Performance analysis of an evolutionary LM algorithm to model the load-settlement response of steel piles embedded in sandy soil

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Abstract

This study was implemented to examine pile load-settlement response and to develop a rapid, highly efficient predictive intelligent model, using a new computational intelligence (CI) algorithm. To achieve this aim, a series of experimental pile load tests were performed on steel, closed-ended pile models consisting of three piles with aspect ratios of 25, 17, and 12 in an attempt to make site in-situ pile-load tests unnecessary. An optimised, evolutionary, supervised Levenberg-Marquardt (LM) training algorithm was used for this process due to its remarkably robust performance. The model piles were penetrated and tested in three sand relative densities; dense, medium, and loose. Applied load (P), pile effective length (lc), pile flexural rigidity (EA), pile slenderness ratio (lc/d) and interface friction angle (δ) were identified, based on a comprehensive statistical analysis, as these parameters play a key role in governing pile settlement. To evaluate the efficiency and the generalisation ability of the proposed algorithm, graphical comparisons were made between the proposed algorithm and the experimental results with further comparisons made with conventional prediction approaches. The results revealed outstanding agreement between the targeted and predicted pile-load settlement with a coefficient of correlation of 0.985 and a Pearson's correlation coefficient, P = 2.22 $x10^{-32}$ and root mean square error (RMSE) of 0.059 respectively. This, in parallel with a non-significant mean square error level (MSE) of 0.002, validates the feasibility of the proposed method and its potential in future applications.

Keywords: Sandy soil; steel pile; Levenberg-Marquardt (LM) algorithm; sensitivity analysis; pile load-settlement.

1. Introduction

Pile foundations are structural elements constructed underneath superstructures, frequently utilised as load carrying systems and soil settlement controls at sites with poor soil bearing capacity at sub-soil layers (Nazir and Nasr, 2013; Tomlinson and Woodward, 2014; Tschuchnigg and Schweiger, 2015). Pile bearing capacity and settlement are the most significant factors that influence pile foundation design procedures (Alkroosh and Nikraz, 2011; Alizadeh et al., 2012; Das, 2015; Nejad and Jaksa, 2017), making this a core feature of research in the field of geotechnical engineering. Accordingly, several procedures concerning pile bearing capacity, are available in the open literature, ranging from the application of complicated nonlinear numerical procedures to empirical relationships (Nasr, 2014). In the absence of in-situ pile load carrying capacity tests, ultimate pile bearing capacity and associated settlement have traditionally been determined separately. However, it has been reported by Fellenius (1989) that: " *…pile allowable load should be influenced by combined procedures taking into consideration soil mobilised resistance and soil settlement performing together and influencing the value of each other*". A definition of ultimate pile bearing capacity is still open to discussion, but a number of criteria have been suggested by scholars to precisely identify pile capacity from full pile load-settlement distribution curves. For instance, ultimate pile capacity can be defined according to a pile settlement equal to 10% of the pile diameter (d), the obtained load then divided by two (the general factor of safety to determine the pile working load). Murthy (2003) and Shahin (2014), however, suggest that if the latter criteria is introduced to certain piles under specific soil properties (e.g., large diameter piles driven in cohesive soil), the settlement calculated will be excessive. The precise assessment of pile bearing capacity can be determined by conducting in-situ, full-scale, pile load-settlement tests. The fact that this is expensive and time consuming have been cited as major drawbacks associated with performing such tests (Momeni et al., 2014; Alkroosh et al., 2015). Alternatively, pile capacity and settlement can be predicted using several suggested design procedures (i.e., Das (1995) and Poulos (1999)). Although those approaches have been frequently used, it has been cited that the aforementioned methods are in-accurate and fail to achieve consistent success due to the many pre-conditions and arbitrary assumptions involved in the factors affecting pile capacity and settlement (i.e., soil-pile interaction, nonlinearity of the soil stress-strain relationship, driving system and initial boundary conditions) (Alkroosh et al., 2015).

Applications of CI techniques based on the concept of artificial neural networks (ANNs), have been highly recommend as a superior alternative approach (Strąkowski et al., 2018). ANN has been applied effectively in a wide range of geotechnical studies, as an efficient computing tool to fully represent and capture pile load-settlement behaviour with an acceptable degree of accuracy (Chang et al., 2018). ANN technology has the capacity to deal with complexity and to map nonlinear complex functions, adopting substantial computer capacity to implement extremely iterated work (Di Santo et al., 2018; Li et al., 2018; Naderpour et al., 2018). In essence, the complex non-linear patterns between the individual variables (IVs) and the model target are precisely addressed, identified and mapped with high dimensional input space (Sun et al., 2014). Furthermore, the advantages of LM modelling includes its ability to resample the complex relationship between pile load-settlement and the parameters affecting it, without the need for any assumptions (Sharma et al., 2017).

With the development of machine learning technologies, the feasibility of the computational approach applied to pile foundation research, has been emphasised recently by many researchers. Alkroosh and Nikraz (2014) conducted a study to review and model pile dynamic capacity based on SPT tests. The dataset was divided into two subsets; a training set for model development and a validation set for evaluation of the performance of the model under the training process. The authors concluded that the trained model had the ability to predict pile capacity with remarkable agreement between the targeted and predicted values, giving a correlation coefficient (R) of 0.94.

Shahin (2014) suggested the application of the recurrent neural network (RNN) to model load-displacement responce of model piles penetrated in layered soil. The RNN model was developed and trained with six model input parameters using cone penetration test (CPT) data. It was concluded that the RNN designed and trained model was an effective approach to accurately simulate pile load-settlement with substantial agreement between targeted and predicted results.

Momeni et al. (2014) examined the feasibility of a new artificial neural network-based model to predict pile bearing capacity. In total, 50 pile load-displacement tests were conducted on concrete piles in order to deliver the essential dataset to develop and train the proposed model. The pile geometrical properties, hammer weight, pile set and drop height have been selected as the most significant input parameters for the pile bearing capacity. The results revelled that good agreement was achieved between the targeted and the predicted values, this was confirmed by a correlation coefficient of 0.99 and a relatively negligible mean square error of 0.002.

Jebur et al. (2018a) addressed the reliability of an enhanced artificial neural network (ANN) as a global search system using the LM training algorithm to develop a reliable predictive model, in an effort to overcome some of the disadvantages associated with the traditional ANN methods, such as poor generalisation ability of the trained network. The developed model input parameters consist of (i) pile applied load, (ii) soil-pile angle of internal friction, (iii) pile length, (iv) pile axial rigidity, and pile slenderness ratio. The results revealed that the proposed model could successfully simulate pile settlement with high efficiency. This was confirmed by a correlation coefficient of about 0.99 and a relatively negligible mean square error.

Nejad and Jaksa (2017) developed a study aimed at exploring pile settlement of model piles based on cone penetration tests. The model input parameters were (i) pile load test type; (ii) material used for the pile; (iii) pile installation method; (iv) tip of pile; (v) pile axial rigidity; (vi) pile tip cross sectional area; (vii) pile effective perimeter; (viii) pile length in the effective zone; (ix) length of pile; (x) the corrected and the average SPT value blow count/300mm along the pile effective depth; (xi) the SPT corrected blow count/300mm at the pile tip and (xii) pile applied load. Settlement of the pile was set to be the model dependent variable (DV). The results revealed that the adopted method had the ability to predict pile settlement with a good level of accuracy.

Jebur et al. (2018b), soon after, developed a novel artificial neural network predictive model to assess the ultimate load-displacement response of steel open-ended pile subjected to compression load. The model piles were driven in sandy soil of different densities, measuring of loose, medium and dense. Five IVs have been underlined at being the most significant parameters influencing the steel pile bearing capacity, these encompassed pile applied load, pile effective length, pile slenderness ratios, pile-soil angle of internal friction, and pile axial rigidity. The results indicated that the model could be employed to predict pile ultimate capacity with substantial accuracy. It should be stated that the common feature in the aforementioned studies is that the steepest descent method used in traditional artificial neural networks, is extremely criticized for its slow rate of convergence towards an optimal solution and for being trapped in a local minima. Moreover, the neural network (NN) internal training parameters need to be user-adjusted parameters at each application before training the developed NN model (Morfidis and Kostinakis, 2017). In this paper, a new methodology has been introduced utilizing a robust, self-tuning artificial intelligence (AI) approach to fully correlate pile load carrying capacity and the associated displacement of rigid and flexible piles. Guided by comprehensive statistical analyses to categorize the effective input parameters, evaluate the contribution of each model input parameters and to check the reliability of the dataset being studied such as the absence of outliers, multicollinearity detection, and dataset size condition.

The current study is structured as follows: the study aim and objectives are given in section 2. The materials and methods, including the sand properties along with the testing procedure used to address the stated study aim, are presented in section 3. The statistical analysis is presented and discussed in section 4. The results are presented and discussed, in depth, in section 5. Graphical comparisons between the suggested LM algorithm and the most conventional design procedures are presented and discussed in detail in section 6. Finally, section 7 gives the concluding remarks about the study and recommendations for future work.

2. Aim and objectives

The current study aims to develop and verify a reliable, cost-effective predictive model to fully capture the load-carrying capacity of steel piles in cohesion-less soil. The specific objectives are to:

- Conduct a series of experimental pile-load tests to evaluate load bearing capacity and the associated settlement of steel, closed-ended piles having three aspect ratios of 25, 17, and 12 to explore the behaviour of flexible and rigid piles in three relative sand densities (D_r): dense, medium and loose.
- Examine the feasibility of an evolutionary supervised Levenberg-Marquardt (LM) training algorithm to develop a rapid, cost effective and reliable predictive model to fully map non-linear, pile load-settlement behaviour, subject to a wide range of axial loads.
- Assess the generalisation ability of the LM algorithm using a dataset cluster (not used in the training process), by comparing the predicted results with the experimental results along with the results from conventional pile-load settlement design approaches.
- Carry out a statistical analysis to highlight the most appropriate model input parameters on the model output (‘Sig’ value) and to identify the contribution of each individual variable (IV) on the model output (‘Beta’ value) using SPSS-23 software.

3. Material and methods

3.1 *Sand properties*

Fine sand was used as a test medium. The sand was consist of sub-rounded particles, as confirmed by scanning electronic microscopy (SEM) observations (Fig. 1). Based on the Unified Soil Classification System (USCS), this sand was classified as poorly graded (SP). The uniformity coefficient (Cu) and the curvature coefficient (Cc) were 1.786 and 1.142, respectively. The sand was prepared in three relative densities (D_r) of 18, 51 and 83%. The minimum and the maximum sand unit weights were 15.33kN/m^3 and 17.5kN/m^3. To overcome scale effect issues and to maintain the influence of grain size distribution on the combined pile-soil interaction, the ratio between the pile diameter (d) to medium diameter (d_{50}) of the sand particles should be 45 (Nunez et al., 1988). Remaud (1999) claimed that *"…the ratio must be 60 times the diameter of the pile"*. Taylor (1995) however, reported that *"…the ratio should be at least 100"*. In the current study, the ratio of the diameter of pile to medium diameter (d/d_{50}) was 133 as indicated in Fig. 2, satisfying the scaling criteria. The relative sand densities were prepared in different stages. For preparing the loose sand bed, the sand particles were poured into the pile testing chamber using a tube delivery system, following the procedure documented by Schawmb (2009). The end of the tube was repeatedly held at a maximum set distance of approximately 40mm between the sand delivery tube and the surface test bed. The medium sand has been prepared utilising *"an air pluviation technique"* as suggested by Ueno (2001). Sand density was controlled by the falling rate, approximately 800mm above the sand surface, with an accuracy of ± 25mm until the desired test depth was achieved. The dense sand was prepared in agreement with the technique suggested by Akdag and Özden (2013).

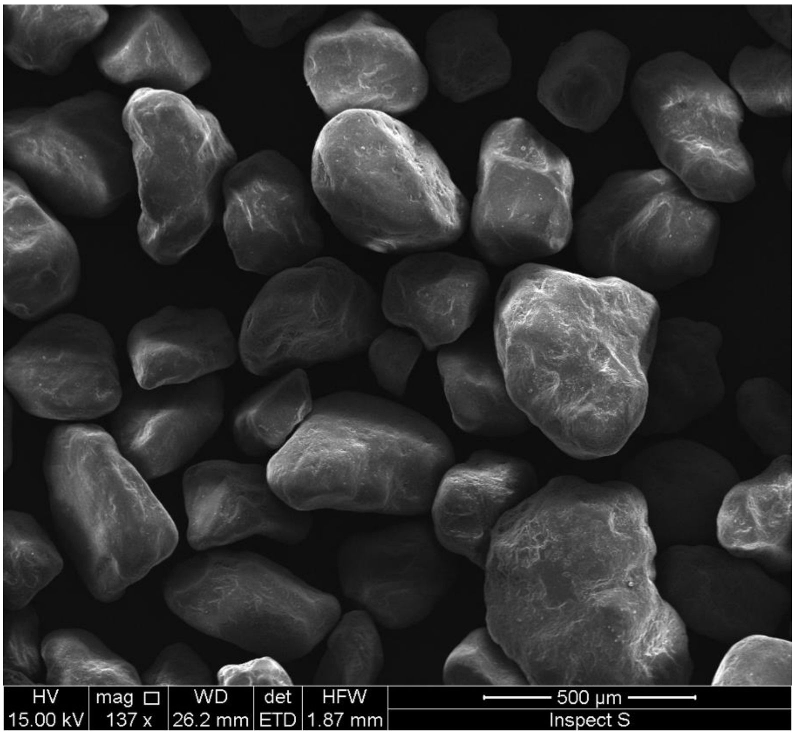

Fig. 1. Scanning electronic microscopy (SEM) view of the sand specimen.

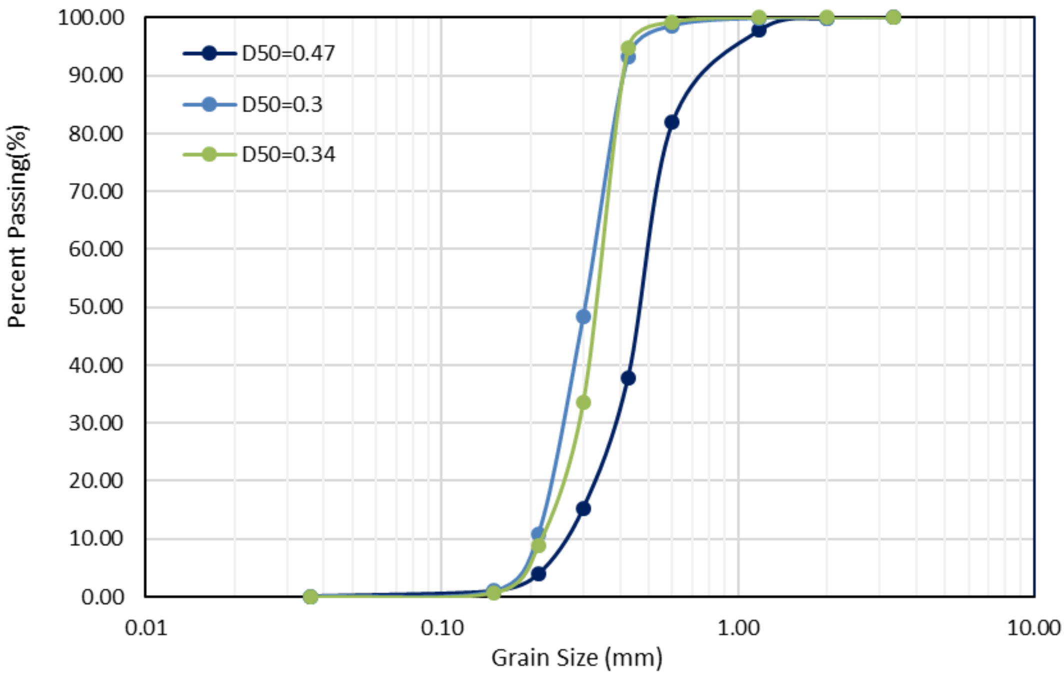

Fig. 2. Particle size gradation in the sand sample.

3.2 Testing procedure

This section details the procedure adopted for pile-load testing. Experimental pile load-tests were performed on steel closed-ended piles with aspect ratios of 12, 17, 25 and 40mm diameter, driven into a calibration pile-testing chamber, as depicted in Fig. 3. During loading applications, the pile point of loading was 50mm above the surface of the sand, the key objective being to minimise sand contact with the pile cap. This is to ascertain that the pile capacity is only as a results of pile-soil interaction. A maintained load test has been adopted at a loading rate equal to 1mm/min as recommended by Bowles (1992) and within the limits stated by BSI (BS EN 8004:1986). A new hydraulic jack system type DBBSM, connected at the top of the load cell, having a maximum capacity of 10Kn has been adopted to drive the pile in the sand. This was fixed between the pile head loading system and the hydraulic ram model (ZE3408E-T). A Polytetrafluoroethylene (PTFE) sheet has been used in the pile testing chamber in an effort to minimize the friction between the sand and the chamber. The PTFE sheet has a friction coefficient of less than 0.04 compared with steel sheet with coefficient of friction of about 0.605 (Young and Freedman, 2000). 20mm thick, sufficiently rigid glass plate, was placed at the front face of the testing chamber and sub-divided into equal segments, as shown in Fig. 3, to provide a clear view of the sand control volume. The loads were applied directly onto an aluminium pile cap with a diameter of 150mm and thickness of 25mm. A spherical steel ball bearing was used on the top of the pile cap to avoid and/or minimize eccentricity during the loading application. Pile load tests were conducted at 1 G-stress conditions, the low effective stress resulting in some differences compared with full-scale tests. The pile head displacement was monitored using a data acquisition system instrument with two linear variable differential transformers (LVDTs) of very high resolution, 0.01mm, with 150mm travel distance to record the corresponding pile settlement, using magnetic stands. The LVDTs were placed on the top of the pile cap in pairs so that the effect of bending could be accurately measured.

Fig. 3. Schematic view and dimensions of the experimental test setup.

4. Statistical analysis

4.1 Model inputs and output

Identifying the parameters controlling pile load-settlement behaviour is essential in order to develop an accurate predictive model (Yadav et al., 2014; Ahmadi et al., 2016; Nejad and Jaksa, 2017). Most of the traditional approaches comprise of (i) pile material; (ii) pile geometry; (iii) applied load and (iv) properties of the soil. Nejad et al. (2009) reported that there are other additional parameters that have substantially lower effects on pile settlement such as pile installation method, pile load test type and water table level. These make a minimal contribution to pile settlement and do not therefore need to be taken into consideration. That said, the selection of the model input variables is one of the vital steps to develop a reliable, predictive model. In this research, an innovative statistical significance analysis (*Sig.*) was developed using SPSS-23 software to identify the model individual input variables (IIVs). This technique has been used because it has many attractive merits (Pathak, 2011) the main one being that it has the ability to explore the relationship between one individual variable (IV) with a set of other individual variables (IVs) (Hashim et al., 2017c). Based on the statistical study, five factors with different levels of contribution, were identified as the most influential input parameters affecting pile-load settlement with a (*Sig.*) value of < 0.05, matching the statistical criteria stated by Field (2008). These parameters are (i) applied load (P), (ii) pile slenderness ratios (lc/d), (iii) pile axial rigidity (EA), (iv) pile embedded length (lc) and (v) the sand-pile friction angle (δ). The model output was pile settlement. The optimal structure of the ANN model had been selected at a topology of 5:10:1 as shown in Fig. 4. The LM algorithm was trained using the parameters listed in Table 1. The *"TANSIG"* transfer function has been utilised between the input parameter *"layers one"* and the hidden layer *"layer two"*, while the *"PURELIN"* transfer function was adopted to link *"layers two"* and three as listed in Eqs. 1 and 2 respectively.

Table 1: The LM internal training parameters.

Parameters	Value
Epochs maximum number	1000.0
Efficiency goal	0.0
Learning rate	0.01
Increase learning rate ratio	1.05
Decrease learning rate ratio	0.7
Validation failure limit	6.0
Minimum performance increase	1.04
Constant of momentum	0.9
Minimum performance gradient	1e-5
Epochs between displays	25.0

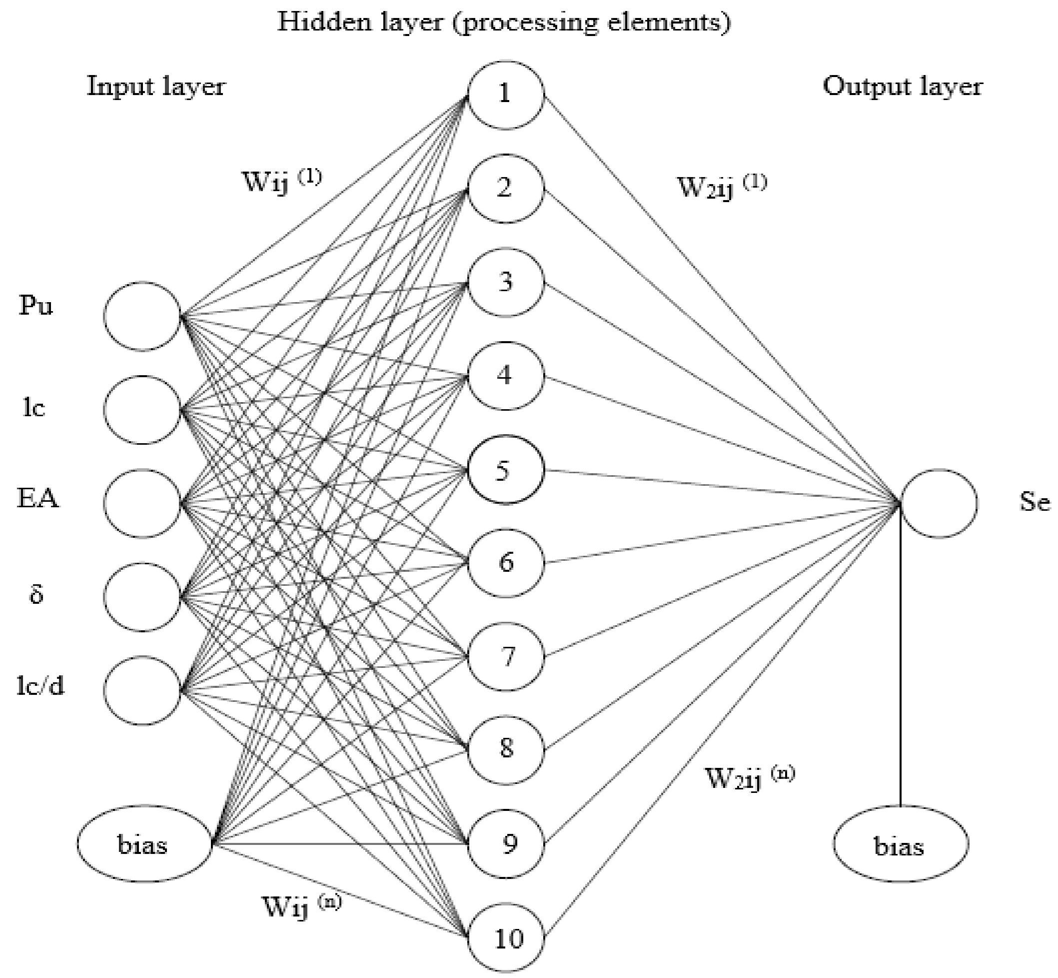

Fig. 4. Structure of ANN model inputs and output parameters.

$$Z_j = \frac{1}{1 + \exp\ (\sum_{i=1}^{n} \pm w_{ij}^{(1)}\ x_i \pm\ b_j^{(1)})} \tag{1}$$

$$y = \sum_{i=1}^{n} w_j^{(2)}\ \ z_j\ \pm\ b^{(2)} \tag{2}$$

The factors $w_{ij}^{(1)}$ and $b_j^{(1)}$ are the weights and biases from the inputs and output (hidden) layer; $b_i^{(1)}$ and $b_i^{(2)}$ the bias for layers one and two.

4.2 Dataset size condition

The reliability of the experimental dataset must be assessed as it plays a substantial role for model efficiency (Hashim et al., 2017a). In an attempt to optimise the relationship between the targeted and predicted model and to develop efficient model performance, the minimum data size required to produce a generalised optimum model must be calculated using a given formula (Eq. 3). Based on the number of individual variables (IVs), the minimum data points required to perform modelling is 90 (Pallant, 2011; Hashim et al., 2017b). In this study, there were 277 experimental dataset points meaning that the condition of dataset size has been satisfied.

$$N \geq 50 + 8 * I \tag{3}$$

where N and I are the size of the sample and individual variable parameters.

4.3 Outliers

Tabachnick and Fidell (2013) reported that "…*An outlier can be defined as a case with an extreme value on one variable (a univariate outlier) or a strange combination of scores on two or more variables (multivariate outlier) that statistically distorts the data"*. Given that the conclusions drawn from simulations may be influenced by the existence of outliers (Hashim et al., 2017c), all independent variables (IVs) and dependent variables (DV) must be statistically tested to identify such extreme values by determining the Mahalanobis distance parameters (MDs). In the current investigation, it was found that the maximum MDs for the five input parameters was 17.01. To check whether this exerted any influence on the results of the LM training algorithm as a whole, Tabachnick and Fidell (2013) recommend calculating Cook's Distance (COO_1), as any point with COO_1 greater than 1.0 is a potential problem. The results in Table 2 show that Cook's Distance (COO_1) for the described point was 0.0091, this confirming that the output results and the efficiency of the model will not be subject to influence as there are no outliers in the data.

4.4 Multi-collinearity

To describe the data screening process, the variance inflation factor (*VIF*) for each IV was determined in an attempt to investigate for the presence of multi-collinearity in the total dataset. Reddy and Ayothiraman (2015) reported that any IV with a VIF higher than 10 may affect the performance of the proposed model. According to the results in Table 2, it was found that the maximum VIF factor for each IV was less than 10, which confirms the validity of the dataset used to develop and train the LM algorithm.

4.5 Statistical significance and the relative importance of each independent variable (IV)

Selection of the most effective model input parameters and the contribution level, or strength, of each independent variable (IV) to the model output has been ascertained through the statistical analysis of parameters (the relative importance parameter "*Beta value*", and the statistical significance "*Sig value*") using SPSS-23. It has been documented by many scholars that any IV at a p value > 0.05 can be omitted from the input layer as it has no significant impact on the model target (Field, 2008; Hashim et al., 2017c). Statistically, the closer the absolute Beta value is to one, the more significant the impact of that IV on the model (Pallant, 2011; Hashim et al., 2017b; Hashim et al., 2017a). According to the results in Table 3, the applied load and the sand-pile angle of interface friction, had a substantial influence on pile settlement, at Beta values of 0.84 and 0.718, respectively. Pile slenderness ratio, pile length and flexural rigidity have comparatively less influence, with different strengths on pile settlement, at Beta values of 0.235, 0.026 and 0.15, respectively. The maximum Sig value was 0.015, which confirms that all IVs have a significant influence on the trained model output.

Table 2: Summery of the statistical analysis results.

Parameters	*Beta. value*	*MDs*	*VIF*	*COO_1*
Applied load, (P)	0.840	21.84	4.01	0.0091
Slenderness ratio, (lc/d)	0.238		1.23	
Flexural rigidity, (EA)	0.015		3.21	
Pile length, (l)	0.026		6.12	
Sand-pile angle of interface friction, (δ)	0.718		3.76	

Table 3: Statistical parameters for model input and output parameters.

Data Set	Statistical Parameters	Input Variables					Output
		Load (kN)	Slenderness ratio, Lc/d	Pile length, (m)	Pile axial rigidity, EA (MN)	Sand-pile friction angle, δ°	Settlement, (mm)
Training Set	Max.	4.426	25	1	251.18	19	14.461
	Min.	0.001	12	0.48	251.18	17	0.0015
	Mean	1.454	17.01	0.711	251.18	17.91	6.097
	S.D.*	1.363	1.345	0.211	0.00	1.05	4.591
	Range	4.425	2.08	0.52	0.00	2	14.49
Testing Set	Max.	4.350	25	1	251.18	19	14.215
	Min.	0.193	12	0.48	251.18	17	0.022
	Mean	0.683	18.323	0.767	251.18	17.783	5.860
	S.D.*	1.260	1.369	0.226	0.00	1.044	4.586
	Range	4.349	13	0.52	0.00	2	14.192
Validation Set	Max.	3.660	25	1	251.18	19	13.861
	Min.	0.084	12	0.48	251.18	17	0.002
	Mean	1.275	17.35	0.724	251.18	17.827	5.727
	S.D.*	1.098	1.347	0.213	0.00	1.049	4.521
	Range	3.576	13	0.52	0.00	1.117	13.814

5. Results and discussion

5.1 The LM model development

The Levenberg-Marquardt (LM) training algorithm can be defined as a data driven computing method based on artificial intelligence (AI) concepts, which, more specifically, is able to correlate inversely and numerically, the nonlinear relationships between a set of individual variables (IVs) and outputs via their characteristic mathematical topology (Nguyen-Truong and Le, 2015; Ahmadi et al., 2016; Jaeel et al., 2016). The basic concept behind the LM method is to correlate the connections between IVs and model output, without assuming a prior formula defining this correlation (Sharma et al., 2017). In this study, supervised training involving feed-forward, multi-layer perceptions (MLPs) using a back-propagation learning process based on a MATLAB (R2017a) environment, was built, and used to fully capture pile load-settlement. Individual predictor parameters (input variables) in the data vector are multiplied by associated scalar weights and activation thresholds “biases” prior to their summation and then processed via non-linear transfer functions in hidden layer processing neurons (PNs). The stated themes were followed by multiplying the output resulting from the hidden layer via their optimised W_{ij}

then summed before processing by the next layer (the output layer). This methodology was repeated during the iteration process, the error propagated backwards with each single epoch, the connection weights (Wij) adjusted during the training process, until a minimum error is achieved (Deo and Şahin, 2015). To summarise, a mathematical illustration of the individual input variables (IIVs) and related output is presented in Eq. 4. The training flow chart for the proposed algorithm is described in Fig. 5.

In total, 277 points were divided into three clusters, two of which called training (70%) and cross- validation (15%). The goal of the training dataset is to create the most appropriate ANN network and fit the model by updating the network connections weights (W_{ij}) and biases (b_{ij}) at each iteration during the process of learning. The cross-validation set is piloted to deliver an independent check of network performance, to avoid the model overfitting and to terminate the training process at a minimum MSE error (Nguyen-Truong and Le, 2015). The third cluster testing dataset was used to evaluate the models' ability to generalise and the validity of the optimum ANN model via using the last 15% of unseen data, after selecting the appropriate network weights and biases (Ahmadi et al., 2016; Shahin, 2016). This dataset is not involved in the learning algorithm process (Millie et al., 2012; Sun et al., 2014). Running the optimisation of interconnected biases and weights was continued until specific measuring performance indicators were met as described in the following sections. Finally, a principal ANN trained network with 5 individual neurons in the input layer, 10 neurons in the hidden layer and 1 neuron in the output layer, was identified as the optimum model.

$$O' = W_0 + \sum_{j=1}^{m} W_{jg} \left(W_{0j} + \sum_{i=1}^{n} W_{ij} X_i \right) \pm b_i \tag{4}$$

where O' is an output layer, X_i to X_n individual input variables, Wij and $\pm b_i$ are network connection weights and biases either added or subtracted.

Database values were scaled to fall between 0.0 and 1.0 using a formula (Eq. 5) given by Sharma et al. (2017) before being processed in the training stage. This allows each (IV) to receive equal attention during the training process as well as avoiding network ill-conditioning (Masters, 1993; Cho, 2009; Majeed et al., 2013; Shahin, 2013).

$$x_{normalised} = \frac{x_{actual} - x_{min}}{x_{max} - x_{min}} \quad (5)$$

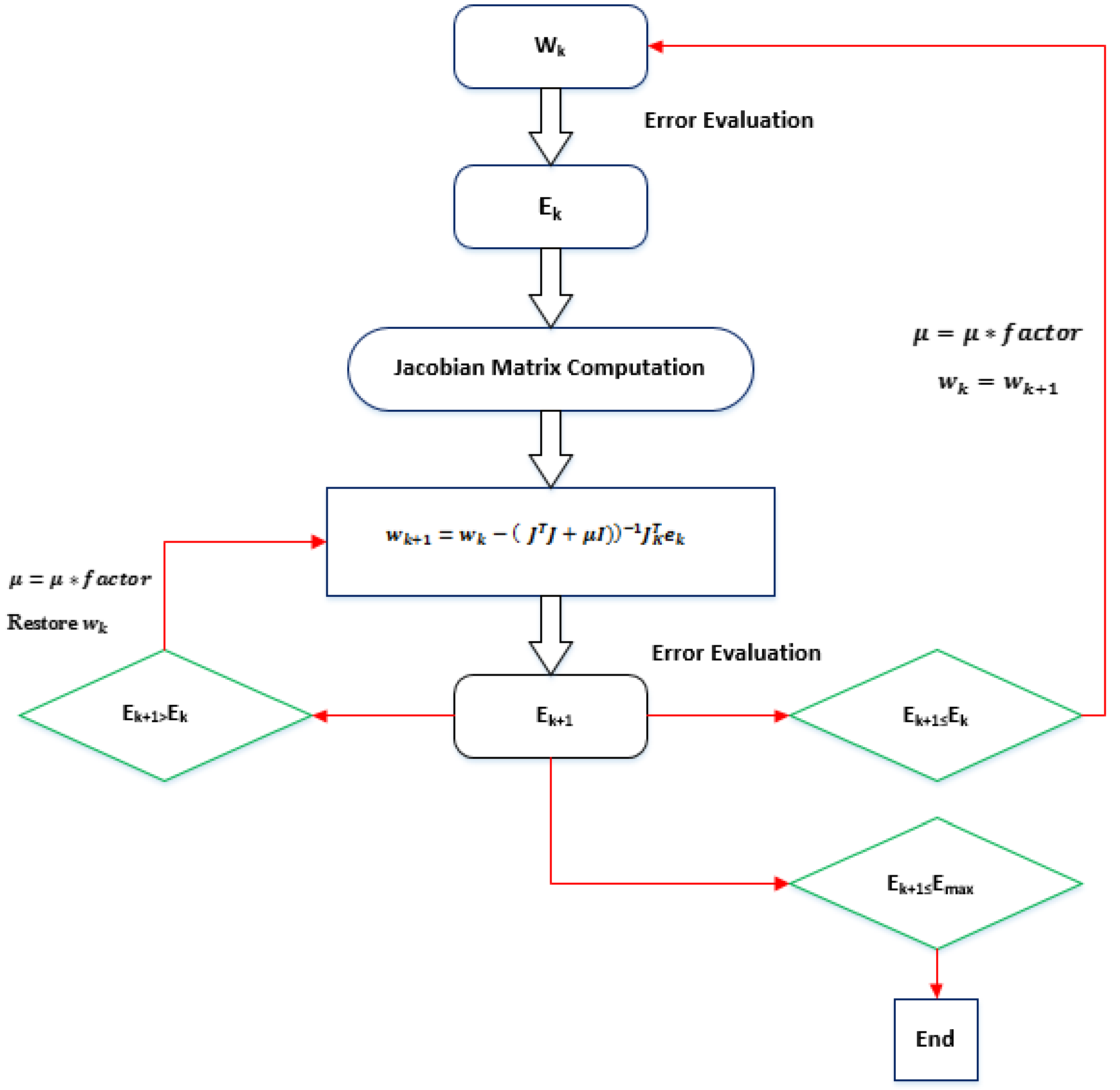

Fig. 5. Block diagram shows the process of training utilising the Levenberg Marquardt (LM) algorithm.

W_k denotes the existing weight, w_{k+1} is the subsequent weight, E_{k+1} *and* E_k are the current and last total error respectively.

5.1.1 The LM model performance

In this study, a LM training algorithm has been applied as it is the most efficient and reliable method in comparison to all other CT approaches, as noted by Jeong and Kim (2005); and Mohammadi et al. (2016). To clarify the effectiveness of the algorithm, various performance indicators as suggested in the research literature can be utilised. In the current study, statistical performance indicators, i.e., the mean square error (MSE), correlation coefficients (R and P) and root mean square error (RMSE) functions were used, as listed in Eqs. 6, 7 and 8, to characterise model performance, with an error goal set at 0.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}|O_i - P_i|}{N}} \tag{6}$$

$$R = \frac{\sum_{i=1}^{n}(O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{n}(O_i - \bar{O})^2 \sum_{i=1}^{n}(P_i - \bar{P})^2}} \tag{7}$$

$$MSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - \bar{O}_i)^2} \tag{8}$$

where N is the number of the dataset used to develop the model, O_i and P_i the targeted and predicted values. $\bar{P}$ and Ō are the mean of the predicted and observed values and R is the coefficient of correlation.

After training the LM algorithm, the results indicated that the optimum ANN model comprised of three layers; the input layer, one hidden layer with 10 neurons or processing elements (PEs) and an output layer. It is worth noting that $2N+1$ is the maximum limit of neurons required to map any ANN network with "*N*" being the number of model input parameters. The network performance under the process of training is shown in Fig. 6, the results revealing that the plot of validation depicts a substantial fall in minimum square error (MSE) with increasing iterations. The optimum network performance is identified with a relatively negligible MSE of 0.0025 at an epoch of 215. It could be inferred that the learning process stopped thus avoiding overfitting, once the cross-validation error started to increase; this can also be defined as early stopping criteria to avoid data overfitting (Shahin, 2014).

The variation in error-gradient, the Marquardt adjustment parameter (m_u) and validation checks are exhibited in Fig. 7, where it can be seen that the gradient error between the targeted and predicted values is a negligible value (0.0003), the m_u factor and validation check numbers being 1 x 10^{-05} and 6 respectively, at an epoch of 221. An error histogram graph (EHG) is revealed in Fig. 8 to provide additional confirmation of the proposed model performance. An EHG gives an indication of outliers "*an observation of data features that seems to be inconsistent* *with other observations in the dataset*” (Jebur et al., 2018a) . As the conclusions drawn from the training process could be strongly affected by outliers (Tabachnick and Fidell, 2013; Hashim et al., 2017c), the training process was stopped once the minimum error started to increase. In Fig. 8, the red, blue, and green bars signify testing, training and validation dataset. It should be stated that the majority of dataset coincides with the zero-error line, which was the original aim.

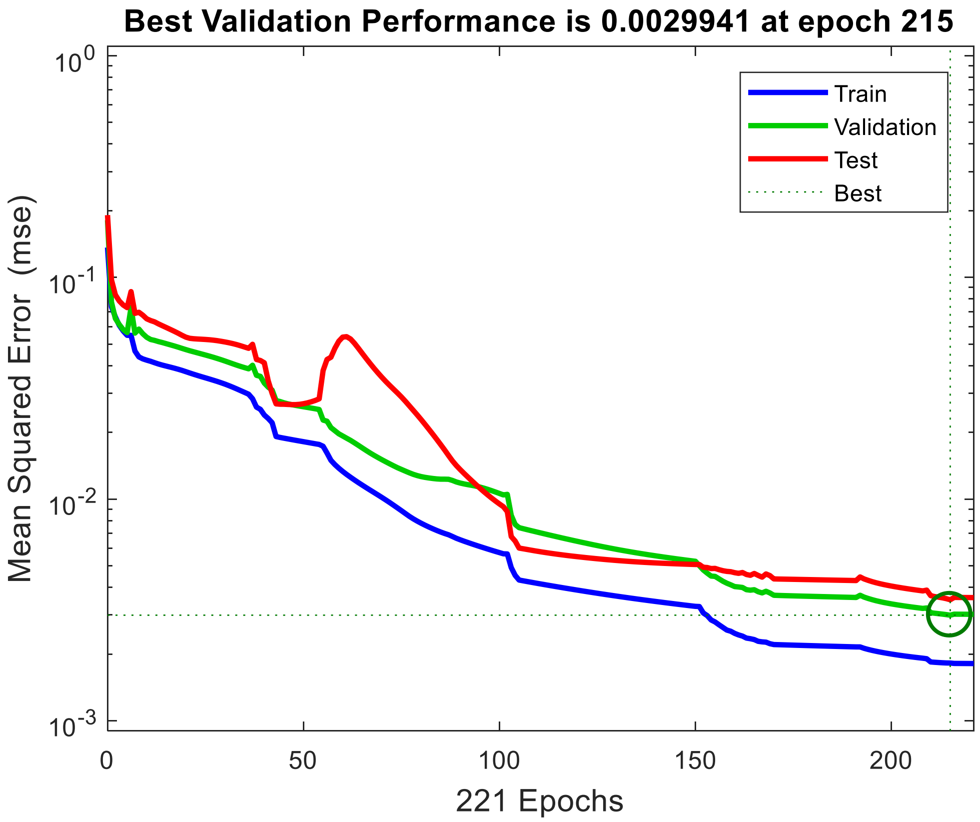

Fig. 6. Graph showing the optimum mean square error (MSE) selected during the training process with configuration of 5-10-1.

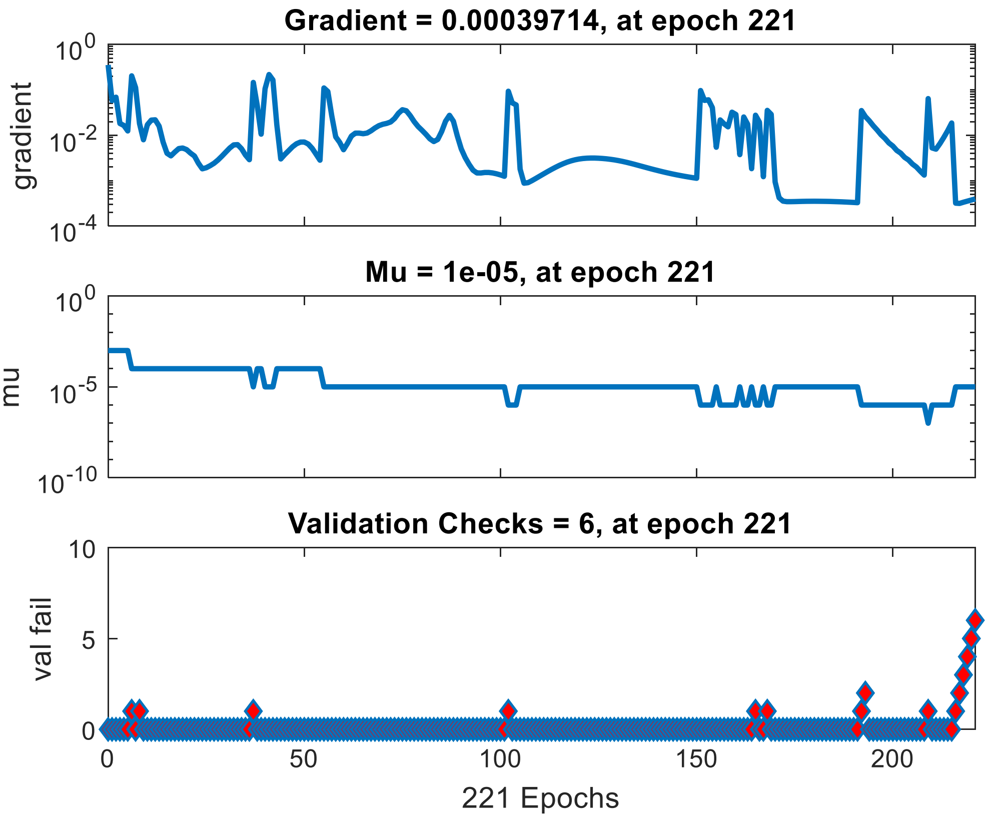

Fig. 7. Performance profiles for the trained ANN network.

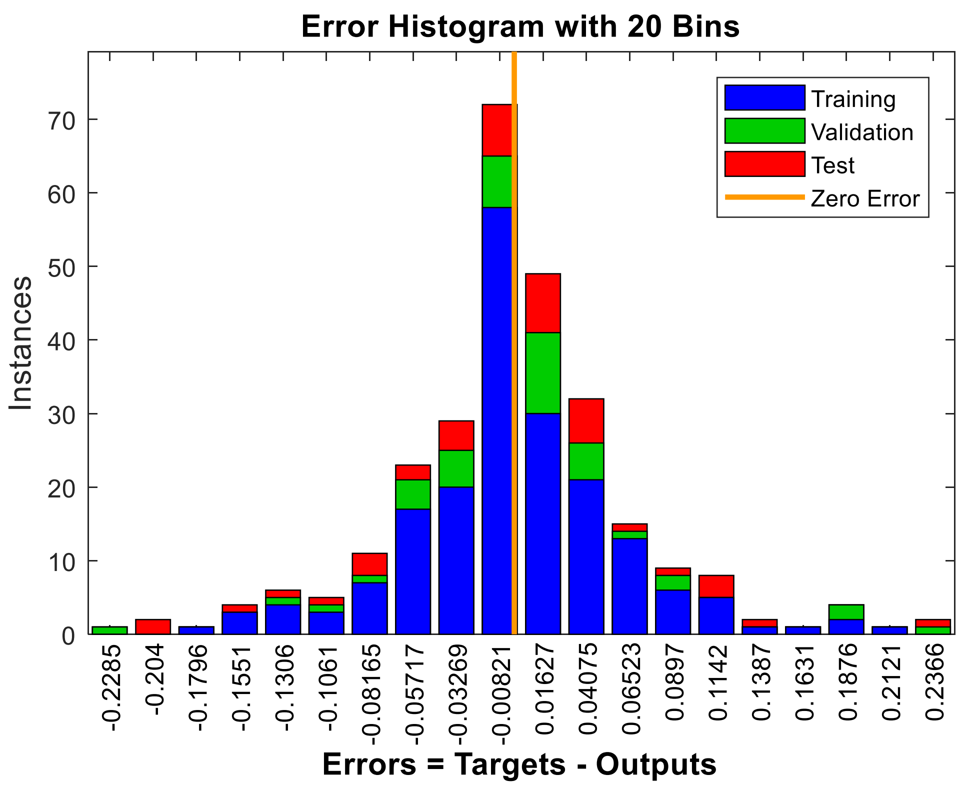

Fig. 8. Error histogram during training, testing and validation.

5.1.2 Assessment of the LM model robustness

In this section of the paper, the results of the experimental load-settlement (Q-S) behaviour tests were compared with the predicted values generated by the LM trained optimum network. A series of experimental pile load tests were conducted on steel, closed-ended pile models. The experimental testing program used three piles with slenderness ratios (Lc/d) of 25, 17, and 12 where Lc is the effective pile depth with a diameter (d) of 40mm, to examine the behaviour of flexible and rigid piles. 277 points in total were recorded from the experimental pile load test data, which used a P3 strain indicator as illustrated in the experimental setup (Fig. 3). The pile head settlement was closely monitored using two linear variable differential transformers (LVDTs), with a 50mm travel distance. Figs. 9, 10 and 11 show the extent of the fit between the experimental and predicted normalised load-carrying capacity of steel piles, subject to axial loads at different stages of mechanical loading. The experimental axial load variations are typical for canonical pile foundations subject to axial mechanical loading systems, i.e., reducing from pile head to pile toe due to the increase in developed shaft resistance in the effective soil zone adjacent to the pile. The results demonstrated that a soil yielding effect for axial applied loads greater than 200N was identified in the upper part of the foundation, where local nonlinearity is marked. It can also be observed that the mobilised pile bearing capacity (end bearing and mobilised skin friction resistance) increases as sand stiffness and pile effective depth increase. Plastic mechanisms in the soil surrounding the pile are the leading cause for the non-linearity of the load-settlement response; as the applied load increases, the pile response shows nonlinearity until reaching a maximum capacity at about 10% of pile diameter (BSI, BS EN 8004:1986). Based on the graphical comparisons, there was an excellent consistency between the predicted and targeted values, with a correlation coefficient of 0.988 for all data, which demonstrates that the developed approach is a superior method to use to predict pile load settlement curves for the range investigated.

The results drawn, using an evolutionary LM trained approach, were also exhibited graphically with the corresponding experimental settlement in the form of a regression calibration curve (Fig 12). The results revealed that, the introduced training algorithm satisfies the robustness test. All the predicted and measured points are matched well and close to the best-fit line with coefficients of correlation of 0.99088, 0.98436, 0.9854 and 0.98861 for training, validation, testing and all data, validating that the application of the LM algorithm as an efficent predictive tool that acts in a fashion that would be expected.

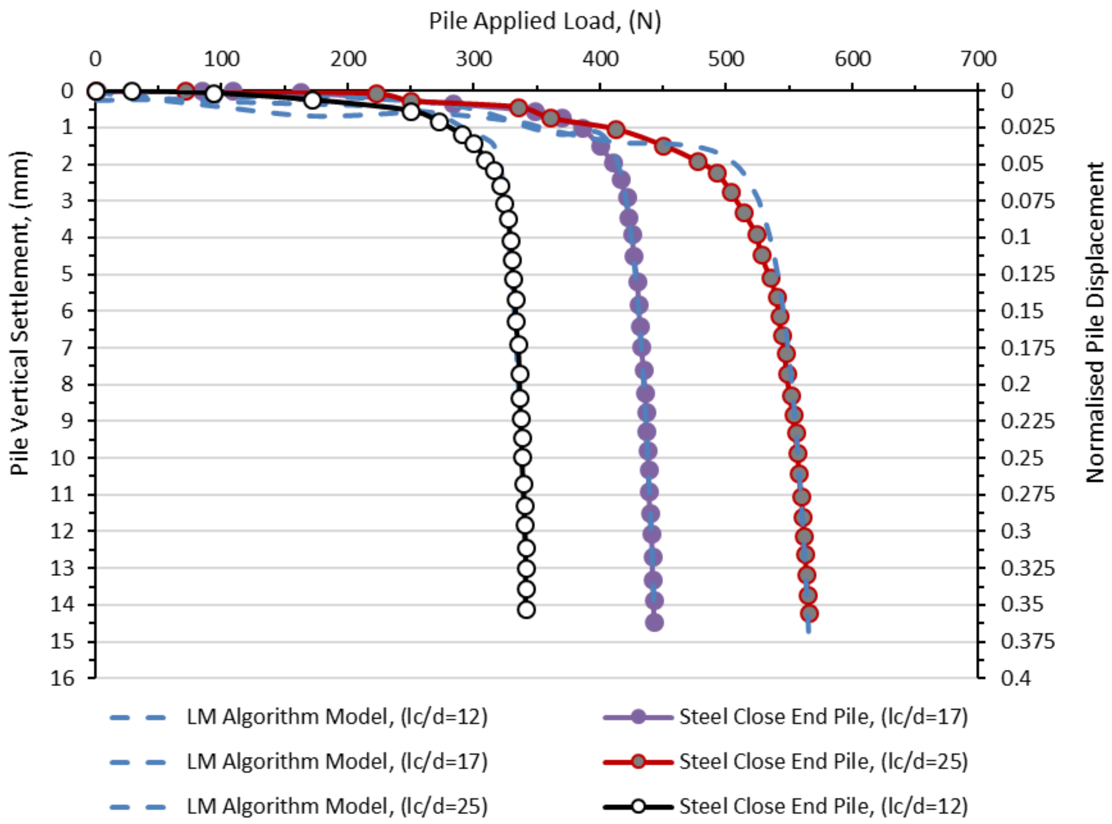

Fig. 9. Profiles of measured versus predicted pile load tests for model piles embedded in loose sand.

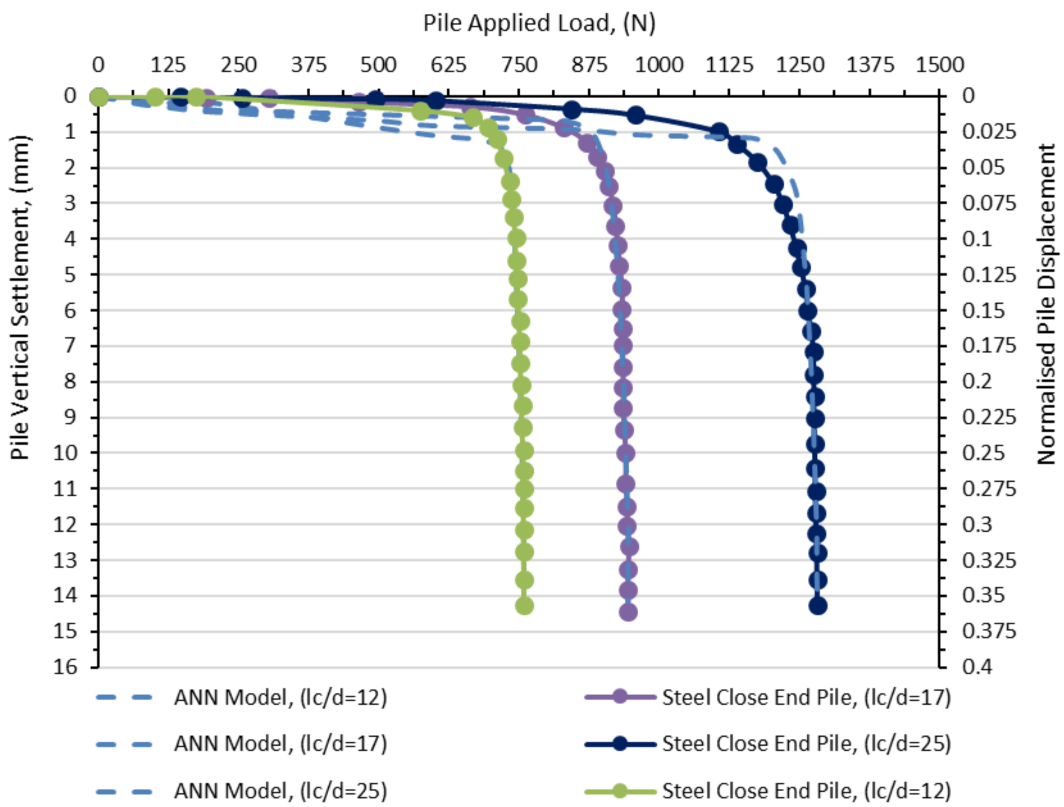

Fig. 10. Profiles of measured versus predicted pile load tests for model piles embedded in medium sand.

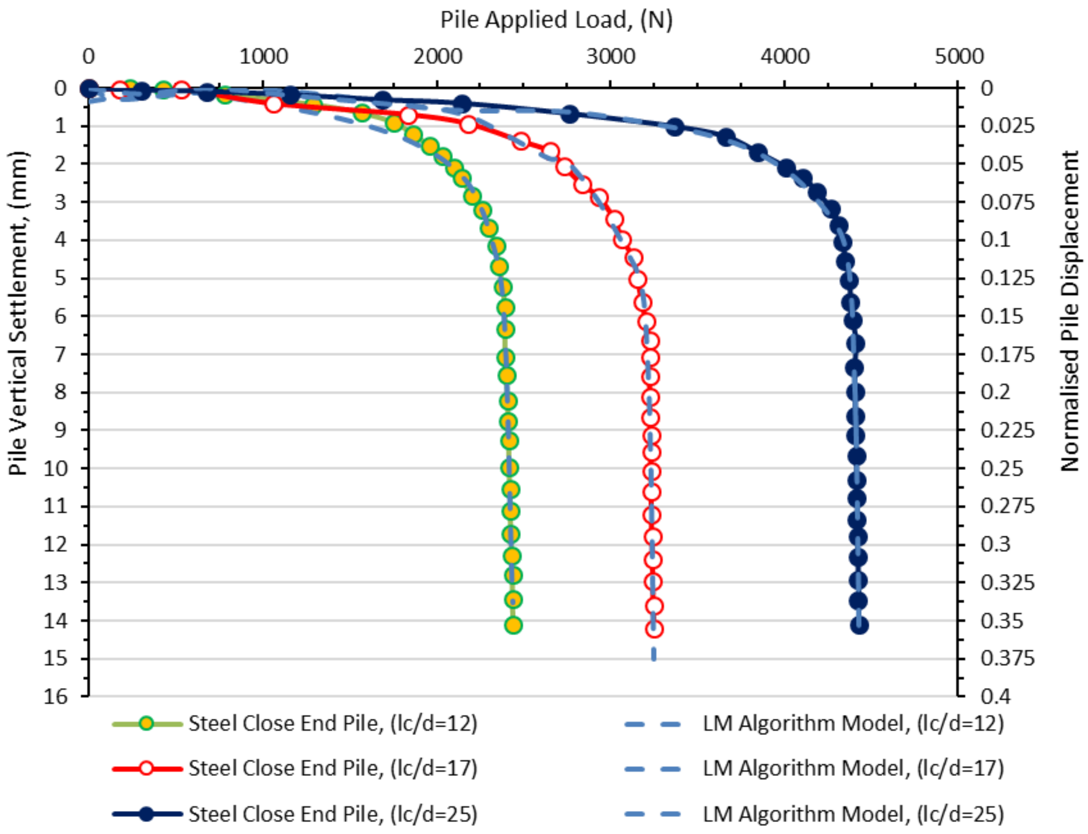

Fig. 11. Profiles of measured versus predicted pile load tests for model piles embedded in dense sand.

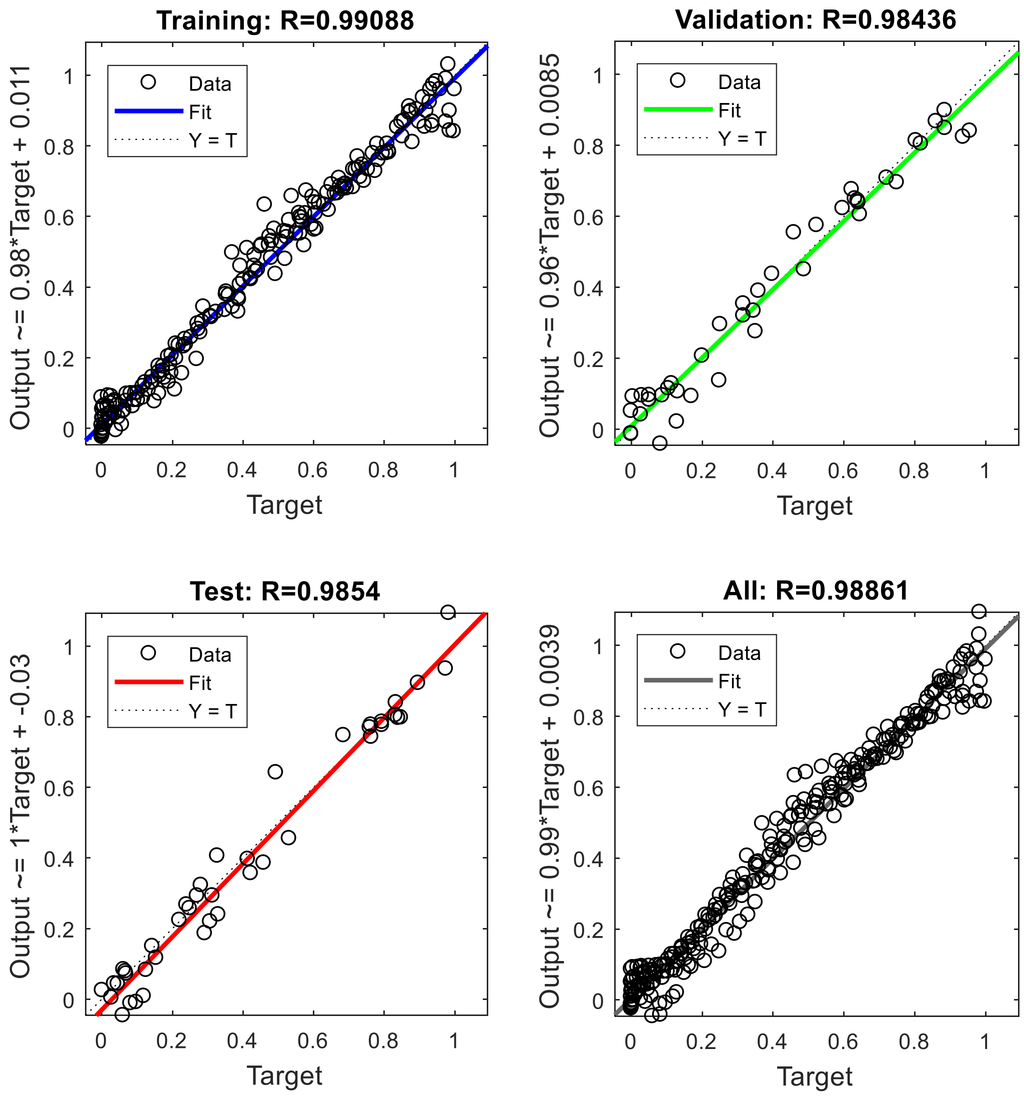

Fig. 12. Regression graphics of the experimental set against predicted pile settlement for the training, validation, testing and all data.

Comparing the experimental and the predicted values, the performance of the adopted algorithm was further examined graphically using the testing dataset. It should be pointed out that this dataset was not used during the training process and it is normally used to assess the generalisability of the algorithm (Millie et al., 2012; Sun et al., 2014). As such, the testing dataset was used to plot a regression calibration curve between the measured versus predicted results, with a 95% confidence interval (CI). A new MATLAB algorithm was used for this comparison. As shown in Fig. 13, there were high levels of consistency between the targeted and predicted values, with a root mean square error (RMSE) and correlation coefficients (R and P) of 0.095, 0.985 and 2.22 $*10^{-32}$, respectively.

This is a clear indication of the ability of the LM approach to successfully reproduce the results of the experimental values accurately.

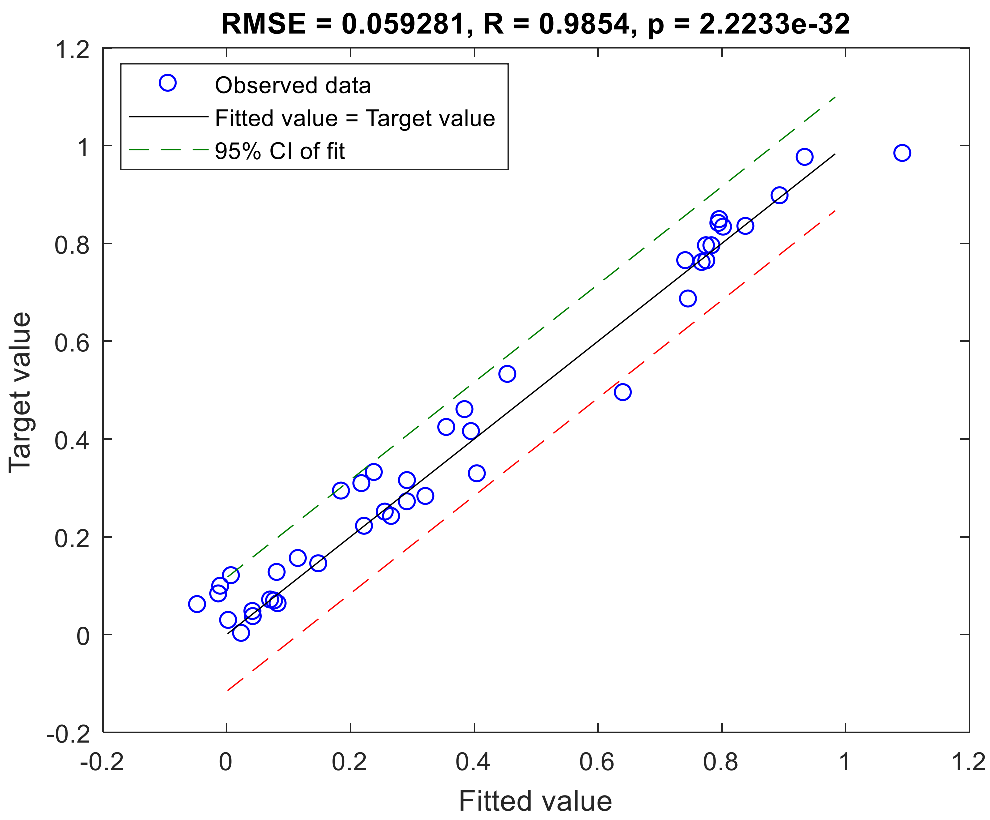

Fig. 13. Profiles of fitted versus observed settlement for the validation dataset at a 95% confidence interval.

6. Comparison between the LM optimum model with the various traditional methods

In this section, comparisons have been made between the experimental and the predicted pile settlement results obtained from the most traditional methods proposed by Poulos and Davis (1980), Vesic (1977) and Das (1995). As stated previously, the testing data subset was allocated to investigate the predictive ability of the LM approach. The testing data subset was also used to evaluate the superiority and the generalisation ability of the LM training algorithm when compared with the aforementioned traditional methods.

6.1 Polous and Davis' (1980) methodology

Poulos and Davis (1980) suggested that the following imperial equations (Eqs. 9 and 10) can be used to predict pile settlement for model piles subjected to axial load:

$$s = \frac{PI}{E_s D} \tag{9}$$

$$I = I_0 R_k R_h R_v \tag{10}$$

P, E_s and D are pile applied load, soil modulus of elasticity and diameter of pile. I is the influence factor of pile settlement, which involves the layer effect of soil depth, pile compressibility and Poisson' ratio. R_h is the influence factor for finite-depth and R_v is the Poisson's ratio correction factor. Such factors can be determined from design charts recommended by Poulos and Davis (1980). Using this approach, regarding a rigid pile driven into a semi-infinite soil with a 0.5 Poisson' ratio, I_0 is the only influencing parameter requiring consideration (Baziar et al., 2015).

6.2 Vesic's (1977) approach

Vesic (1977) suggested that pile settlement can be determined from the summation of three components, S_1, S_2, and S_3, using the following simplified formulas (Eqs. 11, 12 and 13):

$$S_1 = \frac{\left(P_{wp} + \xi P_{ws}\right)L}{A_{tip}E} \tag{11}$$

$$S_2 = C_p \frac{P_{wp}}{dq_p} \tag{12}$$

$$S_3 = C_s \frac{P_{ws}}{Lq_p} \tag{13}$$

P_{wp} is the working load applied at the pile head, P_{ws} is the load supported by the skin resistance and $\boldsymbol{\xi}$ is the skin friction distribution influence factor. C_p is an empirical factor. The coefficients q_p and C_s can be determined via Eqs. 14 and 15:

$$q_p = 40 \frac{N_{tip} l}{d} \leq 400 N_{tip} (kPa) \tag{14}$$

$$C_s = \left(0.93 + 0.16\sqrt{\frac{l}{d}}\right) C_p \tag{15}$$

The factor $\boldsymbol{\xi}$ can be assumed to equal 0.5 and the parameter C_p is equal to 0.09, as recommended for cohesion-less soil.

6.3 Das's (1995) procedure

The method proposed by Das (1995) is similar to that proposed by Vesic (1977) with some modifications when calculating S2 and S3. These modifications can be summarised by Eqs. 16, 17 and 18:

$$S_2 = \frac{P_{wp} D}{A_{tip} E_s}(1 - \upsilon^2) I_p \tag{16}$$

$$S_3 = \left(\frac{P_{ws}}{Ol_{penetarted}}\right)\left(\frac{d}{E_s}\right)(1 - \upsilon^2) I_{ps} \tag{17}$$

$$I_{ps} = 2 + 0.35\sqrt{\frac{l_{penetrated}}{d}} \tag{18}$$

Where I_p is equal to 0.88 as recommended by Nejad et al. (2009).

With the aim of further verifying the validity of the proposed model, Figs. 14 and 15 characterise comparisons between simulated and predicted pile settlement with respect to applied load and those estimated by the most traditional methods used in the absence of the pile load-test. It can be observed that the conventional design procedures are not as reliable when modelling load-distribution curves. As cited by many researchers; they tend to either underestimate or overestimate the predicted pile-load settlement. The comparative results indicate that the LM training algorithm performed well, and is in substantial agreement with the fitted line, suggesting that this new methodology is an expeditious approach, which offers obvious advantages.

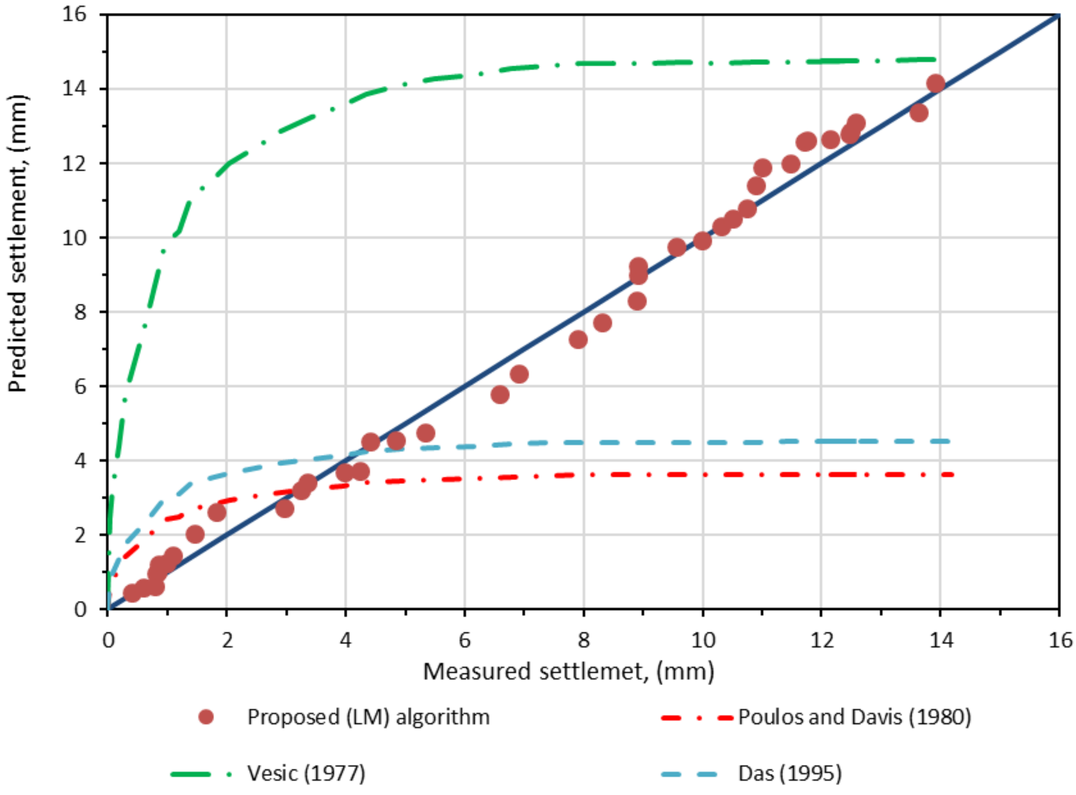

Fig. 14. Profiles of observed versus predicted pile settlement compared with traditional methods.

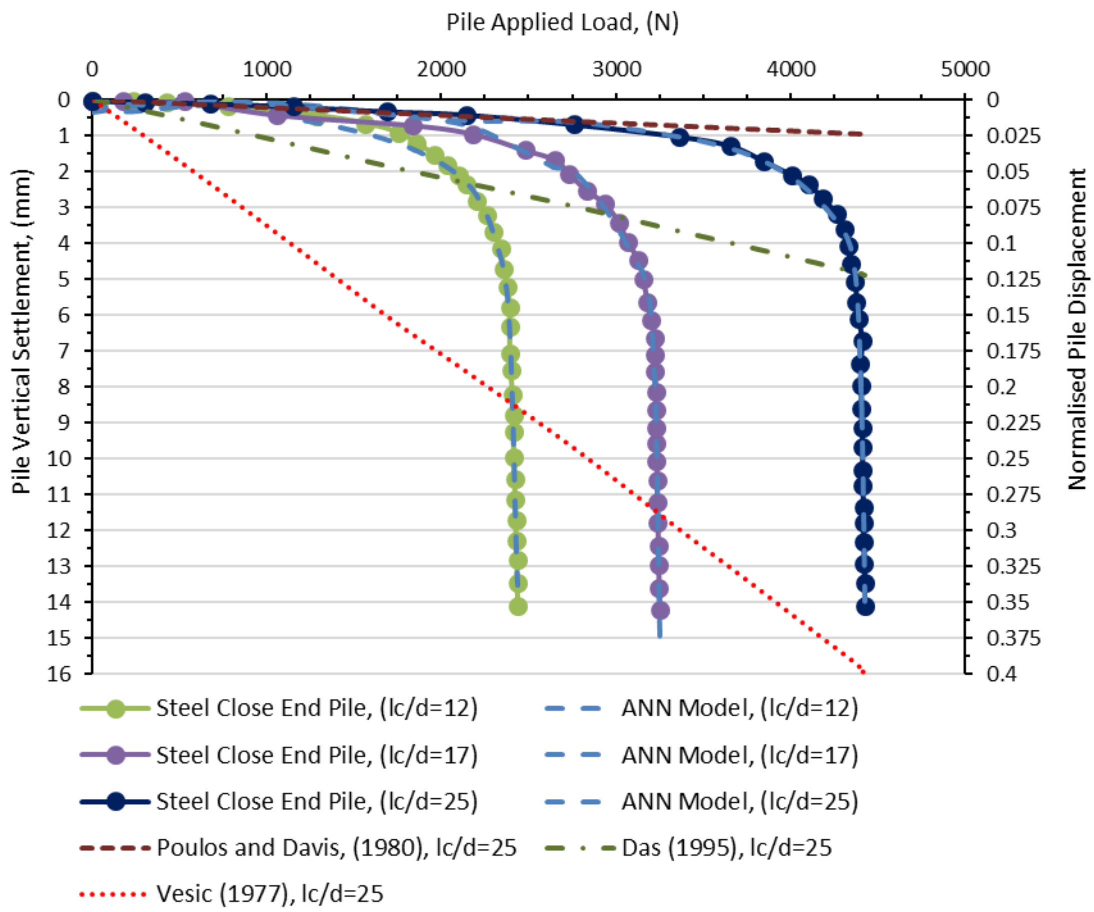

Fig. 15. Profiles of measured versus predicted pile load-settlement for the proposed LM compared with other conventional methods.

7. Concluding remarks and future research

This study examined pile-bearing capacity, and explored the feasibility of an expeditious artificial intelligence tool to develop and apply highly efficient, predictive models of pile-load settlement for steel model piles embedded in cohesion-less soil with three sand densities of 83%, 51%, and 18% using the supervised backpropagation Levenberg-Marquardt (BPLM) algorithm. The results demonstrated that pile-bearing capacity in dense sand is substantially higher than for those embedded in loose and medium sand. This can be assigned to an increase in the end bearing point and integrated shaft resistance developed in the radial effective soil-pile penetration depth. The statistical analysis outcomes indicate that the most influential parameters on pile load-settlement curves are the applied load (P) and the sand-pile friction angle, (δ). Conversely, pile slenderness ratio (lc/d), pile axial rigidity, (EA), pile effective length, (lc) have been identified as having the least impact on load distribution behaviour. It is demonstrated that the proposed LM training algorithm has favourable features; it is simple, easy to use, less vulnerable to overfitting issues and highly efficient. It established that the MSE becomes a negligible value with increases in the learning process, with comparable characteristics between the validation and testing set errors. In essence, the results of the graphical comparisons between measured and predicted pile settlement confirmed that the proposed algorithm could be used as an efficient data-driven approach to capture full pile load-settlement responses with a RMSE and R, of 0.0591 and 0.9854, with non-significant mean square error (MSE). Graphical comparisons were made to authenticate the reliability of the deterministic proposed method. These results revealed that the LM algorithm outperformed traditional approaches, confirming the successful application of the proposed technique. Conventional methods currently in use to predict pile settlement need to be updated, if employed, in future applications. Future study is recommended to focus on concrete piles penetrated in clay soil to examine pile load carrying capacity and to develop a further predictive model using the LM training algorithm.

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