Use of Fuzzy Risk Assessment in FMEA of Offshore Engineering Systems

Zaili Yang[1], Jin Wang
School of Engineering, Technology and Maritime Operations,
Liverpool John Moores University, Liverpool, L3 3AF, UK.

[1] Corresponding author: z.yang@ljmu.ac.uk; 0044 (0) 151 231 2531

Abstract - This paper proposes a novel framework for analysing and synthesising engineering system risks on the basis of a generic Fuzzy Evidential Reasoning (*FER*) approach. The approach is developed to simplify the inference process and overcome the problems of traditional fuzzy rule based methods in risk analysis and decision making. The framework, together with the *FER* approach has been applied to model the safety of an offshore engineering system. The benchmarking between the new model and a well-established Rule based Inference Methodology using the Evidential Reasoning (*RIMER*) is conducted to demonstrate its reliability and unique characteristics. It will facilitate subjective risk assessment in different engineering systems where historical failure data is not available in their safety practice.

Keywords – FMEA, fuzzy rule base, evidential reasoning, maritime risk, maritime safety.

Acronym[2]

AHP	Analytic hierarchy process
ALARP	As low as reasonably practical
CPP	Controllable pitch propeller
DP	Dynamic positioning
D-S	Dempster-Shafer
ER	Evidential reasoning
FPSO	Floating, production, storage and offloading
FRBER	Fuzzy rule based evidential reasoning
FST	Fuzzy set theory
IDS	Intelligent decision system via evidential reasoning
MADM	Multiple attribute decision making
PRS	Position reference system
RCO	Risk control option
RIMER	Rule based inference methodology using the evidential reasoning
TOPSIS	Technique for order preference by similarity to an ideal solution

Notation

A_i^k	The linguistic variables used to describe X_k ($i \in M$; $k \in L$).
X_k	The k^{th} child attribute in a parent-child link.
D_j	The linguistic variables used to describe Y ($j \in N$).
Y	The parent attribute in a parent-child link.

[2] The singular and plural of an acronym are always spelled the same.

L	The total number of the child attributes.
M	The total number of linguistic variables used to describe the X_k child attribute.
N	The total number of linguistic variables used to describe the parent attribute.
F	A logical function representing the relationship between A_i^k and D_j.
$\beta_{i,j}^k$	The belief degree to which D_j is believed to be the parent attribute if the k^{th} child attribute is only described by the i^{th} linguistic variable.
w_k	The weight of the k^{th} child attribute in the X-Y link.
p_i^k	The subjective probability of the input information from real observations belonging to A_i^k.
β_j^k	The belief degree to which D_j is believed to be the parent attribute if the input satisfies the k^{th} child attribute linguistic vector A_i^k ($i \in M$).
m_D^k	The probability mass unassigned to any individual output variables D_j, when synthesising β_j^k ($k \in L$).
u_{S_H}	The utility values of S_H.
RI	Failure ranking index.

1. INTRODUCTION

The growing technical complexity of large engineering systems, together with the intense public concern over their safety has stimulated the research and development of novel risk analysis and safety assessment procedures. However, the process of developing cost-effective, timely and flexible risk assessment methods for complex and large engineering systems has been the subject of considerable debate in recent years. It is mainly due to the fact that modelling and analysing complex risk scenarios increasingly needs to acquire the historical failure data/information as sufficient and precise as possible and to treat relevant uncertainties as flexible and comprehensive as possible.

It may be difficult to use typical safety assessment approaches such as a probabilistic risk assessment approach in situations where there is a lack of information and past experience, or in ill-defined situations in risk analysis [26]. Under prevailing circumstances, little numerical data of any statistical significance may be available to sufficiently support traditional "objective" risk analysis in complex engineering systems. For example, to analyse system safety in the design and operation of large offshore installations with a high level of innovation, it is highly possible that there is a lack of historical failure data. The uncertainty encountered in such engineering system safety analysis does not fit the axiomatic basis of probability theory. It is usually caused by not only randomness but also the inherent fuzziness of parameters and the incompleteness of input. In engineering safety analysis, fuzziness exists due to ill-defined concepts in observation, or the inaccuracy and poor reliability of instruments used to make observations [14]. Incompleteness may be caused due to weak implication which occurs when an expert is unable to establish a strong correlation between premise and conclusion [14]. This means that intrinsically vague information may coexist with conditions of "lack of specificity" originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or credibility [3], [37].

One realistic way to express fuzziness is to employ subjective/linguistic assessment based on fuzzy set theory (*FST*) [5]. One of the most popular fuzzy logic approaches is developed based on the fuzzy *IF-THEN* rules, where conditional parts and/or conclusions contain linguistic variables [41]. It can model the qualitative aspects of human knowledge and thus does not require an expert to provide a precise point at which a risk factor exists. This actually provides a tool for working directly with the linguistic information, which is commonly used in representing risk factors and carrying out safety assessment [1], [2], [9], [13], [18], [21], [25], [28], [29], [33]. The Dempster-Shafer (*D-S*) theory of evidence [8], [20] based on the concept of belief function is well suited to modelling subjective credibility induced by partial evidence [23]. Based on the *D-S* theory, an Evidential Reasoning (*ER*) approach [32], [35] has been developed to describe and handle uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. The kernel of this approach is an *ER* algorithm, which requires modelling the narrowing of the hypothesis set with the accumulation of evidence [35]. The approach shows great potential in multiple attribute decision making (*MADM*) under uncertainty and its wide applications have been documented in the literature [15], [21], [24], [29], [34], [35].

In recognition of the need to handle the hybrid of both fuzziness and incompleteness in complex engineering system safety, it may be desirable and beneficial to extend the fuzzy logic framework to cover credibility uncertainty based on *FST* and *ER*. The benefit of combining fuzzy logic and belief models may become substantial when a lack of specificity in data is prevalent [13]. Several studies have been conducted to investigate the relationship between fuzzy sets and *D-S* theory and suggest different ways of integrating them [7], [10], [17], [30], [31], [33]. Based on fuzzy logic and *ER*, a generic Rule based Inference Methodology using the Evidential Reasoning (*RIMER*) has been proposed [33] and applied to engineering system safety analysis [13], [14]. The model is designed on the basis of a fuzzy rule base with a belief structure. Such a rule base can function on the solutions of the non-linear causal relationships as well as incompleteness and vagueness associated with risk parameters/decision attributes. It can be presented as a belief rule expression matrix, which forms a basis in the inference mechanism of *RIMER* and provides a framework for representing expert knowledge in a compact format [33]. The feature of the rule base (the difference from the original *IF-THEN* rule bases) lies in the fact that the rules include various belief degrees distributed into the multiple linguistic variables of the conclusion attribute. The main advantage of using belief degrees is to capture uncertainty and non-linear casual relationships in safety assessment. However, such a merit could also lead to some debates. The corresponding complex uncertainty inference may be not friendly enough to mathematically unsophisticated users. A Fuzzy Rule-based Bayesian Reasoning (*RuRBaR*) approach was developed to simplify the uncertainty inference of belief rule bases [37]. However, this approach, like *RIMER*, still needs to construct and establish the complex rule bases, which may be error-prone. There is a high possibility to assign incompatible belief degrees in a fuzzy rule base when a large scale scheme with hundreds of rules is defined entirely subjectively. "Incompatible" means contradictory belief degree distributions in two/more relevant rules, in which one rule which should have a better risk evaluation has actually been assigned belief degrees presenting a worse safety level. For example, in the safety rule base developed by Liu et al. [13], [14], Rule 85 and Rule 86 with their *IF* parts as the sets of {"reasonable low" failure occurrence likelihood (*L*), "moderate" failure consequence severity (*C*), "highly unlikely" failure consequence probability (*P*)} and {"reasonable low" *L*, "moderate" *C*, "unlikely" *P*} have been assigned their *THEN* parts as the sets of {(0.5, "good"), (0.5, "average"), (0, "fair"), (0, "poor")} and {(0.6, "good"), (0.4, "average"), (0, "fair"), (0, "poor")}. Furthermore, when new attributes are identified in the *IF* parts, it may be difficult to update the original rule base and to produce the output without the reestablishment of the whole fuzzy rule base systems.

This paper proposes a novel Fuzzy Evidential Reasoning (*FER*) approach, being capable of eliminating the "incompatible" belief degree distributions in traditional fuzzy IF-THEN rule based risk assessment methods while making subjective risk assessment more rational and visible. The new method is generated by extending the *RIMER* approach on the basis of the combination of several different theories and techniques, such as the *FST* [40], an *AHP* technique [19] and the *ER* approach [36], etc. It shares the same philosophy with the divorce approach in Bayesian networks [11], showing its superiority in modelling incompleteness of subjective judgement and accommodating additional attributes compared to the *RIMER*. The main feature of the new method is first to analyse the conditional belief degree distributions of the parent (conclusion) attributes given the individual child (antecedent) attributes in a fuzzy hierarchical link, then use all conditional belief degree distributions as an effective link to transfer input (based on child attributes) to output (based on a common parent attribute) and finally synthesise all output together using the *ER* approach. By doing so, the complex work of constructing fuzzy rule bases can be replaced through establishing relatively straightforward fuzzy link diagrams. Contributions drawn from such a generic method are examined by a case study of collision risk between a floating, production, storage and offloading (*FPSO*) unit and a shuttle tanker due to technical failure during a tandem offloading operation. It can be applied to facilitate subjective risk assessment in different sectors when subjective data collection is needed for risk prioritisation of the identified hazards in engineering systems.

2. EXPERT SYSTEM STRUCTURE BASED ON THE NEW FUZZY EVIDENTIAL REASONING

The generic structure of fuzzy link based expert systems is described in this section. Being the extension of fuzzy rule based systems, fuzzy link based expert systems can be generally considered as a multiple attribute decision making tool. The starting point for investigating a fuzzy link based system is to construct a hierarchy of representing the relationship among all decision attributes from the lowest level to the top one. A knowledge base and an inference engine are then designed to infer useful conclusions from observable facts. Unlike rule based systems, the knowledge base is presented by belief links between a single pair of parent-child attributes in two neighbouring levels instead of *IF-THEN* rules (between multiple antecedent-conclusion attributes). This section mainly concentrates on the development of belief links and the generation of the knowledge base, in part because they are the relatively novel and unsatisfactorily developed in the literature. The inference engine based on an *ER* approach for capturing the non-linear relationship between attributes is briefly illustrated.

A fuzzy link base model can be established to deal with imprecision using linguistic assessments with belief degrees. The model employing fuzzy belief links where the parent-child attributes contain linguistic variables, can model the qualitative aspects of human knowledge and the reasoning process. It can be represented as follows.

$$R= <X, A, Y, D, F, \beta, w> \qquad (1)$$

In the above, $X=\{X_k, k=1, \ldots, L\}$ is the set of child attributes, with each of them taking values from an array of fuzzy sets $A=\{A_i^k, i=1, \ldots, M\}$. A_i^k represents a set of fuzzy values (linguistic variables) used to describe the attribute X_k ($k=1, \ldots, L$). The array $\{X_1, \ldots, X_L\}$ defines a list of finite conditions, representing the elementary states of a decision problem domain. $D=\{D_j, j=1, \ldots, N\}$ is the set of linguistic variables used to describe the parent attribute Y in a parent-child link, representing a utility decision space. F is a logical function, representing the relationship between A_i^k and D_j using belief links β_j^k ($\sum_{i=1}^{M} \beta_{i,j}^k \leq 1$). $w=\{w_k, k=1, \ldots, L\}$ is the set of child attribute weights. The combination of all such parameters constitutes one basic element of a multiple level hierarchical belief link based model R, given $Y \in X'$ (the set of child attributes of the next higher level analysis). Consequently, all the β_j^k at different levels can be analysed and connected/multiplied to represent the relationship between the bottom level attributes and the top level one.

Suppose input information associated with each X_k can be obtained and represented by A_i^k with appropriate subjective probabilities p_i^k. A fuzzy link based model given by Eq. (1) can then be extended to emphasise the relationship between the bottom level A_i and top level D_j and transform the input associated with X_k (k = 1, …, L) (observation facts) into the output based on a utility decision making space D_j (j = 1, …, N) as follows.

$$\beta_j^k = \sum_{i=1}^{M} p_i^k \beta_{i,j}^k \qquad (2)$$

To capture the non-linear relationship between decision attributes X_k (k = 1, …, L), the *ER* approach [32], [36] is used to combine all output β_j^k (j = 1, …, N; k = 1, …, L) associated with X_k and generate final conclusions. The window-based and graphically designed intelligent decision system (*IDS*) has been developed based on the *ER* approach [35], which facilitates the combination [14]. Having represented belief degrees β_j^k transformed from input for each decision attribute X_k, the *ER* approach can be directly implemented as follows. First, transform the degrees of belief β_j^k for all j = 1, …, N, k = 1, …, L into basic probability masses using the following equations [14], [32], [36].

$$m_j^k = w_k \beta_j^k, \; m_D^k = 1 - \sum_{j=1}^{N} m_j^k = 1 - w_k \sum_{j=1}^{N} \beta_j^k,$$

$$\overline{m}_D^k = 1 - w_k, \text{and } \tilde{m}_D^k = w_k\left(1 - \sum_{j=1}^{N} \beta_j^k\right), \text{ for all } j = 1, \ldots, N \text{ and } k = 1, \ldots, L. \qquad (3)$$

where $m_D^k = \overline{m}_D^k + \tilde{m}_D^k$ for all k = 1, …, L and $\sum_{k=1}^{L} w_k = 1$. The probability mass assigned to the top level attribute D, which is unassigned to any individual output variables D_j, is split into two parts, one caused by the relative importance of the k^{th} attribute (or $\overline{m}_D^k$), and the other due to the incompleteness of the subjective probabilities/ transformed belief degrees associated with A_i^k (or $\tilde{m}_D^k$).

Next, aggregate all the output from X_k (k = 1, …, L) to generate the combined degree of belief in each possible top level linguistic variable D_j of D. Suppose $m_j^{I(k)}$ is the combined belief degree in D_j by aggregating all the output from the k child attributes and $m_D^{I(k)}$ is the remaining belief degree unassigned to any top level output variable. Let $m_j^{I(1)} = m_j^1$ and $m_D^{I(1)} = m_D^1$. Then the overall combined belief degree in D_j is generated as follows [14].

$$\{D_j\}: m_j^{I(k+1)} = K_{I(k+1)}[m_j^{I(k)} m_j^{k+1} + m_j^{I(k)} m_D^{k+1} + m_D^{I(k)} m_j^{k+1}]$$

$$m_D^{I(k)} = \tilde{m}_D^{I(k)} + \overline{m}_D^{I(k)} \quad k = 1, \ldots, L-1;$$

$$\{D\}: \tilde{m}_D^{I(k+1)} = K_{I(k+1)}[\tilde{m}_D^{I(k)} \tilde{m}_D^{k+1} + \tilde{m}_D^{I(k)} \overline{m}_D^{k+1} + \overline{m}_D^{I(k)} \tilde{m}_D^{k+1}]$$

$$\overline{m}_D^{I(k+1)} = K_{I(k+1)}[\overline{m}_D^{I(k)} \overline{m}_D^{k+1}]$$

$$K_{I(k+1)} = \left[1 - \sum_{j=1}^{N} \sum_{\substack{t=1 \\ t \neq j}}^{N} m_j^{I(k)} m_t^{k+1}\right]^{-1}, \; k = 1, \ldots, L-1;$$

$$\{D_j\}: \beta_j = \frac{m_j^{I(L)}}{1 - \overline{m}_D^{I(L)}} \; (j = 1, \ldots, N), \text{ and } \{D_j\}: \beta_D = \frac{\tilde{m}_D^{I(L)}}{1 - \overline{m}_D^{I(L)}} \qquad (4)$$

where β_D represents the remaining belief degrees unassigned to any D_j.

3. USE OF THE NEW *FER* TO SUBJECTIVE RISK ASSESSMENT

The proposed framework for modelling system safety consists of eight major steps required for subjective risk analysis using the *FER* approach.

1. Identify the attributes of risk analysis (associated with X and Y).
2. Establish a hierarchy to represent the relationship between the attributes (related to w).
3. Define the linguistic variables used to describe the attributes (associated with A and D).
4. Construct a fuzzy link base with belief structures (associated with F and β).
5. Input transformation (associated with p)
6. Reasoning mechanism based on an *ER* algorithm.
7. Risk ranking

8. Validation using benchmarking.

Each step of the framework is described in detail in the following sections.

Step 1: Having stated the framework above, the first step to develop a general risk analysis model is to identify appropriate risk attributes. After the study of traditional quantitative risk methods like failure modes, and effects (FMEA) analysis, it can be seen that there are three basic attributes - failure likelihood (L), consequence severity (C) and failure consequence probability (P) (i.e. the probability that possible consequences happen, given the occurrence of the failure), which are used in assessing the safety (S) associated with each failure mode of a component and in determining safety level of a whole system [13], [14] , [29].

Step 2: As a result, a hierarchy to represent the relationship between the three basic risk attributes and the safety level of a complex engineering system (and its components) can be developed and shown in Figure 1. It has several levels and the three basic attributes at the bottom level. In such a hierarchical structure, it is usually the case that risk analysis at a higher level makes use of the information produced at lower levels. It is therefore important to synthesise the evaluations of the risk attributes and the components with appropriate weight distributions (between them) so as to obtain the risk estimation of the whole system. When considering a group of attributes for evaluation, an *AHP* technique can provide judgements on the relative importance of these attributes and also to ensure that the judgements are quantified to an extent, which permits a quantitative interpretation of the judgement among these attributes [18]. Using an *AHP* technique to calculate the relative importance of each attribute has been well accepted and evidenced by many publications and applications in the literature [26] and thus, it is used to calculate the weight distributions between the parameters and components in the hierarchy.

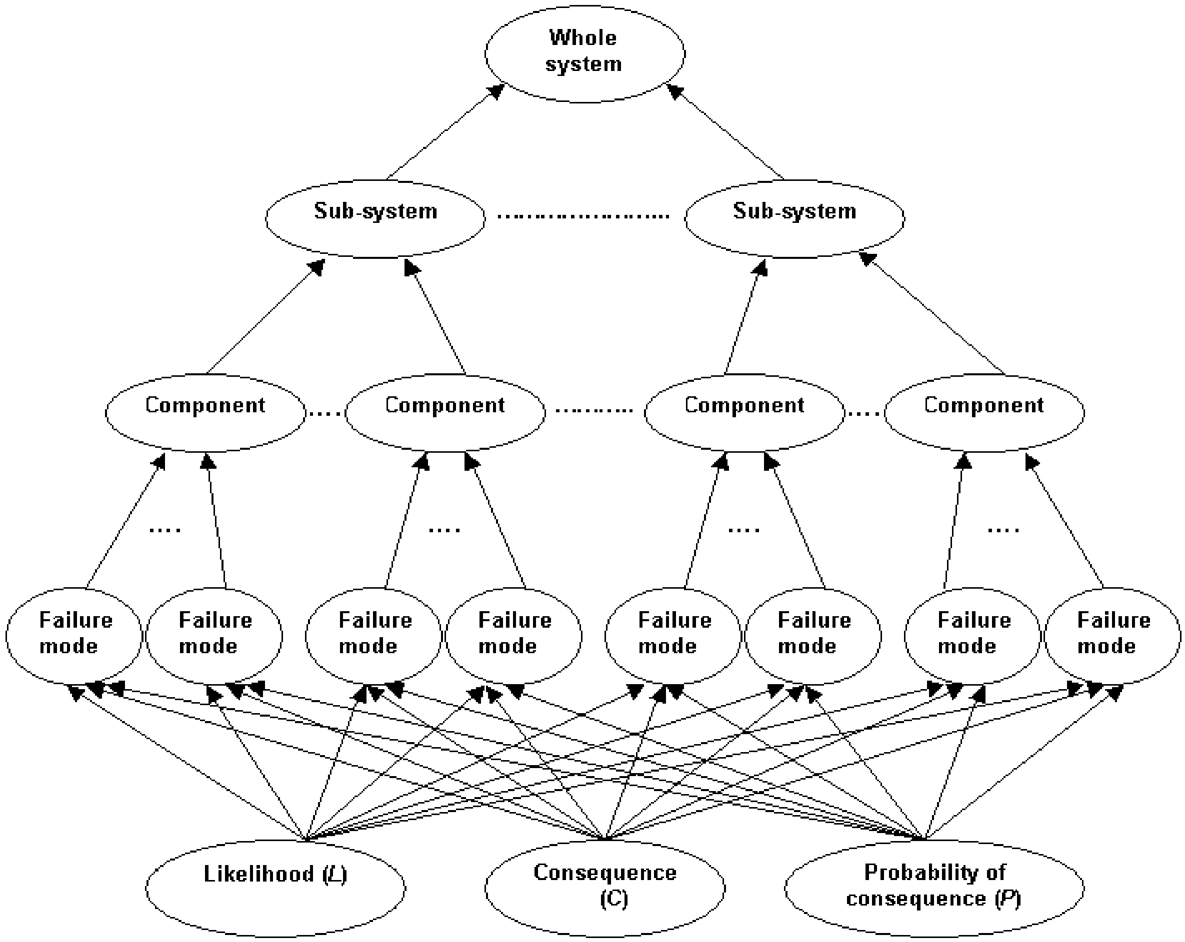

Figure 1. A hierarchical diagram of system safety analysis [29]

Step 3: Using fuzzy linguistic assessments allows the representation of information in a natural and adequate form when it is difficult to express the information with precision. It is commonly used by human experts to represent risk factors in risk analysis [2], [13], [14], [18], [21], [28], [29]. The linguistic variables for describing each attribute are decided according to the situation of the case of interest. The literature survey indicates that the linguistic variables and their membership functions used to describe L, C and P of a particular item may be defined on a unified utility space, in which 0 indicates the lowest risk contribution and 10 means the highest risk contribution [22], [28], [29], as shown in Table I.

Table I. Linguistic variables of risk attributes and their fuzzy memberships

Safety parameters	Linguistic variables	Fuzzy memberships
Likelihood (***L***)	Very low (L_1)	(0, 0, 3, 4)
	low (L_2)	(3, 4, 5)
	Reasonably low (L_3)	(4, 5, 6)
	Average (L_4)	(5, 6, 7)
	Reasonably frequent (L_5)	(6.5, 7, 9.5)
	Frequent (L_6)	(7, 9, 9.5)
	Very frequent (L_7)	(9, 9.5, 10)
Consequence (***C***)	Negligible (C_1)	(0, 0, 1, 3)
	Marginal (C_2)	(1, 2, 3, 4)
	Moderate (C_3)	(3, 4, 6, 7)
	Critical (C_4)	(6, 7, 8, 9)
	Catastrophic (C_5)	(8, 9, 10, 10)
Probability (***P***)	Highly unlikely (P_1)	(0, 0, 1, 2)
	Unlikely (P_2)	(1, 2, 3, 4)
	Reasonably unlikely (P_3)	(3, 4, 5)
	Likely (P_4)	(4, 5, 6)
	Reasonably likely (P_5)	(5, 6, 7, 8)
	Highly likely (P_6)	(7, 8, 9)
	Definite (P_7)	(8, 9, 10, 10)
Safety (***S***)	Good (S_1)	(0,0,2,4)
	Average (S_2)	(2,4,6)
	Fair (S_3)	(4,6,8)
	Poor (S_4)	(6, 8, 10, 10)

Step 4: This step is to establish the belief links between each risk attribute's linguistic variables (L_I, C_J, P_K) and the variables (S_H) used to describe the safety levels of failure modes and the whole system. Generally, the belief degrees between the variables can be assigned based on multiple expert judgements. In this process, a fuzzy mapping approach based on utility theory is used with reference to the work by Li and Liao [12] to assist in obtaining the belief degrees. The philosophy of the approach is to calculate the belief degrees by referring to the comparison of the similarity between the fuzzy membership functions on a common utility (risk contribution) space of the linguistic variables, which are used to describe a pair of child-parent attributes (e.g. L and S). Specifically speaking, it can be symbolised as follows.

Let u_{L_I} and u_{S_H} be the fuzzy membership functions of L_I (I = 1, …, 7) and S_H (H = 1, …, 4), respectively. The similarity degree between u_{L_I} and u_{S_H} is calculated using Eq. (5) due to its simplification [12], despite the availability of the other similarity degree measurement methods (e.g. distance-based).

$$S(u_{L_I}, u_{S_H}) = \frac{\int_{-\infty}^{+\infty}\left(\min\{u_{L_H}(x), u_{S_H}(x)\}\right)\mathrm{dx}}{\int_{-\infty}^{+\infty} u_{L_I}(x)\mathrm{dx}} \tag{5}$$

If two membership functions are the same, that is $u_{Li} = u_{S_H}$, then $S(u_{L_I}, u_{S_H})$ =1. If two membership functions do not have any overlap, the similarity degree is zero. For other situations, the higher the percentage of the overlap is, the higher the similarity degree. In this process, it is particularly noteworthy that the fuzzy membership functions can be obtained using a fuzzy Delphi method [4] and the membership functions u_{Li} and u_{S_H} are developed on the basis of one common utility space (i.e. the scale [0–10] indicating risk contributions). After all the similarity degrees between u_{L_I} and u_{S_H} are computed, the belief degree $\beta^L_{L_I, S_H}$ for all I = 1, …, 7 and H = 1, …, 4, can be computed as follows:

$$\beta^L_{L_I, S_H} = \frac{S(u_{L_I}, u_{S_H})}{\sum_{H=1}^{4} S(u_{L_I}, u_{S_H})} \tag{6}$$

From Eqs. (5) and (6), it can be noted that the more similar u_{Li} is to u_{S_H} , the closer L_I is to S_H and the larger $\beta^L_{L_I, S_H}$ is. It can also be noted that the sum of $\beta^L_{L_I, S_H}$ is equal to 1. Thus, $\beta^L_{L_I, S_H}$ may be viewed as a conditional degree of risk contribution to which L_I contributes to S_H (conditioned on that the risk contributions from C_J and P_K to S_H are unknown). In this way, all fuzzy membership functions of the risk attributes (i.e. u_{L_I} , u_{C_J} and u_{P_K}) can be transformed into the belief structures with the fuzzy set of a utility decision conclusion space, S_H.

Having defined the belief links between the linguistic variables of the three risk attributes and the safety levels of the failure modes, it is straightforward to identify the belief links between the safety levels of the failure modes, components, sub-systems and the whole system as follows.

$$\beta^{S}_{S_H, S_{H'}} = \begin{cases} 1(H = H') \\ 0(H \neq H') \end{cases} \qquad (7)$$

where $S_{H'}$ indicates the linguistic safety variables of the parent attribute and S_H represents the linguistic safety variables of its child attributes.

<u>Step 5:</u> Before starting the safety analysis process, observations from experts should be analysed to determine their relationship with each (L, C, P) at the bottom level of the hierarchy. Four kinds of possible observations may be represented using membership functions to suit conditions under this study. They are either a single deterministic value with 100% certainty, a closed interval, a triangular distribution or a trapezoidal distribution [22].

Having defined the membership functions of the three parameters as u_{L_I}, u_{C_J} and u_{P_K}, a transforming or matching function method [13] can be employed to perform the observation transformation and determine the subjective probabilities to which actual observations match each linguistic variable of the three safety parameters. The matching function method chooses the *Max-Min* operation to show the relationship between the real input fuzzy set u_r and the corresponding fuzzy linguistic variables u_{L_I}, u_{C_J} and u_{P_K}, because it is a classical tool to set the matching degree between fuzzy sets [41]. Therefore, the subjective probabilities, $p_r^{L_I}$ to which real observations belong to u_{L_I} can be defined as follows [13].

$$\hat{p}_r^{L_I} = \tau(u_r, u_{L_I}) = \max[\min_x (\mu_r(x), \mu_{L_I}(x))] \qquad (8)$$

where x covers the domain of the input u_r. Each $\hat{p}_r^{L_I}$ represents the extent to which u_r belongs to the defined linguistic variables in L. In order to keep the completeness, each $\hat{p}_r^{L_I}$ is normalised as $p_r^{L_I}$ so that their sum equals one.

Step 6: As a result, an appropriate five-level belief link based safety analysis model can be represented. Using Eq. (2), all the input can be transformed into the relative output based on the linguistic variables S_H of the safety level of the whole system at the top. The *ER* approach introduced in Eqs. (3) and (4) can be implemented to combine all the output and generate final conclusions, which are expressed using the four linguistic variables S_H at the top level of the hierarchy.

Step 7: In order to rank the safety estimates expressed by S_H, the fuzzy linguistic variables require to be defuzzified by giving each of them an "appropriate" utility value (U_{S_H}). Since the membership functions of S_H, u_{S_H} have been defined in Table I, U_{S_H} can be obtained using defuzzification methods such as a *centroid* approach [17]. Then, the ranking order index *RI* associated with system safety can be obtained by

$$RI = \sum_{H=1}^{4} \beta_H U_{S_H} \tag{9}$$

where the numerical values of β_H (H = 1, 2, …, 4) represent the belief degrees associated with the linguistic variables S_H in the final output (safety level of the whole system) synthesised by using the *ER* approach.

Step 8: When a new method is developed, it requires testing to ensure its soundness. It may be especially important and desirable when subjective elements are involved in the methodology generated [38]. There are several well-accepted validation methods available before the methodology can be broadly accepted for use in practice. In this study, a benchmarking technique will be used to compare the *FER* method and the *RIMER* approach [13], [33] through the analysis of their correlation.

4. A NUMERICAL STUDY OF AN OFFSHORE ENGINEERING SYSTEM

In order to compare with the well-established *RIMER* method, the case study in the work [13] is re-visited and investigated through the new *FER* method here. In this section, failure criticality analysis is carried out on risk introduced by the collision of a *FPSO* system with a shuttle tanker during tandem offloading operations. Only technical failure caused risk is assessed, though the operation failure has also been recognised as one of the major causes of collision. According to the literature survey, a technical failure that might cause collisions between the *FPSO* and a shuttle tanker during tandem offloading operations is a malfunction of the propulsion system. The four major causes to this technical failure are controllable pitch propeller (*CPP*), thruster (*T*), position reference system (*PRS*) and dynamics positioning system (*DP*) failures [6].

As a result, a hierarchy to represent the relationship between the three basic risk attributes and the safety levels of a malfunction of the propulsion system (and its four failure modes) can be developed. Next, the weights of the three basic safety parameters and the four failure modes require to be calculated using an *AHP* method as follows [18].

$$M_S = \begin{array}{c} \\ L \\ C \\ P \end{array} \begin{array}{c} \begin{array}{ccc} L & C & P \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1.98 \\ 0.5 & 0.505 & 1 \end{bmatrix} \end{array}$$

$$[w_L, w_C, w_P] = [0.4, 0.399, 0.201]$$

$$M_{FM} = \begin{array}{c} \\ CPP \\ T \\ PRS \\ DP \end{array} \begin{array}{c} \begin{array}{cccc} CPP & T & PRS & DP \end{array} \\ \begin{bmatrix} 1 & 0.75 & 0.8 & 0.9 \\ 0.33 & 1 & 1.2 & 1.1 \\ 1.25 & 0.83 & 1 & 1.1 \\ 1.11 & 0.91 & 0.91 & 1 \end{bmatrix} \end{array}$$

$$[w_{CPP}, w_T, w_{PRS}, w_{DP}] = [0.213, 0.286, 0.257, 0.243]$$

A measure of the consistency of the two pair-wise comparisons has been conducted by computing their individual consistency ratios. Both of the values have been validated to be less than 0.1 and thus, the pair-wise comparisons are considered consistent and reasonable.

After the fuzzy membership functions of all linguistic variables are acquired, the belief links between the basic parameters and the safety levels of failure modes may be calculated using the information of these variables. Generally, the evaluation of the belief links may be conducted in the common space of safety level S_H, the sets described by the linguistic variables in Table I.

In order to evaluate $\beta^{L}_{L_I,S_H}$, $\beta^{C}_{C_J,S_H}$ and $\beta^{P}_{P_K,S_H}$, it is desirable to use the fuzzy mapping technique in Eqs. (5) and (6). For example, the similarity degree between u_{C_2} and u_{S_H}, (H = 1, 2, 3, 4) (see Figure 2) is calculated as follows.

$$S(u_{C2}, u_{S1}) = \frac{Area1}{Area1 + Area2 + Area3} = 0.75,$$

$$S(u_{C2}, u_{S2}) = \frac{Area2}{Area1 + Area2 + Area3} = 0.33,$$

$$S(u_{C2}, u_{S3}) = \frac{0}{Area1 + Area2 + Area3} = 0,$$

$$S(u_{C2}, u_{S4}) = \frac{0}{Area1 + Area2 + Area3} = 0.$$

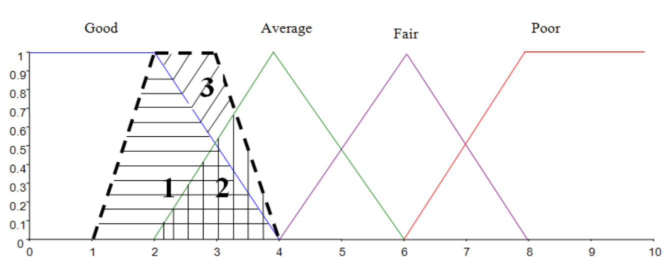

Figure 2. Example of the belief degree calculation between C_2 and S_H (H = 1, 2, 3, 4)

Furthermore, using Eq. (6), $S(F_{C2}, u_{S_H})$ can be normalised as follows.

$$\beta^{C}_{C_2,S_1} = \frac{S(F_{C2}, u_{S1})}{\sum_{h=1}^{4} S(F_{C2}, u_{Sh})} = \frac{0.75}{1.08} = 0.69, \quad \beta^{C}_{C_2,S_2} = \frac{S(F_{C2}, u_{S2})}{\sum_{h=1}^{4} S(F_{C2}, u_{Sh})} = \frac{0.33}{1.08} = 0.31,$$

$$\beta^{C}_{C_2,S_3} = \frac{S(F_{C2}, u_{S3})}{\sum_{h=1}^{4} S(F_{C2}, u_{Sh})} = \frac{0}{1.08} = 0, \quad \beta^{C}_{C_2,S_4} = \frac{S(F_{C2}, u_{S4})}{\sum_{h=1}^{4} S(F_{C2}, u_{Sh})} = \frac{0}{1.08} = 0.$$

In this way, fuzzy number C_2 can be transformed into the belief structure with the safety level expressions (S_H, H = 1, 2, 3, 4) and shown in the following form.

F_{C2} = {(0.69, “good”), (0.31, “average”), (0, “fair”), (0, “poor”)}

Using a similar calculation process, F_{C_J} (J =1, …, 5) can be obtained respectively. The belief links between C_J and S_H of the failure modes and further the top level (the propulsion systems) can be constructed and shown in Figure 3 using Eq. (7). Similarly, the belief links between L_I or P_K and S_H can be established for the fuzzy safety analysis model of the propulsion system.

Having constructed the belief links, the next step is to calculate the input p_i^k from observations. It is assumed that each input parameter (L, C or P) will be fed into the proposed safety model in terms of any of the four input forms described in Step 5 of Section 3 to address different levels of uncertainty. In this case study, the same subjective data in [13] from a panel of five experts from different disciplines is used here, as the one shown in Table II for the failure mode *CPP*. The five experts include a technical manager of more than 20 years experience in the offshore industry from the UK HSE, a senior inspector of more than 20 years experience in the offshore industry from the UK HSE, a team leader of more than 20 years experience in the offshore safety sector from Shell, a senior engineer of more than 20 years experience in the offshore safety sector from Shell and a senior academic researcher of more than 15 working experience in marine and offshore fields.

In order to conduct safety estimation using the new *FER*, the expert judgements obtained above are transformed into the forms, which can be accepted by the belief link based inference mechanism. Therefore, Eq. (8) and a normalisation process can be used to compute p_i^k, which is associated with L_I, C_J or P_K, respectively. For example, Expert #1 used the triangular distribution input form to address the inherent uncertainty associated with the data and information available while assessing the three basic input attributes. The failure consequence severity *C* is described triangularly as (7.5, 8.5, 9.5). Using the fuzzy Max-Min operation in Eq. (8), these input values are transformed and then normalised into the distributed representation on the linguistic variables as follows.

$\hat{p}_r^{C_J}$ = {0 C_1, 0 C_2, 0 C_3, 0.75 C_4, 0.78 C_5}

$p_r^{C_J}$ = {0 C_1, 0 C_2, 0 C_3, 0.49 C_4, 0.51 C_5}

In a similar way, the following is obtained.

$p_r^{L_I}$ ={0 L_1, 0 L_2, 0 L_3, 0.108 L_4, 0.378 L_5, 0.378 L_6, 0.135 L_7}

$p_r^{P_K}$ = {0 P_1, 0 P_2, 0 P_3, 0.1 P_4, 0.5 P_5, 0.3 P_6, 0.1 P_7}

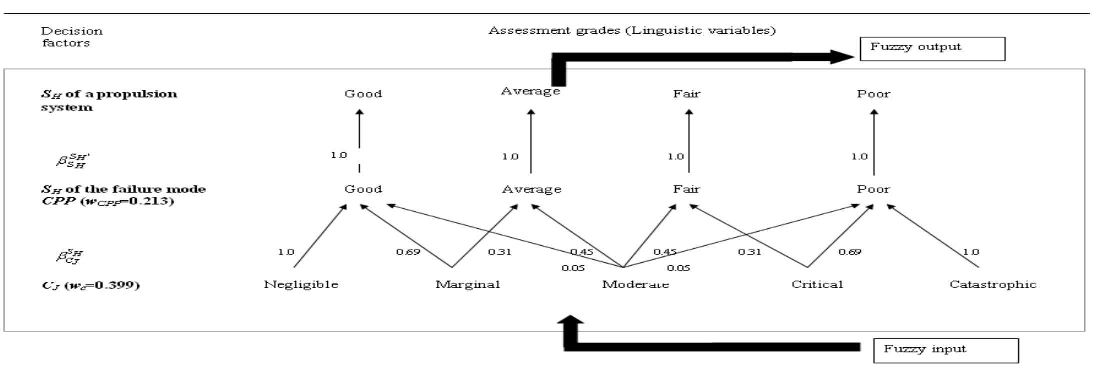

Figure 3. An example of transforming fuzzy C_J to S_H

Table II. Expert judgement of the three input parameters for the *CPP* failure

Expert	**Input form**	***L***	***C***	***P***

E#1	Triangular	(6.5, 8, 9.5)	(7.5, 8.5, 9.5)	(5.5, 7, 8.5)
E#2	Triangular	(5.5, 7.5, 9)	(7, 8.5, 10)	(5, 7.5, 9.5)
E#3	Closed interval	[6, 8]	[7, 9]	[6.5, 9]
E#4	Trapezoidal	{5.5, 6.5, 9, 10}	{5.5, 7, 8, 10}	{5, 7, 8, 8.5}
E#5	Deterministic	7.75	8.25	7.6

Next, using Eq. (2) and Fig. (3), such input can be transformed into output and expressed by S_H (H = 1, 2, 3, 4) as follows.

$$\beta_{S_H}^{C} = \sum_{J=1}^{5} p_{C_J}^{C} \beta_{C_J, S_H}^{C} = \{0\ S_1, 0\ S_2, 0.152\ S_3, 0.848\ S_4\}$$

where 0.152 = (0.49 × 0.31) and 0.848 = (0.49 × 0.69 + 0.51 × 1). In a similar way, the following can be obtained.

$$\beta_{S_H}^{L} = \{0\ S1, 0.0135\ S2, 0.2133\ S3, 0.7722\ S4\}$$

$$\beta_{S_H}^{P} = \{0\ S1, 0.085\ S2, 0.412\ S3, 0.503\ S4\}$$

Based on such output, together with the weight set obtained above, the *ER* based *IDS* is used to implement the combination of the three pieces of output and generate the safety estimate for the propulsion system due to the failure *CPP* by Expert #1. The assessment result is generated as follows and shown in Figure 4 where the generated result is {0 *S1*, 0.0147 *S2*, 0.1858 *S3*, 0.7996 *S4*}

This result can be interpreted in a way that the safety estimate of the propulsion system due to *CPP* by Expert #1 is "*Average*" with a belief degree of 0.0147, "*Fair*" with a belief degree of 0.1858 and "*Poor*" with a belief degree of 0.7996.

The similar computations are performed by the other four experts for *CPP*. The safety estimates generated for *CCP* by the five experts are summarised in Table III. Similarly, the safety estimates the propulsion system due to the other three potential failures (*T*, *PRS* and *DP*) can be calculated. Tables IVa – IVb show the results of multi-expert safety synthesis on collision risk between the *FPSO* and a shutter tanker due to *CPP*, *T*, *PRS* and *DP* caused technical failure using the new *FER* approach respectively. The synthesis is carried out using both equal and different (i.e. $\{w_{E\#1}; w_{E\#2}; w_{E\#3}; w_{E\#4}; w_{E\#5}\} = \{0.3, 0.5, 0.9, 0.8, 1\}$) relative weights among the experts.

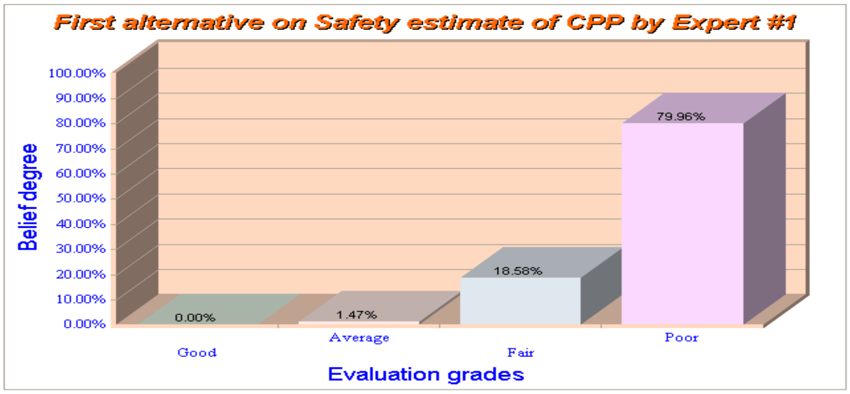

Figure 4. The safety estimate for the failure CPP by Expert #1

Table III. Safety estimate by five experts on the collision risk due to *CPP*

Expert	***L***	***C***	***P***	***S1***	***S2***	***S3***	***S4***
E#1	(6.5, 8, 9.5)	(7.5, 8.5, 9.5)	(5.5, 7, 8.5)	0	0.0147	0.1858	0.7996
E#2	(5.5, 7.5, 9)	(7, 8.5, 10)	(5, 7.5, 9.5)	0	0.0335	0.2321	0.7344
E#3	[6, 8]	[7, 9]	[6.5, 9]	0	0.0177	0.235	0.7473
E#4	{5.5, 6.5, 9, 10}	{5.5, 7, 8, 10}	{5, 7, 8, 8.5}	0.0044	0.0794	0.2751	0.6411
E#5	7.75	8.25	7.6	0	0.0036	0.2003	0.7961

Table IVa. Multi-expert safety synthesis based on the contributions from the four failures with equal expert weights

Safety contributions from the technical failure modes	Safety synthesis			
	S1 (Good)	*S2* (Average)	*S3* (Fair)	*S4* (Poor)
Safety estimate with all contributions from *CPP*, *T*, *PRS* and *DP*	0.0013	0.0259	0.2083	0.7645
Safety estimate with the only contribution from *CPP*	0.0006	0.0222	0.1857	0.7915
Safety estimate with the only contribution from *T*	0.0029	0.0549	0.3099	0.6323
Safety estimate with the only contribution from *PRS*	0.0015	0.0286	0.2712	0.6988
Safety estimate with the only contribution from *DP*	0.0016	0.0238	0.1934	0.7812

Table IVb. Multi-expert safety synthesis based on the contributions from the four failures with different expert weights

Safety contributions from the technical failure modes	Safety synthesis			
	S1 (Good)	*S2* (Average)	*S3* (Fair)	*S4* (Poor)
Safety estimate with all contributions from *CPP*, *T*, *PRS* and *DP*	0.0019	0.0365	0.2358	0.7258
Safety estimate with the only contribution from *CPP*	0.0011	0.0303	0.2045	0.7642

Safety estimate with the only contribution from *T*	0.0045	0.0849	0.3562	0.5544
Safety estimate with the only contribution from *PRS*	0.0016	0.0304	0.2788	0.6892
Safety estimate with the only contribution from *DP*	0.0023	0.0305	0.2125	0.7548

To calculate risk ranking index values associated with various failure modes, it is required to describe the four safety linguistic variables S_H (H = 1, …, 4) using utility values. The *centroid* defuzzification method is used to assign appropriate utility values of S_H. In the *centroid* method, the utility/crisp value is computed by finding the value of the centre of gravity of the membership function for the fuzzy number. From Table I, U_{S_H} can be computed and obtained as follows.

$$\{U_{S1}, U_{S2}, U_{S3}, U_{S4}\} = \{1.556, 4, 6, 8.444\}$$

where the ordering of the utility values of S_H indicates that the larger the risk index values are, the higher the associated potential risk. The risk ranking index value RI_{CCP} associated with the failure *CPP* is calculated based on the multi-expert safety synthesis shown in Table (IVa) with the equal weight and using Eq. (9) as follows.

$$RI_{CCP} = \sum_{H=1}^{4} \beta_H U_{SH} = 0.0006 \times 1.556 + 0.0222 \times 4 +$$
$$0.1875 \times 6 + 0.7915 \times 8.444 = 7.888$$

Based on the results shown in Tables 4a and 4b, the risk ranking index values associated with *CPP*, *T*, *PRS* and *DP* can be calculated and the results are summarised in Table V.

Table V. Risk ranking using the new *FER* method

Ranking items	Expert weights	Ranking			
		CPP	*T*	*PRS*	*DP*
Risk ranking index value	Equal weight	7.888	7.423	7.645	7.877
	Different weights	7.818	7.165	7.617	7.774
Risk ranking order	Equal weight	1	4	3	2
	Different weights	1	4	3	2

From Table V, it can be noted that for the two sets of weights assigned to the experts, the potential risk caused by *CPP* is always the highest and thus, it deserves more attention to reduce its potential risk to ALARP (As Low As Reasonably Practical). *T* is always ranked as the safest among the four failures and the potential risks caused by *DP* and *PRS* are ranked second and third, respectively. It is also noteworthy that the output with a small difference between the risk index values of the four failure modes reflects the similar input between the four failure modes from the five experts. The correlation between input and output may reveal that the model is sensitive and be capable of effectively dealing with the approximation associated with subjective judgement.

The benchmarking between the new model and the well-established *RIMER* method is also conducted to demonstrate the *FER's* reliability and unique characteristics. Because the two approaches are given the same fuzzy input in terms of subjective judgements, the final risk assessment results including safety estimates and ranking order should be kept in harmony to a significant extent in order to validate the feasibility of the proposed *FER* approach. According to [13], the safety estimates with the contributions from all the failure modes and the risk ranking orders of the four modes with different expert weights are presented in Table VI.

Table VI. Safety estimates and risk ranking using *RIMER*

Compared items	Expert weights	Safety estimates			
		S1 (Good)	*S2* (Average)	*S3* (Fair)	*S4* (Poor)
Safety estimates	Equal weight	0	0.0106	0.4698	0.5196
	Different weights	0	0.0115	0.4902	0.4938
		Ranking			
		CPP	*T*	*PRS*	*DP*
Risk ranking order	Equal weight	1	4	3	2
	Different weights	1	4	2	3

Comparing Table IV (both IVa and IVb) and Table VI, it can be found that the highest belief degree is always assigned to the safety linguistic variable “Poor” while the combined belief degrees assigned to “Fair” and “Poor” are more than 95% in both *FER* and *RIMER* methods. However, it is noted that the belief degrees assigned to “Fair” in the *RIMER* method is larger than the ones in the *FER* approach. This is mainly caused by the different ways of assigning belief degrees in the two studies. A sensitivity analysis has therefore been conducted to test the two ways (of degree distribution), with the result showing that impact of the input changes to the safety estimate in *RIMER* is inconsistent (sometimes irrational due to the incompatible rules involved) while it is always consistent in *FER*.

In terms of risk ranking, when the experts' weights are equally assigned, it has been shown that the four failures have been assessed and ranked in the same order in the two studies. When the experts are given different weights, the ranking orders of *DP* and *PRS* are different in the two studies. After a careful investigation, it has been disclosed that the difference ranking orders are caused by the different utility values assigned to the four safety linguistic variables U_{S_H} . It has been tested that if the same utility values are given, the ranking orders in the two cases will be identical. The above shows that the results obtained using the new *FER* and traditional *RIMER* are kept in harmony to a significant extent.

5. CONCLUSION

This paper proposes a fuzzy link based hierarchical group decision making framework for maritime system safety assessment. In the framework, the safety estimates of the basic events/failures are first carried out using a new *FER* approach, in which three belief link bases are developed to model the relationship between risk attributes, *L*, *C*, *P* and the failure safety level, respectively. The *ER* approach is then used to synthesise such estimates to obtain the safety assessment of the failure events and the one of the top level event in the form of linguistics expressions with belief degrees. Finally, such results are defuzzified through assigning utility values to the linguistics expressions for risk ranking and prioritisation. Different from most conventional risk analysis methodologies, the framework introduced is characterised with a unique feature associated with unification of input and output data. In the safety analysis modelling, each input can be represented as a probability distribution on fuzzy linguistic values for the lower level attributes using a belief structure. The main advantage of doing so is that precise data, random numbers and subjective judgements with uncertainty can be consistently modelled under a unified form. The input data transformed by the linked fuzzy belief structures can be unified, taking into account subjective judgements with uncertainties of both probabilistic and possibilistic nature.

More importantly, the new approach also enables the elimination of incompatible rules possibly introduced in the traditional fuzzy rule bases and makes the risk inference more rational through analysing the relationship between each lower level attribute and the upper level one in a parent child link individually. The *ER* approach provides a procedure for aggregating calculation, which can preserve the original features of multiple attributes with various types of information. This provides a solution to updating the safety result when new additional risk attribute(s) are identified at a later stage. The combination of the fuzzy link bases and the *ER* approach offers a powerful tool to conduct engineering system safety assessment and meanwhile, shows significant potential in solving the problems of *MADM* under uncertainty when appropriately tailored. Such advantages of the new approach are demonstrated by the results generated from a series of case studies on collision risk between a *FPSO* and a shuttle tanker. Furthermore due to its generality, the new approach is capable of offering an effective solution to tackling risk data with uncertainty in many engineering systems, including but not limited to those in offshore, marine and transportation sectors.

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REFERENCES

1. Abou, S.C. Fuzzy-logic-based network for complex systems risk assessment: Application to ship performance analysis. Accident Analysis and Prevention, 2012; 45:305-316.
2. An, M., Wang, J. and Ruxton, T. The development of fuzzy linguistic risk levels for risk analysis of offshore engineering products using approximate reasoning approach", OMAE 2000 S&R 6069, Proceedings of OMAE 2000, 19th International Conference on Offshore Mechanics and Arctic Engineering, 14-17 February, New Orleans, USA.
3. Binaghi, E. and Madella, P. Fuzzy Dempster-Shafer reasoning for rule-based classifiers. International Journal of Intelligent Systems, 1999; 4 (2):124-131.
4. Bojadziev, G. and Bojadziev, M. Fuzzy Sets, Fuzzy Logic, Application, World Scientific, Singapore, 1992.
5. Chen, C.T. Extension of the TOPSIS for group decision-making under fuzzy environment. Fuzzy Sets and Systems, 2000; 114(1):1-9.
6. Chen, H. and Moan, T. Collision risk analysis of FPSO-tanker offloading operation. 21st International Conference on Offshore Mechanics & Arctic Engineering, 2002; 23-28 June, Oslo, Norway.
7. Chen, L.H. An extended rule-based inference for general decision-making problems. Information Science, 2002; 102:247-261.
8. Dempster, A.P. A generation of Bayesian inference. Journal of the Royal Statistical Society-Series B, 1968; 13: 205-247.
9. Engemann, K.L., Miller, H.E. and Yager, R.R. Decision making with belief structures: An application in risk management. International Journal of Uncertainty Fuzziness and Knowledge-based Systems, 1996; 4(1): 1-25.

10. Ishizuka, M., Fu, K.S. and Yao, J.T.P. Inference procedure under uncertainty for the problem reduction method. Information Sciences, 1982; 28:179-206.
11. Jensen F.V. Bayesian Network and Decision Graphs, Springer-Verlag, New York, USA, 2001.
12. Li, Y. and Liao, X.W. Decision support for risk analysis on dynamic alliance. Decision Support Systems, 2007; 42(4):2043-2059.
13. Liu, J., Yang, J.B., Wang, J., Sii, H.S. and Wang, Y.M. Fuzzy rule-based evidential reasoning approach for safety analysis. International Journal of General Systems, 2004; 23(2-3):183-204.
14. Liu, J., Yang, J.B., Wang, J. and Sii, H.S. Engineering system safety analysis and synthesis using the fuzzy rule-based evidential reasoning approach. Quality and Reliability Engineering International, 2005; 21(4):387-411.
15. Martinez, L., Liu, J., Yang, J.B. and Herrera, F. A multigranular hierarchical linguistic model for design evaluation based on safety and cost analysis. International Journal of Intelligent Systems, 2005; 20(12):1161-1194.
16. Mizumoto, M. Improvement of fuzzy control methods. In: Fuzzy Logic and Intelligent Systems, Li, H. and Gupta, M. (Eds.) Kluwer Academic Publishers, 1995.
17. Ogawa, H., Fu, K.S. and Yao, J.T.P. An inexact inference for damage assessment of existing structures. International Journal of Man-Machine Studies, 1985; 22:295-306.
18. Pillay, A. and Wang, J. A risk ranking approach incorporating fuzzy set theory and grey theory. Reliability Engineering & System Safety, 2003; 79(1): 61-67.
19. Satty, T.L. The Analytic Hierarchy Process, University of Pittsburgh, USA, 1980.
20. Shafer, G. A Mathematical Theory of Evidence, Princeton University Press, Princeton, USA, 1976.
21. Sii, H.S, Ruxton, T. and Wang, J. A fuzzy-logic-based approach to qualitative safety modelling for marine system. Reliability Engineering & System Safety, 2001; 73(1):19-34.

22. Sii, H.S. and Wang, J. Safety assessment of FPSOs – The process of modelling system safety and case studies. Report of the Project –The Application of Approximate Reasoning Methodologies to Offshore Engineering Design (EPSRC GR/R30624 and GR/R32413), Liverpool John Moores University, Liverpool, UK, 2002.
23. Smets, P. Belief Function, in Non-Standard Logics for Automated Reasoning, (Smets, P., Mamdani, E.H., Dubois, D. and Prade, H. eds.), London, Academic Press, 1988.
24. Sonmez, M., Holt, G.D., Yang, J.B. and Graham, G. Applying evidential reasoning to prequalifying construction contractors. Journal of Management in Engineering, 2002; 18(3):111-119.
25. Tang, D.W., Yang, J.B., Bamford, D., Xu, D.L., Waugh, M., Bamford, J. and Zhang, S.L. The evidential reasoning approach for risk management in large enterprises. International Journal of Uncertainty Fuzziness and Knowledge-based Systems, 2012; 20(S1): 17-30.
26. Vaidya, O.S. and Kumar, S. Analytic hierarchy process: An overview of applications. European Journal of Operational Research, 2006; 169(1):1-29.
27. Wang, J. and Kieran, O. Offshore safety assessment and safety based decision making – the current status and future aspects. Journal of Offshore Mechanics and Arctic Engineering – Transactions of the ASME (American Society of Mechanical Engineers), 2000; 122(2):63-69.
28. Wang, J., Yang, J.B. and Sen, P. Safety analysis and synthesis using fuzzy set modelling and evidential reasoning. Reliability Engineering & System Safety, 1995; 47(3):103-118.
29. Wang, J., Yang, J.B. and Sen, P. Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyses. Reliability Engineering & System Safety, 1996; 52(2):113-128.
30. Yager, R.R. Generalized probabilities of fuzzy events from belief structures. Information Science, 1982; 28:45-62.
31. Yager, R.R. and Filev, D.P. Including probabilistic uncertainty in fuzzy logic controller modelling using

Dempster-Shafer theory. IEEE Transactions Systems, Man Cybernetics, 1995; 25:1221-1230.

32. Yang, J.B. Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties. European Journal of Operational Research, 2001; 131(1):31-61.
33. Yang, J.B., Liu, J, Wang, J., Sii, H.S., A belief rule-base inference methodology using the evidential reasoning approach. IEEE Transactions on Systems, Man, and Cybernetics, 2006; 36(2):266- 285.
34. Yang, J.B. and Sen, P. Preference modelling by estimating local utility functions for multiobjective optimisation. European Journal of Operational Research, 1996; 95:115-138.
35. Yang, J.B. and Xu, D.L. An introduction to an intelligent decision system: IDS. Proceedings of the 42nd Annual Conference of the UK Operational Research Society, Sep. 12-14, Swansea, UK, 2000.
36. Yang, J.B. and Xu, D.L. On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. IEEE Transactions on Systems, Man and Cybernetics – Part A: Systems and Humans, 2002; 32(3):289-304.
37. Yang, Z.L., Bonsall, S. and Wang, J. Fuzzy rule-based Bayesian reasoning approach for prioritization of failures in FMEA. IEEE Transactions on Reliability, 2008; 57:517-528.
38. Yang, Z.L., Bonsall, S. and Wang, J. Use of fuzzy evidential reasoning in maritime security assessment. Risk Analysis, 2009a; 29:95-120.
39. Yang, Z.L., Bonsall, S. and Wang J. Use of hybrid multiple uncertain attribute decision making techniques in safety management. Expert System with Applications, 2009b; 36(2):1569-1586.
40. Zadeh, L.A. Fuzzy sets. Information and Control, 1965; 8(3):338-353.
41. Zimmermann, H.J. Fuzzy Set Theory and its Application, Norwell, MA: Kluwer, 1991.