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# Optimal Selection of Rotor Bar Number for Minimizing Torque and Current Pulsations due to Rotor Slot Harmonics in Three-Phase Cage Induction Motors

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**ABSTRACT** The paper develops a method to choose the number of rotor bars in order to eliminate rotor slot harmonics in stator current spectrum and pulsation torques that are their consequence. Mains-fed, three-phase cage induction motors with the most common number of pole pairs and number of stator slots, that result in integer slot winding, are analyzed. The analysis is based on the recently derived general rule for optimal selection of rotor bars, valid for symmetrical multiphase machine with prime number of phases and integer slot stator winding. As a tool for validation of analytically predicted results, parameterized winding function (PWF) model is used. Electromagnetic torque ripple factor is used as a measure of goodness of the number of rotor bar selection. The practical motivation of the study is an attempt to supersede the many existing rules for rotor bar number selection that, depending on the source, may be different, and provide a unified general approach to the problem. One of the main findings derived in the paper is ascertainment that increasing the number of pole pairs increases the degree of freedom in choosing the proper number of rotor bars. The same applies when the number of motor phases increases.

**INDEX TERMS** Induction machines, multiphase induction machines, rotor slot harmonics, parasitic torques, winding function, design optimization

## I. INTRODUCTION

A mains-fed three-phase cage induction motor still dominates in industrial applications worldwide more than a century after its invention. As the induction motor is a significant consumer of electrical energy globally, more stringent criteria are regularly imposed on its manufacturers, primarily in terms of the efficiency but also in terms of NVH (noise, vibration, harshness) requirements, [1]. One of the issues that, in the authors’ opinion, has not been exhaustively analyzed is the influence of the number of rotor bars on the occurrence of rotor slot harmonics (RSHs) in the stator current spectrum. These mostly unwanted high-frequency current components lead to additional Joule losses and thus directly affect motor efficiency. On the other hand, RSHs existence also implies the existence of parasitic torques and the appearance of unwanted vibrations and noise. It should however be noted that the existence of RSHs in

stator current spectrum may be desirable, for example in sensor-less speed control based on identifying the frequencies of such harmonics, [2].

The influence of rotor bar number on the rotor bar current waveform and other key performance aspects, including rotor cage copper loss, rotor bar current density, average torque, torque ripple, efficiency, etc., has been investigated in detail in [3]. Further, it has been shown that the magnetic saturation of rotor teeth, causing a significant increase in the rotor slot leakage flux, plays a key role in determining the rotor bar current distortion [4].

The problem of choosing the adequate number of rotor bars,  $R$ , in a machine with  $S$  stator slots and  $p$  pole pairs, is commonly known as a ‘slot combination’ problem. Slot combination has significant impact on many different motor performance aspects such as starting torque, torque-speed

curve, vibrations and noise levels. Many different rules have been proposed as guidance for choosing a suitable slot combination from different points of view, [5], [6], [7]. Some of the first rules were proposed as early as in 1931, [8].

The ongoing energy debate and the impulse towards full transport electrification have led to an increased interest in the optimized design and utilization of conventional cage rotor induction motors. The search for an optimal slot combination is one of the tasks (although not the only one) in the optimized design of the motor, [9]-[20]. In most of the cited papers only partial solutions are found, i.e. certain specific numbers of rotor bars are investigated. Additionally, almost exclusively unskewed rotor bars are considered. The reason for such an approach is that most of the existing literature uses commercial software based on finite element (FE) method. Such an approach is very time consuming in terms of both model preparation and simulation time. Additionally, in order to consider skewing of rotor bars in the FE models a 3D approach or several 2D simulations with subsequent post-processing are necessary.

One possible alternative to circumvent the FE-related problems is to use recently developed parameterized winding function (PWF) model, which offers significant advantages when compared to the FE models, [21]. The flexibility and computational efficiency of the PWF model make it a very effective tool to rapidly obtain and compare performance results relating to a wide variety of designs. Namely, PWF model enables the analysis of the induction machine with different number of rotor bars, skewed or not, while the rated power of the machine and its stator design remain invariant. The number of rotor bars and the skewing angle appear in this model as freely selectable variables. It should also be noted that an advantage of this model over the FE techniques is the incomparably smaller time taken to obtain results, as well as the very short time needed for initial model preparation. This model has been validated against accurate time-stepping FE simulations in several previous works, e.g. [22]-[25]. An excellent agreement of results from the two totally independent methods has been always observed, both qualitatively and quantitatively, proving that the PWF model is a means for reliable motor simulation.

The practical motivation of this paper arises from the observation that there exist many different rules for preferred slot combination and guidelines provided by different authors and sources are often in disagreement with one another [22]. The paper analyzes a variety of mains-fed three-phase ( $m=3$ ) induction motors with the most common numbers of stator slots ( $S=24, 36, 48, 54$  and  $72$ ) and pole pairs ( $p=1, 2, 3$  and  $4$ ). Thereby, the most common numbers of stator slots that allow for integral slot winding will be covered, i.e. the numbers of stator slots that satisfy the relation  $S=2pmq$ , where  $q$  is the integer number of slots per pole per phase.

Of course, the treatment proposed does not cover the entire design optimization problem, but rather provides the designer

with a set of slot combinations guaranteeing cancellation or minimization of the RSH-related ripples. The best combination should be then selected considering other optimization targets and constraints, including those related to manufacturing.

## II. OPTIMAL NUMBER OF ROTOR BAR SELECTION

In a recently published paper, [25], general symmetrical  $m$ -phase,  $2p$ -pole cage induction machine, where  $m$  is a prime number ( $m \geq 3$ ), with an integer slot stator winding is studied through a rigorous mathematical analysis with the aim of finding rules for selecting the optimal numbers of rotor bars. Optimal numbers are defined as such numbers of rotor bars that do not lead to appearance of rotor slot harmonics in the stator current spectrum.

In the mentioned work, the  $v^{\text{th}}$  harmonic of  $m$ -phase stator winding rotating flux-density wave is considered, [25],

$$B_v^{(s)}(t, \theta) = B_{\max v}^{(s)} \sum_{k=0}^{m-1} \cos\left(\omega t - vp\theta + k\left(v-1\right)\frac{2\pi}{m}\right), \quad (1)$$

where the  $v^{\text{th}}$  harmonic exists only when the following condition is satisfied:

$$v \in \mathcal{U} = \{2mz + 1 : z \in \mathbb{Z}\}. \quad (2)$$

The flux-density waves, produced by the cage rotor and responsible for rotor slot harmonics, were also analyzed in detail in [25], [26], [27],

$$B_L^{RSH}(t, \theta) = B_{\max, L}^{RSH} \cos\left(\left(1 - \lambda \frac{R}{p}(1-s)\right)\omega t + \left(\frac{\lambda R}{p} - \mu\right)p\theta\right), \quad (3)$$

$$B_U^{RSH}(t, \theta) = B_{\max, U}^{RSH} \cos\left(\left(1 + \lambda \frac{R}{p}(1-s)\right)\omega t - \left(\frac{\lambda R}{p} + \mu\right)p\theta\right), \quad (4)$$

where  $\mu \in \mathcal{U}$ ,  $s$  is slip and  $\lambda$  is an integer, which defines the order of the RSHs ( $\lambda=1, 2, \dots$ ).

It was shown in [25] that, in order for RSHs not to exist in the stator current spectrum, the number of rotor bars must not be a divisor of either  $2p(mz \pm 1)$  or  $2pmz$  for any positive integer  $z$ . By limiting the number of rotor bars to the following range that is practically feasible,

$$0.5S \leq R \leq 1.5S, \quad (5)$$

$R$  should then satisfy the following inequality,

$$R \neq \frac{2p(mz + c)}{\lambda}, \quad (6)$$

where  $z$  and  $c$  are integers:

$$1 \leq z \leq \text{ceil}\left(\frac{3\lambda S}{4mp}\right), \quad (7)$$

$$-1 \leq c \leq 1. \quad (8)$$

If the above conditions are met, all potentially harmful pulsating torques that arise as a consequence of interaction of the RSHs in stator current spectrum and rotor currents are avoided, too.

In any other case, there exist lower or upper RSHs or both of them simultaneously in the stator current spectrum at the following frequencies, [25], [28],

$$f_L^{RSHs} = \left(1 - \lambda \frac{R}{p}(1-s)\right)f_1, \quad (9)$$

$$f_U^{RSHs} = \left(1 + \lambda \frac{R}{p}(1-s)\right) f_1, \quad (10)$$

and associated high frequency pulsating torques at following frequencies, [25]:

$$f_L^{Torques} = \left(2 - \lambda \frac{R}{p}(1-s)\right) f_1, \quad (11)$$

$$f_U^{Torques} = \left(2 + \lambda \frac{R}{p}(1-s)\right) f_1, \quad (12)$$

$$f_M^{Torques} = \lambda \frac{R}{p}(1-s) f_1. \quad (13)$$

In all of the previous expressions  $\lambda$  is an integer that defines the order of harmonics: for  $\lambda=1$  we have the first order RSHs that are also known as PSHs (principal slot harmonics), for  $\lambda=2$  there are second order RSHs, etc.

In the case when the number of rotor bars is a divisor of  $2pmz$ , there simultaneously exist both of the RSHs in the stator current spectrum at frequencies given by (9) and (10), and associated parasitic pulsating electromagnetic torques at frequencies that are the mean values of these frequencies, i.e. at frequencies given by (13).

In the case when the number of rotor bars is a divisor of  $2p(mz+1)$ , lower RSHs exist in the stator current spectrum at frequencies given by (9) and associated lower pulsating torque components at frequencies given by (11).

Similarly, in the case when the number of rotor bars is a divisor of  $2p(mz-1)$ , upper RSHs exist in stator current spectrum at frequencies given by (10) and associated upper pulsating torque components at frequencies given by (12).

It is worth noticing that the rule for number of rotor bars selection does not include the number of stator slots. However, the number of stator slots defines the upper limit for  $z$  in (7), or, in other words, the number of slots per pole per phase and the order of the stator slot harmonics which is given by  $S/p \pm 1 = 2mq \pm 1$ .

### III. RESULTS FOR A GENERAL THREE-PHASE MACHINE

In the following subsections four different (the most common) cases of the numbers of pole pairs ( $p=1, 2, 3$  and  $4$ ) and the most common numbers of stator slots  $S$  in three-phase ( $m=3$ ) cage rotor induction machine will be analyzed, in order to identify the preferred and optimal number of rotor bars in the predefined range (5). Only even numbers of rotor bars will be considered. The reason for this is the fact that odd numbers of rotor bars are commonly avoided due to the associated unbalanced magnetic pull that leads to undesirable NVH problems, [29], [30]. Also, only unskewed rotor bars will be considered. The reason for this is the following: in some cases (especially for large medium voltage machines) bar skewing can introduce manufacturing complications as well as a production cost increase. Furthermore, it is known that rotor bar skewing, in addition to benefits, gives also rise

to possible problems, such as inter-bar currents and occurrence of undesired axial field components, resulting in both core and Joule additional losses [31]. Therefore, the possibility to obtain very small torque pulsations without skewing, i.e. through a proper selection of the number of rotor bars, can represent a significant advantage, [25]. On the other hand, skewing of rotor bars leads to drastic reduction of electromagnetic torque pulsations for almost any choice of an even number of rotor bars, [22], [23], [25].

#### A. TWO-POLE ( $p=1$ ) MOTORS

The two-pole induction motor is a rather specific motor – it differs from all other induction motors with a different number of pole pairs. This motor is characterized by the highest power factor, which follows from the well-known fact that the magnetizing reactance is inversely proportional to the square of the pole pair number, [6], [7]; however, it also exhibits some drawbacks. The first one is the fact that this motor is rather expensive as a great portion of the copper in stator winding is not used for torque production due to the very long winding overhangs. Another disadvantage is the susceptibility of this machine to an unbalanced magnetic pull, [32]. Its third shortcoming will be evident from the analysis that follows.

In fact, it can be inferred from (6) that the first order RSHs cannot be avoided for any even number of rotor bars, regardless of the number of stator slots. This is also the case for higher order RSHs. It can be shown that, when the number of rotor bars is such that the lower first order RSH exists, then the second order lower RSH exists as well, but not the upper one and vice versa. Some characteristic cases are considered in detail further on.

##### 1) $p=1, S=24$

When the stator has  $S=24$  slots, the number of rotor bars of interest is in the range  $R \in [12, 36]$ . In order to avoid existence of RSHs of the first order, i.e. PSHs, the following numbers of rotor bars are “forbidden”, from (6) - (8):

$$R \notin \{12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36\} \quad (14)$$

Obviously, all even numbers of rotor bars are “forbidden”. This means that one cannot identify the preferred number of rotor bars, as any number from the list in (14) will produce at least one of the PSHs: the lower one for  $R=14, 20, 26, 32$ , the upper one PSH for  $R=16, 22, 28, 34$ , or both of them simultaneously for  $R=12, 18, 24, 30, 36$ .

One of the possible solutions is to search for the number of rotor bars from the set given by (14) that exhibits the smallest electromagnetic torque ripple pulsations in steady state conditions, according to the following torque ripple factor definition, [22], [25],

$$r(\%) = \frac{T_{em,AC,RMS}}{T_{em,DC}} \cdot 100, \quad (15)$$

where  $T_{em,DC}$  is the average (useful) torque computed by integration over a period  $T$ ,

$$T_{em,DC} = \frac{1}{T} \int_{t_0}^{t_0+T} T_{em}(t) dt, \quad (16)$$

and  $T_{em,AC,rms}$  is the RMS value of the torque:

$$T_{em,AC,RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} (T_{em}(t) - T_{em,DC})^2 dt}. \quad (17)$$

The value of this performance indicator will be calculated using the PWF model.

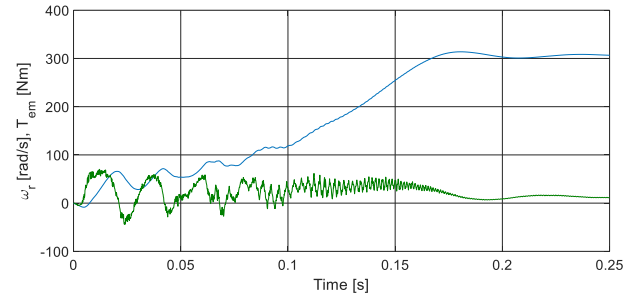
To illustrate the outputs of the PWF model, Figs. 1-8 shows some results for the different numbers of rotor bars. Fig. 1 shows the rotor speed and developed electromagnetic torque during acceleration of a fully loaded 4kW two-pole induction motor with  $S=24$  stator slots and  $R=20$  rotor bars. The motor details are listed in Table I. Fig. 2 gives the stator phase current and rotor bar current during the same transient. Fig. 3 shows the stator phase current and electromagnetic torque in steady-state condition, while Fig. 4 shows their spectral content.

According to the previous discussion, there exists a first order lower RSH at 924.5 Hz and a second order upper RSH at 2000 Hz in the stator current spectrum, for slip  $s = 2.55\%$ . The corresponding harmonics in electromagnetic torque spectra are the first order lower harmonic at 874.5 Hz and the second order upper harmonic at 2050 Hz. These harmonics are clearly visible in the spectral plots in Fig. 4. It should be noted that the useful (dc) torque component (here equal to 13 Nm) is excluded from the torque spectrum in Fig. 4, as well as in all the subsequent torque spectrum illustrations.

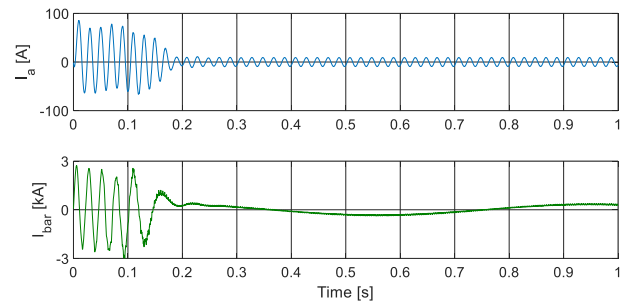
As a second example, Figs. 5 and 6 show the stator current and electromagnetic torque in steady-state condition and their spectral contents for the same motor but now with  $R=22$  rotor bars. According to the above mentioned rules, the first order upper RSH in the stator current is at 1122.5 Hz and the second order lower RSH is at 2095 Hz for  $s=2.51\%$ . The harmonics in electromagnetic torque spectrum are the first order upper, at 1172.5 Hz, and the second order lower harmonic at 2045 Hz. All these harmonics can be seen in the plots of Fig. 6.

As the third example, Figs. 7 and 8 show the stator phase current and electromagnetic torque in the steady-state condition and the corresponding spectral contents for the same motor but now with  $R=12$  rotor bars.

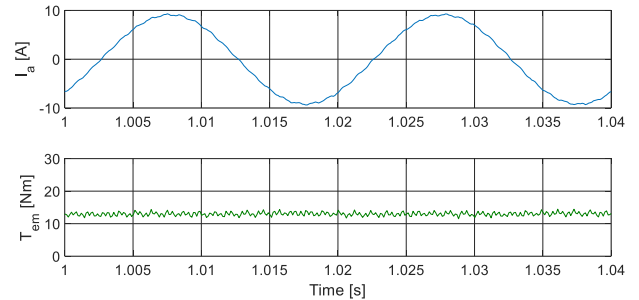
According to the defined rules, both RSHs of all orders now exist simultaneously, with those of the second and the fourth order being particularly pronounced. For slip  $s = 2.6\%$  there are current harmonics at the following frequencies: 534.4 Hz and 634.4 Hz, 1119 Hz and 1219 Hz, 1703 Hz and 1803 Hz, 2288 Hz and 2388 Hz. Corresponding torque harmonics are at the following frequencies: 584.4 Hz, 1169 Hz, 1753 Hz and 2338 Hz.



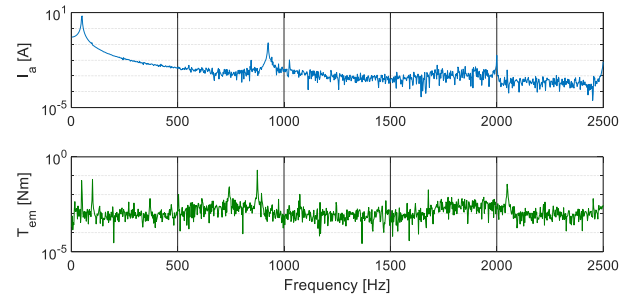
**FIGURE 1.** Rotor speed and electromagnetic torque during acceleration of the fully loaded motor:  $p=1$ ,  $S=24$ ,  $R=20$ ,  $T_r=13\text{Nm}$ ,  $P_r=4\text{kW}$ .



**FIGURE 2.** Stator phase current and rotor bar current during acceleration of the fully loaded motor:  $p=1$ ,  $S=24$ ,  $R=20$ ,  $T_r=13\text{Nm}$ ,  $P_r=4\text{kW}$ .

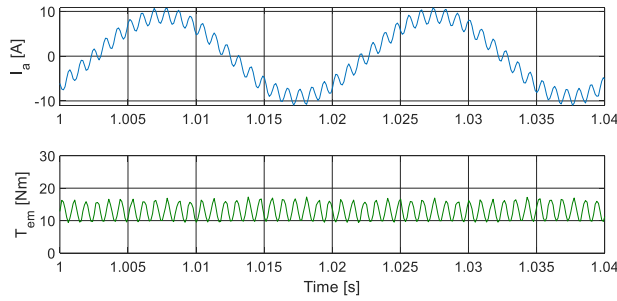


**FIGURE 3.** Stator phase current and electromagnetic torque waveforms in the full-load steady-state condition:  $p=1$ ,  $S=24$ ,  $R=20$ ,  $T_r=13\text{Nm}$ ,  $P_r=4\text{kW}$ .

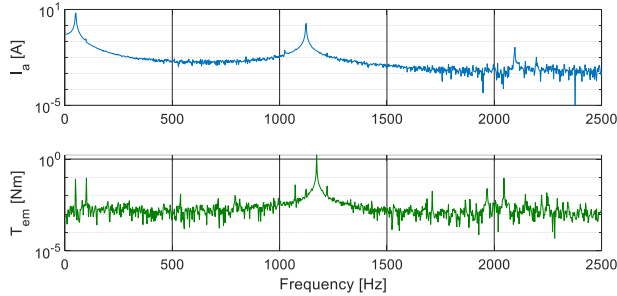


**FIGURE 4.** Stator phase current and electromagnetic torque spectra:  $p=1$ ,  $S=24$ ,  $R=20$ ,  $T_r=13\text{Nm}$ ,  $P_r=4\text{kW}$ ,  $s=2.55\%$ .

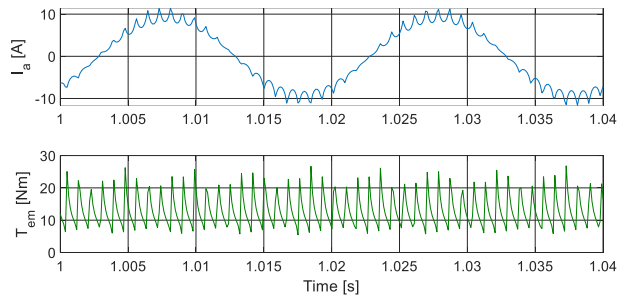
Table I shows the results for torque ripple factor  $r$  obtained from the PWF model for different even numbers of rotor bars belonging to the set given in (14). Torque ripple factor has the maximal value for  $R=S$ , which is in accordance with the well-known common experience.



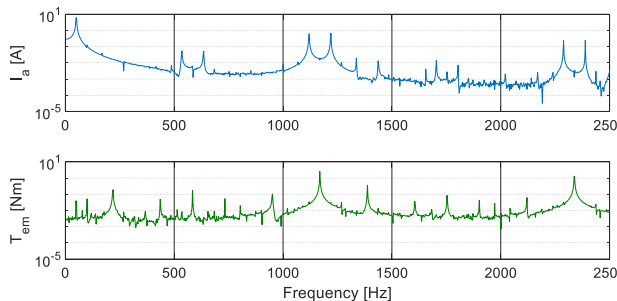
**FIGURE 5.** Stator phase current and electromagnetic torque waveforms in the full-load steady-state condition:  $p=1$ ,  $S=24$ ,  $R=22$ ,  $T_r=13\text{Nm}$ ,  $P=4\text{kW}$ .



**FIGURE 6.** Stator phase current and electromagnetic torque spectra:  $p=1$ ,  $S=24$ ,  $R=22$ ,  $T_r=13\text{Nm}$ ,  $P=4\text{kW}$ ,  $s=2.51\%$ .



**FIGURE 7.** Stator phase current and electromagnetic torque waveforms in the full-load steady-state condition:  $p=1$ ,  $S=24$ ,  $R=12$ ,  $T_r=13\text{Nm}$ ,  $P=4\text{kW}$ .



**FIGURE 8.** Stator phase current and electromagnetic torque spectra:  $p=1$ ,  $S=24$ ,  $R=12$ ,  $T_r=13\text{Nm}$ ,  $P=4\text{kW}$ ,  $s=2.6\%$ .

Moreover, a motor with this slot combination cannot start at all when it is directly connected to the grid. The torque ripple factor for this number of rotor bars was therefore obtained from the PWF model by setting the initial condition for the speed close to the rated motor speed. Rather high values for torque ripple factor are also obtained for the cases when  $R=S \pm 2p$  and when  $R=0.5S$ , which is also in accordance with the common knowledge.

TABLE I

TORQUE RIPPLE FACTOR FOR TWO-POLE ( $p=1$ ) MOTOR WITH  $S=24$  STATOR SLOTS AND DIFFERENT NUMBERS OF EVEN ROTOR BARS

S24R22P1: 4 kW, 400V, Y, 50 Hz, 2925 rpm, $\cos\phi=0.88$ , $\eta=0.88$ , $y/\tau=10/12$ , 8 coils per phase, 17 turns per coil, $J=0.008\text{kgm}^2$ , $R_s=1.054\Omega$ , $L_{\sigma s}=5.44\text{mH}$ , $R_b=69.2\mu\Omega$ , $R_e=1.44\mu\Omega$ , $L_b=223.45\text{nH}$ , $L_e=3.27\text{nH}$ , $D_{\sigma s}=180\text{mm}$ , $D_b=96.2\text{mm}$ , $D_{\sigma r}=95.3\text{mm}$ , $D_{ir}=28\text{mm}$ , $l_{Fe}=121\text{mm}$				
R	12	14	16	18
r (%)	39.08	7.63	16.19	9.95
R	20	22	24	26
r (%)	4.61	18.72	50.99	16.04
R	28	30	32	34
r (%)	3.35	5.58	7.07	4.28

The minimum value for torque ripple factor occurs for  $R=28$  bars and it is considered as an optimal solution with respect to the adopted optimization goal – minimal value of the torque ripple factor in steady-state conditions.

2)  $p=1$ ,  $S=36$

When the number of stator slots is equal to  $S=36$ , the range of numbers of rotor bars of interest is  $R \in [18, 54]$ . In the same manner as before, it can be inferred that there does not exist an even number in this range that does not produce some of the first order RSHs. Therefore, again, in order to find the best possible solution, PWF model should be employed and the torque ripple factor calculated in the full load steady state condition. The results as well as motor parameters are listed in Table II.

The minimal value of the torque ripple factor occurs with  $R=20$  bars.

3)  $p=1$ ,  $S=48$

When the number of stator slots is equal to  $S=48$ , the range of rotor bar numbers of interest is  $R \in [24, 72]$ . In the same manner as before, it can be concluded that there does not exist an even number in this range that does not produce

TABLE II

TORQUE RIPPLE FACTOR FOR TWO-POLE ( $p=1$ ) MOTOR WITH  $S=36$  STATOR SLOTS AND DIFFERENT NUMBERS OF EVEN ROTOR BARS

S36R30P1: 11 kW, 400V, Y, 50 Hz, 2940 rpm, $\cos\phi=0.88$ , $\eta=0.88$ , $y/\tau=15/18$ , 12 coils per phase, 7 turns per coil, $J=0.039\text{ kgm}^2$ , $R_s=0.32\Omega$ , $L_{\sigma s}=2.18\text{mH}$ , $R_b=72.9\mu\Omega$ , $R_e=0.83\mu\Omega$ , $L_b=307.4\text{nH}$ , $L_e=3.1\text{nH}$ $D_{\sigma s}=230\text{mm}$ , $D_b=126.1\text{mm}$ , $D_{or}=124.9\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=158.4\text{mm}$					
<b>R</b>	<b>18</b>	<b>20</b>	<b>22</b>	<b>24</b>	<b>26</b>
$r\%$	30.71	2.51	3.96	12.26	3.80
<b>R</b>	<b>28</b>	<b>30</b>	<b>32</b>	<b>34</b>	<b>36</b>
$r\%$	2.82	4.02	2.80	17.57	36.11
<b>R</b>	<b>38</b>	<b>40</b>	<b>42</b>	<b>44</b>	<b>46</b>
$r\%$	12.67	4.31	4.19	2.85	5.91
<b>R</b>	<b>48</b>	<b>50</b>	<b>52</b>	<b>54</b>	<b>56</b>
$r\%$	4.44	2.76	2.53	8.20	2.85



TABLE III

TORQUE RIPPLE FACTOR FOR TWO-POLE ( $p=1$ ) MOTOR WITH  $S=48$  STATOR SLOTS AND DIFFERENT NUMBERS OF EVEN ROTOR BARS

S48R30P1: 15 kW, 400V, Y, 50 Hz, 2937 rpm, $\cos\phi=0.88$ , $\eta=0.88$ , $y/\tau=20/24$ , 16 coils per phase, 5 turns per coil, $J=0.045 \text{ kgm}^2$ , $R_s=0.245\Omega$ , $L_{os}=1.9\text{mH}$ , $R_b=61.37\mu\Omega$ , $R_e=0.7\mu\Omega$ , $L_b=323.7\text{nH}$ , $L_e=3.19\text{nH}$ , $D_{os}=246\text{mm}$ , $D_b=137.75\text{mm}$ , $D_{or}=136.5\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=173.1\text{mm}$					
<b>R</b>	<b>24</b>	<b>26</b>	<b>28</b>	<b>30</b>	<b>32</b>
<b>r (%)</b>	27.01	2.50	1.42	1.81	10.51
<b>R</b>	<b>34</b>	<b>36</b>	<b>38</b>	<b>40</b>	<b>42</b>
<b>r (%)</b>	2.03	4.71	2.64	1.04	2.76
<b>R</b>	<b>44</b>	<b>46</b>	<b>48</b>	<b>50</b>	<b>52</b>
<b>r (%)</b>	1.60	14.17	27.23	11.14	1.59
<b>R</b>	<b>54</b>	<b>56</b>	<b>58</b>	<b>60</b>	<b>62</b>
<b>r (%)</b>	2.20	1.26	2.40	0.77	1.48
<b>R</b>	<b>64</b>	<b>66</b>	<b>68</b>	<b>70</b>	<b>72</b>
<b>r (%)</b>	2.33	1.17	0.71	1.57	6.00

some of the first order RSHs. Once more, in order to find the best possible solution, the PWF model has to be used and the torque ripple factor calculated in full load steady state condition. The results are given in Table III.

The best solution in terms of the torque ripple factor in steady state conditions is a rotor with  $R=68$  bars. Two other good solutions are rotors with  $R=60$  and  $R=40$  bars.

## B. FOUR-POLE ( $p=2$ ) MOTORS

Four-pole motors are certainly the most common type of induction motors in use and, consequently, manufactured in electrical machines companies. These motors have smaller power factor than their two-pole counterparts, but they are cheaper due to the copper saving that results from the shorter winding overhangs.

An additional advantage is the following: from expression (6) it can be easily inferred that choosing the proper even number of rotor bars results in the elimination of the first order RSHs in the stator current spectrum, regardless of the number of stator slots. This was not possible in the case of two-pole motors. In the following, some of the most common cases are considered.

### 1) $p=2$ , $S=24$

As before, when the stator has  $S=24$  slots, the number of rotor bars of interest is in the range  $R \in [12, 36]$ . In order to avoid existence of RSHs of the first order, i.e. PSRs, for  $\lambda=1$ , it is easy to observe that values of  $R$  should satisfy the following:

$$R \notin \{12, 16, 20, 24, 28, 32, 36\}, \quad (18)$$

Accordingly, the preferred number of rotor bars belongs to the following set:

$$R_{\text{preferred}} \in \{14, 18, 22, 26, 30, 34\}, \quad (19)$$

If one applies more stringent conditions, namely that the number of rotor bars should be such that none of the RSHs of the first and of the second order exist, the following condition is obtained:

$$R \notin \{12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32\}, \quad (20)$$

i.e. the even number of rotor bars leading to no RSHs of the first and the second orders simultaneously does not exist in the defined range of preferred  $R$  values.

In order to verify and illustrate the previous discussion, Figs. 9 and 10 show the steady-state stator current and electromagnetic torque and their spectral content for the machine with  $S=24$  stator slots and  $R=30$  rotor bars in  $p=2$  machine. In accordance with the previous analysis, none of the first order RSHs exist in the spectrum. Otherwise these components should appear at the 675.8 and 775.8 Hz. However, that is not the case with the RSHs of the second order. Both RSHs of the second order exist at 1402 and 1502 Hz, as well as the associated pulsating torque at 1452 Hz.

For the purpose of illustrating the rotor bar skewing effect, Figs. 11 and 12 show steady-state stator current and electromagnetic torque and their spectral content for the same machine. The angle of skewing corresponds to one stator slot pitch,  $\gamma=2\pi/S=2\pi/24$ . The improvement in current and torque waveforms is obvious by comparing Figs. 9 and 11. This fact is additionally underpinned by the comparison of the spectrum content, Figs. 10 and 12. The main idea behind skewing rotor bars by one stator slot pitch is of course the elimination of the stator slot harmonics, as the most prominent higher harmonics in stator flux density wave, from rotor bar currents, [33], [34].

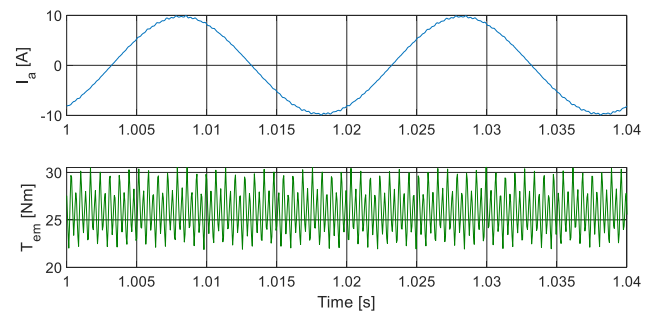


FIGURE 9. Steady-state stator phase current and electromagnetic torque:  $p=2$ ,  $S=24$ ,  $R=30$ ,  $T_f=26\text{Nm}$ ,  $P_f=4\text{kW}$ .

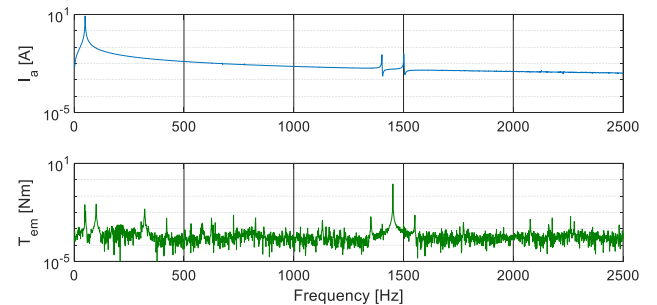
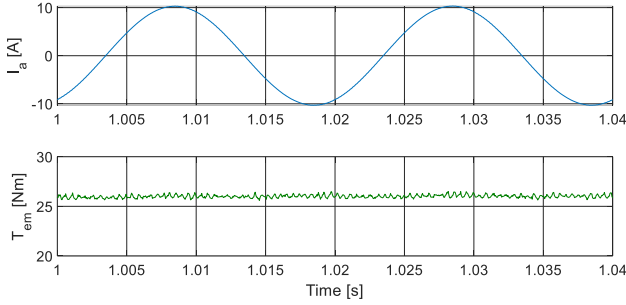
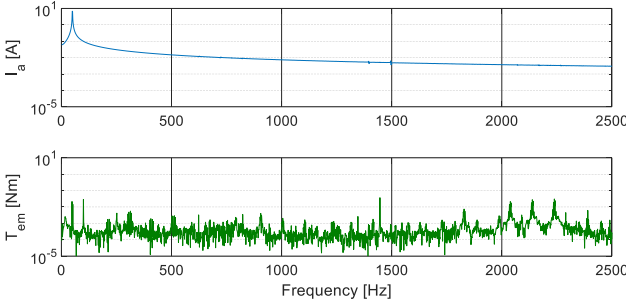


FIGURE 10. Stator phase current and electromagnetic torque spectra:  $p=2$ ,  $S=24$ ,  $R=30$ ,  $T_f=26\text{Nm}$ ,  $P_f=4\text{kW}$ ,  $s=3.23\%$ .



**FIGURE 11.** Steady-state stator phase current and electromagnetic torque - skewed rotor bars:  $p=2$ ,  $S=24$ ,  $R=30$ ,  $T_r=26\text{Nm}$ ,  $P_r=4\text{kW}$ ,  $\gamma=2\pi/S$



**FIGURE 12.** Stator phase current and electromagnetic torque spectra - skewed rotor bars:  $p=2$ ,  $S=24$ ,  $R=30$ ,  $T_r=26\text{Nm}$ ,  $P_r=4\text{kW}$ ,  $s=3.23\%$ ,  $\gamma=2\pi/S$

TABLE IV

TORQUE RIPPLE FACTOR FOR FOUR-POLE ( $p=2$ ) MOTOR WITH  $S=24$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S24R30P2: 4 kW, 400V, Y, 50 Hz, 1452 rpm, $\cos\phi=0.81$ , $\eta=0.84$ , $\gamma/\tau=5/6$ , 8 coils per phase, 23 turns per coil, $J=0.011\text{ kgm}^2$ , $R_s=0.96\Omega$ , $L_{\sigma s}=9.93\text{mH}$ , $R_b=69.82\mu\Omega$ , $R_r=1.56\mu\Omega$ , $L_b=436.85\text{nH}$ , $L_r=3.6\text{nH}$ , $D_{\sigma s}=200\text{mm}$ , $D_b=110.63\text{mm}$ , $D_{or}=110\text{mm}$ , $D_{ir}=28\text{mm}$ , $l_{Fe}=130.3\text{mm}$			
<b>R</b>	<b>14</b>	<b>18</b>	<b>22</b>
$r(\%)$	40.54	19.24	13.07
<b>R</b>	<b>26</b>	<b>30</b>	<b>34</b>
$r(\%)$	11.58	8.28	10.23

Therefore, the optimal solution is one of the numbers of rotor bars that belongs to the set (19) and it can be identified in the same manner as before, by evaluation of the torque ripple factor obtained from the PWF model, Table IV.

Obviously, the optimal solution in the analyzed range is  $R=30$  rotor bars.

## 2) $p=2$ , $S=36$

When stator has  $S=36$  slots, the number of rotor bars of interest is in the range  $R \in [18, 54]$ . In order to avoid existence of RSHs of the first order, i.e. PSHs for  $\lambda=1$ , one must avoid the following numbers of rotor bars,

$$R \notin \{20, 24, 28, 32, 36, 40, 44, 48, 52\}, \quad (21)$$

i.e. the preferred numbers of rotor bars are:

$$R_{\text{preferred}} \in \{22, 26, 30, 34, 38, 42, 46, 50\}, \quad (22)$$

If more stringent conditions are applied, that number of rotor bar should be such that none of the RSHs of the first and of the second order exist simultaneously, it can be easily inferred that such number of rotor bars does not exist in the

defined range. Therefore, the optimal solution is one of the numbers of rotor bars that belongs to the set (22) and it can again be identified by calculating the torque ripple factor, Table V. The optimal solution in the analyzed range is  $R=50$  rotor bars.

## 3) $p=2$ , $S=48$

When the stator has  $S=48$  slots, the number of rotor bars of interest is in the range  $R \in [24, 72]$ . In order to avoid the existence of RSHs of the first order, it is easily determined that the following numbers of rotor bars are undesirable,

$$R \notin \{24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72\}, \quad (23)$$

i.e. that the preferred numbers of rotor bars are:

$$R_{\text{preferred}} \in \{26, 30, 34, 38, 42, 46, 50, 54, 58, 62, 66, 70\}, \quad (24)$$

However, if one applies the more stringent condition that the number of rotor bars should be such that none of the RSHs of the first and of the second order exists simultaneously, it can be inferred that such an even number of rotor bars does not exist in the defined range. Therefore, the optimal solution is one of the numbers of rotor bars that belongs to the set (24) and it is revealed again by the calculation of the torque ripple factor, Table VI. The optimal solution in the analyzed range is  $R=54$  rotor bars.

## 4) $p=2$ , $S=72$

In the case of stator with  $S=72$  slots, the number of rotor bars of interest is in the range  $R \in [36, 108]$ . In order to avoid

TABLE V

TORQUE RIPPLE FACTOR FOR FOUR-POLE ( $p=2$ ) MOTOR WITH  $S=36$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S36R30P2: 11 kW, 400V, Y, 50 Hz, 1464 rpm, $\cos\phi=0.83$ , $\eta=0.91$ , $\gamma/\tau=7/9$ , 12 coils per phase, 9 turns per coil, $J=0.068\text{ kgm}^2$ , $R_s=0.296\Omega$ , $L_{\sigma s}=2.92\text{mH}$ , $R_b=64.49\mu\Omega$ , $R_r=1.55\mu\Omega$ , $L_b=398.58\text{nH}$ , $L_r=5.59\text{nH}$ , $D_{\sigma s}=250\text{mm}$ , $D_b=146.36\text{mm}$ , $D_{or}=145.56\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=172.4\text{mm}$				
<b>R</b>	<b>22</b>	<b>26</b>	<b>30</b>	<b>34</b>
$r(\%)$	2.40	6.46	3.10	8.08
<b>R</b>	<b>38</b>	<b>42</b>	<b>46</b>	<b>50</b>
$r(\%)$	7.65	2.20	2.32	0.98

TABLE VI

TORQUE RIPPLE FACTOR FOR FOUR-POLE ( $p=2$ ) MOTOR WITH  $S=48$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S48R30P2: 11 kW, 400V, Y, 50 Hz, 1462 rpm, $\cos\phi=0.83$ , $\eta=0.91$ , $\gamma/\tau=10/12$ , 16 coils per phase, 7 turns per coil, $J=0.068\text{ kgm}^2$ , $R_s=0.32\Omega$ , $L_{\sigma s}=2.52\text{mH}$ , $R_b=60.64\mu\Omega$ , $R_r=1.44\mu\Omega$ , $L_b=401.88\text{nH}$ , $L_r=5.44\text{nH}$ , $D_{\sigma s}=250\text{mm}$ , $D_b=146.36\text{mm}$ , $D_{or}=145.56\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=172.4\text{mm}$				
<b>R</b>	<b>26</b>	<b>30</b>	<b>34</b>	<b>38</b>
$r(\%)$	14.31	2.17	1.85	1.91
<b>R</b>	<b>42</b>	<b>46</b>	<b>50</b>	<b>54</b>
$r(\%)$	1.85	7.22	5.53	1.18
<b>R</b>	<b>58</b>	<b>62</b>	<b>66</b>	<b>70</b>
$r(\%)$	1.50	1.38	1.47	5.52

existence of RSHs of the first order, it is easy to find that the following numbers of rotor bars are undesirable choices,

$$R \notin \left\{ 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, \right. \\ \left. 76, 80, 84, 88, 92, 96, 100, 104, 108 \right\}, \quad (25)$$

i.e. that the preferred numbers of rotor bars are:

$$R_{\text{preferred}} \in \left\{ 38, 42, 46, 50, 54, 58, 62, 66, 70, \right. \\ \left. 74, 78, 82, 86, 90, 94, 98, 102, 106 \right\}, \quad (26)$$

As before, by applying more stringent conditions, one can conclude that such an even number of rotor bars does not exist in the defined range.

Therefore, the optimal solution is among the numbers of rotor bars that belong to the set (26) and it is identified as before, Table VII. The optimal solution in the analyzed range is  $R=82$  rotor bars.

### C. SIX-POLE ( $p=3$ ) MOTORS

#### 1) $p=3$ , $S=36$

In the case of stator with  $S=36$  stator slots, the number of rotor bars of interest is in the range  $R \in [18, 54]$ . To avoid the existence of RSHs of the first order, the following numbers of rotor bars are to be avoided:

$$R \notin \{18, 24, 30, 36, 42, 48, 54\}. \quad (27)$$

With the more stringent condition, that the number of rotor bars should be such that not a single one of the RSHs of the first and of the second order exists, one obtains the identical set as (27). If one goes further and tries to eliminate the third set of RSHs, it appears that this is not possible for any number of the rotor bars from the set of even numbers between  $R=18$  and  $R=54$ . Therefore, the preferred number of rotor bars belongs to the following set,

$$R_{\text{preferred}} \in \{20, 22, 26, 28, 32, 34, 38, 40, 44, 46, 50, 52\}, \quad (28)$$

and any number of the rotor bars from the previous set guarantees that none of the first and the second order RSHs will appear in the stator current spectrum.

TABLE VII

TORQUE RIPPLE FACTOR FOR FOUR-POLE ( $p=2$ ) MOTOR WITH  $S=72$  STATOR SLOTS AND DIFFERENT EVEN NUMBERS OF ROTOR BARS

S72R50P2: 15 kW, 400V, Y, 50 Hz, 1464 rpm, $\cos\phi=0.84$ , $\eta=0.92$ , $y/\tau=15/18$ , 24 coils per phase, 4 turns per coil, $J=0.084 \text{ kgm}^2$ , $R_s=0.223\Omega$ , $L_{cs}=3\text{mH}$ , $R_b=95.2\mu\Omega$ , $R_e=0.835\mu\Omega$ , $L_b=521.33\text{nH}$ , $L_e=3.6\text{nH}$ , $D_{os}=280\text{mm}$ , $D_{is}=158.68\text{mm}$ , $D_{or}=157.82\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=187\text{mm}$				
<b>R</b>	<b>38</b>	<b>42</b>	<b>46</b>	<b>50</b>
$r(\%)$	9.22	1.63	1.44	1.11
<b>R</b>	<b>54</b>	<b>58</b>	<b>62</b>	<b>66</b>
$r(\%)$	1.31	1.03	1.07	1.17
<b>R</b>	<b>70</b>	<b>74</b>	<b>78</b>	<b>82</b>
$r(\%)$	4.96	2.28	1.78	0.71
<b>R</b>	<b>86</b>	<b>90</b>	<b>94</b>	<b>98</b>
$r(\%)$	1.31	2.69	1.63	0.86
<b>R</b>	<b>102</b>	<b>106</b>	<b>110</b>	<b>114</b>
$r(\%)$	1.34	1.89	1.11	0.82

In order to prove the previous discussion, Fig. 13 shows the stator current and electromagnetic torque spectra for the machine with  $S=36$  stator slots and  $R=40$  rotor bars in a  $p=3$  machine. In accordance with the previous analysis, none of the first and the second order RSHs exist in the spectrum. Otherwise these components should appear at 603.6 Hz, 703.6 Hz, 1257.2 Hz and 1357.2 Hz. However, the same does not apply to the third order RSHs. Upper RSH of the third order is clearly visible in the spectrum at 2011 Hz, as is the corresponding torque pulsation component at frequency that is 50 Hz higher in the torque spectrum.

Hence one concludes that the optimal solution is in the set (28) and it is again determined as before, by evaluating the torque ripple factor obtained using the PWF model of the machine, Table VIII. Obviously, the optimal solution in the analyzed range is either  $R=28$  or  $R=40$  rotor bars; both lead to the same value of the torque ripple factor.

#### 2) $p=3$ , $S=54$

In a machine with  $S=54$  stator slots, the number of rotor bars of interest is in the range  $R \in [28, 80]$ . In order to avoid existence of RSHs of the first order, i.e. PSHs, for  $\lambda=1$ , it is simple to find that the following number of rotor bars are "forbidden":

$$R \notin \{30, 36, 42, 48, 54, 60, 66, 72, 78\}. \quad (29)$$

Using again the condition that the number of rotor bars should be such that none of the RSHs of the first and of the second order exist, one obtains the identical set, (29). Going

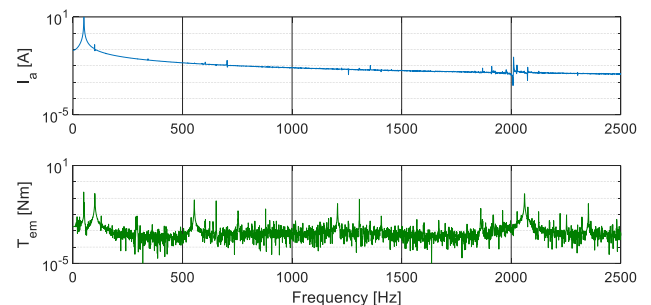


FIGURE 13. Stator phase current and electromagnetic torque spectra:  $p=3$ ,  $S=36$ ,  $R=40$ ,  $T_e=107.2\text{Nm}$ ,  $P=11\text{kW}$ ,  $s=1.96\%$

TABLE VIII

TORQUE RIPPLE FACTOR FOR SIX-POLE ( $p=3$ ) MOTOR WITH  $S=36$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S36R22P3: 11 kW, 400V, Y, 50 Hz, 980 rpm, $\cos\phi=0.77$ , $\eta=0.88$ , $y/\tau=5/6$ , 12 coils per phase, 7 turns per coil, $J=0.12 \text{ kgm}^2$ , $R_s=0.204\Omega$ , $L_{cs}=2.09\text{mH}$ , $R_b=59.5\mu\Omega$ , $R_e=4.69\mu\Omega$ , $L_b=448.6\text{nH}$ , $L_e=13.1\text{nH}$ , $D_{os}=290\text{mm}$ , $D_{is}=184.68\text{mm}$ , $D_{or}=183.9\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=183.7\text{mm}$				
<b>R</b>	<b>20</b>	<b>22</b>	<b>26</b>	<b>28</b>
$r(\%)$	1.90	6.86	6.29	0.86
<b>R</b>	<b>32</b>	<b>34</b>	<b>38</b>	<b>40</b>
$r(\%)$	0.90	3.50	2.99	0.87
<b>R</b>	<b>44</b>	<b>46</b>	<b>50</b>	<b>52</b>
$r(\%)$	1.76	3.02	2.13	0.90



further and trying to eliminate the third set of RSHs, it appears that this cannot be done for any even number of rotor bars from the set between  $R=28$  and  $R=80$ . Therefore, the preferred number of rotor bars belongs to the following set,

$$R_{preferred} \in \left\{ \begin{matrix} 32, 34, 38, 40, 44, 46, 50, 52, \\ 56, 58, 62, 64, 68, 70, 74, 76 \end{matrix} \right\}, \quad (30)$$

and any number of rotor bars from this set guarantees that not a single one of the first and the second order RSHs will appear in the stator current spectrum.

The optimal solution thus belongs to the set (30) and it is identified by evaluating the torque ripple factor by means of the PWF model of the machine, Table IX. The optimal solution in the range of interest is  $R=50$  rotor bars.

$$3) p=3, S=72$$

In a machine with  $S=72$  stator slots, the number of rotor bars of interest is in the range  $R \in [36, 108]$ . In order to avoid existence of the first order RSHs, it can be found that the following numbers of rotor bars are “forbidden”:

$$R \notin \{36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108\}. \quad (31)$$

If the number of rotor bars should be such that none of the RSHs of the first and of the second order exist, the identical set as (31) results. Trying to eliminate the third set of RSHs shows that this is not possible for any number of the rotor bars from the set of even numbers between  $R=36$  and  $R=108$ . Therefore, an optimal number of rotor bars belongs to the following set,

$$R_{preferred} \in \left\{ \begin{matrix} 38, 40, 44, 46, 50, 52, 56, 58, 62, 64, 68, 70, \\ 74, 76, 80, 82, 86, 88, 92, 94, 98, 100, 104, 106 \end{matrix} \right\} \quad (32)$$

and any number of rotor bars in this set guarantees that none of the first and the second order RSHs will appear in the stator current spectrum.

Optimal solution is again identified in the same manner, with the results given in Table X. A few numbers of rotor bars can be observed as the preferred ones:  $R=38, 76, 82$  and  $100$ , as all of them have rather small value of the torque ripple factor. The optimal solution in the range of interest is  $R=38$  rotor bars.

#### D. EIGHT-POLE ( $p=4$ ) MOTORS

$$1) p=4, S=24$$

This kind of machine is rare as number of stator slots per pole per phase is equal to one,  $q=1$ . However, some small-power induction motors can be found with this number of stator slots, as it was the case with small laboratory motor on which experimental results are recorded (in paragraph IV) and that was the reason to cover this case, too.

In this case the number of rotor bars of interest is in the range  $R \in [12, 36]$ . In order to avoid existence of RSHs of the first order, the following numbers of rotor bars are to be avoided:

TABLE IX

TORQUE RIPPLE FACTOR FOR SIX-POLE ( $p=3$ ) MOTOR WITH  $S=54$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S54R36P3: 11 kW, 400V, Y, 50 Hz, 980 rpm, $\cos\phi=0.77$ , $\eta=0.88$ , $y/\tau=7/9$ , 18 coils per phase, 5 turns per coil, $J=0.12 \text{ kgm}^2$ , $R_s=0.213\Omega$ , $L_{\sigma s}=1.56\text{mH}$ , $R_b=94\mu\Omega$ , $R_e=2.86\mu\Omega$ , $L_b=508\text{nH}$ , $L_e=8.16\text{nH}$ $D_{os}=290\text{mm}$ , $D_{is}=184.68\text{mm}$ , $D_{or}=183.9\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=183.7\text{mm}$				
<b>R</b>	<b>32</b>	<b>34</b>	<b>38</b>	<b>40</b>
<i>r</i> (%)	0.80	3.88	3.48	0.80
<b>R</b>	<b>44</b>	<b>46</b>	<b>50</b>	<b>52</b>
<i>r</i> (%)	2.51	1.41	0.74	1.62
<b>R</b>	<b>56</b>	<b>58</b>	<b>62</b>	<b>64</b>
<i>r</i> (%)	1.31	0.89	4.44	1.04
<b>R</b>	<b>68</b>	<b>70</b>	<b>74</b>	<b>76</b>
<i>r</i> (%)	1.48	1.75	1.55	0.87

TABLE X

TORQUE RIPPLE FACTOR FOR SIX-POLE ( $p=3$ ) MOTOR WITH  $S=72$  STATOR SLOTS AND DIFFERENT EVEN NUMBERS OF ROTOR BARS

S72R50P3: 11 kW, 400V, Y, 50 Hz, 979 rpm, $\cos\phi=0.78$ , $\eta=0.88$ , $y/\tau=10/12$ , 24 coils per phase, 4 turns per coil, $J=0.12 \text{ kgm}^2$ , $R_s=0.23\Omega$ , $L_{\sigma s}=1.45\text{mH}$ , $R_b=117\mu\Omega$ , $R_e=1.84\mu\Omega$ , $L_b=614.14\text{nH}$ , $L_e=5.5\text{nH}$ $D_{os}=280\text{mm}$ , $D_{is}=180\text{mm}$ , $D_{or}=179.2\text{mm}$ , $D_{ir}=42\text{mm}$ , $l_{Fe}=179\text{mm}$				
<b>R</b>	<b>38</b>	<b>40</b>	<b>44</b>	<b>46</b>
<i>r</i> (%)	0.85	1.00	2.85	3.41
<b>R</b>	<b>50</b>	<b>52</b>	<b>56</b>	<b>58</b>
<i>r</i> (%)	2.14	1.14	1.43	1.03
<b>R</b>	<b>62</b>	<b>64</b>	<b>68</b>	<b>70</b>
<i>r</i> (%)	3.31	1.15	2.57	1.86
<b>R</b>	<b>74</b>	<b>76</b>	<b>80</b>	<b>82</b>
<i>r</i> (%)	2.13	0.94	1.12	0.92
<b>R</b>	<b>86</b>	<b>88</b>	<b>92</b>	<b>94</b>
<i>r</i> (%)	1.54	3.08	2.75	1.83
<b>R</b>	<b>98</b>	<b>100</b>	<b>104</b>	<b>106</b>
<i>r</i> (%)	1.18	0.94	1.15	2.17

$$R \notin \{16, 24, 32\} \quad (33)$$

If the number of rotor bars is to be such that none of the RSHs of the first and of the second order exist, one obtains the following set:

$$R \notin \{12, 16, 20, 24, 28, 32, 36\} \quad (34)$$

If one goes one step further and tries to eliminate the third set of RSHs, the same set as (34) results. The fourth order RSHs cannot be eliminated for any number of the rotor bars from the set of even numbers between  $R=12$  and  $R=36$ . Therefore, the preferred number of rotor bars belongs to

$$R_{preferred} \in \{14, 18, 22, 26, 30, 34\} \quad (35)$$

and any number of rotor bars from this set guarantees that none of the first three orders of RSHs will appear in the stator current spectrum. The best possible solution among them indicates torque ripple factor values obtained from the PWF model, Table XI.

The number of rotor bars that for a result has the smallest torque ripple factor is  $R=34$ . Other two rather good solutions are cage rotors with  $R=22$  and  $R=26$  rotor bars.

### 2) $p=4, S=48$

In a machine with  $S=48$  stator slots, the number of rotor bars of interest is in the range  $R \in [24, 72]$ . In order to avoid existence of RSHs of the first order, the following numbers of rotor bars are to be avoided:

$$R \notin \{24, 32, 40, 48, 56, 64, 72\} \quad (36)$$

If the number of rotor bars is to be such that none of the RSHs of the first and of the second order exist, one obtains the following set:

$$R \in \{24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72\} \quad (37)$$

Trying to additionally eliminate the third set of RSHs, the same set as (37) results. The fourth order RSHs cannot be eliminated for any number of the rotor bars from the set of even numbers between  $R=24$  and  $R=72$ . Therefore, the preferred number of rotor bars belongs to the following set,

$$R_{preferred} \in \{26, 30, 34, 38, 42, 46, 50, 54, 58, 62, 66, 70\} \quad (38)$$

and any number of rotor bars from this set guarantees that none of the first three orders of RSHs will appear in the stator current spectrum.

In order to prove the previous discussion, Fig. 14 shows the stator current and electromagnetic torque spectra for the machine with  $S=48$  stator slots and  $R=30$  rotor bars in a  $p=4$  machine. In accordance with previous analysis none of the first, the second and the third order RSHs exist in the spectrum. However, that is not the case with the fourth order RSHs. Both fourth order RSHs are clearly visible in the spectrum at 1413 and 1513 Hz, as is the accompanying torque pulsation component at a frequency that is in the middle of these two, at 1463 Hz.

The optimal solution hence belongs to the set (38) and it is once more identified by evaluation of the torque ripple factor, the results being those given in Table XII. The optimal solution in the range of interest is  $R=54$  rotor bars.

### 3) $p=4, S=72$

In a machine with  $S=72$  stator slots, the number of rotor bars of interest is in the range  $R \in [36, 108]$ . In order to avoid existence of RSHs of the first order, the following numbers of rotor bars are eliminated:

$$R \notin \{40, 48, 56, 64, 72, 80, 88, 96, 104\} \quad (39)$$

By requiring that the number of rotor bars ensures that none of the RSHs of the first and of the second order exist, one obtains the following set:

$$R \notin \left\{ \begin{array}{l} 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, \\ 76, 80, 84, 88, 92, 96, 100, 104, 108 \end{array} \right\} \quad (40)$$

Trying to eliminate the third set of RSHs, the same set as (40) results. The fourth order RSHs cannot be eliminated for

TABLE XI  
TORQUE RIPPLE FACTOR FOR EIGHT-POLE ( $P=4$ ) MOTOR WITH  $S=24$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S24R22P4: 1.1 kW, 400V, Y, 50 Hz, 732 rpm, $\cos\phi=0.6$ , $\eta=0.7$ , $y/\tau=2/3$ , 8 coils per phase, 41 turns per coil, $J=0.095 \text{ kgm}^2$ , $R_s=2.8\Omega$ , $L_{\sigma s}=29.7\text{mH}$ , $R_b=82.9\mu\Omega$ , $R_e=8.94\mu\Omega$ , $L_b=195.46\text{nH}$ , $L_e=10.14\text{nH}$ , $D_{\sigma s}=200\text{mm}$ , $D_b=127\text{mm}$ , $D_{or}=126.5\text{mm}$ , $D_{ir}=28\text{mm}$ , $l_{Fe}=124.7\text{mm}$			
<b>R</b>	<b>14</b>	<b>18</b>	<b>22</b>
<b>r (%)</b>	9.96	8.73	3.46
<b>R</b>	<b>26</b>	<b>30</b>	<b>34</b>
<b>r (%)</b>	3.38	4.68	2.40

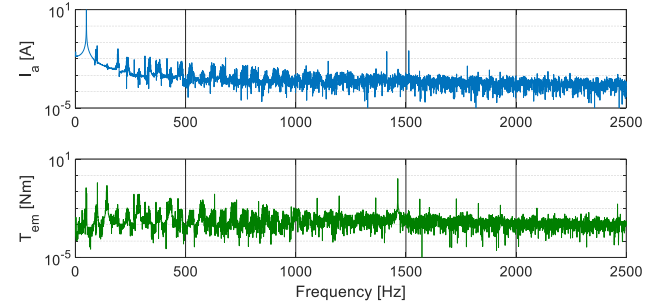


FIGURE 14. Stator phase current and electromagnetic torque spectra:  $p=4, S=48, R=30, T=143\text{Nm}, P=11\text{kW}, s=2.46\%$ .

TABLE XII  
TORQUE RIPPLE FACTOR FOR EIGHT-POLE ( $P=4$ ) MOTOR WITH  $S=48$  STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S48R30P4: 11 kW, 400V, Y, 50 Hz, 732 rpm, $\cos\phi=0.7$ , $\eta=0.9$ , $y/\tau=5/6$ , 16 coils per phase, 6 turns per coil, $J=0.24 \text{ kgm}^2$ , $R_s=0.22\Omega$ , $L_{\sigma s}=2.16\text{mH}$ , $R_b=77.4\mu\Omega$ , $R_e=4.57\mu\Omega$ , $L_b=445.1\text{nH}$ , $L_e=12\text{nH}$ , $D_{\sigma s}=320\text{mm}$ , $D_b=204.38\text{mm}$ , $D_{or}=203.6\text{mm}$ , $D_{ir}=48\text{mm}$ , $l_{Fe}=200.65\text{mm}$				
<b>R</b>	<b>26</b>	<b>30</b>	<b>34</b>	<b>38</b>
<b>r (%)</b>	4.64	0.89	2.74	2.28
<b>R</b>	<b>42</b>	<b>46</b>	<b>50</b>	<b>54</b>
<b>r (%)</b>	1.12	1.56	0.91	0.79
<b>R</b>	<b>58</b>	<b>62</b>	<b>66</b>	<b>70</b>
<b>r (%)</b>	0.84	1.08	1.13	1.49

any number of rotor bars belonging to the set of even numbers between  $R=36$  and  $R=108$ . Therefore, the preferred number of rotor bars belongs to the following set,

$$R_{preferred} \in \left\{ \begin{array}{l} 38, 42, 46, 50, 54, 58, 62, 66, 70, \\ 74, 78, 82, 86, 90, 94, 98, 102, 106 \end{array} \right\} \quad (41)$$

and any number of rotor bars from this set guarantees that none of the first three orders of RSHs will appear in the stator current spectrum.

The optimal solution belongs to the set (41). Torque ripple factor is used again as a measure of goodness and the values are given in Table XIII. The optimal solution in the range of interest is  $R=50$  rotor bars.

## IV. EXPERIMENTAL RESULTS

Although the analytically predicted results are in accordance with the results from the mathematical model, in

TABLE XIII

TORQUE RIPPLE FACTOR FOR EIGHT-POLE (P=4) MOTOR WITH S=72 STATOR SLOTS AND PREFERRED EVEN NUMBERS OF ROTOR BARS

S72R50P4: 11 kW, 400V, Y, 50 Hz, 733 rpm, $\cos\phi=0.7$ , $\eta=0.9$ , $y/\tau=7/9$ , 24 coils per phase, 4 turns per coil, $J=0.24 \text{ kgm}^2$ , $R_s=0.216\Omega$ , $L_{\sigma s}=1.55\text{mH}$ , $R_b=133.44\mu\Omega$ , $R_e=2.89\mu\Omega$ , $L_b=633\text{nH}$ , $L_e=7.26\text{nH}$ $D_{os}=320\text{mm}$ , $D_{is}=204.38\text{mm}$ , $D_{or}=203.6\text{mm}$ , $D_r=48\text{mm}$ , $l_{Fe}=200.65\text{mm}$				
R	38	42	46	50
r (%)	2.01	1.17	1.27	0.95
R	54	58	62	66
r (%)	1.28	1.18	1.69	1.85
R	70	74	78	82
r (%)	1.96	1.80	2.09	1.22
R	86	90	94	98
r (%)	1.57	2.48	1.74	1.46
R	102	106		
r (%)	3.38	1.81		

order to fully validate the results, four experiments were performed on four different three-phase cage induction motors. What distinguishes them is the fact that they have different numbers of pole pairs.

Fig. 15 shows recorded stator phase current spectrum of a two-pole cage induction motor whose data are: 30 kW, 400V, 53 A,  $\Delta$ , 50 Hz,  $\cos\phi = 0.87$ ,  $n_r = 2955 \text{ rpm}$ ,  $S = 36$ ,  $R = 22$ . As analytically predicted, this motor has upper PSH of the first order at frequency:

$$f_{1,upper}^{RSH} = (1 + 22(1 - 0.0158)) \cdot 50 \cong 1133 \text{ Hz} \quad (42)$$

This harmonic component is easily observable in Fig. 15 as one of the most prominent harmonics in the higher frequency part of the spectrum.

Fig. 16 shows recorded stator phase current spectrum of a four-pole laboratory motor whose rated data are: 3 kW, 380V, 6.8 A, Y, 50 Hz,  $\cos\phi = 0.81$ ,  $n_r = 1415 \text{ rpm}$ . The motor has  $S = 36$  stator slots and  $R = 32$  rotor bars. Motor was lightly overloaded during the experiment.

As analytically predicted, this motor has upper RSHs of the first order at the following frequency, for slip  $s=6.62\%$ :

$$f_{1,upper}^{RSH} = \left(1 + \frac{32}{2}(1 - 0.0662)\right) \cdot 50 \cong 797 \text{ Hz} \quad (43)$$

This stator current component is the most prominent in the higher frequency part of the spectrum in Fig. 16.

Fig. 17 shows recorded stator phase current spectrum of a small six-pole laboratory motor whose data are: 0.75 kW, 380V, 2.2A, Y, 50 Hz,  $\cos\phi=0.73$ ,  $n_r = 940\text{rpm}$ ,  $S=36$ ,  $R=33$ . This was one of the rather unusual examples of a motor with an odd number of rotor bars. Such motor develops upper RSH of the second order at following frequency:

$$f_{II,upper}^{RSH} = \left(1 + 2 \frac{33}{3}(1 - 0.09)\right) \cdot 50 = 1051 \text{ Hz} \quad (44)$$

This stator current component can be observed in the spectrum in Fig. 17.

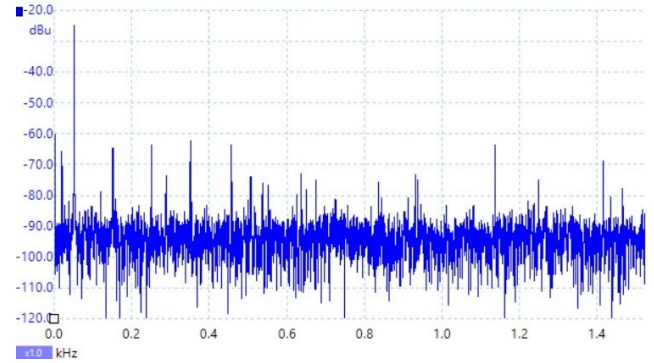


FIGURE 15. Stator phase current spectrum:  $P_r=30\text{kW}$ ,  $p=1$ ,  $S=36$ ,  $R=22$ ,  $s=1.58\%$ .

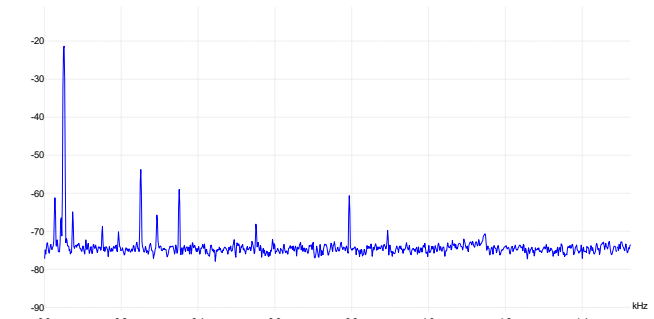


FIGURE 16. Stator phase current spectrum:  $P_r=3 \text{ kW}$ ,  $p=2$ ,  $S=36$ ,  $R=32$ ,  $s=6.62\%$ .

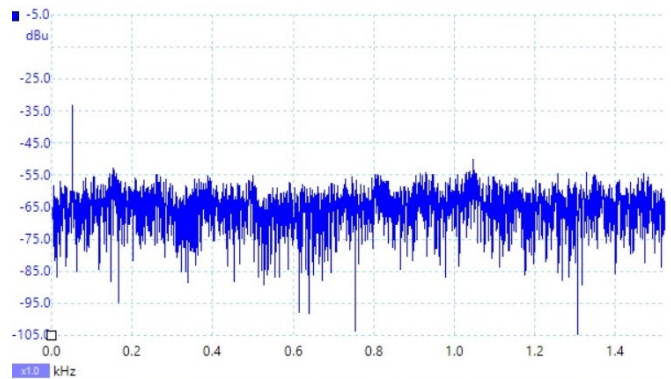


FIGURE 17. Experimentally recorded stator phase current spectrum:  $P_r=0.75\text{kW}$ ,  $p=3$ ,  $S=36$ ,  $R=33$ ,  $s=8.7\%$ .

Fig. 18 shows recorded stator phase current spectrum of a small eight-pole laboratory motor whose data are: 0.25kW, 380V, 1.6A, Y, 50Hz,  $\cos\phi = 0.5$ ,  $n_r = 685 \text{ rpm}$ ,  $S=24$ ,  $R=22$ . Motor was fully loaded during the experiment. As analytically predicted, this motor does not have any of RSHs of the first three orders, (35). The first RHS that can exist in the stator current spectrum is the fourth order upper RSH at frequency:

$$f_{IV,upper}^{RSH} = \left(1 + 4 \frac{22}{4}(1 - 0.0867)\right) \cdot 50 \cong 1055 \text{ Hz} \quad (45)$$

However, this frequency component cannot be observed in the spectrum. The main reason is the fact that at this rather high frequency stator phase leakage reactance, that is already

of high value at fundamental frequency in such small machines, is rather high so stator current component at this frequency is significantly attenuated.

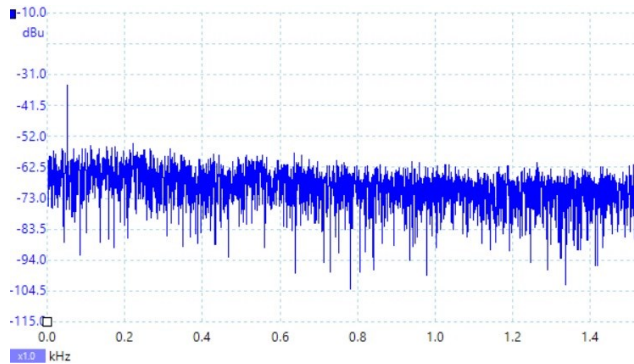


FIGURE 18. Stator phase current spectrum:  $P=0.25\text{kW}$ ,  $p=4$ ,  $S=24$ ,  $R=22$ ,  $s=8.67\%$ .

## V. DISCUSSION

The authors are aware that the paper may appear as tedious to follow. This is due to the nature of the problem and also due to the authors' intention to cover all the cases of the numbers of pole pairs and numbers of stator slots occurring in practice. Hence the paper provides in one place, concisely, unambiguous results – the preferred numbers of rotor bars for each analyzed case and, among them, the optimal number of rotor bars in terms of minimization of the electromagnetic torque ripple in steady-state conditions – i.e. elimination of the RSHs in stator current spectrum and associated high frequency pulsating torques. The results are summarized in Table XIV. This should result in minimizing additional Joule losses and enable electrical motor manufacturers, who are faced with ever-increase NVH requirements on a daily basis, to design better motors.

By comparing Table XIV with a similar table given in [5] and reproduced here for convenience (Table XV; skewed rotor bars only), it can be concluded that some of the numbers of rotor bars identified in this paper can be found in [3] but most of them do not appear in Table XV. Table XV from [3] also gives some odd numbers of rotor bars as preferred.

One of the main contributions of this paper is showing that the degree of freedom in choosing the number of rotor bars that leads to the elimination of the RSHs in the stator current spectrum increases with an increase in the number of pole pairs. It can be said that the degree of freedom in three phase machines is  $p-1$ : in a two-pole machine ( $p=1$ ) none of the even numbers of rotor bars leads to elimination of the first order RSHs; in a four-pole machine ( $p=2$ ) degree of freedom is one, i.e. there are some numbers of rotor bars that lead to the elimination of the first order RSHs but not those of the higher orders; in a six-pole machine ( $p=3$ ) the degree of freedom is equal to two – there are some numbers of rotor bars that lead to the elimination of the first and the second

order RSHs but not those of the higher orders; in an eight-pole machine ( $p=4$ ) degree of freedom is equal to three – there are some numbers of rotor bars that lead to the elimination of the first, the second and the third order RSHs simultaneously, but not those of the higher orders.

Another valuable information is the following: in the general rule (6) a number appears that is equal to the product of the number of phases and the number of pole pairs. It therefore follows that multiphase induction machines, i.e. machines with a number of phases greater than three, have higher degree of freedom for the same number of pole pairs than their three-phase counterparts. This will be illustrated by the following example, already analyzed in [25] but given here with a more details. A five phase ( $m=5$ ), four-pole ( $p=2$ ) machine with  $S=40$  stator slots is considered. In order for the first order RSHs to disappear from the stator phase currents, the number of rotor bars must not belong to the following set:

$$R \notin \{20, 24, 36, 40, 44, 56, 60\} \quad (46)$$

TABLE XIV

PREFERRED AND OPTIMAL NUMBER OF STRAIGHT ROTOR BARS AS A FUNCTION OF THE NUMBER OF STATOR SLOTS AND NUMBER OF POLE PAIRS

	$S$	$R_{\text{preferred}}$
$p=1$	24	20, <b>28</b> , 30, 34
	36	<b>20</b> , 28, 32, 44, 50, 52, 56
	48	28, 30, 40, 44, 52, 56, 60, 62, 66, <b>68</b>
$p=2$	24	26, <b>30</b> , 34
	36	22, 30, 42, 46, <b>50</b>
	48	30, 34, 38, 42, <b>54</b> , 58, 62, 66
	72	50, 58, 62, <b>82</b> , 98
$p=3$	36	<b>28</b> , 32, <b>40</b> , 52
	54	32, 40, <b>50</b> , 58, 64, 76
	72	<b>38</b> , 40, 58, 64, 76, 80, 82, 100
$p=4$	24	22, 26, <b>34</b>
	48	30, 50, <b>54</b> , 58
	72	42, <b>50</b> , 58, 82

TABLE XV

PREFERRED NUMBER OF SKEWED ROTOR BARS AS A FUNCTION OF THE NUMBER OF STATOR SLOTS AND THE NUMBER OF POLE PAIRS [3]

	$S$	$R_{\text{preferred}}$
$p=1$	24	18, 20, 22, 28, 30, 33, 34
	36	25, 27, 28, 29, 30, 43
	48	30, 37, 39, 40, 41
$p=2$	24	16, 18, 20, 30, 33, 34, 35, 36
	36	28, 30, 32, 34, 45, 48
	48	36, 40, 44, 57, 59
	72	42, 48, 54, 56, 60, 61, 62, 68, 76
$p=3$	36	20, 22, 28, 44, 47, 49
	54	34, 36, 38, 40, 44, 46
	72	44, 46, 50, 60, 61, 62, 82, 83
$p=4$	48	26, 30, 34, 35, 36, 38, 58
	72	42, 46, 48, 50, 52, 56, 60



If the first and the second order RSHs are to be eliminated from the stator phase currents, the number of rotor bars must not belong to the following set:

$$R \notin \left\{ \begin{array}{l} 20, 22, 24, 28, 30, 32, 36, 38, 40, \\ 42, 44, 48, 50, 52, 56, 58, 60 \end{array} \right\} \quad (47)$$

To eliminate the first, the second and the third order RSHs from the stator phase currents, the same result as in (47) is obtained. It can be shown that even number of rotor bars in the earlier defined range, (5), does not exist if the goal is to eliminate the first four orders of RSHs.

Therefore, in a five-phase four-pole machine there are three degrees of freedom, in comparison with one degree of freedom in the three-phase machine. This fact can be seen as an additional advantage of the use of multiphase induction machines. Hence, the preferred number of rotor bars that results in elimination of the first three orders of RSHs in the analyzed example is,

$$R_{\text{preferred}} = \{26, 34, 46, 54\} \quad (48)$$

This is in accordance with results already presented in [25].

As a final remark, it is emphasized that the design optimization of an induction motor (and any other electric machine) is intrinsically a multi-objective constrained problem in which several aspects need to be taken into account to fully relate the machine geometry and its performance. This work does not claim to propose a complete optimization approach, but intends to provide the designer with a set of slot combinations which are favorable in terms of minimizing current and torque ripples resulting from RSHs. This can be helpful as it may reduce the range of the design configurations to be considered and compared in the search for an optimum. It is obvious that the designer is expected to select the most appropriate slot combination (presumably among those indicated as preferred in the paper) also considering other aspects (targets and constraints), which pertain to both motor performance and manufacturing.

## VI. CONCLUSION

This paper addresses the problem of determining the optimal number of rotor bars  $R$  of a three-phase induction motor to cancel current and torque ripples related to the RSHs of different orders. For this purpose, the general rule, derived in a previous work for multiphase cage induction motors, is applied. The main finding of the work is that the possibilities to find the optimal number of rotor bars leading to elimination of the RSH-related pulsations increase as the number of poles increases. In other words, it has been shown that as the number of poles increases, it is possible for the designer to select values of  $R$  that eliminate the RSHs of increasing order and, therefore, lead to better torque and current waveforms. The result has been proven by considering the cases of 2-, 4-, 6- and 8-pole motors equipped with the number of stator slots leading to integer

slot windings. All the cases are analyzed using the PWF model to simulate the motor performance at steady-state. For low-pole-count machines, it has been shown that some RSH-related current and torque pulsations cannot be cancelled through an appropriate selection of the number of rotor bars  $R$ . In this case, the PWF model has proved to be an effective tool to numerically compare the motor performance for different choices of  $R$  in order to identify the value or values which leads to the lowest torque ripple amplitudes.

Finally, some consideration has also been given to multiphase motors. It has been shown that, for a given number of pole pairs, the higher the number of phases, the more possibilities the designer has to choose  $R$  such that RSH-related pulsations are cancelled. In other words, for any given number of poles, the higher the phase number the higher the RSH order whose effect can be cancelled through an optimal selection of  $R$ .

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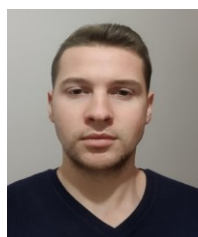


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