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Application of a multiple attribute group decision making (MAGDM) model for selecting appropriate maintenance strategy for marine and offshore machinery operations

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Abstract

The process of selecting appropriate maintenance strategy to enhance the operational efficiency of marine and offshore machinery under an uncertain environment is challenging due to the many criteria that need to be considered and modelled. In addition, the design of such complex machinery on-board a vessel consists of many subjective and imprecise parameters contained in different quantitative and qualitative forms. This paper proposes a strategic multi-attribute group decision-making (MAGDM) methodology for the concise and straightforward selection of an appropriate maintenance strategy. The decision support structure allows the use of multiple decision makers to incorporate and aggregate their subjective opinions transparently. In the analysis, a Technique for Order Preference by Similarity to Ideal Situation (TOPSIS) was employed to rank the maintenance strategies with respect to costs and benefits for their subsequent implementation. The purpose of using MAGDM in this paper is to aggregate and synthesise opinions of experts, thus, guiding them in decision making when they are planning to implement a cost effective maintenance investment.

Keywords: Maintenance strategy, multi-attribute group decision making, run-to-failure maintenance, preventive maintenance, condition based maintenance, reliability-centred maintenance.

1 Introduction

In 1979, the Massachusetts Institute of Technology (MIT) carried out an extraordinary milestone study in which it estimated that over $200 billion was spent annually on maintenance in North America. Moreover, approximately one third of this expenditure was determined to be unnecessary. Maintenance, and in particular the effect of mal-lubrication, is still one of the few remaining areas of a company's expenditure that can be significantly improved upon. Many modern engines contain a number of complex systems and thus require a variety of maintenance procedures for reliable, cost effective operation. The increasing cost, complexity of maintenance, other uncertainties, and their effect on production has initiated a need for adequate and proper planning, management, and omission of the maintenance process (Toms and Toms, 2008). Almost all modern maintenance programs include a variation of one or more of the following general maintenance procedures: Run-to-Failure Maintenance, Preventive Maintenance, Condition-Based Maintenance, and Reliability Centred Maintenance.

Therefore, the assessment of the cost of the planned maintenance (PM) strategies may require an advanced cost benefit analysis and a powerful tool for risk management methodology to aid in decision-making. Decision-making can be characterised as a process of selecting a sufficient alternative from a set of alternatives to attain a goal. Many decisions involve uncertainty. In order to overcome the uncertainty and risk that threatens the maintenance, it is important to design a robust expert system that will cater for all the above maintenance procedures.

In this paper, decision makers' opinions are expressed through a process of multi-attribute group decision making and aggregated to obtain the performance rating with respect to all of the attributes for each maintenance procedure alternative. Decision makers' decision matrixes are used and converted into an aggregated decision matrix to determine the most preferable choice among all possible alternatives. Multi-attribute decision-making (MADM) is a tool that is suitable for group decision making under a uncertain environment (Li, 2007). There are a number of Multiple Criteria Decision Making (MCDM) methods in literature, such as Technique for Order Preference by Similarity to Ideal Situation (TOPSIS) (Chen, 2000). A novel method for multi-attribute group decision-making (MAGDM) will be proposed in this paper. In this method, the linguistic terms will be used during the evaluation process, and then TOPSIS is used to rank the alternatives. This novel MAGDM technique can efficiently resolve the uncertain information by decreasing its uncertainty level, is capable of reducing the computation time, and can provide reasonable and robust ranking results.

2 Literature Review

Over the past few decades, the decision making process has evolved into an increasingly more sophisticated approach that includes expert judgements, cost-benefit analysis, risk analysis, and many other methods for collaborative modelling of complex socio-technical systems under a uncertain environment. This evolution has led to an improved range of decision making aids, which have resulted in the development of MCDA tools that offer a scientifically sound decision analysis framework for group decision making in a ductile approach.

According to Reichert *et al.* (2007) as cited in John *et al.* (2014), decision analysis techniques were originally developed to support individual decision makers in carefully considering all aspects of the decision making process. Nonetheless, Ananda and Herath (2003) and Marttunen and Hamalainen (1995) are of the view that because these techniques are used to structure the problem under consideration and to make clear the expectations about outcomes and preferences, they can also be used to support group decisions as well as communicating decisions.

The significant issues described in literature for the effective application of MCDM revolve around the information and data available to characterize a piece of equipment, and the related uncertainties that affect the models and parameters supporting the decision process. Several decision-making problems involve uncertainty; thus, methods that facilitate better and optimum management decisions must account for variations in decision makers' preferences for attributes and conflicting interests in a systematic fashion. As the complexity of decisions increases in complex machineries, it becomes more challenging for decision makers to identify appropriate alternatives. As a result, robust but flexible analytical tools that can account for these difficulties are required to consider the numerous criteria and decision outcomes (John *et al*., 2014).

Several methods of MCDM have been developed, with even small variations to existing methods causing the creation of new branches of research (Velasquez and Hester, 2013). Among these methods are: Multiple Attribute Decision Making (MADM), Multiple Objective Decision Making (MODM), Multi-Attribute Utility Theory (MAUT), Analytic Hierarchy Process (AHP), Fuzzy Set Theory (FST), Case-based Reasoning (CBR), Data Envelopment Analysis (DEA), Simple Multi-Attribute Rating Technique (SMART), Goal Programming (GP), Elimination and Choice Expressing Reality (ELECTRE), Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), Simple Additive Weighting (SAW), TOPSIS, *etc*. These methods are the most important branches of decision making under uncertainty and involve decision makers' subjective judgements.

2.1 Analysis of Multiple Attribute Group Decision Making Methods

MADM methods are designed to evaluate and select the desired alternative from a set of alternatives, which are characterised by multiple criteria. If more than one person is interested in the same MADM problem, it then becomes a MAGDM problem (Yang *et al.*, 2014). For both MADM and MAGDM problems, consistency among the preference relations is crucial to the result of the final decision. Guo (2013) perceives MAGDM as one of the most common activities in modern society, involving the selection of the optimal option, from a finite set of alternatives with respect to a collection of predefined criteria, by a group of experts with a high collective knowledge level on these particular criteria.

Moreover, as stated in Bozóki (2008), the determination of attribute weight is also a key issue to be considered when using the MAGDM approach. In many decision cases, some attributes are considered to be more important in the experts' judgment than the others. However, for these vital attributes, the preference relation provided by experts may be quite similar for all alternatives. Even for the attribute with the highest weight, the degree of influence on the final decision could be very small. Consequently, Wang and Fan (2007) regard this kind of attribute as being unimportant to the final decision. Thus, during the multiple attribute group decision process, the following five guidelines should be noted:

1. Different assessment types need to be considered concurrently.
2. Experts' preference relations that have been provided need to be consistent.
3. Diverse expert's opinions need to be taken into consideration.
4. The weight of each attribute needs to be determined.
5. All alternatives need to be carefully ranked.

MADM is an algorithm deployed to solve problems involving selection from a list of alternatives. It specifies how criteria or attribute information can be processed in order to arrive at a choice suitable for investment. MADM methods generally require comparisons of criteria with respect to alternatives for efficient trade-offs. In a MADM process, each decision table (also called decision matrix) has four main parts; these can be summarised as follows:

- Alternatives.
- Criteria or Attributes.
- Weight of experts or relative importance of each attribute.
- Performance measure of alternatives with respect to criteria.

Based on the analysis of MCDA methods, the basic information in a MADM model can be represented in the matrix as presented in Equation 1.

$$Z = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \\ (w_1 & w_2 & \cdots & w_m) \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} & \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,m} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,m} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n,1} & y_{n,2} & \cdots & y_{n,m} \end{bmatrix} \end{array} \tag{1}$$

where A_i $(i = 1, 2, \dots n)$ is the i^{th} alternative; C_i $(i = 1, 2, \dots m)$ is the i^{th} set of criterion with which each alternative's performances can be measured; $y_{i.j}$ $(i = 1, 2, \dots n)$; $(j = 1, 2, \dots m)$ is the measure of performance of the i^{th} alternative with respect to the m^{th} criterion; and w_j $(j = 1, 2, \dots m)$ is the j^{th} criterion weight. It is important to stress here that all the elements in the decision matrix must be normalised to the same units, so that all the possible attributes in the decision problem can be dealt with easily to eliminate any computational difficulty.

There are four means of normalisation in a MADM problem (Lavasani *et al.,* 2012). The two most popular methods are summarised as follows:

- Linear Normalisation: This method divides the rating of n attribute by its maximum value. Usually, the normalised value of $p_{i,j}$ can be obtained using Equation 2.

$$p_{i,j} = \frac{y_{i,j}}{y_j^*}, i = 1, 2, \dots m \tag{2}$$

where y_j^* is the maximum value of $y_{i,j}$. $p_{i.j}$ values range between 0 to 1 $(0 \le p_{i,j} \le 1)$.

- Vector normalisation: This method divides the ratings of each attribute by its average, so that each normalised rating of $y_{i,j}$ can be obtained by Equation 3.

$$p_{i,j} = \frac{y_{i,j}}{\sqrt{\sum_{i=1}^{n} y_{i,j}^2}}, i = 1, 2, \dots n; j = 1, 2, \dots m \tag{3}$$

Both Equations 2 and 3 are used for cost and benefit criteria respectively. Normally, an alternative in a MADM problem is often described using qualitative variables expressed by decision makers. However, when no criteria evidence or information is available, the preferred approach is to use fuzzy set theory, which has the capability of handling such a situation under varying constraints (John *et al*., 2014).

One of the theoretical approaches to preference relations used for MADM problems is fuzzy preference relations. The majority of real-life complex problems have fuzzy information about the alternatives with respect to criteria, and it is usually difficult for crisp numerical values to be provided by the subjective opinions of decision makers due to their inadequate knowledge, and the intrinsic complexity and uncertainty within the decision-making environment. The Fuzzy Multiple Attribute Decision Making (FMADM) technique can then be used to handle these complex decision making problems, which are incomplete and unquantifiable. FMADM is an attractive approach, as it is able to actualise decision-making processes for complex equipment that has uncertainty in its operational procedures.

Hypotheses, approximations and judgments of experts are very often required in studies involving complex machineries, in order to handle the imprecision and vagueness associated with making strategic decisions about the operations of these machineries under uncertain conditions. Obviously, criteria values information is presented in the form of linguistic variables, which are generally calibrated from fuzzy scales. According to Yang *et al.* (2011), the calibration of this information from fuzzy scales is due to the fact that fuzzy logic provides the needed flexibility to represent vague information that results from a lack of data or knowledge of the piece of equipment under investigation.

Several studies have been conducted on how to handle the aforementioned challenges in managing incomplete information and to linguistically model these machineries in a systematic manner (Cabrerizo *et al.*, 2009). Although the results of these studies have shown some advantages in dealing with imprecise and incomplete information for MCDM problems, the research has been widely criticised because of the complex computational algorithm involved, which makes it difficult to use for analysts who are not knowledgeable in modern computational analysis. Therefore, there is a need for a user-friendly fuzzy decision support algorithm that can guide effective decisions in a simplified manner.

A FMCDM problem can be defined as follows:

Let A = {A_i, for i = 1, 2, 3..., *m*} be a (finite) set of decision alternatives and G = {g_j, for j = 1, 2, 3…, *n*} be a (finite) set of goals according to which the desirability of an action is judged. Determine the optimal alternative A^+ with the highest degree of desirability with respect to all relevant goals g_j (Zimmermann, 1991).

According to Hipel *et al*. (1993), a decision problem is said to be complex and difficult where the following conditions apply:

1. Multiple criteria exist, which can be both quantitative and qualitative in nature.
2. There may be multiple decision makers.
3. Uncertainty and risk is involved.
4. Decision (input) data may be vague, incomplete or imprecise.

The FMCDM is applied in this model due to the fact that the decision-making process for the selection of an ideal maintenance strategy for a piece of equipment in a marine and offshore environment involves a subjective analysis of uncertain and/or incomplete data.

2.2 Fuzzy TOPSIS

Fuzzy TOPSIS (FTOPSIS) is a fuzzy extension of TOPSIS. It was modelled to efficiently handle the fuzziness of the data to be applied in the decision-making process. According to Sodhi and Prabhakar (2012), the FTOPSIS method can help in objective and systematic evaluation of alternatives on multiple criteria. It has been demonstrated to be a robust tool for handling complex and real-life problems (Vahdani *et al.,* 2011) for collaborative modelling and decision-making processes in an uncertain environment. A fuzzy approach to TOPSIS is useful because it assigns the relative importance of attributes using fuzzy numbers instead of precise numbers (Kabir and Hasin, 2012).

Linguistic preferences can easily be converted to fuzzy numbers and TOPSIS allows for the use of these fuzzy numbers in the calculation (Pam, 2013). FTOPSIS is widely applied in many areas such as landfill site selection (Beskese *et al.,* 2015), failure mode and effect analysis (FMEA) (Shan and Shao, 2015), and fire and explosion in the process industry (Yazdi *et al.,* 2018). In order to apply FTOPSIS to a MCDM problem, selection criteria have to be monotonic. A monotonic relationship is a relationship that does one of the following: (1) as the value of one variable increases, so does the value of the other variable; or (2) as the value of one variable increases, the other variable value decreases. Monotonic criteria could be classified as either benefits (B) or costs (C). A criterion can be classified as a benefit if the more desirable the candidate, the higher its score versus this criterion. On the contrary, cost criteria see the least desirable candidate scoring at the lowest. In FTOPSIS, the cost criteria are defined as the most desirable candidates scoring at the lowest, while the benefit criteria are described as the most desirable candidate scoring at the highest.

Other advantages of the FTOPSIS technique as highlighted in Bottani and Rizzi (2006), Kore *et al.* (2017) and Puente *et al.* (2015) include the fact that:

1. Fuzzy logic is conceptually easy to understand.
2. The mathematical concepts behind fuzzy TOPSIS are simple.
3. It is a realistic compensatory modelling method which includes or excludes alternative solutions based on hard cut-offs.
4. With any given system, it is easy to layer on more functionality without starting again from scratch.

Trapezoidal fuzzy numbers are applied in the FTOPSIS used in this study. This is because it is intuitively easy for the decision-makers to use and calculate

(Dagdeviren *et al*., 2009). Secondly, modelling using trapezoidal fuzzy numbers has proven to be an effective way to formulate the decision making problem where the information is subjective and inaccurate (Dagdeviren *et al*., 2009).

Based on Figure 1, a trapezoidal fuzzy membership function can be described as follows:

$$\tilde{\mu}_A(x) = \begin{cases} 0, x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, x \in [a_1, a_2] \\ 1, x = [a_2, a_3] \\ \frac{a_4-x}{a_4-a_3}, x \in [a_3, a_4] \\ 0, x \geq a_4 \end{cases} \tag{4}$$

where a_2 and a_3 are the modal, and the upper and lower boundaries are a_1 and a_4 respectively.

Insert Figure 1 here

While the uncertainty issue is tackled by means of fuzzy logic, the application of TOPSIS makes it possible to appraise the distances of each decision option from the positive ideal solution and the negative ideal solution (Nadaban *et al.,* 2016). The capability and efficiency of FTOPSIS in handling complex engineering solutions, simultaneously considering positive and negative ideal solutions, having flexibility in computational analysis, and providing systematic and logical results' evaluation, make it useful for strategic decisions to select the most ideal maintenance strategy for marine and offshore machinery.

Moreover, the way linguistic ratings and weights are given is very straightforward. A Fuzzy-TOPSIS approach has been applied in this study in order to support the evaluation of decision-making criteria and attributes. Thus, the FTOPSIS framework is being incorporated and presented in the following section.

3 Methodology

The proposed methodology and hierarchical structure describing the decision making process of selecting an ideal maintenance strategy for marine and offshore machinery is graphically illustrated in Figure 2. The first stage is the identification of decision making alternatives for marine equipment maintenance. The decision alternatives and evaluation criteria are literature-based and have been derived from various literature reviews. The evaluation process is conducted by decision analysts based on their subjective knowledge and judgment on marine equipment maintenance practice.

The second stage in the methodology is the identification of the evaluation criteria for the identified proto-type maintenance strategies. In the third stage, the AHP methodology is applied to obtain the importance weights of the evaluation criteria. In the fourth stage, FTOPSIS is applied to obtain performance ratings of the various decision alternatives. The importance weights obtained through the AHP are incorporated into the FTOPSIS analysis to obtain performance ratings of the decision alternatives.

A Microsoft Windows Application (Excel) is used to compute the performance ratings of these alternatives. Results of the decision analysis are ranked in their order of preference by the analysts for a final selection and adaptation by the decision-makers (*e.g.* Maintenance Engineer on-board) or end-users within the marine and offshore industry.

Insert Figure 2 here

3.1 Identification of Decision-Making Alternatives (**Step one**)

The four decision making alternatives (Run-to-failure, preventive maintenance, condition-based maintenance, and reliability centred maintenance) described below have been identified and applied in this model. The maintenance strategies have been selected from the operations and maintenance best practices, as well as the machinery oil analysis, methods, automation, and benefits recommended by Sullivan *et al.* (2010) and Toms and Toms (2008), respectively.

3.1.1 Run-to-failure maintenance (RTFM)

Run-to-failure maintenance is basically the “run it till it breaks” maintenance approach. It is also known as reactive maintenance (Sullivan *et al.,* 2010) or corrective maintenance (Toms and Toms, 2008). In this type of maintenance approach, no actions or efforts are taken to maintain the equipment, as the designer originally anticipated the use of the equipment until the design life is reached. However, Toms and Toms (2008) believe that this type of maintenance is the action of affecting repairs when some part breaks down or ceases to function properly.

3.1.2 Preventive maintenance (PM)

Preventive maintenance can be defined as an action performed on a time or machine-run-based schedule that detects, prevents, or mitigates degradation of a component or system, with the aim of sustaining or extending its useful life through controlling degradation to an acceptable level (Sullivan *et al.,* 2010). It is a periodic component replacement. Preventive maintenance is not the optimum maintenance program, but it does have several advantages over that of a purely reactive program. By performing the preventive maintenance as the equipment designer projected, the life of the equipment can be extended nearer to design. Preventive maintenance that involves lubrication, a filter change, etc. will generally ensure the efficient running of the equipment and will result in cost savings (Sullivan *et al.,* 2002; Kimos, 2009). While catastrophic equipment failures cannot be prevented, the number of failures can be decreased. Thus, minimizing failures can translate into maintenance and capital cost savings.

3.1.3 Condition-based maintenance (CBM)

Condition-based maintenance is a type of maintenance used in determining the optimum time at which to perform specific maintenance by monitoring the operation and condition of each component in a given application (Toms and Toms, 2008). According to Sullivan *et al.* (2010), CBM is also known as "Predictive Maintenance", and can be described as an attempt to refine maintenance activities to only those times when they are functionally necessary, based on data collection, analysis, and (negative) trend determination from an established "healthy" base level. Condition-based maintenance is best used in situations where equipment is critical to operations and the appropriate monitoring system is reliable and economical.

3.1.4 Reliability centred maintenance

Reliability centred maintenance (RCM) is a systematic approach to evaluate a facility's equipment and resources to a high degree of facility reliability and cost-effectiveness (Sullivan *et al.,* 2010). The philosophy of RCM employs the three maintenance strategies mentioned above in an integrated manner to increase the probability that a piece of equipment / component will function as expected over its design life cycle with minimum maintenance. The goal of this philosophy is to provide the stated function of the facility, with the required reliability, and at the lowest cost. One of the prerequisites of RCM is that maintenance decisions be based on maintenance requirements supported by rigorous technical and economic justification.

3.2 Identification of Evaluation Criteria (**Step two**)

ABB (2016) and Toms and Toms (2008) identify reliability, cost effectiveness, operational safety, availability, and equipment downtime as the main attributes critical to enhancing the selection of an ideal maintenance strategy in an uncertain environment. These five attributes, described below, have been applied in this model as evaluation criteria to reduce the elicitation process and to serve as a check for completeness and transparency.

3.2.1 Reliability

The study of component and process reliability is the basis of many efficiency evaluations in operation management (Carlo, 2015). Reliability has long been considered to be one of the three related attributes that must be taken into consideration when making, buying, or using a piece of equipment or component. It describes the ability of a system or component to function under stated conditions for a specified period of time. However, Toms and Toms (2008) identify reliability as the probability that an equipment system will operate at a specified performance level for a specific period. ABS (2016) also perceives reliability as the probability that an item will perform its intended function for a specified interval under stated conditions.

3.2.2 Cost

This cost includes equipment capital cost, cost due to unplanned downtime of equipment, labour cost, and cost involved with repair or replacement of equipment. An independent study conducted by Forrester Consulting on behalf of ABB Turbocharging reveals that organisations are under pressure to reduce cost, and that the three quarterly reports always consider the cost implications of parts and services (ABB, 2016). History, however, reveals that not all equipment operators utilize maintenance strategy in the most cost effective manner (Taylor, 1995). Decreasing unplanned downtime, and costs of maintenance, availability, and reliability are therefore significant considerations for investing in capital-intensive machinery.

3.2.3 Safety

There are numerous definitions of safety among professionals and researchers in the safety and risk fields. For example, Leveson (1995, 2004) cited in Aven (2013) defines safety as "the absence of accidents, where an accident is defined as an event involving an unplanned and unacceptable loss". Safety is also linked to risk and uncertainty as Moller *et al*. (2006) views safety as the opposite of risk, while, Aven (2013) considers epistemological uncertainty of great importance when discussing safety and safety matters, but argues that this uncertainty aspect is not reflected in many perceptions of risk.

Safety can also refer to the control of recognized hazards in order to achieve an acceptable level of risk. Safe operation of marine and offshore equipment is very important, thus, the general safety guidance for equipment is to be adhered to at all times. Potential hazards of operating machines and equipment are numerous, and thus, machine and equipment operators are encouraged to become familiar with the standards for safe machine and equipment operations relevant to their work (Toms and Toms, 2008). With this, it is envisaged that risks associated with the machines / equipment can be reduced to a feasible and acceptable level.

3.2.4 Availability

Availability, according to Carlo (2015) may be defined as the percentage of time that a repairable system is in an operating condition. Toms and Toms (2008) view equipment availability as the degree to which the machine / equipment in context is in a specified operable and committable state at the start of operation, when the operation is called for at an unknown (*i.e*. a random) time. This basically means that the machine / equipment is suitable and ready for use when needed. However, in literature, equipment availability depends on the reliability and maintainability of that equipment, and availability itself therefore, depends on the time between two consecutive failures, and how long it will takes to restore the system. The ability to measure and control costs of equipment deterioration has an obvious direct impact on equipment availability and operational costs (Toms and Toms, 2008).

3.2.5 Equipment downtime

A period during which an equipment or machine is not functional or cannot work is referred to as downtime. Downtime can occur due to technical failure, machine adjustment, maintenance, or non-availability of inputs such as materials, labour, and power (ABB, 2016), (Toms and Toms, 2008). An independent study for ABB Turbocharging found that 87 percent of organizations work only or mostly with Original Equipment Manufacturers (OEMs) for maintenance support and spare parts procurement (ABB, 2016). Key benefits cited were reduced downtime and better parts availability according to the Forrester Consulting Technology Adoption Profile.

3.3 Rating Phase - Determination of Importance Weights (**Step three**)

As indicated in the model hierarchy for decision making, the rating phase deals with the determination of importance weights (which includes experts' weights, the criteria's weights with respect to the alternatives), defuzzifying the weights and normalising the decision matrix with respect to the goal. In the next step, the experts allocate linguistic variables to the criteria and the alternatives, respectively. The linguistic terms are calibrated into fuzzy triangular numbers for their fuzzy numbers. Then, FTOPSIS is adopted to aggregate the criteria and the alternative ratings to generate an overall score of the alternatives for ranking.

In fuzzy set theory, conversion scales are applied to transform the linguistic terms into fuzzy numbers for system modelling and analysis. In this study, a conversion scale proposed by Chen and Hwang (1992), is being adopted to rate the evaluation criteria with respect to the decision alternatives. As presented in Figure 3, both the performance score (x) and the membership degree (μ_x) are in the range of 0 and 1.

Insert Figure 3 here

Considering Figure 3, when the performance score is between 0 and 0.2, the linguistic assessment is considered to be Very Low, and 0 to 0.1 performance score is considered to be 100% Very Low. Between 0.1 and 0.4 the linguistic assessment is considered to be Low, and 0.25 is considered to be 100% Low. Between 0.3 and 0.7 the linguistic assessment is considered to be Medium, and 0.5 is considered to be 100% Medium. Between 0.6 and 0.9, the linguistic assessment is considered to be High, and 0.75 is considered to be 100% High. Between 0.8 and 1, the linguistic assessment is considered to be Very High, with the range from 0.9 to 1 considered to be 100% Very High.

The triangular fuzzy numbers in Figure 3 are converted to trapezoidal fuzzy numbers for easy computational analysis in this section, so that information can be represented in a concise and precise manner, as shown in Table 1.

At this stage, a series of calculations are conducted on weights of the alternatives and experts used during the collaborative modelling process. To establish a decision matrix for the evaluation process, as shown in Figure 3, expert opinions on the decision alternative with respect to each criterion can be made using linguistic variables. Linguistic variables are often used when describing situations that are too complex and fuzzy to be analysed quantitatively (Vahdat *et al.,* 2014a). Human judgements, including preferences, are often vague and their preferences cannot be indicated by an exact numerical value (Vahdat *et al.,* 2014b), therefore, a more realistic approach may be to use linguistic assessments such as “very good”, “medium good” and “good” instead of numerical values.

Insert Table 1 here

3.3.1 Estimating weights of experts

The weight of the expert can be determined in a simplified manner using established methods such as simple rating methods or more elaborate methods based on the weighting scores and factors. For this study, the weights of the experienced experts used are considered to be equal.

3.3.2 Estimating weights of criteria

The weights of criteria have played a vital role in measuring the overall preference values of the alternatives in many MCDM models. Based on the different assumptions on $U(Z(x))$ or $U(R(x))$, MCDM models have different aggregation rules that allow the use of the criteria weights in different ways. Moreover, distinct methods for assessing criteria weights are designed for different aggregation rules (Choo *et al.,* 1999). In this study, the weights of the five criteria proposed are considered to be equal.

3.3.3 Aggregation of experts' opinions

When carrying out collaborative modelling of large and sophisticated engineering machinery, experts may have different opinions; thus, it is essential to aggregate these opinions in a logical, systematic, and simplified manner. In line with the modelling approach presented in Hsu and Chen (1994), consider that each expert E_u $(u = 1, 2, 3, \dots M)$ expresses their opinions on a particular criterion based on their expertise by a set of linguistic variables that are described by fuzzy numbers. The aggregation of the experts' judgement can be obtained as follows:

1. Calculate the degree of agreement (degree of similarity) $S_{uv}(\check{\delta}_u \check{\delta}_v)$ of the opinions $\check{\delta}_U$ and $\check{\delta}_v$ of a pair of experts E_u and E_v where $S_{uv}(\check{\delta}_u \check{\delta}_v)$ ∈ (0, 1). Based on this approach, $\tilde{X}$ = (a_1, a_2, a_3, a_4) and $\tilde{Y}$ = (b_1, b_2, b_3, b_4) are trapezoidal fuzzy numbers. The degree of similarity between these two fuzzy numbers can be evaluated by the similarity function S defined as follows (Hsu and Chen, 1994):

$$S(\check{X}, \check{Y}) = 1 - \frac{1}{4} \sum_{i=1}^{4} |a_i - b_i| \quad (5)$$

where $S(\check{X}, \check{Y})$ ∈ (0, 1). It is important to mention that the larger the value of $S(\check{X}, \check{Y})$, the greater the similarity between two fuzzy numbers of $\check{X}$ and $\check{Y}$ respectively.

2. Calculate the degree of average agreement (AA) of expert E_u; this can be obtained using Equation (6).

$$AA(E_u) = \frac{1}{N-1} \sum_{\substack{u \neq v \\ v=1}}^{N} S(\check{\delta}_u, \check{\delta}_v) \quad (6)$$

3. Calculate the relative agreement (RA) degree $RA(E_u)$ of experts E_u; this can be obtained as follows:

$$RA(E_u) = \frac{AA(E_u)}{\sum_{u=1}^{N} AA(E_u)} \quad (7)$$

4. Calculate the consensus coefficient degree CC of experts E_u $(u = 1, 2, \dots M)$; this can be analysed as follows:

$$CC(E_u) = \beta . w(E_u) + (1 - \beta) . RA(E_u) \quad (8)$$

where $\beta (0 \leq \beta \leq 1)$ is a relaxation factor of the proposed approach. It highlights the importance of weight of expert $w(E_u)$ over $RA(E_u)$. It is important to note that when $\beta = 0$, no importance has been given to the weight of experts and, thus, a homogeneous group of experts is used. When $\beta = 1$, then the consensus degree of an expert is the same as its importance weight. The consensus coefficient degree of each expert is a good measure for evaluating the relative worthiness of judgement of all experts participating in the decision making process. John *et al.*, (2014) believe that it is the responsibility of the decision maker to assign an appropriate value of β. Moreover, a sensitivity analysis can be applied by varying β.

5. The expert aggregation judgement $\breve{R}_{AG}$ can be obtained as follows:

$$\breve{R}_{AG} = CC(E_1) \times \tilde{R}_1 + CC(E_2) \times \tilde{R}_2 + \cdots + \cdots CC(E_m) \times \tilde{R}_n \quad (9)$$

where $\tilde{R}_i (i = 1, 2, \dots n)$ is the subjective rating of a given criterion with respect to alternative by expert $E_u (u = 1, 2, \dots m)$.

3.3.4 Defuzzification of the aggregated fuzzy results

In order to rank the alternatives of the decision problem, all aggregated fuzzy numbers must be defuzzified. Each element of matrix $\tilde{x}_i = (a_1, a_2, a_3, a_4)$ can be converted to a crisp value using Equation 10 proposed by Sugeno (1999) using the centre of area defuzzification technique. Equation 10 is adapted within this study because of the ease in the computation process compared to other techniques in the literature, such as Chen (2000).

$$X^* = \frac{\int_{a_1}^{a_2} \frac{x-a}{a_2-a_1} x dx + \int_{a_2}^{a_3} x dx + \int_{a_3}^{a_4} \frac{a_4-x}{a_4-a_3} x dx}{\int_{a_1}^{a_2} \frac{x-a}{a_2-a_1} dx + \int_{a_2}^{a_3} dx + \int_{a_3}^{a_4} \frac{a_4-x}{a_4-a_3} dx} = \frac{1}{3} \frac{(a_4+a_3)^2 - a_4 a_3 - (a_1+a_2)^2 + a_1 a_2}{a_4+a_3-a_1-a_2} \quad (10)$$

3.4 Selection Phase - Application of FTOPSIS Approach to Obtain Performance Rating of Decision Alternatives (**Step four**)

Selection of best maintenance strategies often requires analysts to provide both quantitative and qualitative assessments for determining the performance of each alternative with respect to each criterion. A modelling approach that will handle uncertain, imprecise, indefinite, and subjective data that often result from such assessments in a flexible manner is required. As a consequence of that, this study utilises a FTOPSIS algorithm (Yang *et al.*, 2009), (Lin and Chang, 2008), (Wang and Lee, 2007), (Jahanshahloo *et al.*, 2006), and (Chen, 2000) due to the fact that fuzzy sets might provide the needed flexibility to represent the vague information resulting from the lack of data or knowledge. TOPSIS can reasonably deal with the multiplicity of the criteria in order to rank the alternatives based on the aggregated decision matrix and weight vector analysis. To carry out the assessment, consider x possible alternatives $A_1, A_2, A_3 \dots A_x$ from which E_u decision-makers $E_u =$

$(1, 2, 3, \dots m)$ have to make a credible decision on an appropriate maintenance strategy on the basis of n sets of criteria C_1, $C_2, C_3, \dots$ C_n. The decision support procedure is achieved through the following steps:

3.4.1 Fuzzy decision matrix construction

This step involves choosing appropriate linguistic variables for the alternatives with respect to criteria. Suppose the aggregation rate of alternative $A_1 (i = 1, 2, \dots x)$ for criteria $C_1 (j = 1, 2, \dots n)$ is (t_{ij}). Therefore, TOPSIS can be expressed in a matrix format as follows:

$$Z = (t_{ij})_{y \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ t_{x1} & t_{x2} & \cdots & t_{xn} \end{bmatrix} \end{array} \quad i = 1, 2, \dots., x; \; j = 1, 2, \dots., n \tag{11}$$

where, matrix Z is composed of x alternatives and n criteria.

In the proposed model, the process for the estimation of the values for the best maintenance strategy for marine and offshore machinery will depend on expert knowledge and judgement of the decision analysts.

3.4.2 Fuzzy decision matrix normalisation

After producing the decision matrix for the alternatives, the fuzzy data obtained in the matrix are normalised in order to eliminate the units of criteria scores, so that numerical comparisons often associated with MCDM problems can be brought to the same perception. The process involves dividing the score within each criterion by the root-sum-of-squares for all the decision-making criteria. Normalisation has two main aims:

1. For the comparison of heterogeneous criteria.
2. To ensure that all triangular fuzzy numbers are ranged within the interval, 0 and 1 (Wang and Chang, 2007).

Since n criteria may be measured in different ways, the decision matrix Z needs to be normalised. This step transforms various criteria dimensions into non-dimensional units, which allows for comparisons across the criteria. The normalised decision matrix can be obtained by using Equation 3.

$$R = (r_{ij})_{y \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{x1} & r_{x2} & \cdots & r_{xn} \end{bmatrix} \end{array} \tag{12}$$

3.4.3 Construction of weighted normalisation fuzzy decision matrix

The weighting factors are a set of percentages that add up to 100%, with the most important alternative receiving the highest weighting factor. The process involves multiplying the importance weights of the alternative by the values in the normalised fuzzy decision matrix. Considering the different importance of each criterion, the weighted normalized fuzzy-decision matrix $\tilde{V}$ can be constructed using Equations 13 and 14.

$$V = \lfloor v_{ij} \rfloor_{m\times n}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \tag{13}$$

$$v_{ij} = r_{ij} \times w_j \tag{14}$$

where, w_j denotes the importance weight of criterion C_j.

3.4.4 Determination of the fuzzy positive ideal reference point (FPIRP) and fuzzy negative ideal reference point (FNIRP)

The FPIRP is obtained by identifying the best score in a criterion. Similarly, the worst score of a criterion is identified and recorded as the FNIRP. The FPIRP (A^+) [the benefit criterion] and FNIRP (A^-) [the cost criterion) are defined as follows:

$$A^+ = (v_1^+, v_2^+, \ldots, v_n^+) \tag{15}$$

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-) \tag{16}$$

where,

$$\tilde{v}_j^+ = \{Max\, v_{ij}, i \in j_1; Min\, v_{ij}, i \in j_2\} \tag{17}$$

$$\tilde{v}_j^- = \{Max\, v_{ij}, i \in j_1; Min\, v_{ij}, i \in j_2\} \tag{18}$$

where j_1 and j_2 are associated with the sets of benefit and cost criteria respectively.

The distance of each alternative (maintenance strategy) from the FPIRP (D_i^+) and FNIRP (D_i^-) with respect to each criterion can be obtained by utilising Equations 19 and 20 respectively.

$$D_i^+ = \sqrt{\textstyle\sum_{i=1}^{x} (v_{ij} - v_i^+)^2}, \quad j - 1, 2, \ldots, n \tag{19}$$

$$D_i^- = \sqrt{\textstyle\sum_{i=1}^{x} (v_{ij} - v_i^-)^2}, \quad j - 1, 2, \ldots, n \tag{20}$$

The obtained D_i^+ and D_i^- values can then be used in obtaining the Closeness Coefficient (CC_i) of each alternative for ranking purposes.

3.4.5 Obtaining the closeness coefficient of each alternative

The ranking of the alternative can be determined after the obtaining CC_i. This allows the decision making experts to choose the most rational and appropriate alternative. To calculate the CC_i Equation 21 is used.

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad i = 1, 2, \dots, m \tag{21}$$

3.4.6 Ranking the alternatives

The different alternatives are ranked according to the closeness coefficient CC_i in decreasing order. It is important to note that the best alternative is closest to the FPIRP and farthest from the FNIRP. This means that the larger the CC_i, the better the associated alternative.

3.5 Perform Sensitivity Analysis (**Final**)

Conducting a sensitivity analysis (SA) is an important aspect of the novel hybrid methodology presented in Section 3, as it is meant to provide a reasonable amount of confidence in the overall result of the study. Given that the final output result is dependent on the subjective judgements of the decision makers, it is essential to perform SA based on a set of scenarios that reflect different views on the relative importance of the attributes, in order to observe the stability and ranking order of the model's output. Then, managerial attention is focused during implementation of the maintenance strategies for the decision making process.

4 Application of Methodology to a Test Scenario

The proposed model will be demonstrated in a decision making analysis of the selection of an on-board machinery (crane) maintenance strategy for ships operating under an uncertain environment, as presented in Section 3. The hierarchical model of this decision-making analysis process is as illustrated in Figure 4, with the goal of the decision problem in level 0, decision alternatives in level 1, and evaluation criteria in level 2. It is important to note that the proposed model is applied for decision making in the selection of appropriate maintenance strategies for marine and offshore machineries.

This representation is made to simplify the computational complexity associated with the analysis and to provide managerial insight to decision makers in a reasonable manner prior to their subjective evaluation of criteria with respect to alternatives. The analysis will be conducted through a robust literature review and brainstorming session with the experts.

Insert Figure 4 here

The positions of the experts and their degree of competency in the industry are as shown in Table 7. The primary objective of the decision-making analysis is to identify the best, most appropriate and acceptable maintenance strategy to be adopted by the engineer on-board ships and offshore installations.

4.1 Identification of Decision Making Alternatives (**Step one**)

This involves the identification of the decision making alternatives through a literature review of the machinery maintenance strategies on-board ships. As presented in Section 3.1, four (4) alternatives were established for this analysis.

4.2 Identification of Evaluation of Criteria (**Step two**)

Based on the expert opinions, the criteria or attributes that are critical to enhancing the selection of the best maintenance strategy in uncertain situations are stated in Section 3.2. It is evident that the criteria used for the selection procedure consist of two main categories: cost (C) (the lower the value, the more effective the alternative) and benefit (B) (the higher the value, the more robust or effective the alternative). As a consequence, the cost type criteria include the cost (equipment capital cost, labour cost, repair/replacement cost), downtime, and availability, while the benefit type criteria consist of safety (operational safety, environmental safety) and reliability. The assigned criteria are described in Table 2. Based on this, it is worth mentioning that maintenance strategy selection can be carried out with respect to three cost and two benefit criteria.

Insert Table 2 here

4.3 Rating Phase - Determination of Importance Weight (**Step three**)

In order to show the relative important of each criterion, it is necessary to assign a weight to each (Reliability, Cost Effectiveness, Safety, Availability, and Downtime). There are two types of criteria for a selection problem involving complex networks of decision making. If an assessment of the criteria is made with respect to alternatives from field data or a literature review, the criteria are called ‘objective’; when such information is obtained using expert judgement in the form of fuzzy linguistic estimates, then the criteria are called ‘subjective’. The assessment type used for all the criteria in this model is fuzzy linguistic estimates, thus, the criteria are subjective. Based on this, each subjective criterion is assessed with respect to each alternative by a group of three experts or decision makers (DMs), and their assessments are presented in Tables 3, 4, and 5, respectively. The experts’ backgrounds are presented as follows:

1. A senior maintenance engineer with a PhD who has been involved with marine and offshore machinery maintenance and services for over 25 years.
2. A ship chief engineer officer with a class 1 marine certificate of competency (COC) who has been involved with machinery maintenance and operations on-board ship for over 25 years.
3. A senior port maintenance engineer with a master's degree who has been involved with the port equipment's safety and operational services for over 30 years.

Insert Tables 3, 4 and 5 here

4.3.1 Estimating weights of experts

The weights of the experts are determined based on the available information in Section 3.3.1. Three experts were employed in this study, and the weights of their judgements are considered to be equal (0.333).

4.3.2 Estimating weights of criteria

For this model, equal weight values are assigned to the five identified evaluation criteria, as shown in Table 6. These weight values will then be applied in the assessment process to establish the fuzzy performance ratings of the model's evaluation alternatives.

Insert Table 6 here

4.3.3 Aggregation of experts' opinions

This stage of the analysis involves a series of aggregation calculations of criteria ratings with respect to alternatives. Since decision making on maintenance strategies involves complex networks of group decision making in a fuzzy environment, it is important to emphasise that three experts are employed for this strategic evaluation; for this study, their weights are considered to be equal. When conducting the Fuzzy-TOPSIS process as applied in this model, the knowledge and judgement of analysts involved are to be considered. The four decision alternatives and five evaluation criteria shown in Table 9 will be used to develop the fuzzy TOPSIS decision matrix.

Table 7 shows the decision alternatives and the evaluation criteria and their corresponding nomenclature respectively, while Tables 8, 9, and 10 show the corresponding fuzzy numbers of the alternatives with respect to the criteria by the three experts. The figures obtained are based on the membership functions of the linguistic variables developed in Figure 3 and the scale for the measurement of the evaluation criteria, as shown in Table 1.

Aggregation calculations are conducted using Equations 5, 6, 7, 8, and 9 for the experts' judgement on reliability with respect to run-to-failure maintenance, as seen in Table 11. Similar, calculations were conducted on the other attributes and their fuzzy estimates are presented in Tables 12a and 12b.

Insert Tables 7, 8, 9, 10, 11, 12a and 12b here

4.3.4 Defuzzification of the aggregated fuzzy results

Based on the aggregation results presented in Tables 12a and 12b, the fuzzy numbers are converted into crisp values using Equation 10 and the results are presented in Table 13.

Insert Table 13 here

4.4 Selection Phase - Application of FTOPSIS Approach to Obtain Performance Rating of Decision Alternatives (**Step four**)

In order to obtain the performance rating for the decision alternatives, the FTOPSIS algorithm is applied in this section, as follows:

4.4.1 FTOPSIS decision matrix construction

Based on crisp values obtained for the four decision-making alternatives (A1 – A4) and five evaluation criteria (C1 – C5) obtained in Table 13, a Fuzzy-TOPSIS decision matrix, shown in Tables 14 is constructed.

Insert Table 14 here

4.4.2 Fuzzy decision matrix normalisation

Based on Equation 3, the fuzzy decision matrix presented in Table 14 is normalised. The results are presented in Table 15. As an example, the normalised reliability (C1) with respect run-to-failure maintenance (A1) is presented as follows:

$$\frac{0.675}{[(0.675^2+0.75^2+0.925^2+0.925^2)]^{\frac{1}{2}}} = 0.409$$

Insert Table 15 here

4.4.3 Construction of weighted normalisation fuzzy decision matrix

The normalized fuzzy numbers obtained in Table 15 are multiplied by the important weight values of the evaluation criteria given in Table 6. For example, the weighted normalized fuzzy number for A1 of C1 is given as follows:

$v_{1,1}$ = 0.409 x 0.2 = 0.082

Similarly, the weighted normalized fuzzy numbers for other alternatives are calculated and presented in Table 16.

Insert Table 16 here

4.4.4 Determination of the fuzzy positive ideal reference point (FPIRP) and fuzzy negative ideal reference point (FNIRP)

Determination of the FPIRP can be made by taking the largest element of each benefit criterion and the smallest element of each cost criterion. Ultimately, FNIRP is the reverse of the FPIRP in relation to this representation, as presented in Table 17.

Insert Table 17 here

The distances of each maintenance strategy from FPIRP and FNIRP values with respect to each criterion are calculated using Equations 17 and 18. As an example, the distance of alternative A1 to A^+ is calculated as follows:

$$D^+ = \ [(0.112 - 0.082)^2 + (0.029 - 0.041)^2 + (0.168 - 0.048)^2 + (0.065 - 0.065)^2 + (0.021 - 0.192)^2]^{1/2} = 0.211$$

$$D^- = \ [(0.082 - 0.082)^2 + (0.172 - 0.041)^2 + (0.014 - 0.048)^2 + (0.119 - 0.065)^2 + (0.192 - 0.192)^2]^{1/2} = 0.146$$

Similarly, and by applying Equations 17 and 18, the distances of other decision alternatives to FPIRP and FNIRP were determined and the results are presented in Table 18.

Insert Table 18 here

4.4.5 Obtaining the closeness coefficient and ranking of alternatives

Based on the results obtained in Section 4.4.4, the closeness coefficient of each alternative can be calculated using Equation 21. The calculation of the CC value for alternative A1 is described as follows:

$D_1^+ = 0.211$, and $D_1^- = 0.146$

$$CC_1 = \frac{0.146}{0.211+0.146} = 0.408$$

Similarly, the CC values for alternatives A2 to A4 can be calculated and the results are presented in Table 19.

Insert Table 19 here

It can be observed in Table 19 that each instance of the hybrid approach produces different values for each maintenance strategy that correspond to the strategic decisions of experts. Obviously, the result of the calculations revealed that A3 and A2 scored the highest CC values compared to the remaining alternatives or strategies. The detailed results of the fuzzy TOPSIS analysis are presented in Table 20.

Insert Table 20 here

4.4.6 Ranking the alternatives

Based on the evaluation of closeness coefficient above, by comparing the values of the four alternatives, as shown in Table 20, the ranking order of the maintenance strategies is given as A3 > A2 > A4 > A1. Additionally, Figure 5 is obtained based on the analysis result presented in Table 20. The graph depicts the sensitivity of the analytical result as being non-linear. It is noteworthy that the procedure outlined in the proposed framework revealed that A3 and A2 seem reasonable and appropriate choices for investment in the ship crane under investigation, in order to improve the performance of the crane's operations. These maintenance strategies have closeness coefficient values of 0.659 and 0.597 respectively.

Insert Figure 5 here

4.5 Perform Sensitivity Analysis (**Final**)

In order to validate and test the robustness of this model, a sensitivity analysis is conducted. The analysis is necessary in order to test the suitability and sensitivity of the model for decision analysis of the maintenance strategies (as decision alternatives), and for the interpretation and communication of results based on a sensitivity study so that managerial insight can correctly provide guidance for investment in maintenance strategies. Based on the input data presented in Section 4.4.1 (Table 14), the crisp values of each attribute are slightly varied while the resulting change and the final ranking of the alternatives are observed. This process of analysis is useful in situations of high uncertainties concerned with many factors that need to be modelled when investing in machinery maintenance strategies. Apparently, due to the vagueness surrounding the strategic decision making process, it is usually very challenging to predict and analyse the delivery of the analytical result in a fuzzy environment.

The analysis is conducted under five conditions, as tabulated in Table 21. The first step in the sensitivity analysis process involves an increment of the cost values (*i.e*. C2, C4 and

C5) of each decision alternative by 10% and decreasing the benefit values (*i.e*. C1 and C3) by the same 10%. The next step is to determine the distance of each alternative to FPIRP and FNIRP, then obtain the CC values and observe the results of the final ranking, as described in Sections 4.4.4 and 4.4.5.

Insert Table 21 here

Based on Table 14 in Section 4.4.1, the values for 10% increment on the cost and 10% decrement on the benefit criteria are shown in Table 22, and their normalised and weighted normalised values are shown in Tables 23 and 24, respectively.

Insert Tables 22, 23 and 24 here

From Table 24, the distances of each alternative to the FPIRP (*i.e*. D^{+}) and FNIRP (*i.e*. D^{-}), and their corresponding CC values are obtained. The results of the sensitivity analysis (*i.e*. when the input values of the criteria are changed by 10%) are presented in Table 25.

Insert Table 25 here

5 Discussion of Results

In this study, sensitivity analysis is implemented to see the effect in the output data given a slight change in the input data. From the results of the sensitivity analysis (Table 25), it can be observed that the ranking order of the four decision alternatives maintained a consistency when the cost category of the criteria (C2, C4, C5) was increased by 10%, and the benefit category (C1, C3) decreased by the same 10%. The analysis reveals that almost all the changes in the criteria input data do not change the final ranking of the maintenance strategies. For this model to be validated, this pattern in the results is to be expected. It can therefore be deduced that the model is reasonable and capable of being applied to the analysis of machinery maintenance strategy decision-making alternatives.

Based on the result obtained from this analysis, the marine machinery (crane) under investigation can be enhanced by implementing A3 (*i.e*. condition based maintenance) strategy. However, implementing A2 (*i.e*. preventive maintenance), especially during follow-up analysis (when improving maintenance process), can promote continuous improvement and enhance the crane's performance under high uncertainty. Experience has shown that investing in maintenance selection strategies seems to be an important strategy to mitigate cost issues under a fuzzy environment. Therefore, the result of the analysis would help improve the decision-making process, thus allowing for a flexible response to operational uncertainties through a systematic approach.

The analysis result reflecting on A3 (condition based maintenance) as the recommended strategy certainly shows that expert judgement was based on increase in machinery operational life/availability, increase in machinery reliability, increase in cost for parts and labour, and decrease in machinery downtime. Minimizing maintenance costs seems to be an effective way to build up efficient maintenance, especially when one is required to work within a limited budget. When investments in maintenance have to be made to reduce the overall costs (*i.e.*, operations costs), it seems logical to consider the minimization of total cost of ownership or the life-cycle costs instead. However, Goossens (2015) ascertained that the ultimate goal of maintenance cannot be cost reduction only and must be maintaining functionality (at the lowest cost). Nevertheless, how costs can best be interpreted in relation to the selection process of the best maintenance strategy remains to be further explored.

The proposed methodology has been somehow validated through a sensitivity analysis. The difficulty in validating engineering related analysis methods has been well recognised. A ‘validation square’ has been proposed, consisting of the phases of method consistency, accepting example problems, accepting usefulness and accepting usefulness beyond examples (Pederson *et al.*, 2000). In this study, the first three phases have been verified through the development of the proposed model and the described case study in this paper. The last phase may only be verified by conducting more case studies.

6 Conclusion

This paper presented a collaborative modelling and strategic FMADM method that can be adopted for the selection of appropriate machinery maintenance strategies in a concise, logical, and transparent manner against multiple scenarios where the data available is subjective and imprecise. The strength of this strategic decision making approach is in the fact that both heterogeneous and homogeneous groups of experts can be utilised and their subjective opinions can be aggregated simply, with partial or incomplete information available.

In the evaluation process, a fuzzy TOPSIS algorithm is implemented to rank the machinery maintenance strategies or alternatives in a way that is flexible and straightforward. To support a strategic decision on machinery maintenance strategy selection, fuzzy AHP (Anagnostopoulos *et al.,* 2007) and fuzzy TOPSIS (Sodhi and Prabhakar, 2012) need to be utilised to handle multiple organisational objectives, complex decision making, and long term condition of machineries in an uncertain environment. The proposed approach can be applied to situations where both qualitative and quantitative data has to be integrated and synthesized for evaluation processes during complex and multiple decision making involving marine and offshore machinery operations. Since the result of the calculations is sensitive to criteria and the number of experts engaged, these should be carefully chosen by maritime maintenance and safety analysts to avoid misrepresentation and information loss during the interpretation process.

During this study, five factors – *reliability, cost, safety, availability, and downtime* – have converged to create a succinct and effective meaning. However, in practice, the interpretation and relations of these factors differ depending on which experts are involved. The research described in this paper can serve as a basis to further explore the roles of these factors for selection of an appropriate maintenance strategy. The relation between availability and reliability needs elaboration. Although clear definitions for both are presented in the literature, practitioners seem to have varying interpretations and views of what these two terms mean to their specific situation, and how they are related. To gain a better understanding of the interpretation differences and origin, and how these influence maintenance strategy selection, a structured experimental investigation needs to be considered.

The role of safety within the maintenance strategy selection can also be misinterpreted since, according to Goossens (2015), by definition, absolute safety is impossible. As such, safety is considered to act as a pre-condition for maintenance strategy selection. Nevertheless, a balance between safety and availability or reliability can be desirable (or even possible). The exact role that safety currently has within machinery maintenance strategy selection, as well as the role it should have, is worth further investigation. The model developed in this study is by no means conclusive. It is subject to further modification, given the acquisition of new data or before its utilization by end-users in the industry. A sensitivity analysis was conducted to partially validate the developed model and establish its ability to respond to changes in input variables.

The study conducted attempts to highlight a comprehensive analysis of the marine and offshore machinery planned maintenance strategy in relation to improvement of the machinery operations. The experience and knowledge proficiencies of domain experts are vital when the hierarchical framework of maintenance strategy selection is applied to real industrial case studies, as described in this work, and thus, careful selection of such experts is necessary in order to achieve reasonable outcomes. In other words, if domain experts who do not have the requisite knowledge or experience are selected and used for the analysis, the framework may produce poor outcomes, which may defeat the purpose of selecting an appropriate maintenance strategy and render the methodology ineffective.

This study has provided a conceptual hierarchical model for selecting an appropriate maintenance strategy for efficient operation of offshore marine machinery under highly uncertain environments. The following avenues to further enhance the implementation of the model that can be applied in a different context have been identified:

- Application of this methodology to other machinery in complex and high reliability industries (*e.g*. nuclear, aviation, health care, *etc*.) could give rise to useful results that may further enrich the deficient literature of maintenance strategy selection for critical machinery. It can also give greater confidence and insight into the uses and limitations of this methodology.
- Within this research, three experts were employed to conduct the assessment. However, it is recommended that the number of experts be increased for collaborative modelling of the system, from a range of different marine and offshore industries, to include maintenance engineers, researchers, marine superintendents, and machinery operators. This will further enhance the collaborative design and effectiveness of the result obtained for use in its wider perspective.
- Combination of diverse but powerful intelligent tools and algorithms from other fields and concepts will open promising new pathways for effective maintenance strategy selection for machinery operations under uncertainty.

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Table 1: Fuzzy Linguistic Scale for Alternative Rating

Linguistic Variables	Corresponding Trapezoidal Fuzzy Numbers
Very Low	(0, 0, 0.1, 0.2)
Low	(0.1, 0.25, 0.25, 0.4)
Medium	(0.3, 0.5, 0.5, 0.7)
High	(0.6, 0.75, 0.75, 0.9)
Very High	(0.8, 0.9, 1, 1)

Source: Hypothetical data [Chen and Hwang (1992)]

Table 2: Criteria for Maintenance Strategy Selection

Criteria	Criteria Description	Assessment Type	Category
C1	Reliability	Linguistic Assessment	B
C2	Cost	Linguistic Assessment	C
C3	Safety	Linguistic Assessment	B
C4	Availability	Linguistic Assessment	C
C5	Downtime	Linguistic Assessment	C

Source: Test case data

Table 3: Linguistic Assessment of the Alternatives with Respect to Criteria by Expert 1

	EXPERT 1			
	RTFM	**PM**	**CBM**	**RCM**
Reliability	H	H	VH	VH
Cost	L	VH	M	VL
Safety	L	VH	M	VL
Availability	M	VH	VH	VH
Downtime	H	L	VL	VL

Source: Test case data

Table 4: Linguistic Assessment of the Alternatives with Respect to Criteria by Expert 2

	EXPERT 2			
	RTFM	**PM**	**CBM**	**RCM**
Reliability	H	H	VH	VH
Cost	VL	H	L	VL
Safety	L	VH	M	VL
Availability	M	H	VH	VH
Downtime	H	VL	VL	VL

Source: Test case data

Table 5: Linguistic Assessment of the Alternatives with Respect to Criteria by Expert 3

	EXPERT 3			
	RTFM	**PM**	**CBM**	**RCM**
Reliability	M	H	VH	VH
Cost	L	H	M	L
Safety	L	H	M	VL
Availability	M	M	VH	H
Downtime	M	VL	VL	L

Source: Test case data

***Note:** VL, L, M, H and VH stand for Very Low, Low, Medium, High, and Very High, respectively.*

Table 6: Weights of Criteria

Criteria	Assigned Weights
Reliability	0.2
Cost	0.2
Safety	0.2
Availability	0.2
Downtime	0.2

Source: Test case data

Table 7: Decision Alternatives and Evaluation Criteria

	Decision Alternatives		Evaluation Criteria
A1	Run-to-Failure Maintenance	C1	Reliability
A2	Preventive Maintenance	C2	Cost
A3	Condition Based Maintenance	C3	Safety
A4	Reliability-Centred Maintenance	C4	Availability
		C5	Downtime

Source: Test case data

Table 8: Fuzzy Numbers for Alternatives with Respect to Criteria by Expert 1

	Expert 1			
	A1	**A2**	**A3**	**A4**
C1	0.6, 0.75, 0.75, 0.9	0.6, 0.75, 0.75, 0.9	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1
C2	0.1, 0.25, 0.25, 0.4	0.8, 0.9, 1, 1	0.3, 0.5, 0.5, 0.7	0, 0, 0.1, 0.2
C3	0.1, 0.25, 0.25, 0.4	0.8, 0.9, 1, 1	0.3, 0.5, 0.5, 0.7	0, 0, 0.1, 0.2
C4	0.3, 0.5, 0.5, 0.7	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1
C5	0.6, 0.75, 0.75, 0.9	0.1, 0.25, 0.25, 0.4	0, 0, 0.1, 0.2	0, 0, 0.1, 0.2

Source: Test case data

Table 9: Fuzzy Numbers for Alternatives with Respect to Criteria by Expert 2

	Expert 2			
	A1	**A2**	**A3**	**A4**
C1	0.6, 0.75, 0.75, 0.9	0.6, 0.75, 0.75, 0.9	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1
C2	0, 0, 0.1, 0.2	0.6, 0.75, 0.75, 0.9	0.1, 0.25, 0.25, 0.4	0, 0, 0.1, 0.2
C3	0.1, 0.25, 0.25, 0.4	0.8, 0.9, 1, 1	0.3, 0.5, 0.5, 0.7	0, 0, 0.1, 0.2
C4	0.3, 0.5, 0.5, 0.7	0.6, 0.75, 0.75, 0.9	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1
C5	0.6, 0.75, 0.75, 0.9	0, 0, 0.1, 0.2	0, 0, 0.1, 0.2	0, 0, 0.1, 0.2

Source: Test case data

Table 10: Fuzzy Numbers for Alternatives with Respect to Criteria by Expert 3

	Expert 3			
	A1	**A2**	**A3**	**A4**
C1	0.3, 0.5, 0.5, 0.7	0.6, 0.75, 0.75, 0.9	0.8, 0.9, 1, 1	0.8, 0.9, 1, 1
C2	0.1, 0.25, 0.25, 0.4	0.6, 0.75, 0.75, 0.9	0.3, 0.5, 0.5, 0.7	0.1, 0.25, 0.25, 0.4
C3	0.1, 0.25, 0.25, 0.4	0.6, 0.75, 0.75, 0.9	0.3, 0.5, 0.5, 0.7	0, 0, 0.1, 0.2
C4	0.3, 0.5, 0.5, 0.7	0.3, 0.5, 0.5, 0.7	0.8, 0.9, 1, 1	0.6, 0.75, 0.75, 0.9
C5	0.3, 0.5, 0.5, 0.7	0, 0, 0.1, 0.2	0, 0, 0.1, 0.2	0.1, 0.25, 0.25, 0.4

Source: Test case data

Table 11: Aggregation Calculation for Reliability with Respect to RTFM

Expert 1	H	0.6, 0.75, 0.75, 0.9
Expert 2	H	0.6, 0.75, 0.75, 0.9
Expert 3	M	0.3, 0.5, 0.5, 0.7
S(Expert 1 & 2) = 1 - $\frac{(0.6-0.6)+(0.75-0.75)+(0.75-0.75)+(0.9-0.9)}{4} = 1$		
S(Expert 1 & 3) = 1 - $\frac{(0.6-0.3)+(0.75-0.5)+(0.75-0.5)+(0.9-0.7)}{4} = 0.75$		
S(Expert 2 & 3) = 1 - $\frac{(0.6-0.3)+(0.75-0.5)+(0.75-0.5)+(0.9-0.7)}{4} = 0.75$		
AA(Expert 1) = $\frac{1+0.75}{2} = 0.875$	AA(Expert 2) = $\frac{1+0.75}{2} = 0.875$	AA(Expert 3) = $\frac{0.75+0.75}{2} = 0.75$
RA(Expert1) = $\frac{0.875}{0.875+0.875+0.75} = 0.35$	RA(Expert2) = $\frac{0.875}{0.875+0.875+0.75} = 0.35$	RA(Expert3) = $\frac{0.75}{0.875+0.875+0.75} = 0.3$
Expert aggregation Result $\tilde{R}_{AG}$	0.35(0.6, 0.75, 0.75, 0.9) + 0.35(0.6, 0.75, 0.75, 0.9) + 0.3(0.3, 0.5, 0.5, 0.7) = (0.51, 0.675, 0.675, 0.84)	

Source: Test case data

Table 12a: Aggregation Results of Criteria Ratings with Respect to Alternatives

	C1	**C2**	**C3**
A1	0.51, 0.675, 0.675, 0.84	0.067, 0.167, 0.2, 0.333	0.1, 0.25, 0.25, 0.4
A2	0.6, 0.75, 0.75, 0.9	0.662, 0.797, 0.828, 0.931	0.738, 0.853, 0.922, 0.969
A3	0.8, 0.9, 1, 1	0.233, 0.417, 0.417, 0.6	0.3, 0.5, 0.5, 0.7
A4	0.8, 0.9, 1, 1	0.035, 0.088, 0.153, 0.270	0, 0, 0.1, 0.2

Source: Test case data

Table 12b: Aggregation Results of Criteria Ratings with Respect to Alternatives

	C4	C5
A1	0.3, 0.5, 0.5, 0.7	0.51, 0.675, 0.675, 0.84
A2	0.573, 0.722, 0.754, 0.871	0.031, 0.078, 0.147, 0.262
A3	0.8, 0.9, 1, 1	0, 0, 0.1, 0.2
A4	0.738, 0.853, 0.922, 0.969	0.035, 0.088, 0.153, 0.270

Source: Test case data

Table 13: Transformation of the Fuzzy Numbers into Crisp Values

	C1	C2	C3	C4	C5
A1	(0.51+ 0.675 + 0.675 + 0.84) / 4 = 0.675	(0.067 + 0.167 + 0.2 + 0.333) / 4 = 0.192	(0.1 + 0.25 + 0.25 + 0.4) / 4 = 0.25	(0.3 + 0.5 + 0.5 + 0.7) / 4 = 0.5	(0.51+ 0.675 + 0.675 + 0.84) / 4 = 0.675
A2	(0.6 + 0.75 + 0.75 + 0.9) / 4 = 0.75	(0.662 + 0.797 + 0.828 + 0.931) / 4 = 0.805	(0.738 + 0.853 + 0.922 + 0.969) / 4 = 0.871	(0.573 + 0.722 + 0.754 + 0.871) / 4 = 0.730	(0.031 + 0.078 + 0.147 + 0.262) / 4 = 0.130
A3	(0.8 + 0.9 + 1 + 1) / 4 = 0.925	(0.233 + 0.417 + 0.417 + 0.6) / 4 = 0.417	(0.3 + 0.5 + 0.5 + 0.7) / 4 = 0.5	(0.8 + 0.9 + 1 + 1) / 4 = 0.925	(0 + 0 + 0.1 + 0.2) / 4 = 0.075
A4	(0.8 + 0.9 + 1 + 1) / 4 = 0.925	(0.035 + 0.088 + 0.153 + 0.270) / 4 = 0.136	(0 + 0 + 0.1 + 0.2) / 4 = 0.075	(0.738 + 0.853 + 0.922 + 0.969) / 4 = 0.871	(0.035 + 0.088 + 0.153 + 0.270) / 4 = 0.136

Source: Test case data

Table 14: Fuzzy-TOPSIS Decision Matrix

	C1	C2	C3	C4	C5
A1	0.675	0.192	0.25	0.5	0.675
A2	0.75	0.805	0.871	0.730	0.130
A3	0.925	0.417	0.5	0.925	0.075
A4	0.925	0.136	0.075	0.871	0.136

Source: Test case data

Table 15: Normalised Decision Matrix

	C1	C2	C3	C4	C5
A1	0.409	0.205	0.241	0.323	0.958
A2	0.454	0.859	0.839	0.471	0.184
A3	0.560	0.445	0.482	0.597	0.106
A4	0.560	0.145	0.072	0.563	0.193

Source: Test case data

Table 16: Weighted Normalized Decision Matrix

	C1	C2	C3	C4	C5
A1	0.082	0.041	0.048	0.065	0.192
A2	0.091	0.172	0.168	0.094	0.037
A3	0.112	0.089	0.096	0.119	0.021
A4	0.112	0.029	0.014	0.113	0.039

Source: Test case data

Table 17: Representation of FPIRP and FNIRP Values

	Positive Ideal Solution	Negative Ideal Solution
Reliability (C1)	0.112	0.082
Cost (C2)	0.029	0.172
Safety (C3)	0.168	0.014
Availability (C4)	0.065	0.119
Downtime (C5)	0.021	0.192

Source: Test case data

Table 18: Distance of each Alternative to the FPIRP and FNIRP

	A1	A2	A3	A4
D^+	0.211	0.148	0.108	0.162
D^-	0.146	0.220	0.209	0.212

Source: Test case data

Table 19: CC Results and Ranking Order of the Maintenance Strategies

	A1	A2	A3	A4
CC	0.408	0.597	0.659	0.566
Ranking	4	2	1	3

Source: Test case data

Table 20: Results of Fuzzy TOPSIS Analysis

	Decision-Making Attributes	D^+	D^-	CC Values	Ranking
A1	Run-to-Failure Maintenance	0.211	0.146	0.408	4
A2	Preventive Maintenance	0.148	0.220	0.597	2
A3	Condition Based Maintenance	0.108	0.209	0.659	1
A4	Reliability-Centred Maintenance	0.162	0.212	0.566	3

Source: Test case data

Table 21: Conditions for Changing Input Values by Percentages

Condition	Percentage
1	Decrease C1 by 10%
2	Increase C2 by 10%
3	Decrease C3 by 10%
4	Increase C4 by 10%
5	Increase C5 by 10%

Source: Test case data

Table 22: Fuzzy-TOPSIS Decision Matrix when Criteria are changed by 10%

	10% Decrement	10% Increment	10% Decrement	10% Increment	10% Increment
	C1	**C2**	**C3**	**C4**	**C5**
A1	0.575	0.292	0.15	0.6	0.775
A2	0.65	0.905	0.771	0.83	0.23
A3	0.825	0.517	0.4	1.025	0.175
A4	0.825	0.236	-0.025	0.971	0.236

Source: Test case data

Table 23: Normalised Decision Matrix when Criteria Values are changed

	C1	C2	C3	C4	C5
A1	0.395	0.264	0.17	0.344	0.901
A2	0.447	0.817	0.874	0.476	0.267
A3	0.567	0.467	0.454	0.588	0.203
A4	0.567	0.213	-0.028	0.557	0.274

Source: Test case data

Table 24: Weighted Normalised Decision Matrix when Criteria Values are changed

	C1	C2	C3	C4	C5
A1	0.079	0.053	0.034	0.069	0.180
A2	0.089	0.163	0.175	0.095	0.053
A3	0.113	0.093	0.091	0.118	0.041
A4	0.113	0.043	-0.006	0.111	0.055

Source: Test case data

Table 25: Sensitivity Analysis Results

	Decision-Making Attributes	D^{+}	D^{-}	CC Values	Ranking
A1	Run-to-Failure Maintenance	0.212	0.131	0.383	4
A2	Preventive Maintenance	0.143	0.214	0.599	2
A3	Condition Based Maintenance	0.115	0.189	0.622	1
A4	Reliability-Centred Maintenance	0.183	0.192	0.512	3

Source: Test case data

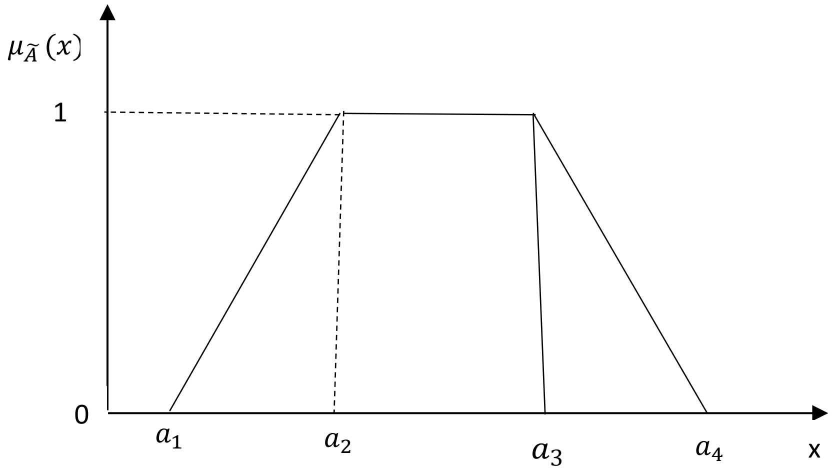

Figure 1: Membership Function of Trapezoidal Fuzzy Number

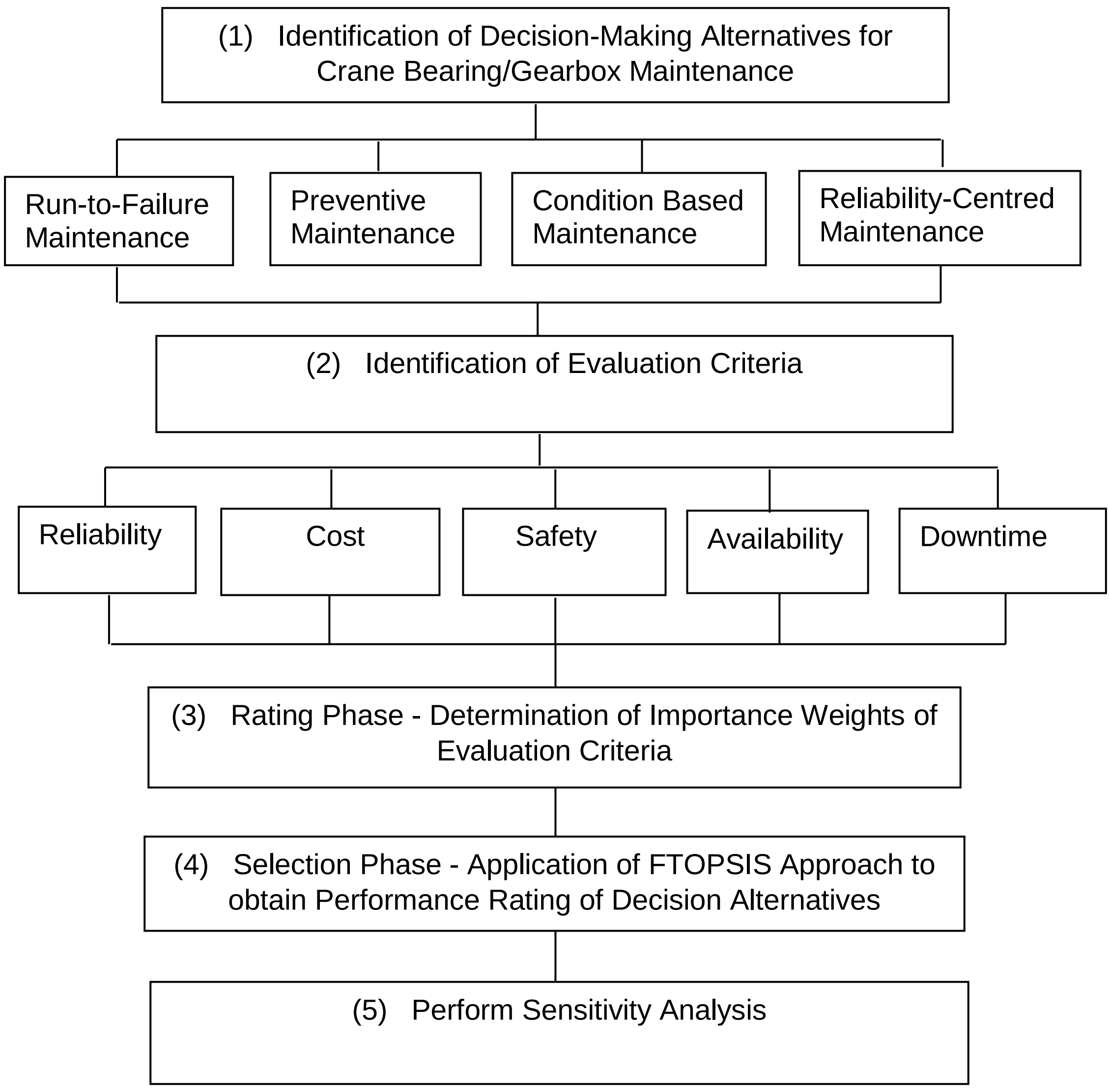

Figure 2: Hierarchical Model of Decision Making Analysis for Equipment

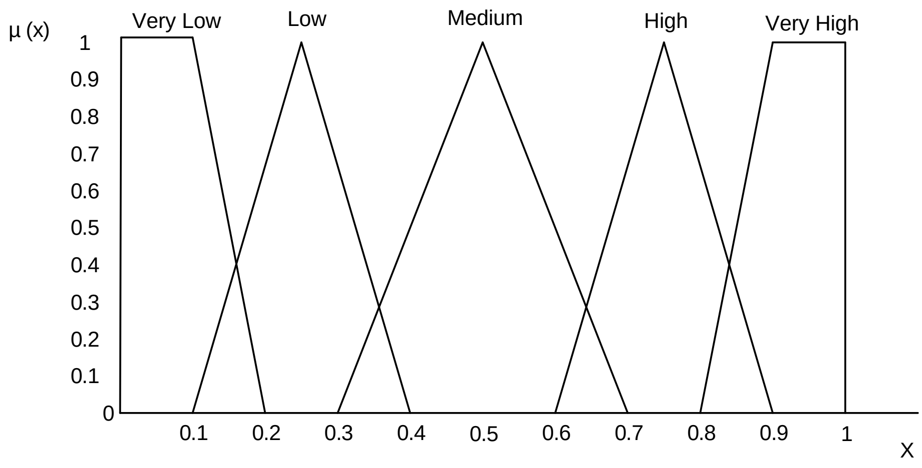

Figure 3: Membership Degree for Linguistic Ratings [Chen and Hwang(1992)]

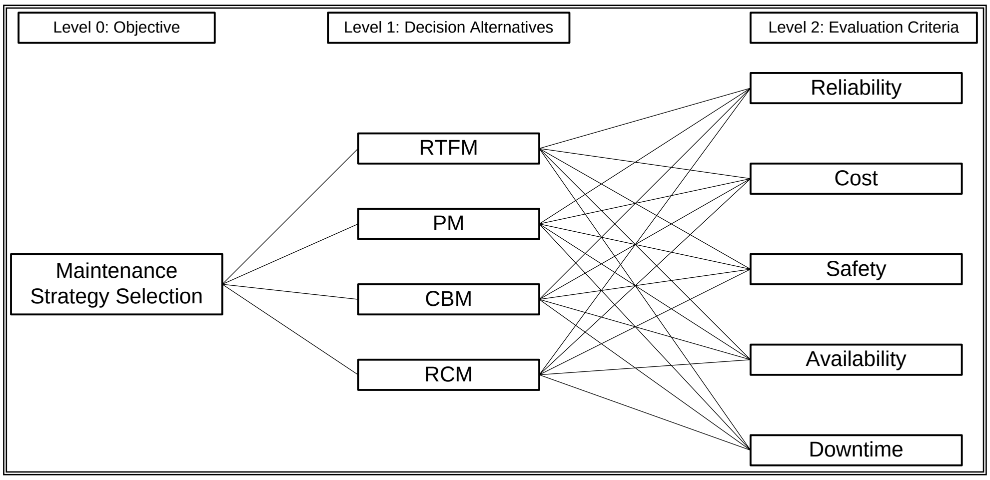

Figure 4: Hierarchical Structure of Maintenance Strategy Selection

Note: *RTFM, PM, CBM, and RCM stand for Run-to-Failure Maintenance, Preventive Maintenance, Condition Based Maintenance, and Reliability Centred Maintenance, respectively*.

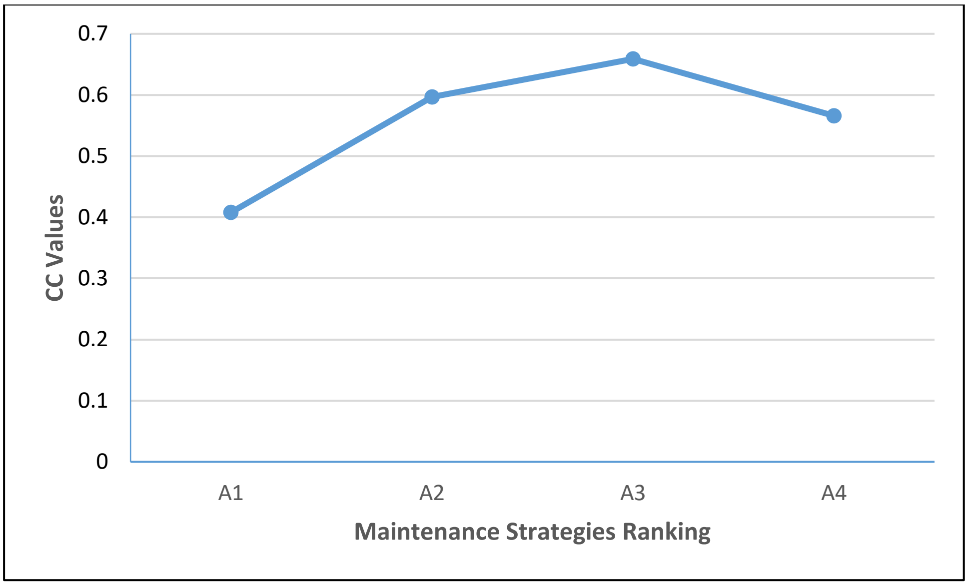

Figure 5: Ranking Order of the Maintenance Strategies