loss of accidents: C_{SW}

When an accident happens, it will lead to the loss of social welfare. This type of loss includes environmental pollution, salvage cost, recovery cost and so on. Port authorities should take these losses into their account when calculating social welfare loss. Similar to accident loss of ship owners, different vessel safety levels will cost differently.

- C_{SW1}: social welfare loss of standard vessels
- C_{SW2}: social welfare loss of sub-standard vessels

(5) Inspection cost C_I

When making the decision to inspect a vessel, port authorities need to spend money and human resources. At the same time, it will incur a cost to ship owners as well.

- C_{I1}: inspection cost of port authorities
- C_{I2}: inspection cost of ship owners

(6) Maintenance cost of ship owners: $C\ (P_i,\ i)$

In order to pass inspections and avoid maritime accidents, ship owners will spend a certain amount of money and resources, including technological, operational and preventive costs. The more they invest the higher probability they pass the inspections and avoid the occurrence of accidents. This type of cost is presented as $C\ (P_i,\ i),\ i = m,\ or\ M$

- $C\ (P_M,\ M)$: cost to maintain standard vessels
- $C\ (P_m,\ m)$: cost to maintain sub-standard vessels

(7) Port charges: C_{PC}

When a vessel arrives at a port, it will face some different types of charges from port, e.g. tonnage dues, harbour dues, pilotage dues, berth hire charges and anchorage fee. Unlike detention cost and inspection cost, costs in this part is indispensable for all vessels, no matter the vessel is standard or sub-standard, detained or not detained.

3.2.3 The payoff matrix

In order to determine the optimal strategy for each player, a payoff matrix is applied in this study. It is an $m \times n$ matrix that gives the possible payoff of a two-person game when player 1 has m strategies and player 2 has n strategies. This visual representation approach can describe the payoff of each player under different strategy profiles in Table 2.

Table 2. An example of a payoff matrix

	C	*D*
A	w_1, w_2	y_1, y_2
B	x_1, x_2	z_1, z_2

Player 1's strategies are identified with the rows and player 2's with the columns. The two numbers in each cell are the players' payoffs when player 1 chooses the row strategy and player 2 chooses the column one. For example, the two numbers w_1 and w_2 in first cell means when player 1 chooses strategy *A* and player 2 chooses strategy *C*, the payoff of player 1 is w_1 and the payoff of player 2 is w_2.

When formulating the payoff matrix, the primary work is to figure out the payoffs under different strategy combinations. Based on the identified parameters and the information provided above, the payoff functions of port authorities and ship owners under different situations are provided in equation (1) and equation (2). For the different pairs of strategy combinations, the payoff of each stakeholder can be obtained via inserting the values of parameters reflecting the investigated situation into corresponding function.

Payoff of port authorities = (Expected social welfare increase due to detention – Expected social welfare loss of accident - inspection cost + port charges) (1)

Payoff of ship owners = - (expected detention cost + expected accident loss + inspection cost + maintenance cost + port charges) (2)

Scenario 1: Inspection (port authorities) and standard vessel (ship owners)

(1) Payoff of port authorities

There are two possible results: detention or no detention. From equation (1), there are four components to form the payoff.

Expected social welfare increase exists only when detention occurs, hence it is C_{D2} when the vessel is detained, otherwise it is 0.

Expected social welfare loss of an accident always exists whatever the inspection result is. In this scenario, the accident rate is P_M; and the social welfare loss when an accident happens is C_{SW1}. Therefore, the expected social welfare loss is $P_M \times C_{SW1}$. Other components, the inspection cost and port charges, can be easily obtained as C_{I1} and C_{PC}, respectively.

In summary, the payoff of port authorities is:

- If the vessel is detained (D_M)
  $C_{D2} - P_M \times C_{SW1} - C_{I1} + C_{PC}$
- If the vessel is not detained (1- D_M)

$$0 - P_M \times C_{SW1} - C_{I1} + C_{PC}$$

Overall payoff:

$$D_M \times C_{D2} - P_M \times C_{SW1} - C_{I1} + C_{PC}$$

(2) Payoff of ship owners

According to equation (2), the payoff of ship owners consists of five parts.

Similar to the expected social welfare increase, the expected detention cost is also influenced by the inspection results. If the vessel is detained, the detention-related cost is C_{D1}. If not, ship owners do not need to pay anything.

Since the vessel is at a standard safety level, the probability it encounters an accident is P_M, while the consequence of the maritime accident for ship owners is C_{A1}. Hence, the expected accident loss is $P_M \times C_{A1}$.

In addition, to ensure the vessel's compliance with regulation standards, it will cost ship owners $C\ (P_M, M)$ to maintain the vessel. Furthermore, the inspection cost C_{I1} and the port charges C_{PC} are important expenditure of ship owners. In summary, the payoff of ship owners is:

- If the vessel is detained (D_M)
  $-(C_{D1} + P_M \times C_{A1} + C_{I2} + C(P_M, M) + C_{PC})$
- If the vessel is not detained (1- D_M)
  $-(0 + P_M \times C_{A1} + C_{I2} + C(P_M, M) + C_{PC})$

Overall payoff:

$$-(D_M \times C_{D1} + P_M \times C_{A1} + C_{I2} + C(P_M, M) + C_{PC})$$

Scenario 2: Inspection (port authorities) and sub-standard vessel (ship owners)

In this scenario, the way to calculate the payoffs of each player is similar to scenario 1. The components of payoff need to change to the corresponding values of sub-standard vessels based on the information provided in parameter identification section, e.g. D_M to D_m, P_M to P_m, $C\ (P_M, M)$ to $C\ (P_m, m)$, C_{A1} to C_{A2} and C_{SW1} to C_{SW2}.

However, when a sub-standard vessel is detained, it is asked to repair the deficiencies until the vessel complies with the regulations of the port. This process will improve the safety level of the vessel and reduce the accident probability. In this study, in order to simplify the model, the accident rate is set as P_M for the sub-standard vessel after its detention. At the same time, the expected accident loss and expected social welfare loss also change as follows.

$$\text{Expected accident loss} = \begin{cases} P_M \times C_{A1}, & \text{detention} \\ P_m \times C_{A2}, & \text{no detention} \end{cases}$$

$$\text{Expected social welfare loss}=\begin{cases} P_M \times C_{SW1}, & \text{detention} \\ P_m \times C_{SW2}, & \text{no detention} \end{cases}$$

(1) Payoff of port authorities

- If the vessel is detained (D_m)
  $C_{D2} - P_M \times C_{SW1} - C_{I1} + C_{PC}$
- If the vessel is not detained (1- D_m)
  $0 - P_m \times C_{SW2} - C_{I1} + C_{PC}$

Overall payoff:

$$D_M \times C_{D2} - P_m \times C_{SW2} - C_{I1} + C_{PC} - D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2})$$

(2) Payoff of ship owners

- If the vessel is detained (D_m)
  $-(C_{D1} + P_M \times C_{A1} + C_{I2} + C(P_m, m) + C_{PC})$
- If the vessel is not detained (1- D_m)
  $-(0 + P_m \times C_{A2} + C_{I2} + C(P_m, m) + C_{PC})$

Overall payoff:

$$-(D_m \times C_{D1} + C_{I2} + P_m \times C_{A2} + C(P_m, m) + C_{PC} + D_m \times (P_M \times C_{A1} - P_m \times C_{A2}))$$

If port authorities do not inspect the vessel, the detention will not occur and the values of the inspection-related parameters will be *0*, including the expected detention cost, the expected social welfare loss and the inspection cost. Meanwhile, the risk of being detained is free. As a result, the payoff equation is simplified as:

Payoff of port authorities = (– Expected social welfare loss of accident + port charges) (3)

Payoff of ship owners = - (Expected accident loss + maintenance cost + port charges) (4)

Scenario 3: No inspection (port authorities) and standard vessel (ship owners)

In this scenario, the payoffs of the port authority and ship owners are described as follows respectively.

(1) Payoff of port authorities

$$-P_M \times C_{SW1} + C_{PC}$$

(2) Payoff of ship owners

$$-(P_M \times C_{A1} + C(P_M, M) + C_{PC})$$

Scenario 4: No inspection (port authorities) and sub-standard vessel (ship owners)

In this scenario, the payoffs of port authorities and ship owners are described as follows respectively.

(1) Payoff of port authorities

$$-P_m \times C_{SW2} + C_{PC}$$

(2) Payoff of ship owners

$$-(P_m \times C_{A2} + C(P_m, m) + C_{PC})$$

Summarizing the scenarios above, the payoff matrix of the strategic inspection game is presented in Table 3.

Table 3 Payoff matrix of PSC inspection game

	Standard vessel	***Sub-standard vessel***
Inspection	$D_M \times C_{D2} - P_M \times C_{SW1} - C_{I1} + C_{PC}$ $-(D_M \times C_{D1} + P_M \times C_{A1} + C_{I2} + C(P_M, M) + C_{PC})$	$D_M \times C_{D2} - P_m \times C_{SW2} - C_{I1} + C_{PC} - D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2})$ $-(D_m \times C_{D1} + C_{I2} + P_m \times C_{A2} + C_{PC} + C(P_m, m) + D_m \times (P_M \times C_{A1} - P_m \times C_{A2}))$
No inspection	$-P_M \times C_{SW1} + C_{PC}$ $-(P_M \times C_{A1} + C(P_M, M) + C_{PC})$	$-P_m \times C_{SW2} + C_{PC}$ $-(P_m \times C_{A2} + C(P_m, m) + C_{PC})$

3.2.4 Nash equilibrium solution

Nash equilibrium is the most commonly used solution concept in the game theory. It captures a steady state of the play of a strategic game in which each player holds the correct expectation about the other players' behaviour and acts rationally. When it comes to a Nash equilibrium, no players have another action yielding a better outcome given that every other player chooses his/her equilibrium action.

For the inspection game in this study, the choices of players are not deterministic but are regulated by probabilistic rules. The Nash equilibrium under this condition is called mix strategy Nash equilibrium, and the aim of this section is to find out the mix strategy Nash equilibrium for the PSC inspection game.

According to Osborne and Rubinstein (1994), there is a useful way to calculate mixed strategy Nash equilibrium.

For a finite strategic game G, α is a mixed strategy Nash equilibrium of G if and only if for

every player i in the game, every pure strategy in the support of α_i is the best response to α_{-i}.

($\boldsymbol{\alpha_i}$ means the mixed strategy of player i in the mixed Nash equilibrium, while $\boldsymbol{\alpha_{-i}}$ means the mixed strategies of players without player i.)

In other words,

Every action in the support of any player's equilibrium mixed strategy yields the same payoff for that player.

Based on this principle, the mixed strategy Nash equilibrium solution in this study is obtained in Table 4.

Table 4. The simplified payoff matrix

	Standard vessel(Y)	*Sub-standard vessel(1-Y)*
Inspection(X)	PA_{11}, SO_{11}	PA_{12}, SO_{12}
No inspection(1-X)	PA_{21}, SO_{21}	PA_{22}, SO_{22}

Table 4 presents the simplified payoff matrix. In terms of the payoffs in this table, the equation set is shown as follows

$$\begin{cases} Y \times PA_{11} + (1-Y) \times PA_{12} = Y \times PA_{21} + (1-Y) \times PA_{22} \\ X \times SO_{11} + (1-X) \times SO_{21} = X \times SO_{12} + (1-X) \times SO_{22} \end{cases}$$

Where PA means port authority, SO means ship owner. The first number in each cell represents the payoff of port authorities, while the second represents the one of ship owners.

The equilibrium point is:

$$\begin{cases} X = \dfrac{SO_{22} - SO_{21}}{SO_{11} + SO_{22} - SO_{12} - SO_{21}} \\ Y = \dfrac{PA_{22} - PA_{12}}{PA_{11} + PA_{22} - PA_{12} - PA_{21}} \end{cases} \tag{5}$$

After plugging the payoffs into the corresponding places in equation (5), the Nash equilibrium of the strategic game between port authorities and ship owners is presented in Equation 6 and Equation 7, respectively:

$$X^* = \begin{cases} X_0: \dfrac{P_M \times C_{A1} - P_m \times C_{A2} + C(P_M, M) - C(P_m, m)}{C_{D1} \times (D_M - D_m) + D_m \times (P_M \times C_{A1} - P_m \times C_{A2})} \\ 0, \quad Y^* > Y_0 \\ X_0, \quad Y^* = Y_0 \\ 1, \quad Y^* < Y_0 \end{cases} \tag{6}$$

$$Y^* = \begin{cases} Y_0: \dfrac{D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2}) - D_m \times C_{D2} + C_{I1}}{C_{D2} \times (D_M - D_m) + D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2})} \\ 1, \quad X^* > X_0 \\ Y_0, \quad X^* = X_0 \\ 0, \quad X^* < X_0 \end{cases} \tag{7}$$

This means that if $Y^* > Y_0$, port authorities will not inspect the vessel; if $Y^* < Y_0$, port authorities will inspect the vessel. Only when $Y^* = Y_0$ will port authorities choose the mix strategy $X^* = X_0$. The same goes to ship owners.

According to assumption (6) in Section 3.2.1, the accident loss under standard and sub-standard conditions are set the same ($C_{A1} = C_{A2} = C_A{}^0$, where $C_A{}^0$ is a constant no matter the vessel is standard or not). Therefore, the final Nash equilibrium solution is defined as follows.

$$X^* = \begin{cases} X_0: \dfrac{(P_M - P_m) \times C_A{}^0 + C(P_M, M) - C(P_m, m)}{C_{D1} \times (D_M - D_m) + D_m \times C_A{}^0 \times (P_M - P_m)} \\ 0, \quad Y^* > Y_0 \\ X_0, \quad Y^* = Y_0 \\ 1, \quad Y^* < Y_0 \end{cases} \tag{8}$$

$$Y^* = \begin{cases} Y_0: \dfrac{D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2}) - D_m \times C_{D2} + C_{I1}}{C_{D2} \times (D_M - D_m) + D_m \times (P_M \times C_{SW1} - P_m \times C_{SW2})} \\ 1, \quad X^* > X_0 \\ Y_0, \quad X^* = X_0 \\ 0, \quad X^* < X_0 \end{cases} \tag{9}$$

4. Empirical study and result analysis

To characterize the optimal inspection policy for bulk carriers with respect to the Paris MoU, Nash equilibrium solutions need to be analysed through a numerical case. However, it is very difficult, if not impossible, to acquire the data information of all parameters. Previous scholars chose to simulate the parameter values or discuss them by empirical data (e.g. Florens & Foucher, 1999). Nevertheless, there exists too much noisy vessel data, which requires a screening process before using them in this study. In this paper, data come from three different databases: basic vessel information database (mainly from World Shipping Encyclopedia), casualty database (mainly from the International Maritime Organization and Lloyd's Register of Shipping), and PSC Inspection database of the Paris MoU. The objective is to find out the optimal inspection policy for port authorities.

4.1 BN for calculating the detention rate of PSC inspections

A dataset consisting of 49328 PSC inspection records in 2015-2017 based on the Paris MoU is developed to construct the BN. Among them, 10000 inspection records related to bulker carriers and from nine major members of the Paris MoU are selected to form a bulk carrier dataset used in this work.

Figure 2 shows the result of detention analysis based on the BN model. It indicates that the detention rate of bulk carriers is estimated as 3.25% given the input data covering the period of 2015-2017. If we calculate the detention rate from database directly[1], the result is 3.23%, which shows a harmony with the result delivered by the model. The model is verified in terms of prediction of detention rate of bulk carriers.

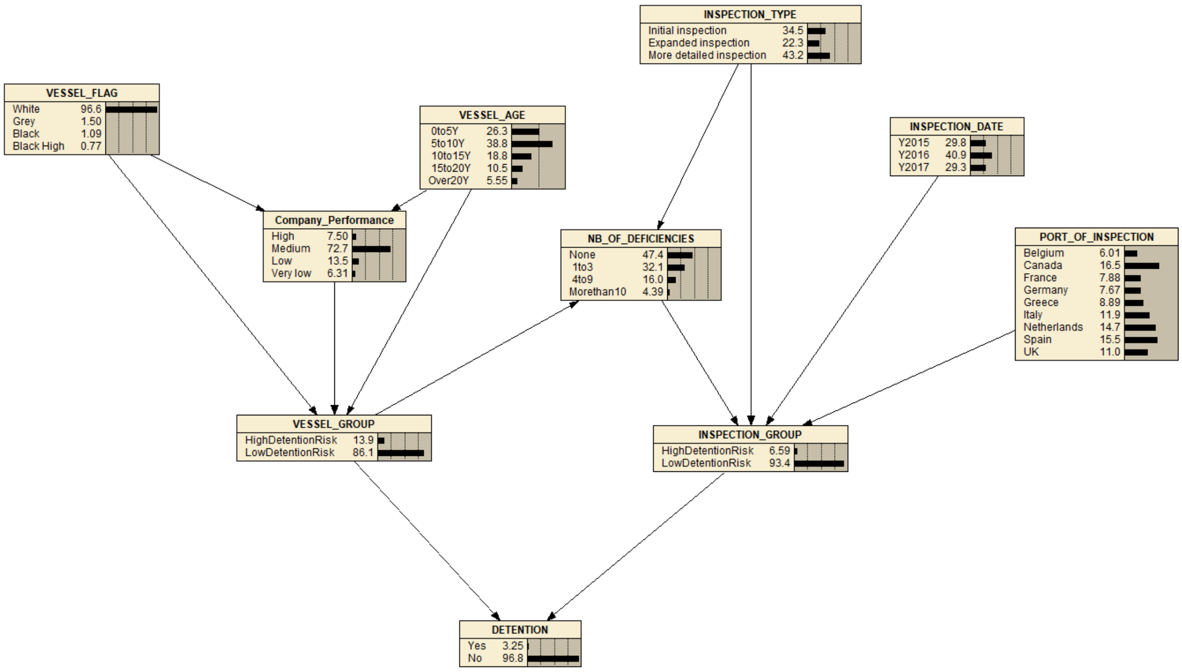

Figure 2. Result of the PSC BN

Since the model developed is proved reliable (Yang et al., 2018), it can be used to predict the detention rate of PSC inspection when any new evidence is observed and collected. Based on the function, the detention rates of different safety levels of any investigated vessel can be obtained.

(1) Standard vessels

If a ship owner makes high intensity effort in maintaining his/her vessel, the vessel will be maintained at a standard safety level and reach the criteria of inspection regulations. During an inspection, the detention risk of the vessel is relatively low, which means the two comprehensive factors 'inspection group' and 'vessel group' that represent two aspects of detention risk are both at a low level. From the BN reasoning, the detention rate is calculated as 0.46% (decrease from the average 3.25%).

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[1] Although based on the database we can calculate the average detention rate of a bulk carrier, but it cannot be used for any other advanced detention analysis for a specific selected bulk carrier as the BN does.

(2) Sub-standard vessels

Accordingly, a sub-standard vessel is more likely to be caught in detention. It indicates the two major risk factors (i.e. ‘vessel group’ and ‘inspection group’), are at the ‘high detention risk’ state. The result of the BN reasoning reveals that the detention rate of a sub-standard vessel is 58.8% (increased from the average 3.25%).

4.2 Determination of the other parameters

4.2.1 Maintenance cost and accident loss

The maintenance cost is crucial for ship owners and it is affected by a large number of factors, e.g. vessel age, material price, regional differences and damage degree. In addition, the effort of ship owners also needs to be considered as an important factor.

Table 5 shows the maintenance cost under different conditions. It contains the maintenance cost of bulk carriers with different sizes and ages in a certain period. For example, the repair and maintenances cost for a young bulk ship with standard effort is US $200,175, while it is only US $120,105 with sub-standard effort (Drewy Shipping Concultants, 2012).

According to United Nations Conference on Trade and Development (UNCTAD) Review of Maritime Transport 2016, there are five types of bulk carriers: small, handysize, handymax, panamax and capesize. The size of five types of vessels is incremental. Based on this, vessel size in this paper is separated into two states: small, handysize and handymax bulk carriers as ‘Small’, panama and capsize bulk carriers as ‘Large’.

Vessel age is classified into three groups ‘Young 0-5 years’, ‘Medium 6-10 years’ and ‘Old over 10 years’.

Table 5. Estimated approximate repair and maintenances under different conditions (US$)

Vessel size	Small					
Vessel age	Young		Medium		Old	
SO’s effort	Stan	Sub	Stan	Sub	Stan	Sub
Bulk carrier	200175	120105	440385	190166	447057	266900
Vessel size	Large					
Vessel age	Young		Medium		Old	
SO’s effort	Stan	Sub	Stan	Sub	Stan	Sub
Bulk carrier	319650	191790	703230	303667	713885	426200

Source: Drewy Shipping Consultants Ltd.

Meanwhile, as mentioned in the parameter identification, the value of vessels is viewed as the accident loss of ship owners. Therefore, the price of second-hand vessels is used as the accident loss in this work.

Table 6. Estimated accident loss under different conditions (US$M)

Vessel size	Small			Large		
Vessel age	Young	Medium	Old	Young	Medium	Old
Bulk carrier	-31	-28	-11	-67	-53	-20

Source: Drewy Shipping Consultants Ltd

4.2.2 Accident rate

The accident rate is calculated by using a logit model (Li et al.,2014), which is an exponential function of various influencing factors shown below.

$$X\beta_i = \beta_0 + \beta_1 VA + \beta_2 VS + \sum_{i=1}^{5} \beta_{i+2} VT_i + \beta_8 CS + +\beta_9 FS + \sum_{j=1}^{30} \beta_{j+9} Z_j + \mu_i$$

where:

VA: vessel age.

VS: vessel size.

VT: vessel type. *VTi=1* if it is a dry cargo ship, otherwise *VTi=0, i = 1, 2, …, 4* indicating the four different vessel types, namely dry cargo, bulk carrier, tanker and container.

CS: classification society. If the vessel is a member of IACS, *CS=1*; otherwise *CS=0*

FS: flag state. If the vessel's flag is a close registry, *FS=1*; otherwise *FS=0*

Zj: dummy variables representing different geographical zones. In this paper, we divide the world into 31 zones according to the *World Casualty Statistics*. Each zone has its own effect on the accident probability.

μi: stochastic component that follows the logistic distribution.

According to the definition of logistic distribution, the accident rates of different vessels are calculated by:

$$\hat{p}_i = \frac{e^{\Sigma \beta_i X}}{1 + e^{\Sigma \beta_i X}} \tag{10}$$

Through applying the Maximum Likelihood Estimator (MLE) method, the estimation of *βi* is obtained.

Eventually, by inserting the values of corresponding parameters into equation (10), the accident rates of bulker carriers under different situations are obtained and presented in Table 7.

Table 7. Accident rates of bulk carriers

Vessel size	Small					
Vessel age	Young		Medium		Old	
SO's effort	Stan	Sub	Stan	Sub	Stan	Sub
Accident rate	0.106	0.278	0.0959	0.26	0.0643	0.227
Vessel size	Large					
Vessel age	Young		Medium		Old	
SO's effort	Stan	Sub	Stan	Sub	Stan	Sub
Accident rate	0.164	0.299	0.111	0.234	0.0669	0.249

4.2.3 Detention cost

Because of the detention punishment from port authorities, avoiding detention with minimum effort is the primary goal of ship owners. At the same time, detention punishment helps regulate the behaviours of ship owners from the perspective of port authorities. Hence, the detention cost (or the detention punishment) C_D is a focus of both sides.

If C_D is not large enough, ship owners may maintain their vessels at a sub-standard safety level. In order to reduce the social welfare loss, port authorities have to increase the inspection rate or extend the detention time; if C_D is large enough, ship owners will turn to improve the quality of vessels, resulting in less inspection costs and a lower accident rate.

In this paper, we assume that C_D has a liner relationship with the expected accident loss of sub-standard vessels, as the punishment policy aims at dealing with illegal actions and sub-standard safety level of the inspected vessels.

$$C_D = \omega C_A P_m$$

where ω representing the punishment intensity, is a positive value and is set differently according to different port inspection policies.

4.3 Optimal inspection rate

As discussed in sections 4.1 and 4.2, all the parameters in equation (8) are constant values, except the detention cost. The detention cost is a dynamic parameter that varies with the punishment intensity ω. That is to say, the optimal inspection rate is actually a function of the punishment severity ω, denoted as $X(\omega)$.

Because ω is a positive variable related to the port inspection regulations, it is impractical to fix it at a certain value to satisfy all the cases. Hence, in this study, the punishment severity is changed to see the optimal inspection rates in various circumstances.

The following table shows the optimal inspection rates when ω changes from 0 to 20 (ω is integar).

Table 8. Optimal inspection rates with different punishment severity levels

	Small			Large		
	Young	Medium	Old	Young	Medium	Old
$\omega=1$	65.025%	63.184%	64.793%	53.061%	55.913%	67.126%
$\omega=2$	40.343%	39.316%	41.103%	31.518%	33.892%	42.716%
$\omega=3$	29.243%	28.536%	30.098%	22.417%	24.315%	31.325%
$\omega=4$	22.933%	22.396%	23.741%	17.394%	18.958%	24.730%
$\omega=5$	18.863%	18.430%	19.602%	14.210%	15.536%	20.429%
$\omega=6$	16.020%	15.657%	16.691%	12.011%	13.160%	17.403%
$\omega=7$	13.921%	13.610%	14.533%	10.402%	11.414%	15.157%
$\omega=8$	12.309%	12.036%	12.870%	9.173%	10.078%	13.425%
$\omega=9$	11.031%	10.788%	11.548%	8.203%	9.021%	12.048%
$\omega=10$	9.994%	9.775%	10.472%	7.419%	8.165%	10.927%
$\omega=11$	9.135%	8.936%	9.580%	6.772%	7.457%	9.997%
$\omega=12$	8.412%	8.229%	8.827%	6.229%	6.863%	9.213%
$\omega=13$	7.795%	7.626%	8.185%	5.766%	6.356%	8.543%
$\omega=14$	7.262%	7.106%	7.629%	5.367%	5.919%	7.964%
$\omega=15$	6.798%	6.652%	7.144%	5.020%	5.538%	7.458%
$\omega=16$	6.389%	6.252%	6.717%	4.715%	5.203%	7.013%
$\omega=17$	6.027%	5.898%	6.339%	4.445%	4.906%	6.618%
$\omega=18$	5.703%	5.581%	6.000%	4.205%	4.642%	6.265%
$\omega=19$	5.413%	5.297%	5.696%	3.989%	4.404%	5.948%
$\omega=20$	5.151%	5.041%	5.422%	3.794%	4.190%	5.661%

Based on the information in table 8, figure 5 provides a diagram to describe the tendency of optimal inspection rates when the punishment severity changes.

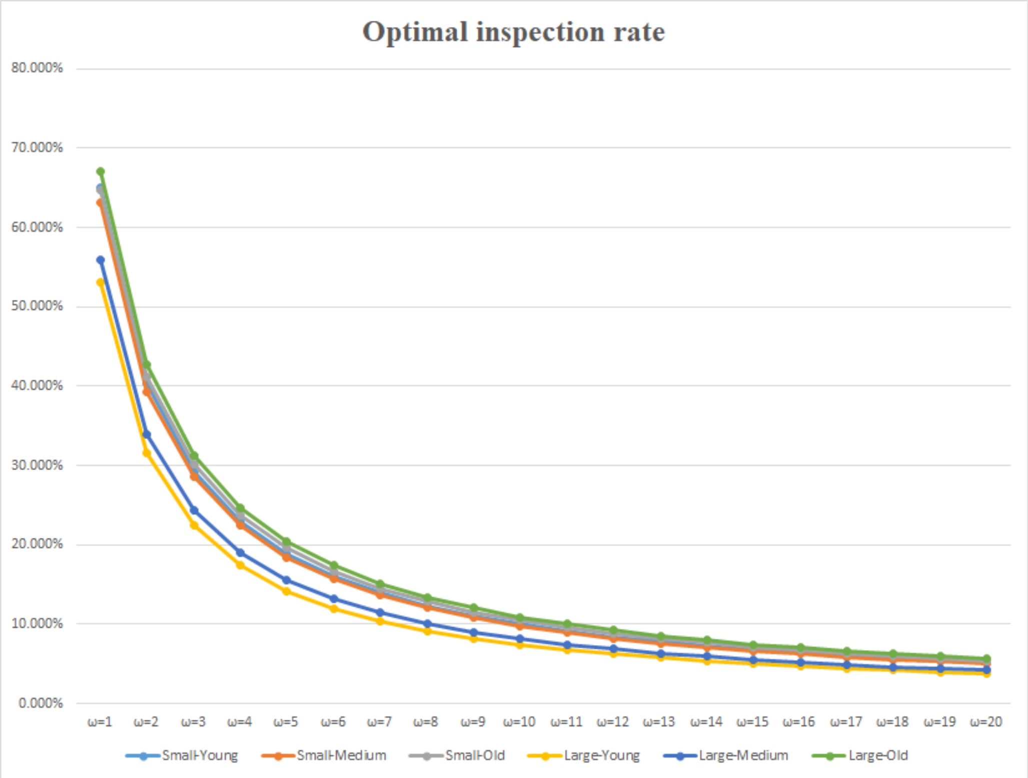

Figure 3. Trend of optimal inspection rate

From table 8 and figure 3, several conclusions are made and research implications are derived.

(1) **With the increase of punishment severity, the optimal inspection rates see a decreasing trend regardless the vessel conditions.**

For example, the optimal inspection rate of small and young bulk carriers at $\omega=1$ is 65.025% and falls to 18.863% when ω *increases to 5*.

Actually, when calculating an optimal inspection rate, the only variable in equation (8) is the severity degree ω. Other parameters, like the accident rate, accident loss, they are all constant. Hence, the function of optimal inspection rate can be written as

$$X(\omega) = \frac{a_3}{a_1\omega + a_2}$$

where a_1, a_2, a_3 are positive constant.

Because of the positive value of ω, therefore, the first derivative test of the optimal inspection rate is

$$X'(\omega) = -\frac{a_1 a_3}{(a_1\omega + a_2)^2}$$

$X'(\omega) < 0$ means the optimal inspection rate is a decreasing function and does not have an extremum. The limiting case lies that when ω is infinitely great, the optimal inspection rate of bulkers is infinitely close to zero regardless the safety condition and characteristics of the vessel.

(2) The declining speed of the optimal inspection rates slows down with the increase of the punishment severity.

It can be explained from *a mathematical perspective:*

The second derivative test of the optimal inspection rate is:

$$X''(\omega) = \frac{2{a_1}^2 a_3}{(a_1\omega + a_2)^3}$$

Because ω is a positive variable,

$$X''(\omega) > 0$$

If the second derivative test of a function is always positive no matter how the variable changes, the first derivative test is an increasing function. When combining it with the result that $X'(\omega) < 0$, we disclose that $X'(\omega)$ is a negative increasing function and $|X'(\omega)|$ is a positive decreasing function.

When represented in the graph, $X'(\omega)$ reflexes the slope of the line $X\,(\omega)$, and $|X'(\omega)|$ measures the steepness or grade of the line. Therefore, a positive decreasing nature of $|X'(\omega)|$ indicates the line of optimal inspection rate is steeper at first and tends to be smooth with the increase of the punishment severity.

In fact, the variation trend reveals that with the increase of the punishment intensity, the sub-standard ship owners' motivation on implementing better safety maintenance policy becomes lower and lower.

(3) **Vessel age has little influence on the optimal inspection rates of small bulk carriers.**

Table 9 illustrate the standard deviation of small bulk carriers. It is obvious that the standard deviation of small bulk carriers is always low no matter how the punishment intensity changes. It means that the dispersion of the data is kept at a low level under all the circumstances. Hence, for small bulk carriers, vessel age has no influence on their optimal inspection rates.

Table 9. The standard deviation of small bulk carriers

	Small			
	Young	Medium	Old	Standard deviation
ω=*1*	65.03%	63.18%	64.79%	0.82%
ω=*2*	40.34%	39.32%	41.10%	0.73%

ω=3	29.24%	28.54%	30.10%	0.64%
ω=4	22.93%	22.40%	23.74%	0.55%
ω=5	18.86%	18.43%	19.60%	0.48%
ω=6	16.02%	15.66%	16.69%	0.43%
ω=7	13.92%	13.61%	14.53%	0.38%
ω=8	12.31%	12.04%	12.87%	0.35%
ω=9	11.03%	10.79%	11.55%	0.32%
ω=10	9.99%	9.78%	10.47%	0.29%
ω=11	9.14%	8.94%	9.58%	0.27%
ω=12	8.41%	8.23%	8.83%	0.25%
ω=13	7.80%	7.63%	8.19%	0.23%
ω=14	7.26%	7.11%	7.63%	0.22%
ω=15	6.80%	6.65%	7.14%	0.21%
ω=16	6.39%	6.25%	6.72%	0.20%
ω=17	6.03%	5.90%	6.34%	0.19%
ω=18	5.70%	5.58%	6.00%	0.18%
ω=19	5.41%	5.30%	5.70%	0.17%
ω=20	5.15%	5.04%	5.42%	0.16%

(4) **Large and old bulk carriers have the highest optimal inspection rates.**

This finding indicates that large and old bulk carriers have higher detention risks than others, prompting port authorities to pay more attention on these types of bulk carriers.

(5) **For young and medium bulk carriers, vessel size has more influential power than vessel age in PSC.**

To compare the effect of vessel size and vessel age on the optimal inspection rate, sensitivity analysis is conducted. When locking one factor and changing the states of another factor (target factor), the change of optimal inspection rate is measured as the effect of the target factor in this scenario.

For example, when $\omega=1$, if the vessel is a small vessel, it can be observed that the optimal rate of young small vessel is 65.03%. On the other hand, the medium small vessel is 63.184%. Therefore, the different value 1.84% is the effect of vessel age on the optimal inspection rate when locking vessel size at 'small' and $\omega=1$. Table 10 shows the individual effect of vessel size and vessel age in different scenarios.

Table 10. Effect of vessel age and vessel size

Target factor	Vessel age		Vessel size	
Locked state	Small	Large	Young	Old
ω=1	1.84%	2.85%	11.96%	7.27%

$\omega=2$	1.03%	2.37%	8.83%	5.42%
$\omega=3$	0.71%	1.90%	6.83%	4.22%
$\omega=4$	0.54%	1.56%	5.54%	3.44%
$\omega=5$	0.43%	1.33%	4.65%	2.89%
$\omega=6$	0.36%	1.15%	4.01%	2.50%
$\omega=7$	0.31%	1.01%	3.52%	2.20%
$\omega=8$	0.27%	0.90%	3.14%	1.96%
$\omega=9$	0.24%	0.82%	2.83%	1.77%
$\omega=10$	0.22%	0.75%	2.58%	1.61%
$\omega=11$	0.20%	0.68%	2.36%	1.48%
$\omega=12$	0.18%	0.63%	2.18%	1.37%
$\omega=13$	0.17%	0.59%	2.03%	1.27%
$\omega=14$	0.16%	0.55%	1.90%	1.19%
$\omega=15$	0.15%	0.52%	1.78%	1.11%
$\omega=16$	0.14%	0.49%	1.67%	1.05%
$\omega=17$	0.13%	0.46%	1.58%	0.99%
$\omega=18$	0.12%	0.44%	1.50%	0.94%
$\omega=19$	0.12%	0.42%	1.42%	0.89%
$\omega=20$	0.11%	0.40%	1.36%	0.85%

It is obvious that vessel size has more influence on the inspection rate than vessel age under various situations. However, the tendency of the impact magnitude gradually decreases when the punishment intensity of port authorities is higher and higher.

4.4 Recommendations for port authorities

This section illustrates how the proposed model and theoretical optimal inspection rates can help port authorities to make their optimal decision in PSC inspections. It is noteworthy that the prerequisite of the suggestion is that port authorities and ship owners make their decisions independently, and both of them are not aware of the choice of the other.

According to the historical inspection data, port authorities can figure out the average detention time of detained bulk carriers under different scenarios, and then calculate the possible social welfare increase per inspection. Based on the proposed optimal inspection rate equation, the optimal inspection rates of vessels under different conditions can be obtained when inserting corresponding values, denoted as X_i (i represents the vessels with different status). Meanwhile, the historical data can tell the numbers of bulker arriving at port per day, denoted as N_i. Therefore, the optimal number of PSC inspections at the port per vessel type per day is X_i N_i, which is useful for port authorities when formulating its inspection regulations.

However, sometimes the resources that port authorities have in reality do not support them to do the exact number of inspections that the Nash solution suggests. On this occasion, port authorities have two strategies:

- Increase the resources for PSC inspection, e.g. PSC inspectors (human resources), funding and operational expenditure.
- If it is not possible to increase the resources, port authorities can use the equation (2) to improve its inspection policy.
  1) Based on the limited resources port authorities have, the maximum number of inspections per day is obtained, which is set as the improved optimal inspection rate X_i'
  2) Input X_i' into equation and use the backward calculation to get the detention loss C_D' and new punishment severity degree ω'. Because $X_i' < X_i$, then $C_D' > C_D$, $\omega' > \omega$. Port authorities can increase the punishment severity degree for the optimal inspection equation.

In general, when port authorities have sufficient resources, they should choose the optimal inspection rate; otherwise they can increase the punishment severity level to tackle the sub-standard effort and illegal actions of ship owners.

5. Conclusion

Despite the fact that PSC inspections came into force for many years, we still see many vessels failing to pass their inspections according to the Paris MoU inspection records. The main reason lies behind this phenomenon is that the interests of port authorities and ship owners are opposite to each other. Meanwhile, since 2009, the performance of ISM companies has been listed as one of the important factors in PSC inspections. There are few studies, based on the authors' best knowledge, investigating the new inspection relationships in recent years.

In this study, a risk-based game model is constructed to figure out the optimal inspection policy for port authorities since the implementation of NIR in 2009. To facilitate the study, a BN is developed based on the inspection data of bulk carriers in 2015-2017 involving nine major countries from the Paris MoU. It is noteworthy that company performance, which is among the most important indicators in NIR, is considered in the BN model to generate the comprehensive detention rate function. The result of BN risk analysis helps determine the detention rate of standard and sub-standard vessels, as well as provides valuable input information for the game model. Through a payoff matrix, the factors influencing the game between the port authorities and ship owners are connected. The incorporation of BN and game model, for the first time, describes the PSC inspection game comprehensively and concretely, presenting the decision-making process of both stakeholders. By calculating the Nash equilibrium solutions of the game model, the optimal inspection rates for port authorities and the optimal maintenance rate for ship owners are obtained.

More precisely, the result reveals the optimal inspection rates of bulk carriers under different conditions from a port authority viewpoint. It can be used as a real-time decision tool for port authorities to respond to ships of different risk profiles under various dynamic situations. The associated variables include the general maintenance cost of the inspection period, the investigated accident rate and accident loss, and the punishment severity of the port.

As to the Nash equilibrium solution, new managerial insights are established and verified through an empirical study investigating the inspections happened in 2015-2017. For example, 1) with the increase of punishment severity, the optimal inspection rates present a decreasing trend regardless the vessel conditions. 2) The declining speed of the optimal inspection rates slows down with the increase of the punishment severity level. 3) Vessel age has little influence on the optimal inspection rates of small bulk carriers. 4) Large and old bulk carriers have the highest inspection rates. 5) For young and medium bulk carriers, vessel size is a factor of more influential power than vessel age in PSC. The above managerial insights can be served as useful information for i) the port authorities when formulating their inspection policy regarding the bulk carrier part and ii) the ship owners when minimizing their ships detention rate given economic constrains.

Based on the findings, there are two suggestions for port authorities when formulating their inspection policies respectively.

- If having sufficient resources for inspections, port authorities can use the calculated optimal inspection rate to determine the number of inspected bulk carriers per day.
- If there are limited inspection resources, port authorities can use the backward calculation function to increase the vessel detention time based on the maximum number of inspections it can afford per day.

Further effort will focus on the improvement of the game model, taking into account the effect of repair at port due to detention, the severity classification of accidents and corresponding accident loss. Data acquisition (i.e. the statistics of total accident loss) presents another issue to investigate in the future research agenda.

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