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### Article

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# On the separation between RR Lyrae and Type II Cepheids and their importance for distance determination: the case of omega Cen<sup>★</sup>

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## ABSTRACT

The separation between RR Lyrae (RRLs) and Type II Cepheid (T2Cs) variables based on their period is debated. Both types of variable stars are distance indicators and we aim to promote the use of T2Cs as distance indicators in synergy with RRLs. We adopted new and existing optical and Near-Infrared (NIR) photometry of  $\omega$  Cen to investigate several diagnostics (colour-magnitude diagram, Bailey diagram, Fourier decomposition of the light curve, amplitude ratios) for their empirical separation. We found that the classical period threshold at 1 day is not universal and does not dictate the evolutionary stage: V92 has a period of 1.3 days but is likely to be still in its core Helium-burning phase, typical of RRLs. We also derived NIR Period-Luminosity relations and found a distance modulus of  $13.65 \pm 0.07$  (err.)  $\pm 0.01$  ( $\sigma$ ) mag, in agreement with the recent literature. We also found that RRLs and T2Cs obey the same PL relations in the NIR. This equivalence gives the opportunity to adopt RRLs+T2Cs as an alternative to classical Cepheids to calibrate the extragalactic distance scale.

**Key words.** Stars: variables: Cepheids, Stars: variables: RR Lyrae, Globular Clusters: individual:  $\omega$  Cen, Stars: distances

## 1. Introduction

Type II Cepheids (T2Cs) are pulsating variable stars of the Cepheid Instability Strip (IS), typically associated to old ( $>10$  Gyr) stellar populations. They are low-mass stars in either the post-horizontal branch (post-HB), asymptotic giant branch (AGB) or post-AGB phase (see, e.g., Gingold 1974; Sweigart et al. 1989; Bono et al. 1997c; Bono et al., 2020, submitted) and are mostly found in stellar systems with extended Blue HBs (BHB). T2Cs can be used as standard candles for distance estimates because they display optical and near-infrared

(NIR) Period-Luminosity (PL) relations, analogous to those of RR Lyrae (RRLs) and Classical Cepheids (CCs). Moreover, PL relations of T2Cs are only minimally affected by metal abundance (Bono et al. 1997c; Matsunaga et al. 2006; Di Criscienzo et al. 2007; Matsunaga et al. 2013; Lemasle et al. 2015).

Despite being much less numerous than RRLs (the Galactic Bulge hosts almost 70,000 RRLs but only around 1,000 T2Cs Soszyński et al. 2014, 2017, 2019), T2Cs are from one to five mag brighter. This means that their detection in high extinction environments (e.g., the Galactic Bulge, Bhardwaj et al. 2017b; Braga et al. 2018a, 2019) is easier, but also that they can be identified and characterized in external galaxies. Indeed, T2Cs have been found near M31 (Kodric et al. 2018) in M101 and M106 (Stetson et al. 1998; Macri et al. 2006; Majaess et al. 2009), and

<sup>★</sup> Tables 1, 2, 3, 7, A.1, B.1 and B.2 are only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via <http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/xxx>

more recently even an RVT in the Seyfert 1 galaxy NGC 4151 (Yuan et al. 2020).

In particular, in the NIR bands ( $JHK_s$ ) T2Cs seem to follow the same PL relations of RRLs (Majaess 2010) although there is still no solid empirical evidence. This means that RRLs and T2Cs may potentially be adopted jointly to calibrate Supernovae Type Ia (SNIa) luminosities and, in turn, to measure the Hubble constant ( $H_0$ ). In the last years, a  $\sim 4\sigma$  level tension between estimates of the local value  $H_0$  from the CCs+SNIa scale ( $H_0=74.03\pm 1.42$  km/s/Mpc Riess et al. 2019), and from CMB analyses ( $H_0=67.39\pm 0.54$  km/s/Mpc Planck Collaboration et al. 2018) has arisen, meaning that either a bias in any of these techniques, or new physics come into play. Note, that the discussion concerning the extent of the current tension is far from being settled. In a recent investigation Majaess (2020) argued that neglected or inaccurate blending corrections may result in an overestimated  $H_0$ . Moreover, by using a new calibration of SNIa based on the luminosity of the Tip of the Red Giant Branch (TRGB, Beaton et al. 2019), alleviates the tension by yielding intermediate values of  $H_0$ , ( $69.6\pm 1.7$  km/s/Mpc Freedman et al. 2020).

An independent estimate of the local  $H_0$  obtained with RRLs+T2Cs could be crucial to either validate or reconsider the tension. To adopt RRLs+T2Cs instead of CCs would also remove a population bias, for the latter variables are only found in late type galaxies, while RRLs and T2Cs are ubiquitous. A calibration of SNIa distances through old population tracers—RRLs and TRGB—has been already proposed (Beaton et al. 2016), however, this approach requires one more intermediate calibrations (that from RRLs to the TRGB), brings into play different physics (TRGB stars are not pulsating variables) and, in turn, different systematics when compared with homogeneous PL relations for RRLs and T2Cs.

The separation between RRLs and T2Cs is a long-standing problem. As a first approximation, it is possible to adopt a period threshold, whose exact value is still a matter of debate. A threshold of  $\sim 0.8$  days was set in the review by (Gautschy & Saio 1996) where type 1 (AHB1) stars above the HB—as defined in Strom et al. (1970) and Diethelm (1983, 1990)—were considered as T2Cs rather than evolved RRLs. This threshold is however obsolete, because a more extended and homogeneous investigation, based on period distribution and on the Fourier parameters of the light curve of RRLs in the Galactic Bulge, has now set the threshold at 1 day (Soszyński et al. 2008, 2014).

RRLs in the Bulge have a primordial (or minimally enhanced) helium abundance (Marconi & Minniti 2018) but there is theoretical evidence that helium enhancement increases the periods of RRLs (Marconi et al. 2018). This means that a 1-day period threshold should be considered a particular case of a more generic chemical- physics- and evolution-dependent threshold.

T2Cs are typically separated into BL Herculis (BLHs), W Virginis (WVs) and RV Tauri (RVTs) stars. Soszyński et al. (2011) investigation with OGLE-III data found two minima—at 5 and 20 days—in the period distribution of 335 T2Cs in the Galactic Bulge. They adopted these values as thresholds between BLHs-WVs and WVs-RVTs, respectively. These thresholds were later validated on the OGLE-IV sample of bulge T2Cs, that is almost three times larger (Soszyński et al. 2017). Note that the BLHs-WVs threshold of the General Catalog of Variable Stars (GCVS, Samus’ et al. 2017), is 4 days, based on the period distribution of T2Cs in the LMC (Soszyński et al. 2008). However, this is based on a small sample ( $\sim 200$  T2Cs) and new LMC data ( $>300$  T2Cs, Soszyński et al. 2018) invalidate the 4-day threshold.

Whether RVTs should be all classified as bona-fide T2Cs is still a pending issue. In fact RVTs are associated to either low- or intermediate-mass (from  $\sim 0.5$  up to  $\sim 3M_\odot$  Dawson 1979) post-AGB stars (Gingold 1985; Wallerstein 2002), belonging to old- and intermediate-age populations, respectively. To further stress the importance of the difference between old and intermediate-age RVTs, a different naming was proposed for the low-mass ones (Catelan & Smith 2015, V2342 Sgr stars). There is empirical evidence that RVTs in Galactic globular clusters (GGCs) which should belong to the V2342 Sgr class, do not share the same properties as field RVTs (Zsoldos 1998, e.g., they are missing the typical alternating deep and shallow minima). Finally, there is no consensus on the use of RVTs as reliable distance indicators. In fact, it is still a matter of debate whether they do follow the PL relation of BLHs and WVs (Matsunaga et al. 2006; Ripepi et al. 2015; Bhardwaj et al. 2017a).

Among nearby coeval stellar systems, the GGC  $\omega$  Cen (NGC5139) is the best workbench for T2Cs. It hosts the largest T2C sample in GGCs (seven T2Cs) after the two more metal rich clusters NGC6388 (twelve) and NGC6441 (eight), as well as long-period ( $>0.7$  days) RRLs. Moreover, three of its T2Cs have periods shorter than two days—which is optimal to investigate the transition between RRLs and T2Cs—while all T2Cs in NGC6388 and NGC6441 have periods longer than two days, with only one exception. While it is true that the Bulge hosts more RRLs and T2Cs, they are not at the same distance, the differential reddening and the stellar crowding are more severe, and NIR time series are only available for the  $K_s$ -band.

Furthermore,  $\omega$  Cen is characterized by a well known spread in metallicity (Johnson & Pilachowski 2010; Johnson et al. 2020), in helium content (Lee et al. 1999; Calamida et al. 2020) and affected by peculiar radial distribution of metal-poor and metal-rich stellar populations (Lee et al. 1999; Calamida et al. 2020, and references therein). This is an advantage because the pulsation properties of RRLs and T2Cs depend on chemical composition, meaning that  $\omega$  Cen is the GGC where these variable stars have more heterogeneous and varied pulsation properties.

Recently, the unprecedented wealth of kinematic data from Gaia DR2 and APOGEE DR14 (Gaia Collaboration et al. 2018; Abolfathi et al. 2018) were employed by Ibata et al. (2019) to validate the existence of the tidal stellar stream of  $\omega$  Cen. Based on its motion, Myeong et al. (2019) argued that  $\omega$  Cen could have been accreted by the Milky Way during the Sequoia merger, while Massari et al. (2019) and Kruijssen et al. (2020), more conservatively, associated  $\omega$  Cen to either Sequoia or to the larger Gaia-Enceladus (Helmi et al. 2018) merger event. On the theoretical side, Bekki & Tsujimoto (2019) validated these hypotheses and speculated that  $\omega$  Cen was a GGC of an accreted galaxy. Based on the properties of the different stellar population of  $\omega$  Cen, Calamida et al. (2020) proposed that, before being accreted by the Milky Way,  $\omega$  Cen might have formed by mergers between clusters and—eventually—the nucleus of a dwarf galaxy.

Therefore,  $\omega$  Cen is not only a GGC showing optimal properties to investigate T2Cs, but also a very interesting object on its own, being the largest and most heterogeneous GGC within the Galaxy. Finally, being close and well populated, its distance was estimated by using several diagnostics (RRLs, T2Cs, TRGB, white dwarfs, eclipsing binaries).

The aim of the paper is to provide more rigorous criteria to differentiate between RRLs and T2C, by viewing them from an evolutionary perspective. This would be a complete reversal of the point of view. In fact, until now, the period threshold has been the most common criterion to separate RRLs and T2Cs.

Almost always, this separation separates HB stars on one side ( $P < 1$  day) and post-HB stars on the other side ( $P > 1$  day). As a consequence, RRLs are considered as HB (core He burning) stars and T2Cs are considered as post-HB (double shell burning) stars. Our claim is, instead, that the leading argument to separate RRLs and T2Cs is their evolutionary stage, and that the empirical separation—only based on pulsation properties—is the consequence.

Moreover, we plan to adopt T2Cs as distance indicators, compare them with RRLs and use them jointly within a common distance diagnostic. This means that, on the one side, we aim for a more solid understanding of the differences in the evolutionary properties of both RRLs and T2Cs and, on the other side, we use them together as a single distance indicator. Seemingly, the two aims are at odds, but this is not the case. In fact, for distance determinations (especially of Local Group galaxies), it would be a threefold advantage to adopt a common PL relation because *i*) RRLs complement the small number of T2Cs; *ii*) T2Cs complement the lower brightness of RRLs, and *iii*) together they provide a wider period range on which to calibrate the relation upon, as it was already suggested by Benedict et al. (2011). Still, RRLs and T2Cs are not the same objects from the evolutionary point of view, and it is important to provide a clear criterion to tell them apart not only for a mere taxonomical purpose, but also for the correct development of pulsation models (Bono et al. 2020, submitted) and to investigate the population ratios of the host stellar system (GCs, nearby galaxies...).

The paper is structured as follows: in Section 2 we discuss the pulsation properties of T2Cs and long-period RRLs in  $\omega$  Cen; moreover, we discuss the transition between the two types of variables, also based on Bulge T2Cs. Section 3 is devoted to the comparison of the Bailey diagram and amplitude ratios of the T2Cs in  $\omega$  Cen, with those in the Galactic Halo and Bulge. We discuss the PL relations of T2Cs, their comparison with the PLs of RRLs, and use them to estimate the distance of  $\omega$  Cen in Section 4. We discuss our results in Section 5.

## 2. Type II Cepheids

According to the catalogue of variable stars in GGCs by Clement et al. (2001),  $\omega$  Cen is the third richest GGC in T2Cs, after the metal-rich clusters NGC 6388 and NGC 6441. In fact,  $\omega$  Cen hosts seven T2Cs: five BLHs (V43, V48, V60, V61, V92), one WV (V29) and one RVT (V1). All these variables were already discovered in the first investigation of variable stars in  $\omega$  Cen (Bailey 1902).

A detailed list of the T2Cs and their pulsation properties is provided in Appendices A and B.

### 2.1. The transition between RRLs and T2Cs

The period threshold between RRLs and T2Cs is a long-standing dilemma in the field of pulsating variables. In the literature, the accepted values for the maximum period of RRLs range from 0.75 days (Wallerstein & Cox 1984) to 2.5 days (Diethelm 1983).

An investigation of a homogeneous and extended sample of RRLs and T2Cs in the LMC with *VI*-band OGLE photometry, allowed to establish a more solid empirical threshold at 1 day (Soszyński et al. 2008). This was based on the period distribution and on the position of the pulsating variables in the  $\log P$ - $\phi_{21}$  plane, where  $\phi_{21}$  is one of the coefficients of the Fourier series fit to the light-curve.

As stated in the Introduction, the threshold at 1 day should be considered as a lower limit in the case of a cosmological helium abundance. In fact, RRL pulsation models (Marconi et al. 2018) predict longer pulsation periods for RRLs with helium-enhanced chemical composition, hence in stellar systems hosting helium-enhanced stellar populations, the 1-day threshold might not be reliable, and the separation between RRLs and BLHs might be less sharp. Recently, Kovtyukh et al. (2018a) hypothesised that the very existence of BLHs is to be ascribed to helium enhancement in a progenitor mass of  $0.8 M_{\odot}$ . This is consistent with the fact that the BHB of the metal-rich GGCs NGC6441 and NGC6388 (the two GGCs hosting the highest number of T2Cs) can be reproduced by helium-enhanced ( $Y=0.35$ - $0.40$ ) HB models (Busso et al. 2007; Bellini et al. 2013). Dalessandro et al. (2011) and Tailo et al. (2019) showed that the initial helium content also affects the mass loss on the RGB and, in turn, the position of the stars on the HB. The more helium-enhanced is the progenitor, the least massive and bluer is the star on the ZAHB, both because the mass loss is higher and because, at fixed age and fixed mass-loss rate, the turn-off mass is smaller. Therefore, helium enhancement would favour an evolution to T2Cs, which are observed only in systems with a well-populated BHB. However, there is no evidence of an extensive helium-enhanced population in the Halo, thick disk and old population of the Galactic Bulge, where Galactic T2Cs are found. This means that, while helium enhancement might favour the formation of T2Cs, it is not the only requirement. Moreover, evolved helium-enhanced RRLs could have periods and luminosities similar to those of BLHs (Marconi et al. 2018).

To this purpose,  $\omega$  Cen is the most appropriate stellar system to inspect the RRL-BLH separation for several reasons.

*i*)  $\omega$  Cen displays a well-defined BHB (Castellani et al. 2007), and T2Cs are associated with stellar systems showing an extended HB, since their progenitors are mainly BHB stars (Beaton et al. 2018; Bono et al. 2020, submitted).

*ii*), It hosts five RRLs with periods between 0.85 and 1 day (Navarrete et al. 2015; Braga et al. 2016) and four BLHs with a period shorter than 3 days (V43, V60, V61 and V92);

*iii*), there is evidence that the HB of  $\omega$  Cen should be, at least in part, enhanced in Helium content (Cassisi et al. 2009; Bellini et al. 2013; Tailo et al. 2016; Latour et al. 2018,  $Y \sim 0.28$ - $0.38$ ).

We investigate the transition between RRLs and T2Cs by using the CMD, Bailey diagram and the Fourier parameters of the light curve fit. Note that the latter two diagnostics are not used for the long-period RRLs, because our sampling of the time series is not optimal for variables with periods too close to 1 day (V263 and NV366) and we have too few *I*-band phase points for the other variables.

Note that the rate of period change is not among the quoted diagnostics, because the sampling of our data does not allow us to provide accurate measures of this parameter. Recent theoretical results about the evolutionary channels producing T2Cs (Bono et al. 2020) show that a significant fraction of T2Cs evolve from the blue to the red side of the HRD. These are low-mass ( $0.495 \leq M/M_{\odot} < 0.55$ ) horizontal branch stars that after the central helium exhaustion evolve towards the AGB. In subsequent evolutionary phases they move back to the blue towards the WD sequence, but this happens at higher luminosities (WVs, RVTs). These objects are characterized in their fainter limit by positive period derivatives. However, these models might also perform several gravo-nuclear loops (Bono et al. 1997a,b; Constantino et al. 2016) in the HRD, either during the AGB phase and/or in their approach to the WD cooling sequence. Some of

these loops take place inside the instability strip, and in turn, the period derivative can attain both positive and negative values.

Evolutionary models also show that long-period RRLs evolve from the blue to the red (see Fig. 5 in Bono et al. 2020). This means that positive period derivatives do not help in discriminating between long-period RRLs and short period T2Cs. On the other hand, both positive and negative period derivatives can be univocally associated to T2Cs. The current empirical evidence indicates that period derivatives for the short-period BLHs of  $\omega$  Cen, are positive (Jurcsik et al. 2001). Therefore, they cannot help us in separating BLHs from RRLs.

### 2.1.1. Long-period RRLs

We inspect five RRLs with  $P > 0.85$  days (V91, V104, V150, V263 and NV366). We do not include NV455 because we do not have data for this star, that is located more than 40 arcmin from the center. There is a clear period gap at  $\sim 0.935$ - $0.995$  days, where no RRLs are found. Unfortunately, there are no strong arguments to assess whether the gap is real or just due to the poor statistics of long-period RRLs.

V91, V104 and V150—These variables are all below the period gap, and their position in the  $V, B - I$  and  $K, B - K$  CMDs (see Fig.B.3) is consistent with being bona-fide RRLs. In the Bailey diagram, they are on the locus derived by performing a linear fit of  $Amp(V)$  vs period of the RRab stars of NGC6388 and NGC6441 ( $Amp(V) = 2.30 - 2.04 \cdot P$ , see Fig.B.4).

These clusters have been defined as Oosterhoff III in the literature, however, we decided to call them Oosterhoff 0 (Oo0, Braga et al. 2016), because these GGCs are very metal-rich ( $[Fe/H] < 1$  dex) and the progression in metallicity is replicated by the numbers (Oo0=metal-rich; OoI=metal-poor; OoII=very metal-poor).

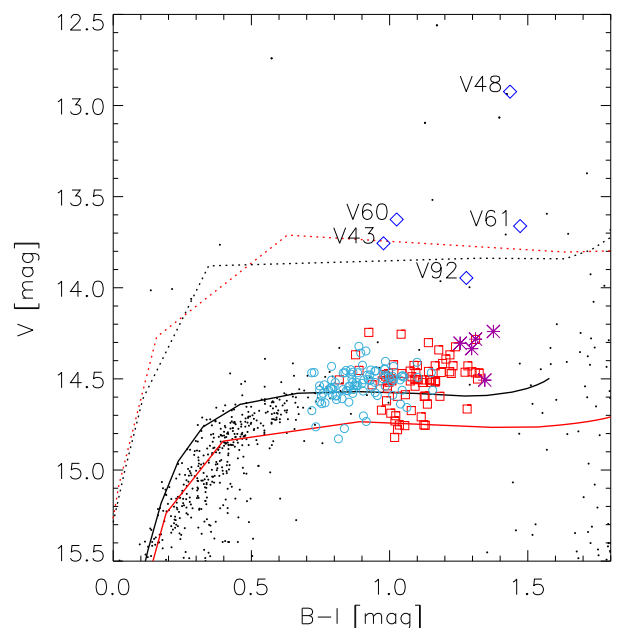
V263—This star is above the period gap ( $P \sim 1.01$  days), but its pulsation amplitude is very small, and seem to agree well with the decreasing trend of RRab amplitudes at long periods (see Fig. B.4). Despite its very long period, this star is consistent with being an RRab, due to its position in the CMDs, well within the magnitude range of HB stars with the same colour.

NV366—The light curve is heavily aliased because its period is 0.9999 days (above the period gap). In the optical bands the phases around minimum are missing. In the NIR bands, only the upper part of the decreasing branch is sampled. Therefore, both the optical and the NIR mean magnitudes which we have derived are probably underestimated, and the amplitudes are not reliable. However, in both CMDs, it is placed within the HB. Despite being located  $\sim 3.05$  arcmin from the cluster center, in a very crowded region, a visual inspection of the images did not outline any sign of blending. However, the crowding probably affects the photometric calibration between our different optical datasets (Braga et al. 2016).

We note from the Bailey diagrams in Fig.B.4, that a sizeable sample of  $\omega$  Cen RRLs, including the five ones discussed above, is placed on the locus of RRab stars in Oo0 clusters. The debate regarding the relation between the high metallicity of Oo0 clusters and the properties of their HB and RRLs is still unsettled. Pritzl et al. (2002) invokes a bimodal distribution of metallicities in the Oo0 clusters, with RRLs and BHB stars belonging to the more metal-poor population. In this scenario, HB stars evolve to the red, thus crossing the instability strip as metal-poor off-ZAHB RRLs. However, low-resolution spectra of RRLs in NGC 6441, provide high metallicities for these stars (Clementini et al. 2005), thus invalidating the scenario by Pritzl et al. (2002).

A similar scenario is described by Tailo et al. (2016) for  $\omega$  Cen: Based on population synthesis, they found that RRLs in  $\omega$  Cen should mostly be metal-poor and evolved (off-ZAHB), with a smaller fraction of metal-rich and fainter ZAHB RRLs; neither of these populations is significantly enhanced in helium ( $Y \leq 0.28$ ).

We have compared the empirical distribution of the RRLs and of the BHB in  $\omega$  Cen with the BaSTI  $\alpha$ -enhanced, helium-standard,  $Z=0.0006$  and  $Z=0.004$  tracks (corresponding to  $[Fe/H] \sim -1.84$  and  $[Fe/H] \sim -1.01$ , respectively, see Fig. 1). The two metallicity values were chosen to reproduce the peak of the metallicity distribution of RRLs (Magurno et al. 2019) and its metal-rich extension, which is the most prominent tail of the distribution. The majority of RRLs overlaps with the most metal-poor ZAHB model. On the other hand, the metal-rich ZAHB better fits the fainter and—presumably—more metal-rich RRLs.



**Fig. 1.** Optical ( $V$  vs  $B - I$ ) CMD of  $\omega$  Cen. Light blue circles: RRc; red squares: RRab; purple asterisks: long-period-RRab; blue diamonds: BLHs. Black and red lines display the ZAHB (solid) and Helium-exhaustion (dotted) sequences for  $\alpha$ -enhanced, helium-normal HB models (Pietrinferni et al. 2006) for two different metal contents:  $Z=0.0006$  ( $[Fe/H] = -1.84$ ) and  $Z=0.004$  ( $[Fe/H] = -1.01$ ).

We have checked that none of the Oo0-like RRLs in  $\omega$  Cen belong to the faint part of the RRL sample. Based on the analysis of high-resolution spectra, Magurno et al. (2019) found indeed a minority of metal-poor RRLs. However, the latter are not significantly fainter, in the  $V$  band, compared to the metal-poor RRLs, therefore, no firm conclusion can be reached.

Therefore, while it is reasonable to assume that evolution is the most important factor in generating Oo0-like RRLs, secondary factors should be also considered to reproduce in detail the observations. Also, their metallicity is not necessarily low.

One factor might be the initial helium abundance. Marconi et al. (2011) investigated the period distribution of the RRLs in  $\omega$  Cen and its correlation with helium enhancement; They concluded that the latter might be a concurrent reason for the long periods of RRLs in  $\omega$  Cen, but also put an upper limit (20%) to the fraction of helium-enhanced ( $Y \geq 0.30$ ) RRLs.



Another factor might be the mass on ZAHB: since mass-loss along the RGB evolution is likely –at least partially– a stochastic process, its correlation with  $Y$  or  $Z$  is not one-to-one. Therefore, stars with the same chemical abundance, might lose less/more than an average amount of mass on the RGB and will have cooler/higher temperatures on the ZAHB, as discussed by (Origlia et al. 2007; van Loon 2008).

### 2.1.2. Short-period BLHs

V43, V60, V61 and V92 are four BLHs with period shorter than 3 days. We have inspected their photometric properties to assess whether they might be better classified as very long-period RRLs, either evolved HB stars or helium-enhanced. To this purpose, we adopt two different diagnostics, namely *evolutionary and pulsation models* and *Fourier coefficients*.

*Evolutionary and pulsation models:* Figure 1 shows that V92 is consistently fainter than the helium-exhaustion track. This is a further argument to reconsider its classification. We also point out that V43 is below the metal-rich helium-exhaustion track. However, we should assume a very high metallicity for V43 ( $[Fe/H] > 1.3$  dex, typical of less than 10% of the RRL population of  $\omega$  Cen, Magurno et al. 2019) to consider its reclassification as candidate RRab, while none of the other diagnostics points to a reclassification as a borderline RRab/BLH star.

*Fourier coefficients:* We have derived the  $\phi_{21}$  and  $\phi_{31}$  Fourier coefficients of their  $V$ - and  $I$ -band light curves, to compare with those from the OGLE survey, which is the largest sample of Fourier coefficients of pulsating variable stars. OGLE provides only the  $I$ -band Fourier coefficients, hence by using OGLE  $V$ -band time series, we derived ourselves the Fourier coefficients of the  $V$ -band light curves of Bulge RRLs and T2Cs.

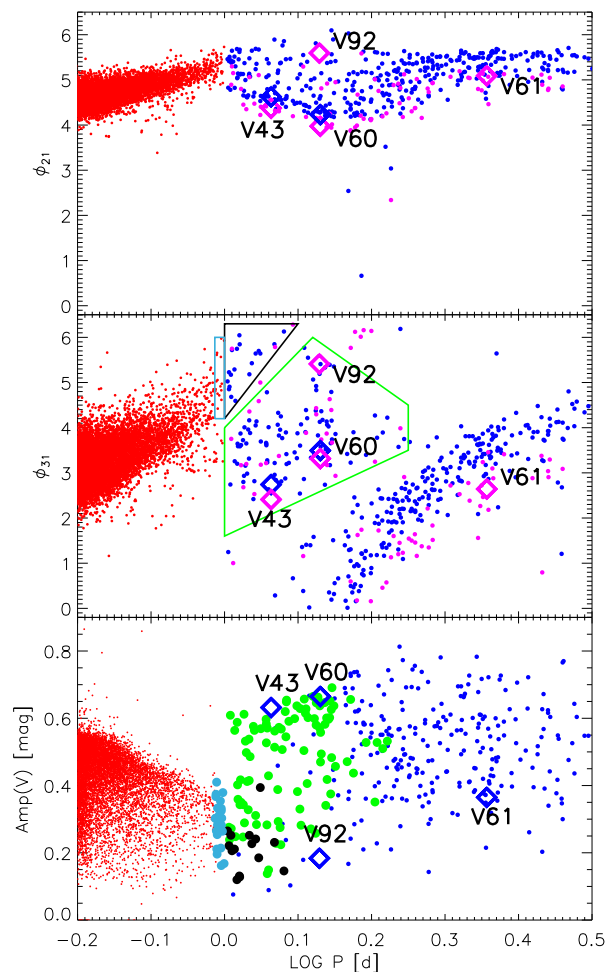
The top and middle panels of Fig. 2 display the  $V$ - (magenta) and  $I$ -band (blue) Fourier coefficients of OGLE bulge RRLs and T2Cs in the  $\phi_{21}$  and  $\phi_{31}$  vs  $\log P$  diagrams.

Larger symbols display the Fourier coefficients of the short-period BLHs in  $\omega$  Cen. Note that we did not derive the Fourier parameters of V61 and V92 in the  $I$  band, given that their light curve is not well sampled. However, we assume that their  $I$ -band coefficients are similar—or slightly higher—than their  $V$ -band coefficients, as for all the other  $\omega$  Cen and Bulge variables in the two planes.

V43, V48 and V61 are well within the typical loci of BLHs with the same period within the  $\phi_{21}$ - $\log P$  and  $\phi_{31}$ - $\log P$  diagrams. On the other hand, V92, especially in the  $\phi_{21}$  vs  $\log P$  plane, is at the edge of the Bulge BLH locus. Moreover, its position is consistent with an extrapolation of the RRLs at longer periods. This triggers a question: Could there be RRLs with periods longer than 1 day within the sample of T2Cs in the Bulge? To check this hypothesis, we have selected a few sub-samples of Bulge RRLs and T2Cs.

First, we selected 16 long-period ( $P > 0.97$  d) RRLs (light blue box in the middle panel of Fig. 2) to build a light curve template of long-period RRLs (see Appendix D).

Second, we selected two sub-groups of T2Cs: One on the extension of the RRL locus in the  $\phi_{31}$ - $\log P$  diagram (black box in the middle panel of Fig. 2), that we name “*candidate RRLs*” (15 objects, see Table 1) and the other at lower  $\phi_{31}$  and longer periods (green box in the middle panel of Fig. 2) that we name “*short-period T2Cs*” (99 objects). Note that the boxes have a purely empirical meaning, only to separate the two groups. The working hypothesis is that the stars in the first sub-group are, indeed, long-period RRLs.



**Fig. 2.** Top:  $\phi_{21}$ - $\log P$  diagram of long-period RRLs and short-period T2Cs in  $\omega$  Cen and Galactic Bulge. Red dots: RRab; blue dots: T2Cs ( $I$ -band Fourier coefficients); magenta dots: T2Cs ( $V$ -band Fourier coefficients). Middle: same as top but for the  $\phi_{31}$  coefficients. The green box contains variables in the *short-period T2Cs* sample; the black box contains variables in the *candidate RRLs* sample, and the light blue box contains variables in the *RRL template* sample. Bottom: Bailey diagram of the same variables as in the top and middle panel. Light blue, black and green circles display variables in the *short-period T2Cs*, *candidate RRLs* and *RRL template* samples, respectively.

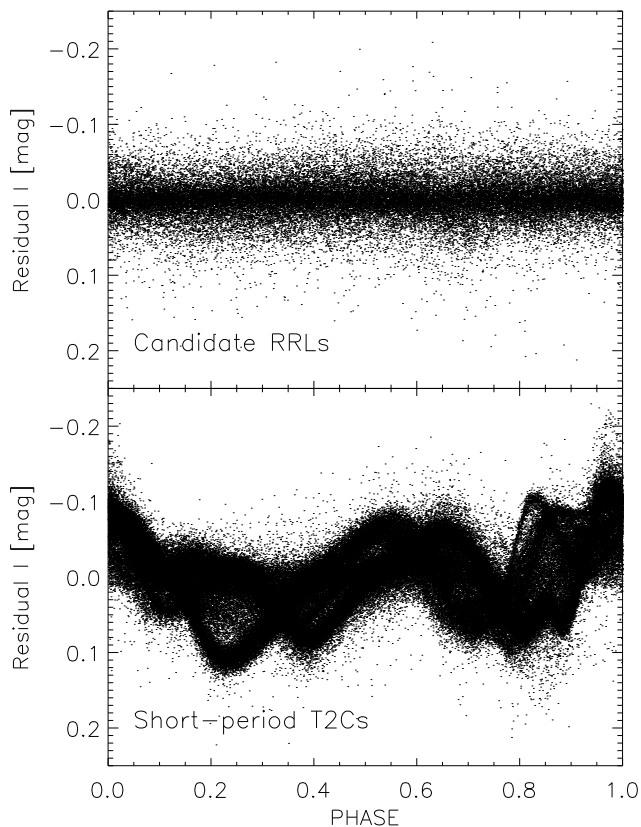
To quantitatively validate this hypothesis, we fitted the  $I$ -band light curves of both sub-groups of variables with the RRL light-curve templates derived before. Indeed, we found that the mean standard deviation from the fit of the light curves of the *candidate RRLs* is  $0.016 \pm 0.011$  mag. For the second sub-group, the mean standard deviation is  $0.045 \pm 0.018$  mag. Moreover, a visual inspection of the residuals of the light curves from the template fit (see Fig. 3) reveals that the light curves of the *candidate RRLs* are much closer to those of bona-fide RRLs than the light curves of *short-period T2Cs*. In fact, while the residuals of the *candidate RRLs* from the template fit do not follow any trend with the phase, those of the *short-period T2Cs* show a clear periodic behaviour, although this is not the same for all the stars. Finally, we found that the candidate RRLs are placed at the lower edge of the T2C distribution in the Bailey diagram (see bottom panel of Fig. 2).

Based on these considerations, we conservatively classify V92 as a candidate RRab variable. We point out that these clas-

**Table 1.** List of candidate RRab among Bulge T2Cs.

ID (OGLE IV) <sup>a</sup>	
0022	0517
0041	0598
0061	0636
0062	0638
0228	0767
0261	0797
0297	0909
0326	

**Notes.** <sup>(a)</sup> The full name is OGLE-BLG-T2CEP-XXXX, where “XXXX” is the ID appearing in the first column.



**Fig. 3.** Top: Residuals of the *I*-band light curve of all *candidate RRLs* from the template fit. All the variables were phased with the period and epoch of maximum provided by OGLE, meaning that phase 0 is the phase of maximum brightness. Bottom: same as top, but for all the *short-period T2Cs*.

sifications—despite being crucial to understand the evolutionary status of the stars—are irrelevant concerning the Period-Luminosity relations and distance estimates, because RRab and T2Cs do follow the same relations in the NIR (Matsunaga et al. 2006; Majaess 2010).

### 3. Comparison with Type II Cepheids in other stellar systems

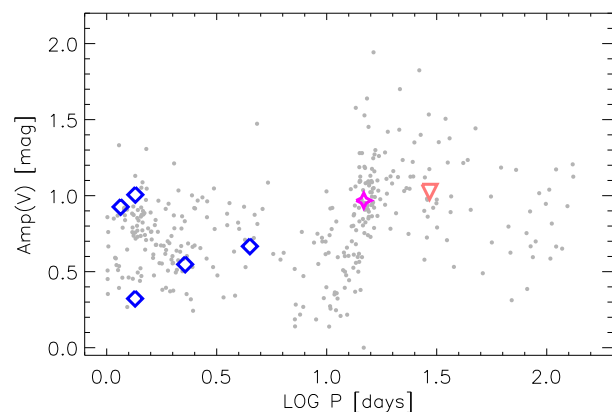
T2Cs are not as numerous as RRLs, but they are found in all the regions of the Galaxy (Bulge, Halo, GGCs), with the exception of the thin disk. They are also found in the Magellanic Clouds. Since their pulsation properties depend on the population (metal-

licity, age) properties of the host system, it is useful to adopt several diagnostics to compare the properties of T2Cs in  $\omega$  Cen with those in other environments.

#### 3.1. Bailey diagram: comparison with the Halo

The Galactic Halo is the most important component of the Galaxy concerning its merging history. Thanks to the Gaia mission (Gaia Collaboration et al. 2016), the investigation in Galactic archaeology is at its peak, with streams and remnants from merged galaxies being found, elucidating the past merging history of the Galaxy (Helmi et al. 2018; Myeong et al. 2019; Kruijssen et al. 2020; Helmi 2020).  $\omega$  Cen itself is, most likely, a former GC of an accreted galaxy (either Gaia-Enceladus or Sequoia Massari et al. 2019; Bekki & Tsujimoto 2019), now orbiting the Galactic halo.

The fact that Halo and GGC T2Cs belong to a different population was already suggested by Woolley (1966) and recently confirmed by Wallerstein & Farrell (2018), based on both kinematics and metal abundance. It is therefore interesting to compare the properties of  $\omega$  Cen T2Cs with those in the Halo. As a diagnostic, we adopt the Bailey diagram, independent of distance and reddening.



**Fig. 4.** Bailey diagram of T2Cs in the Halo (grey points), and in  $\omega$  Cen (same symbols as in Fig. 1, plus magenta star for the WV and orange upside-down triangle for the RVT.)

Figure 4 displays the optical (*V*-band) Bailey diagram of 362 Halo T2Cs and  $\omega$  Cen T2Cs. We adopted the catalogue of Cepheids within Gaia DR2 published by Ripepi et al. (2019). However, we both complemented and corrected this list, by comparing their classification with other surveys and literature data, namely, ASAS (Pojmanski 1997), ASASSN (Shappee et al. 2014; Jayasinghe et al. 2019), GCVS (Samus’ et al. 2017) and Warren & Harvey (1976). We provide in Table 2 the list of T2Cs that we added to the Ripepi et al. (2019) sample and those for which we have changed the classification.

We note that the Halo field hosts T2Cs with periods longer than 100 days. This is a remarkable difference compared to T2Cs in all GGCs (V16 in NGC 6569 with a period of 87.5 days, Clement et al. 2001) and—as we will show in Section 3.2—with those in the Galactic Bulge (the longest period is 84.8 days for T2Cs in the outer Bulge (Soszyński et al. 2017) and 93.5 days for T2Cs in the inner Bulge (Braga et al. 2019)). This is further evidence that T2Cs in the Halo belong to a different population than T2Cs in GGCs. We also note that the four short-period BLHs of  $\omega$  Cen are placed at the edges of the distribution of field T2Cs.



**Table 2.** Complements and changes to the Ripepi et al. (2019) T2C catalogue.

Name	Gaia DR2 ID	class (R19)	new class	Ref. <sup>a</sup>
XX Vir	3640760901131104256	N/A	BLH	2
VZ Aql	4205497393488435200	N/A	BLH	2
V439 Oph	4472449191647245184	N/A	BLH	2
BL Her	4527596850906132352	N/A	BLH	3
V446 Sco	4037438101994829312	N/A	WV	2
AL CrA	4037674673147744384	N/A	WV	2
V564 Sgr	4042147241577658880	N/A	WV	1
V1834 Sgr	4045437774995118976	N/A	WV	2
V1303 Sgr	4052014091545300480	N/A	WV	2
V1185 Sgr	4052361842043219328	N/A	WV	1
V802 Sgr	4073100869046439040	N/A	WV	5
V554 Oph	4117307863590649600	N/A	WV	1
BH Oph	4484791347109688832	N/A	WV	2
HQ Car	5254665166975458944	N/A	WV	2
MR Ara	5954403987593491584	N/A	WV	1
AL Vir	6303152720661307648	N/A	WV	3
BO Tel	6643297500393311616	N/A	WV	2
V347 CrA	6726052960315592576	N/A	WV	1
V383 Sgr	6736147782729787264	N/A	WV	2
SZ Mon	3112344688094507136	N/A	RVT	3
MZ Cyg	1964010169902699648	WV	RVT	2
IU Cyg	2035402872974695936	—	RVT	2
V1831 Sgr	4048985899093631616	N/A	RVT	2
TZ Ser	4161479334488856320	WV	RVT	1
EP Mus	5855676944429471872	WV	RVT	2
CQ Sco	5957918469109353216	N/A	RVT	2
RX Lib	6241789522177233664	N/A	RVT	2
ET Oph	4111880369315900032	RVT	RVT/DCEP	4/2

**Notes.** <sup>(a)</sup> Reference for the new classification 1: ASAS, 2: ASASSN, 3: GCVS, 4: Gaia DR2 (Ripepi et al. 2019), 5: Warren & Harvey (1976)

More precisely, V43 and V60 are at the high-amplitude edge, while V61 and V92 are placed around the low-amplitude edge.

### 3.2. Amplitude ratios: comparison with the Bulge

Within the OGLE survey, more than 1,000 T2Cs were detected in the Galactic Bulge (Soszyński et al. 2018, and further addenda). Although both  $V$ - and  $I$ -band time series are available, only  $Amp(I)$  were published. Therefore, we have downloaded the  $V$ -band time series and derived  $Amp(V)$ , that we publish in Table 3. Note that, for RVTs showing alternating deep and shallow minima, we folded the light curves at their pulsation period (that is, the period between two relative minima). This means that their folded light curves display a wide dispersion around the minimum and our  $Amp(V)$  estimates for these stars are an average between their minimum amplitude (shallow minimum-to-maximum magnitude difference) and maximum amplitude (deep minimum-to-maximum magnitude difference). To compare the Bailey diagrams also in the NIR, we have adopted the  $Amp(K_s)$  of the same variables obtained from VVV data (Braga et al. 2018a).

The Bailey diagrams in all three bands are displayed in Fig. 5. The two optical ones are quite similar. In fact, BLHs display a very shallow and high-dispersion increase in amplitude from 1 to  $\sim 3.2$  days. In the  $K_s$  band, the increase is not only steeper, but also displays a smaller dispersion. Starting from  $\sim 3.2$  days up to the whole period range of T2Cs, all three Bailey diagrams show a minimum at  $\sim 8$  days and a subsequent increase in amplitude, until reaching another maximum at  $\sim 20$  days, which is also the threshold between WVs and RVTs. This feature was recently discussed, from a theoretical perspective, in Bono et al. (2020, submitted). At longer periods, the behaviour is not clear due to a large dispersion, but a general decrease in amplitude is observed in all three bands.

**Table 3.**  $Amp(V)$  of Bulge T2Cs.

ID (OGLE IV) <sup>a</sup>	$Amp(V)$ mag
0001	...
0002	...
0003	...
0004	...
0005	...
0006	$0.894 \pm 0.090$
0007	$0.838 \pm 0.069$
0008	...
0009	$0.561 \pm 0.036$
0010	...

**Notes.** Only the first 10 of the 1068 lines of the table are shown. The full table is shown in the machine-readable version of the paper.

<sup>(a)</sup> The full name is OGLE-BLG-T2CEP-XXXX, where “XXXX” is the ID appearing in the first column.

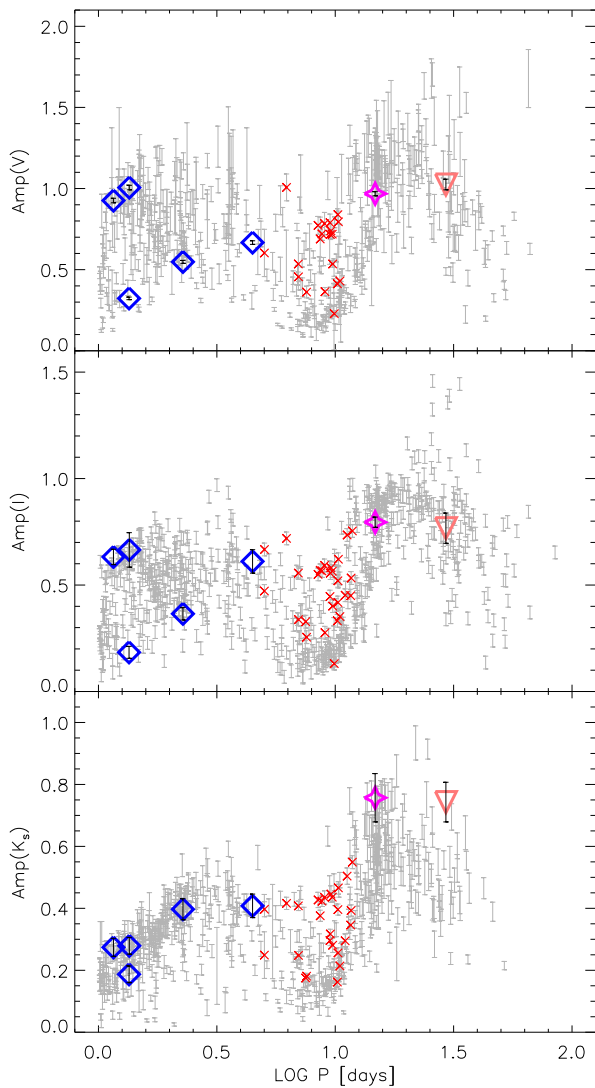
We point out that the short-period BLHs of  $\omega$  Cen are at the lower and upper edge of the distribution in the Bailey diagrams, especially the optical ones. This is the same behaviour that we observed when comparing to Halo T2Cs. In passing, we note that the peculiar WV stars (pWVs) are, at fixed period, brighter than canonical WV and are thought to belong to binary systems (Soszyński et al. 2008; Pilecki et al. 2017, 2018). The pWVs in the Bailey diagram appear to have optical amplitudes ( $Amp(V)$ ,  $Amp(I)$ ) that are either similar or larger than those of canonical WVs. In the NIR the trend is not so clear.

We adopted  $Amp(V)$ ,  $Amp(I)$  and  $Amp(K_s)$  to derive the amplitude ratios of T2Cs (see Fig. 6).  $Amp(I)/Amp(V)$  shows a clear linear trend with the period, and it is also interesting to notice that the zero point (0.62) is almost identical to the  $Amp(I)/Amp(V)$  amplitude ratio of RRLs (Braga et al. 2016,  $0.63 \pm 0.01$ ). It would be tempting to adopt  $Amp(I)/Amp(V)$  to separate RRLs and T2Cs, because of their different behaviour (RRLs display a constant  $Amp(I)/Amp(V)$ ). However, the intrinsic dispersion of the trend and the uncertainties on the amplitudes are larger than the difference of  $Amp(I)/Amp(V)$  between RRLs and T2Cs, especially at short periods ( $< 3$  days), where RRLs disguised as T2Cs are to be searched. The ratios involving the  $K_s$ -band amplitudes display a less clear behaviour, with BLHs+WVs and RVTs following different trends. More precisely, both  $Amp(K_s)/Amp(V)$  and  $Amp(K_s)/Amp(I)$  ratios increase with period in the BLHs range, approach a steady value and then decrease again in the 16–20 days range. However, we conservatively fit their distribution with a simple linear fit, also because the dispersion is quite large compared to the average ratio.

Unfortunately, in the VVV data many RVTs are saturated (Bhardwaj et al. 2017b; Braga et al. 2018a) therefore we have less points than for  $Amp(I)/Amp(V)$ . The ratios of RVTs in the  $K_s$ -band do not show any clear dependence on period and they are lower than those of WVs. We ascribe the different behaviour of the NIR amplitude ratios of RVTs to the presence of circumstellar dust. The long-wavelength excess light could explain the fact that, in the NIR, amplitudes are smaller than expected.

Our referee noted that the dispersion in the optical/NIR amplitude ratios is larger than intrinsic errors and might be caused by a possible dependence on metal content.

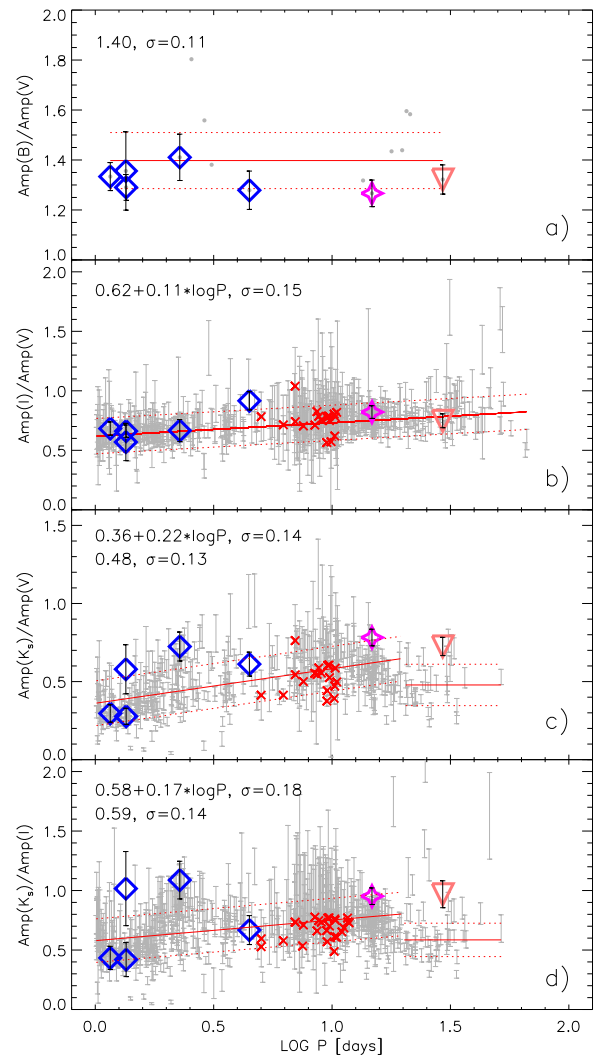
$\omega$  Cen T2Cs follow, within  $2\sigma$ , the trends that we have found, especially for the  $Amp(I)/Amp(V)$  ratio, that has the tightest relation.



**Fig. 5.** Top: Bailey diagram of Bulge and  $\omega$  Cen T2Cs in the  $V$  band. Grey: Bulge BLHs; red crosses: Bulge pWVs; other symbols are for  $\omega$  Cen T2Cs and have the same meaning as in Figs. 1 and 4. The uncertainties of the light curve amplitudes of  $\omega$  Cen T2Cs are displayed as black error bars. Middle: same as top, but for the  $I$  band. Note that the OGLE catalogues do not provide the uncertainties on the  $Amp(I)$ , therefore we assumed an uncertainty of 0.05 mag. This is a conservative assumption, since the median of the uncertainties on  $Amp(V)$  is 0.052 mag, and the  $I$ -band time series of OGLE have about one order of magnitude more points than the  $V$ -band time series. Bottom: same as top, but for the  $K_s$  band.

#### 4. Period-Luminosity relation

T2Cs are distance indicators that are not so widely used as CCs and RRLs, since they are not as numerous. For this reason, the calibration of their PL still lags that of RRLs and CCs. However, T2Cs in GGCs already proved to be crucial for understanding the difference between Population I and Population II stars (Baade 1956).



**Fig. 6.** Panel a:  $B$  over  $V$  amplitude ratios of GGC T2Cs. Black: T2Cs from NGC 6388 and NGC 6441; other symbols have the same meaning as in Figs. 1 and 4. The solid red line denotes the average while the dotted lines denote the  $1\sigma$  dispersion. Panel b:  $I$  over  $V$  amplitude ratios of Bulge and  $\omega$  Cen T2Cs. Here, the red line displays a linear fit. The symbols have the same meaning as in Fig. 5. Panel c: same as panel b, but for  $K_s$  over  $V$  amplitude ratios. Here, the red line displays a linear fit for  $\log P < 1.3$  and the average for  $\log P > 1.3$ . Panel d: same as panel b, but for  $K_s$  over  $I$  amplitude ratios. Here, the red line displays a linear fit for  $\log P < 1.3$  and the average for  $\log P > 1.3$ .

#### 4.1. The NIR PLs of T2Cs in GGCs

Although not all clusters host T2Cs, these variables are quite iconic for GGCs and were named "cluster cepheids" until the 1950s.

Matsunaga et al. (2006) performed an analysis of the NIR PL relations of the T2Cs in 23 GGCs. Our aim is to complement their sample and update the relations. For  $\omega$  Cen, they had data only for V1, V29 and V48. Moreover, they adopted  $E(B-V)$  from the Harris catalogue of GGCs (Harris 1996) and derived the true distance moduli ( $DM_0$ ) from the  $M_V$ -[Fe/H] relation of HB stars, adopting  $V_{HB}$  and [Fe/H] from the Harris catalogue. However, not only this catalogue contains heterogeneous data, mostly based on optical investigations, but the  $M_V$ -[Fe/H] rela-

**Table 4.** Distance moduli and reddening of GGCs.

GC	$\mu$ mag	$E(B-V)$ mag	Ref. <sup>a</sup>
NGC 2808	15.04	0.17	1
NGC 5272	15.07	0.01	2,3
NGC 5904	14.35	0.035	2,4
NGC 5986	15.10	0.28	3
NGC 6093	15.01	0.18	3
NGC 6218	13.43	0.19	3
NGC 6254	13.22	0.28	3
NGC 6256	14.79	1.20	5
NGC 6266	14.11	0.47	5
NGC 6273	14.58	0.40	5
NGC 6284	15.93	0.28	3
NGC 6325	14.51	0.91	3
HP 1	14.17	1.18	6
Terzan 1	14.13	1.99	6
NGC 6402	14.85	0.60	3
NGC 6441	15.65	0.52	5
NGC 6453	15.15	0.69	6
NGC 6569	15.40	0.49	5
NGC 6626	13.73	0.40	3
NGC 6749	14.45	1.50	7
NGC 6779	15.03	0.18	8
NGC 7078	15.13	0.09	2,4
NGC 7089	15.09	0.06	9,3

**Notes.** <sup>(a)</sup> When two references are given, the first is for  $\mu$  and the second for  $E(B-V)$ .

1: Kunder et al. (2013), 2: Sollima et al. (2006), 3: Harris (1996), 4: Ferraro et al. (1999), 5: Valenti et al. (2007), 6: Valenti et al. (2010), 7: Kaisler et al. (1997), 8: Ivanov et al. (2000), 9: Lázaro et al. (2006).

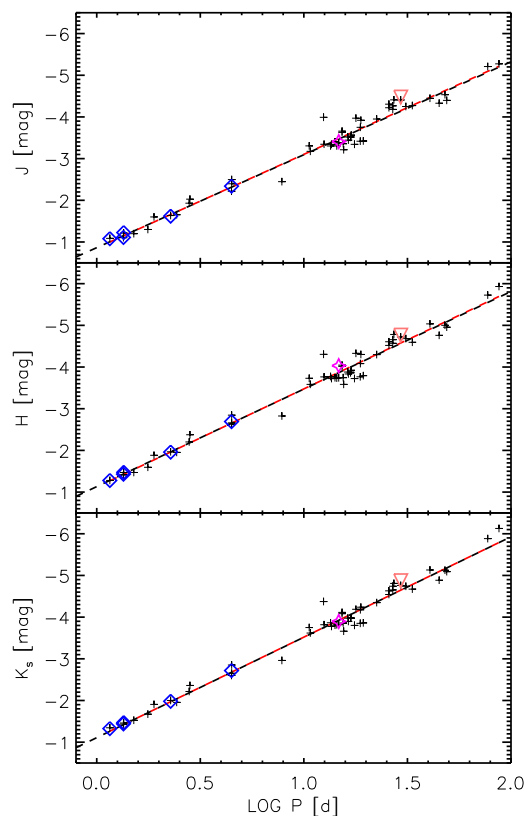
tion is prone to both non-linearity and evolutionary effects (Caputo et al. 2000). Moreover, several GGCs are located in the Galactic Bulge and recent NIR investigations provide more reliable estimates.

Therefore, we have collected  $DM_0$  and  $E(B-V)$  of GGCs from more recent literature to derive the absolute magnitudes. When possible, we have favoured papers providing distance estimates based on PL relations of variable stars and on NIR data. This means that, in the end, our absolute magnitudes are slightly different compared to those of Matsunaga et al. (2006). The degree of homogeneity is not complete, but is higher than that of the Harris catalogue. All references are listed in Table 7 4.

Finally, we have added V43, V60, V61 and V92 to the sample of Matsunaga et al. (2006), and replaced their mean magnitudes of V1, V29 and V48 with our own values, since our light curves are better sampled.

The updated PL relations that we have derived agree very well with those found by Matsunaga et al. (2006) (see Table 5). By comparing our Figure 7 with Figure 3 in Matsunaga et al. (2006), we note that the T2Cs in  $\omega$  Cen that we have in common (V1, V29 and V48), follow even more closely the NIR PL relations. The only relevant difference is the position of NGC 6256 V1: in our figure, it is  $\sim 0.6$  mag brighter in all bands, meaning an offset larger than  $3\sigma$ .

We point out that NGC 6256 is a Bulge GGC, affected by high extinction, with a large uncertainty:  $E(B-V)$  ranges from 0.84 (Harris 1996) to 1.66 mag (Schlegel et al. 1998). Moreover, there is solid empirical evidence to believe that the reddening in NGC 6256 is, most probably, differential (Valenti et al. 2007). We have also checked that V1 is not blended in the IRSF  $JHK_s$  images of Matsunaga et al. (2006). In the end, we cannot exclude the possibility that NGC 6256 V1 is a pWV. This is a kind of



**Fig. 7.** Top:  $J$ -band PL relation of T2Cs in GGCs. Symbols for  $\omega$  Cen are the same as in Figs. 1 and 4; T2Cs from other GGCs are displayed as black crosses. The dashed black line displays the PL found by Matsunaga et al. (2006), while the dashed red line displays our own PL. Middle: same as top but for the  $H$ -band PL relation. Bottom: same as top but for the  $K_s$ -band PL relation.

**Table 5.** Empirical NIR PL relations of T2Cs in GGCs.

Band	a mag	b mag	$\sigma$ mag
—Matsunaga et al. (2006)—			
$J$	$-0.86 \pm 0.08$	$-2.23 \pm 0.07$	...
$H$	$-1.13 \pm 0.07$	$-2.34 \pm 0.06$	...
$K_s$	$-1.11 \pm 0.07$	$-2.41 \pm 0.06$	...
—Our coefficients—			
$J$	$-0.86 \pm 0.06$	$-2.23 \pm 0.05$	0.17
$H$	$-1.11 \pm 0.07$	$-2.36 \pm 0.06$	0.19
$K_s$	$-1.12 \pm 0.05$	$-2.40 \pm 0.05$	0.16

**Notes.**  $M_X = a + b \cdot \log P$ .

object that is slightly brighter than WVVs and that is likely to belong to a binary system (Soszyński et al. 2008). This would be the first pWV found in a GGC, since pWVs have already been identified in the Magellanic Clouds (Soszyński et al. 2008, 2010), in the field ( $\kappa$  Pav, Matsunaga et al. 2009) and in the Bulge (Soszyński et al. 2017).

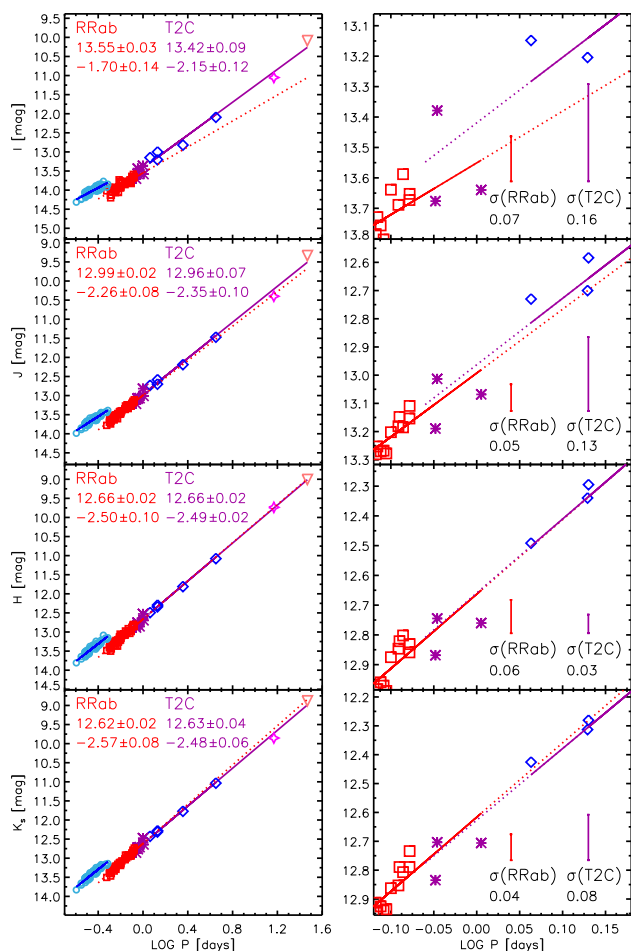
#### 4.2. The PL transition from RRLs and T2Cs and distance determination

Figure 4.2 displays the PL relations of RRLs and T2Cs of  $\omega$  Cen, from the  $I$  to the  $K_s$  band. Given that RRLs do not follow tight PL relations in passbands bluer than  $R$  (Catelan et al.

2004; Braga et al. 2015), we do not display the  $B$  and  $V$  PL relations. The  $R$ -band PL relations are not discussed, because the light curves are hampered by poor/modest sampling. Theory and observations indicate that the PL relations of RRLs are mildly affected by metallicity (Marconi et al. 2015), while T2Cs are not (Di Criscienzo et al. 2007; Lemasle et al. 2015), we decided to rescale the magnitudes of all  $\omega$  Cen RRLs to the value they would have at the average metallicity of the sample. To do so, we have adopted the theoretical metallicity coefficients of the PLs by Marconi et al. (2015, see their Table 6) and the new spectroscopic  $[Fe/H]$  estimates by Magurno et al. (2019). The rescaled magnitudes in a generic filter  $X$  ( $X_{Fe}$ ) have been determined as

$$X_{Fe} = X - c \cdot ([Fe/H] - \langle [Fe/H] \rangle)$$

where  $c$  is the metallicity coefficient and  $\langle [Fe/H] \rangle$  is the mean metallicity—calculated as  $\log(\langle 10^{[Fe/H]} \rangle)$ —of  $\omega$  Cen RRLs. These rescaled magnitudes were adopted to derive the PL relations of RRLs displayed in Fig. 4.2



**Fig. 8.**  $IJHK_s$  PL relations of RRLs and T2Cs in  $\omega$  Cen. The symbols have the same meaning as in Figs. 1 and 4. Left panels display both RRLs and T2Cs, while right panels are a close-up on the period range of the transition between long-period RRLs and BLHs. Red and purple lines show the PL relation of RRLs and T2Cs, respectively. The dotted lines show the extension at longer/shorter periods of the RRLs/T2Cs PL relations, respectively. The dispersions of the relations are displayed as bars in the lower-right corner of the right panels.

We note that, with the only exception of the  $H$ -band, V29 (the WV) is always underluminous compared to the T2C rela-

**Table 6.** Empirical Optical and NIR PL relations of T2Cs in  $\omega$  Cen.

Band	a mag	b mag	$\sigma$ mag
$I$	$13.419 \pm 0.090$	$-2.151 \pm 0.117$	0.160
$J$	$12.963 \pm 0.074$	$-2.354 \pm 0.096$	0.131
$H$	$12.659 \pm 0.018$	$-2.485 \pm 0.023$	0.031
$K_s$	$12.627 \pm 0.044$	$-2.479 \pm 0.058$	0.079

**Notes.**  $X = a + b \cdot \log P$ .

**Table 7.** True Distance Moduli to  $\omega$  Cen

Band	$DM_0$ mag	err mag	$\sigma$ mag
$J$	13.656	0.066	0.137
$H$	13.640	0.073	0.075
$K_s$	13.663	0.061	0.084
$mean_{JHK_s}$	13.649	0.067	0.008

tion, while V1 (the RVT) is always overluminous. If the PL relations of T2Cs were affected by metallicity in the same way as RRLs, we would expect an opposite behaviour, based on the metal abundances by Gonzalez & Wallerstein (1994). This is further evidence that the effect of metallicity on the PL relations of T2Cs is negligible.

We notice that in the  $JHK_s$  bands, the long-period RRL stars follow both the RRL and the T2C relations. Most importantly, the coefficients of the RRL and T2C relations are the same within  $1\sigma$  in the  $J$  and  $K_s$  bands, and are practically identical in the  $H$  band. This means that RRLs and T2Cs obey a common NIR PL relation, as already suggested by Majaess (2010).

To estimate the  $DM_0$  of  $\omega$  Cen, we adopt both our own empirical calibration of the NIR PL relation based on GGC T2Cs (Table 5) and, as a comparison, that by Matsunaga et al. (2006).<sup>1</sup> Finally, we derived the distance moduli, listed in Table 7.

By following the referee's suggestion, we have obtained the  $K_s, J - K_s$  Period-Wesenheit (PW) relation, adopting the total-to-selective extinction ratio both by Cardelli et al. (1989,  $\frac{AK_s}{E(J-K_s)}=0.69$ ) and by Majaess et al. (2016,  $\frac{AK_s}{E(J-K_s)}=0.49$ ), to derive the  $DM_0$ . We found, respectively,  $DM_0=13.571 \pm 0.073 \pm 0.049$  mag and  $DM_0=13.718 \pm 0.075 \pm 0.052$  mag. The use of different reddening laws affects the estimate of the true distance modulus at the level of  $\pm 1\sigma$ , but in opposite directions. Therefore, we will not take into account the DMs derived from the PW and simply adopt the average value from the PLs as our final estimate of  $DM_0$ .

We note that the overall estimate agrees within  $1\sigma$  with previous estimates based on RRLs, but using the same photometry (Braga et al. 2016, 2018b).

## 5. Summary and final remarks

We have adopted Optical ( $UBVRI$ ) and NIR ( $JHK_s$ ) PSF photometry of  $\omega$  Cen (Braga et al. 2016, 2018b) and derived the pulsation properties (periods, mean magnitudes, light amplitudes and light-curve Fourier coefficients) of its seven T2Cs. We have

<sup>1</sup> The coefficients in Table 5 were obtained by including  $\omega$  Cen T2Cs. In principle, for a rigorous estimate of  $\omega$  Cen distance we should re-derive the coefficients of the NIR PL relations by removing  $\omega$  Cen T2Cs from the sample discussed in Section 4.1. However, we have checked that, within the errors, the coefficients are the same, either including or neglecting  $\omega$  Cen T2Cs from the sample.

discussed in detail the transition between RRLs and T2Cs, by using also data from the OGLE-IV survey and adopting several diagnostics (CMD, Bailey diagram, Fourier coefficients and light-curve template). We found that the period threshold at 1 day—commonly adopted to separate RRLs and T2Cs—is not universal. This was already evident in the literature (Sandage et al. 1994) but there has been a long-lasting lack of investigation on this issue. After 25 years, based on an unprecedented amount of data including the OGLE -IV time series (Udalski et al. 2015) we have obtained the following results.

Three main mechanisms severely hamper the reliability of the period threshold as a method to separate between RRLs and T2Cs.

*Evolutionary effects:* RRLs with periods longer than 1 day, overlapping with the shortest-period T2Cs, are predicted by pulsation models for evolved objects approaching helium exhaustion (Marconi et al. 2015, 2018).

*Helium abundance:* Although there is no direct evidence of helium enhancement of the RRLs and T2Cs of  $\omega$  Cen, pulsation models predict that helium-enhanced RRLs have periods longer than 1 day (Marconi et al. 2018). This means that also helium-enhanced RRLs can overlap in period with the shortest-period T2Cs.

*Period aliasing:* The most severe periodicity alias for ground-based observations, when using various types of techniques to estimate periodicity (e.g., Lomb-Scargle, Phase Dispersion Minimization, string length) is at 1 day, because of the daily cycle of telescope activity. The lack of variable stars around this period is, at least in part, due to this alias, which makes more difficult to detect the correct period. Among the largest surveys, only OGLE (thanks to its huge number of observations) and Gaia (Gaia Collaboration et al. 2016), the latter being a space telescope, easily overcome this limitation. In the future, LSST is also expected to be less affected by this period alias, thanks to the extension of its final time series.

To sum up, the period threshold is not universal and, when adopted, it brings to an approximate separation. Based on the diagnostics discussed in Section 2.1 (especially the  $\phi_{31}$ - $\log P$  diagram and the residuals from template light curves), we propose to re-classify V92 ( $P=1.346$  days) as a candidate RRab. We also found that 15 Bulge variables, previously classified as BLHs in the OGLE survey (Soszyński et al. 2017) are more similar to long-period RRab than to other T2Cs. Therefore, we suggest to re-classify these variables as candidate RRab stars.

We therefore suggest a more rigorous—although less immediate—approach to separate RRLs and T2Cs, based on the evolutionary status of the star. We consider as RRLs all the pulsating stars of the RRL and Cepheid IS, that are in their core helium-burning stage, regardless of their period. Only those stars that have exhausted helium in their cores should be classified as T2Cs.

Unfortunately, it is not easy to provide a solid diagnostic to follow this evolutionary criterion, for several reasons. *i)*—The ZAHB and helium-exhaustion tracks in the CMD do depend on metallicity and helium abundance. Moreover, each stellar evolutionary code generates slightly different tracks, depending on model assumptions (e.g. convection efficiency). On top of this, a proper comparison with empirical data would require a supplementary spectroscopic investigation to estimate the  $[\text{Fe}/\text{H}]$  abundance of the T2Cs. *ii)*—Although the  $\phi_{31}$ - $\log P$  diagram is very informative, we have checked that the morphological subclasses

of BLHs (the so-called AHB1, AHB2 and AHB3, where AHB1 are classically associated to RRab) are not well-separated in this diagram. Moreover, one needs at least some tens of phase points to derive a reliable estimate of the Fourier coefficients, which is not always the case for extensive surveys where variability is not among the main science cases.

We have studied the properties of the PL relations of RRLs and T2Cs. We found empirical evidence that RRab and T2Cs obey the same  $JHK_s$ -band PL relations, thus confirming the preliminary working hypothesis by Majaess (2010). This has remarkable consequences for distance estimates and, in turn, the setting of an extragalactic distance scale anchored only to Population II stars. In fact, the most severe limitation to the use of RRLs as distance indicators is their faintness, despite being ubiquitous and very numerous. T2Cs are 1 to 5 mag brighter, meaning that they can be detected in both farther and more reddened environments. On the other hand, the use of T2Cs is hampered by their modest number (at least one order of magnitude smaller than RRLs). Therefore, by virtue of the existence of a common PLs, one could employ RRLs and T2Cs together—as if they were the same class of variable stars—and overcome their respective weaknesses as distance indicators. Despite a more solid calibration—based on more objects—is needed, this assumption opens the path to adopt a RRL+T2C calibration of SNIa, leading to an independent estimate of  $H_0$ .

Although, at the time of writing, there is no possibility to calibrate the RRL+T2C NIR PL relations based on a large sample of both types of variables, instrumentation coming up in the near future will provide this opportunity. First of all, WFIRST and JWST (Gardner et al. 2006) will provide NIR photometry of, respectively, wide/shallow and narrow/deep areas. Also, the next Gaia data releases will provide not only more accurate parallaxes for a geometrical calibration, but also more extended time series. On the other hand, LSST (LSST Science Collaboration et al. 2009) will provide an unprecedented wealth of time series in six passbands (*ugrizy*). These will be crucial to establish more quantitative criteria to separate RRLs from T2Cs, by using both the quoted diagnostics and, eventually, the colour-colour curves (Diethelm 1983) for which an optimal coverage of the light curves in at least three passbands is required.

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## Appendix A: Identification

We did not find any new T2C within our new data, so we will consider only the seven ones in the Clement catalogue (Clement et al. 2001) and classified as Population II Cepheids by Kaluzny et al. (2004). We will discuss the transition between long-period RRLs and BLHs in Section 2.1.

We retrieved all the seven T2Cs in our images by cross-matching our astrometric solution (with an accuracy of  $0.1''$ , see Braga et al. 2016) with the coordinates from the Clement et al. (2001) catalogue. We provide updated RA and DEC of the T2Cs of  $\omega$  Cen in Table A.1. We have matched our coordinates with the Gaia DR2 (Gaia Collaboration et al. 2018) astrometric catalogue and found that they all match within  $0.12''$ , except V43. Its *Gaia* coordinates are  $\sim 2.8''$  away from ours. However, V43 has no measurements in the *Bp* and *Rp* bands, and its *Gaia* astrometric solution has only 2 parameters (RA, DEC instead of the more accurate 5-parameter (RA, DEC,  $\mu_{RA}$ ,  $\mu_{DEC}$ ,  $\pi$ ,) solution. Therefore, we assume that the best coordinates for V43 are ours.

## Appendix B: Light curves

For the optical and NIR light curves we have adopted the photometry by Braga et al. (2016, 2018b). For V1, we have added photometry from the ASAS Survey (Pojmanski 1997).

By following a suggestion of the referee, we have quantitatively checked the impact of blending. Fig. B.2 displays the residuals of the *H*-band magnitudes of T2Cs and long-period RRLs respect to the empirical PL(*H*) derived in Section 4.2, plotted versus the angular distance from the center of the cluster. The small sample and wide dispersion of the residuals do not allow to give firm conclusions. However, there is no clear trend suggesting that, within the errors, blending does not affect our estimates.

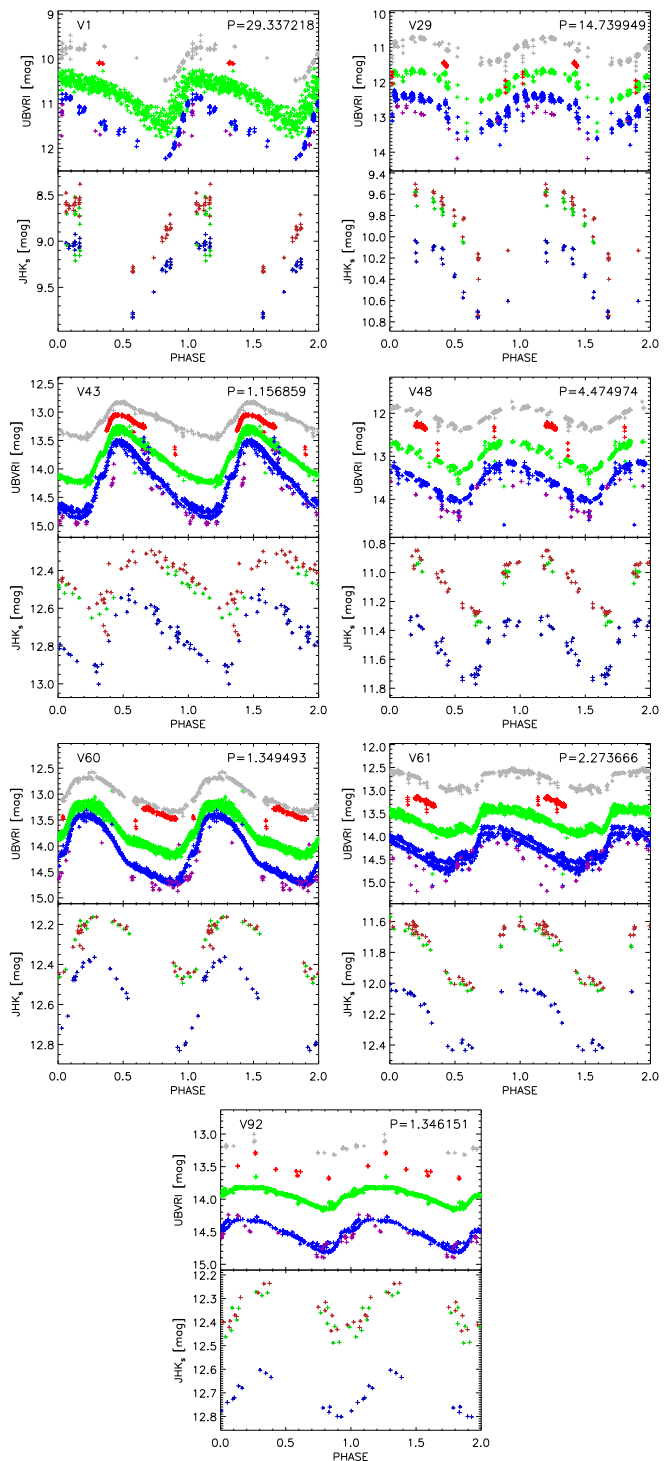
Figure B.1 displays the optical and NIR phased light curves of the T2Cs. Their periods are displayed in Table B.1 and were derived performing the multiband approach described in Stetson et al. (2014), using both optical and NIR data. We did not find any significant difference between our periods and those in the Clement et al. (2001) catalogue, in fact the relative offsets are smaller than  $5 \cdot 10^{-4}$  for all the T2Cs.

V1 does not show clear signs of alternating deep and shallow minima, typical of field—presumably intermediate-mass—RVTs. Moreover, it does follow the same PL and PW relations as BLHs and WVs (see Section 4.1), which are the common features of GGC RVTs. According to Gonzalez & Wallerstein (1994), it is also metal-poor ( $[Fe/H] = -1.77$  dex) and highly enhanced in  $\alpha$  elements ( $[\alpha/Fe] > 0.4$  dex). Therefore, we will consider V1 as a bona-fide (low-mass and old) T2C.

V29 displays an almost sinusoidal light curve, typical of WVs with similar periods. It shows no signs of over-luminosity, therefore it should be classified as a bona-fide WV.

BLHs, on the other hand, display varied light-curve morphologies, also at similar periods (see Fig. B.1). This is a known issue: Diethelm (1990) and Sandage et al. (1994) defined three different classes of T2Cs with periods shorter than 3 days, namely, AHB1, AHB2 and AHB3 (See Appendix C). Due to the wide magnitude range covered by T2Cs, and in turn, the broad range of evolutionary phases they experience, the classification of their light curve morphology and their correlation with physical parameters (stellar mass, chemical composition) is difficult. We inspected the light curve morphologies of the BLHs in  $\omega$  Cen. The results are the following, as summarized in Table A.1:

V43—Its light curve displays a steep rising branch and a shallower decreasing branch, typical of AHB1 stars.



**Fig. B.1.** Top panels: optical light curves of the T2Cs of  $\omega$  Cen. Purple: *U*; blue: *B*; green: *V*; red: *R*; grey: *I*. Names and periods are labelled at the top-left and top-right corners, respectively. Bottom panels: NIR light curves of the T2Cs of  $\omega$  Cen. Blue: *J*; green: *H*; red: *K<sub>s</sub>*.

V48—Its light curve is almost sinusoidal and does not look like any of the quoted morphological types. This is not unexpected, because its period is  $\sim 4.5$  days, which is outside of the period range of the AHBs. It is the brightest BLH in  $\omega$  Cen and the only one for which elemental abundances were derived (Gon-

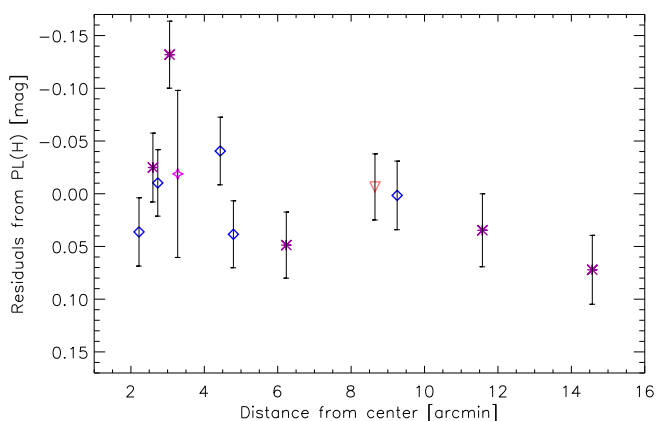


**Table A.1.** Astrometric properties and classification of the T2Cs in  $\omega$  Cen.

ID	type	AHB class	RA deg	Dec deg	RA <sub>Gaia</sub> deg	Dec <sub>Gaia</sub> deg	$\mu_{Gaia}^{RA}$ mas/yr	$\mu_{Gaia}^{Dec}$ mas/yr
$\omega$ Cen <sup>a</sup>	...	...	201.694625	-47.483306	201.697	-47.480	-3.234 ± 0.039	-6.719 ± 0.039
V1	RVT	N/A	201.521542	-47.395194	201.521533	-47.395181	-3.18 ± 0.12	-7.79 ± 0.19
V29	WV	N/A	201.613458	-47.479889	201.613428	-47.479890	...	...
V43	BLH	AHB1	201.643792	-47.449389	201.644919	-47.449480	...	...
V48	BLH	N/A	201.657542	-47.507083	201.657495	-47.507090	-3.57 ± 0.12	-7.11 ± 0.25
V60	BLH	AHB3	201.648708	-47.546806	201.648682	-47.546805	...	...
V61	BLH	AHB2	201.808208	-47.458639	201.808196	-47.458634	-2.48 ± 0.06	-7.08 ± 0.10
V92	Candidate RRab <sup>b</sup>	AHB3	201.561750	-47.354139	201.561742	-47.354131	-2.84 ± 0.07	-6.74 ± 0.10

**Notes.** Col. 1: Name; Col. 2: Variable type; Col. 3: Light curve morphology type (see Section B); Col. 4: Right ascension (our astrometry); Col. 5: Declination (our astrometry); Col. 6: Right ascension (Gaia DR2); Col. 7: Declination (Gaia DR2); Col. 8: Proper motion in Right Ascension (Gaia DR2); Col. 9: Proper motion in Declination (Gaia DR2). The average epoch of our astrometry is 1998.5

<sup>(a)</sup> Average for the whole cluster. RA and Dec average from Braga et al. (2016); Gaia averages from Vasiliev (2019). <sup>(b)</sup> See Section 2.1.2.



**Fig. B.2.** Residuals between measured and predicted  $H$ -band magnitude versus angular distance from the center. Symbols have the same meaning as in Figs. 1 and 4. The bars represent the squared sum of photometric error plus the PL intrinsic dispersion.

zalez & Wallerstein 1994). Its low metallicity ( $[Fe/H]=-1.66$ ) and the enhancement in  $\alpha$  elements suggest that it is of the spectroscopic class UY Eri, recently proposed by (Kovtyukh et al. 2018b).

V60)—The light curve displays a flat maximum, typical of AHB3 stars.

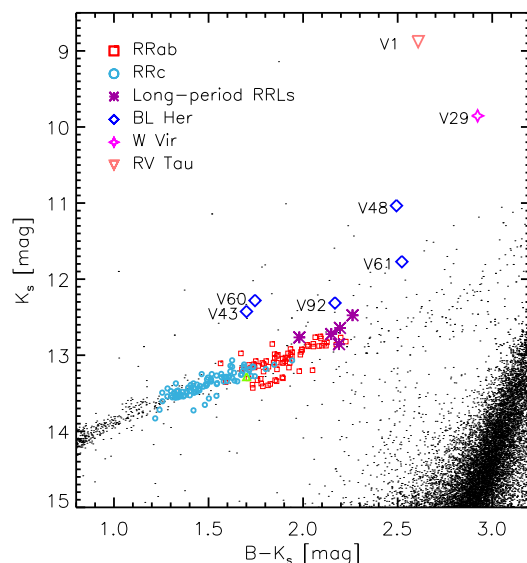
V61)—The light curve displays the typical bump of AHB2 class.

V92)—The light curve displays a flat maximum, typical of AHB3 stars.

Our morphological classification matches that provided by Sandage et al. (1994) except for V60 and V92: they were classified, respectively, as borderline AHB1 and AC (Anomalous Cepheid). We rule out the possibility that V92 is an AC because, based on its proper motion (see Table A.1), it is a cluster member (the proper motion of  $\omega$  Cen is  $\mu_{Gaia}^{RA}=-3.24\pm 0.01$  mas/yr;  $\mu_{Gaia}^{Dec}=-6.73\pm 0.01$  mas/yr, Baumgardt et al. 2019) but does not follow the PL relation of cluster-member ACs, while it does follow the PL relation of cluster-member BLHs (see Section 4.2).

### Appendix C: Above Horizontal Branch variables subclasses

As Diethelm (1983) pointed out, variable AHB stars come in three different flavors, which were later labelled as AHB1, AHB2 and AHB3 by Sandage et al. (1994). Their classifica-



**Fig. B.3.** Optical-NIR ( $K_s, B - K_s$ ) CMD of  $\omega$  Cen (close-up on RRLs and T2Cs). Symbols have the same meaning as in Figs. 1 and 4.

tion is based on the morphology of their optical light curves (see Fig. C).

AHB1 display saw-tooth RRab-like light curves (in fact, they were associated to RRLs by Diethelm (1983); AHB2 display a prominent secondary peak before the rising branch, and AHB3 stars display either a bump on the decreasing branch, or a plateau which covers  $\sim 20\%$  of the pulsation cycle around the maximum light. Note that the prototype BL Her itself is a AHB3 star, with a solar-like metallicity, higher than the majority of T2Cs (Caldwell & Butler 1978; Maas et al. 2007). We have checked that AHB1 and AHB3 variables are not well separated in the Bailey and the  $\phi_{31}$  vs  $\log P$  diagrams, while AHB2 are more easily detectable.

### Appendix D: Light curve template of long-period RRLs

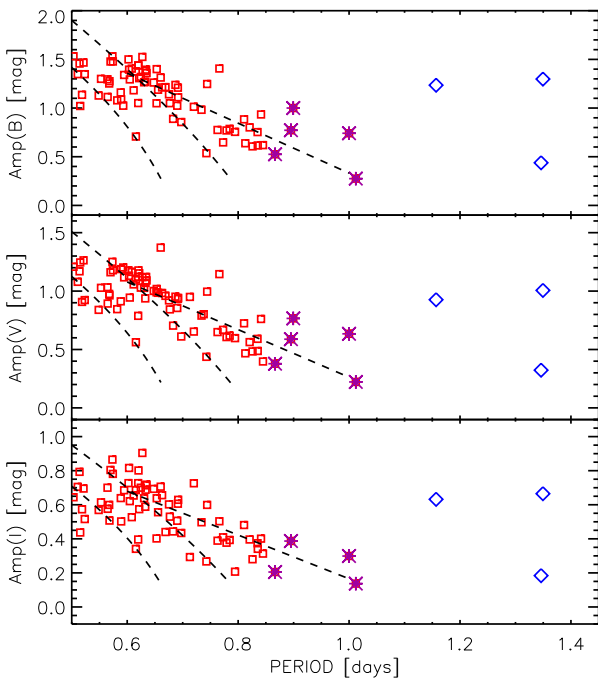
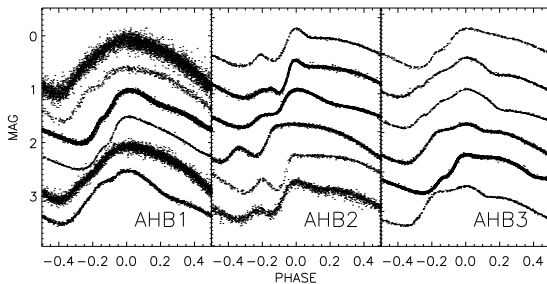
To build the  $I$ -band light curve template of long-period RRLs, we have selected 16 RRLs with periods between 0.97 and 1.00 days, that is, a sample that has no intersection with either the “candidate RRLs” or the “Short-period T2Cs” groups. We normalized and co-phased their  $I$ -band light curves by adopting the  $I$ -band amplitude and epoch of maximum provided by the OGLE

**Table B.1.** Optical mean magnitudes, amplitudes and periods of the T2Cs in  $\omega$  Cen.

ID	Period days	<i>U</i> mag	<i>B</i> mag	<i>V</i> mag	<i>R</i> mag	<i>I</i> mag	Amp( <i>U</i> ) mag	Amp( <i>B</i> ) mag	Amp( <i>V</i> ) mag	Amp( <i>R</i> ) mag	Amp( <i>I</i> ) mag
V1	29.337218	11.923±0.484	11.488±0.011	10.829±0.029	10.102±0.014	10.058±0.012	...	1.355±0.062	1.025±0.033	...	0.767 ± 0.071
V29	14.739949	12.867±0.165	12.776±0.014	12.015±0.007	11.507±0.034	11.049±0.018	...	1.226±0.143	0.968±0.013	...	0.795 ± 0.024
V43	1.1568588	14.467±0.016	14.139±0.024	13.759±0.008	13.175±0.071	13.149±0.031	1.084±0.056	1.235±0.038	0.926±0.014	...	0.632 ± 0.036
V48	4.4749736	13.845±0.014	13.528±0.017	12.924±0.008	12.317±0.037	12.092±0.012	0.855±0.130	0.853±0.032	0.667±0.011	...	0.611 ± 0.055
V60	1.3494930	14.353±0.025	14.028±0.017	13.624±0.007	13.420±0.078	13.001±0.071	0.938±0.150	1.298±0.035	1.006±0.014	...	0.665 ± 0.081
V61	2.2736663	14.487±0.017	14.293±0.004	13.661±0.003	13.254±0.064	12.821±0.013	0.890±0.131	0.773±0.027	0.548±0.009	...	0.365 ± 0.029
V92	1.3461514	14.590±0.016	14.480±0.006	13.946±0.003	13.567±0.037	13.199±0.011	0.482±0.056	0.438±0.027	0.323±0.008	0.207±0.057	0.184 ± 0.028

**Table B.2.** NIR photometric properties of the T2Cs in  $\omega$  Cen.

ID	<i>J</i> mag	<i>H</i> mag	<i>K<sub>s</sub></i> mag	Amp( <i>J</i> ) mag	Amp( <i>H</i> ) mag	Amp( <i>K<sub>s</sub></i> ) mag
V1	9.334±0.022	9.008±0.005	8.879±0.023	0.788±0.083	...	0.743±0.064
V29	10.379±0.013	9.736±0.073	9.854±0.026	0.738±0.054	...	0.757±0.078
V43	12.730±0.013	12.492±0.006	12.426±0.013	0.393±0.040	0.252±0.035	0.274±0.029
V48	11.470±0.013	11.078±0.009	11.034±0.011	0.433±0.037	0.438±0.053	0.408±0.038
V60	12.584±0.005	12.295±0.008	12.281±0.008	0.453±0.053	0.301±0.036	0.279±0.027
V61	12.190±0.007	11.811±0.007	11.771±0.008	0.411±0.038	0.425±0.048	0.397±0.034
V92	12.700±0.004	12.340±0.010	12.313±0.008	0.189±0.025	0.210±0.031	0.187±0.027

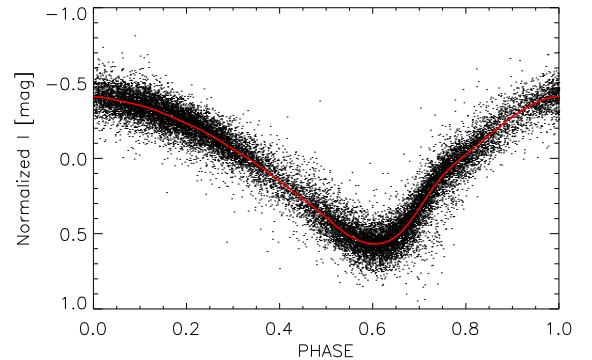

**Fig. B.4.** Bailey diagram of RRab and T2C stars in  $\omega$  Cen. The symbols are the same as in Fig. B.3. Dashed lines represent the Oosterhoff 0 (Oo0, see section 2.1.1), Oosterhoff I (OoI) and Oosterhoff II (OoII) loci of RRab stars. The Oo0 linear relation was derived by us in 2.1.1. OoI and OoII were adopted from Fabrizio et al. (2019). We rescaled the relations by using the amplitude ratios provided by Braga et al. (2016).

**Fig. C.1.** Sample normalized *I*-band light curves of Bulge AHB variables from OGLE-IV (Soszyński et al. 2017).

**Table D.1.** Coefficients of the light-curve template.

$A_0$	0.00915	$\phi_1$	...
$A_1$	0.44528	$\phi_2$	2.74455
$A_2$	0.08866	$\phi_3$	4.74558
$A_3$	0.02648	$\phi_4$	1.19303
$A_4$	0.02067	$\phi_5$	3.33520
$A_5$	0.00940	$\phi_6$	4.90126
$A_6$	0.00279	$\phi_7$	0.06851
$A_7$	0.00221	$\phi_8$	2.19867
$A_8$	0.00346		3.77858

**Notes.**  $F(\phi) = A_0 + \sum_i A_i \cos(2\pi i \phi - \phi_i)$

collaboration. Their cumulated, normalized light curve was fitted with a 8<sup>th</sup>-order Fourier series (see Fig.D).


**Fig. D.1.** Cumulated, normalized light curve of long-period RRLs and the Fourier fit adopted as template light curve (red line).

The coefficients are shown in Table D.1. We adopt this fit as the light-curve template for long-period RRLs.

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