Optimal Currency Composition for China's Foreign Reserves: a Copula Approach

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This paper investigates the optimal currency composition for a country's foreign reserves. In the context of China, we examine the asymmetry fat-tails and complex dependence structure in distributions of currency returns. A skewed, fat-tailed, and pair-copula construction is then built to capture features of higher moments. In a D-vine copula approach, we show that under the disappointment aversion effect, the central bank in our model can achieve sizeable gains in expected economic value from switching from the mean-variance to copula modelling. We find that this approach will lead to an optimal currency composition that allows China to have more space for international currency diversification while maintaining the leading position of the US dollar in the currency shares of China's reserves.

Key words: Foreign reserves; Pair copula construction; Currency structure; International diversification

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1. INTRODUCTION

Management of foreign reserves has been a constant concern for central banks (Nugee, 2000). In the wake of the rapid accumulation of reserves that has taken place since the start of the Asian crisis in the late 1990s, the challenge has become even more acute. According to International Monetary Fund, the amount of global foreign reserves grew from around 2 trillion US dollars in 1999 to more than 10 trillion dollars by the end of 2012, while during the same period, international monetary relations underwent fundamental changes. In a time of the global financial crisis, interest rates of main reserve assets are approaching zero, resulting in a low yield environment for central banks' investment of their foreign reserves. On the domestic front, central banks typically sterilise the accumulation of foreign reserves by issuing domestic debt. The difference between the returns on investment of external assets and the cost of issuing domestic debt represents the social cost of holding reserves, which increases with interest spreads and the size of reserve holdings. If the interest rate on reserve assets is lower than the domestic interest rate, holding reserves incurs quasi-fiscal costs (Dominguez *et al.*, 2012). In an environment of low international yield and with rising levels of reserves, this social cost could be substantial (Walther, 2012).

To compound the situation, the value of the dollar fluctuated widely during the period, with a largely downward trend, so eroding the purchasing power of nations' reserve stocks. The euro, once a promising contender to the dollar (Chinn and Frankel, 2006, 2008), had to fight for its survival in the shadow of the eurozone crisis. The crisis also plunged the world economy into its worst recession since the GreatDepression. In the circumstances, sound and prudent management of foreign reserves has become all the more critical, especially for large reserve holders such as China (Ryan, 2009).

Reserve management involves determination of two essential aspects, i.e. the desired amount and the form of reserve assets a country should hold (Roger, 1993). For larger reserve holders, recent research indicates that the appropriate reserve composition is more critical than the reserve level (Beck and Weber, 2011). Following this insight, the current study concentrates on how to derive the optimalcurrency composition for China while taking the reserve level as exogenously given. As the world's largest reserve holder, China reportedly holds as much as 70% of its total reserves in US dollars. This exposes China to great currency risk. Consequently, it is desirable and necessary for China to hedge against the currency exposure by diversifying the currencies denominating the reserve assets.

Existing literature of reserve management offers two conventional approaches to analysing currency composition, i.e. the mean-variance approach and the transactions approach (Roger, 1993). In the mean-variance approach, the central bank is treated as an investor who is concerned only about the risk and returns on investment of reserves, and the returns are measured in terms of a basket of currencies or commodities. The analyst has to find the currency share (weight) that can maximise the value of the investment portfolio for any given level of risk. The transaction approach argues that the central bank should seek to optimise the currency composition of the net foreign assets rather than of gross foreign reserves, which can be achieved by manipulating the structure of gross assets, gross liabilities or both (Dooley, 1986). While this means that the currency composition can be optimised on the side of either assets or liabilities, Dooley suggests that more considerations should be given to transaction cost on the assets side and to mean-variance on the liabilities side. In a subsequent empirical investigation, Dooley *et al.* (1989) identify some key determinants of the transaction considerations, such as a currency's usage in international trade and financial transactions, the exchange rate regime, and country size.

While it certainly makes sense to optimise reserves on the assets side while taking into account the known foreign exchange liabilities, as suggested by the transactions approach, it is difficult for academic researchers to have access to detailed data on central banks' foreign assets and liabilities, which makes meaningful research in this approach virtually impossible. In contrast, the mean-variance analysis can be conducted using data in the public domain and computationally it is rather tractable. This may partly explain the ready application of the mean-variance approach to analysing optimal currency composition of reserves (Ben-Bassat, 1980; Rikkonen, 1989; Dellas and Yoo, 1991; Murray *et al.*, 1991; Petursson, 1995; Levy and Levy, 1998; and Papaioannou *et al.*, 2006).

However, the mean-variance approach has its weaknesses as a tool for analysing wealth diversification. The essence of the approach assumes that investors maximise the expected returns for a given level of risk. Asset returns are fat-tailed, and variance is not sufficient as a measure of risk if investor preferences are not mean-variance or returns are not normally distributed (Bouye, *et al.*, 2000). Furthermore, it is well known that financial risks are often correlated in a non-Gaussian way (Clemen and Reilly, 1999; Embrechts *et al.*, 1999; Ane and Kharoubi, 2003).

Recent research has highlighted in particular the inadequacy of this approach to take account of influences of asymmetries in individual distributions and in dependence, occurrence of extreme events and the complexity in the dependence structure of asset returns as documented in papers such as Ait-Sahalia and Brandt (2001), Longin and Solnik (2001), Ang and Chen (2002), Bae *et al.* (2003), Hong *et al.* (2007) and Ammann and Suss (2009). These effects can fundamentally affect portfolio performance and the corresponding investment decision. Campbell *et al.* (2001) show that the portfolio efficient frontier is altered by the non-normal marginal distribution.

It turns out that the fundamental difficulties with the mean-variance approach, i.e. the Gaussian assumption and the joint distribution modelling, can be treated as a copula problem. A copula is a function that links univariate marginals to their multivariate distribution. Since the seminal work of Embrechts *et al.* (1999), copulas have found increasing applications in financial research. In the field of portfolio management, copulas have also been applied to modelling multivariate distributions in problems of portfolio optimization (Hennessy and Lapan, 2002; Thorp and Milunovich, 2005; Hong *et al.*, 2007; Natale, 2008; Christoffersen and Langlois, 2011; Garcia and Tsafack, 2011).

Patton (2006) applies the copula function to highlight construction of foreign currency portfolios. Hurd *et al.* (2007) provide a copula-based study of the bilateral exchange rate between the euro-sterling and the dollar-sterling exchange rates. Dias and Embrechts (2010) model exchange rate dependence dynamics at different time horizons in a time-varying setting. Wang *et al.* (2010) estimate risk of foreign exchange portfolio using models including the copula framework. Kitamura (2011) applies the copula approach to investigate the impact of order flow on foreign exchange market.

Despite the fact that the copula literature is large and growing, the great part of the research involves only bivariate modelling and construction of higher dimensional copulas is rather limited (Genest *et al.*, 2009). To extend bivariate copulas to higher dimensions, Joe (1997), Bedford and Cooke (2001, 2002), and Kurowicka and Cooke (2006) have proposed the pair-copula decomposition approach. Aas *et al.* (2009) illustrate how multivariate data with complex patterns of dependence in the tails can be modelled using a cascade of pair-copulas acting on two variables at a time and show that the pair-copula approach is a flexible and intuitive way of extending bivariate copulas to higher dimensions.

This study contributes to the reserve management literature by applying the copular approach that models asymmetric, fat-tail, and multiple dependence to the currency composition of foreign reserves in the context of China. The pair-copula construction method is applied for modelling the dependence structure among international currencies. Specializing in modelling multivariate cases, the pair-copulas are based on a decomposition of higher-dimensional copula densities into bivariate ones, of which some are conditional and unconditional functions of modelled variables (Aas and Berg, 2011).

In conventional extension of a bivariate Archimedean copula to a multivariate case, the dependence parameters will not increase with the number of variables, hence one would end up with an over-simplified dependence structure.As suggested in Demarta and McNeil (2005) the group t copula does not suffer from this inability to increase parameters, it does lack the ability of an Archimedean copula to model asymmetric dependence. This is particularly problematic for currency returns since their modelling requires flexibility in both the high dimensional situation s and in complex dependence features such as asymmetries. The pair copula construction method overcomes this problem by composing multiple variables through layers of bivariate copulas, each with its own different dependence parameters. As such, the pair copula construction represents an efficient technique that allows the construction of flexible and accessible multivariate copula extensions for optimal portfolio formation and quantitative risk management.

Based on their importance in China's trade and financial transactions, twelve currencies are chosen in this research as the possible candidates for the optimal currency composition of China's foreign exchange reserves. With this selection, we form the optimal portfolio based on the pair copula construction, the performance of which is then compared with the outcome obtained under a Gaussian copula approach. Using the performance measure of expected economic value of switching to the vine copula, the pair copula method shows clear advantages. The dominance of the copula method is also manifested under *ad hoc* weight constraints to reflect some common transaction motives, i.e. the international trade needs and foreign financing needs. Taking into account asymmetry, fat-tail and complex dependence, the pair copular approach suggests that China should hold a smaller proportion of US dollars than conventionally thought, around 40% of the total reserves for 2001-2009, the period under examination. The remainder of the article is arranged as follows. Section 2 discusses the methodology of how to build asymmetry marginals and the fat-tailed dependence structure. In addition, we specify a utility function that incorporates disappointment aversion as in Gul (1991), Ang *et al.* (2005) and Hong *et al.* (2007), which enables the portfolio optimization on non-Gaussian distribution. Data analysis and model results are presented in section 3, and we conclude in section 4.

2. METHODOLOGY

a. Distribution building

Two steps are involved in building the multivariate distribution using copulas. The first is to build the single variable distribution for each return series and the second is to build the dependence by copula for joining the separate return distributions together.

A copula function $C(u_1, u_2)$ can be defined in the following way: Let $H(x, y)$ be the joint distribution with margins $X \sim F(x), Y \sim G(y)$, and use "probability integral transforms" to denote $U_1 = F(X), U_2 = F(Y)$. Hence we have equation 1:

$$
\begin{aligned}
H(x, y) &= P(X \leq x, Y \leq y) \\
&= P(F(X) \leq F(x), G(Y) \leq G(y)) \\
&= P(U_1 \leq F(x), U_2 \leq G(y)) \\
&= C(u_1, u_2) \qquad (1)
\end{aligned}
$$

According to Sklar's (1959) theorem, if the margin density functions (d.f.s) and the joint d.f. are continuous, the copula C will be unique. The joint distribution building is simply the reverse of this process. We select models for the single return distribution and the copula for dependence.

Distribution of each return series

For univariate return series, Hansen's skewed Student-t distribution is considered as an option for modelling the residuals from some conditional mean and conditional variance models. This is to reflect the asymmetry features of each currency's returns. The density function of the skewed Student-t distribution is defined by:

$$d(z;\eta,\lambda)=\begin{cases} bc(1+\frac{1}{\eta-2}(\frac{bz+a}{1-\lambda})^2)^{-[(\eta+1)/2]} & if\ z<-a/b \\ bc(1+\frac{1}{\eta-2}(\frac{bz+a}{1+\lambda})^2)^{-[(\eta+1)/2]} & if\ z\geq -a/b \end{cases} \quad (2)$$

where

$$a\equiv 4\lambda c\frac{\eta-2}{\eta-1}, b^2\equiv 1+3\lambda^2-a^2, c\equiv \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})} \quad (3)$$

and η and λ denote the degree-of-freedom parameter and the asymmetry parameter of the distribution. We write $Z\sim ST\ (\eta,\lambda)$, if a random variable Z has the density $d(z;\eta,\lambda)$. Similarly $Z\sim T(\lambda)$ denotes a random variable following a standardised t distribution and $Z\sim N$ means that it follows a standardised normal distribution. The Student t distribution and Gaussian distribution are also deployed to model the residuals.

The conditional mean model of ARMA (u, v) is employed with (u, v) ranging from 0 up to 3 lags. For modelling the conditional volatility, GARCH (p, q) and APARCH (p, q) are used with (p, q) ranging from 0 to 3 are to fit the currency data.

The Akaike information criterion (AIC) is used to determine the lag length, the choice between the GARCH and APARCH volatility model, and the type of residual distribution for the best fit. We have 12 currencies for 9 years' horizon and this method provides a wide range to find the best fit model for each individual currency return. Specifically, we have:

$$r_t = c_0 + \sum_{i=1}^{u} ar_i r_{t-i} + \sum_{j=1}^{v} ma_j \varepsilon_{t-j} + \varepsilon_t, \quad (4)$$

$$\varepsilon_t = \sigma_t z_t, \quad (5)$$

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{p} \alpha_i {\varepsilon_{t-i}}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}, \quad (6)$$

$$\sigma_t^{\delta} = \omega_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}, \quad (7)$$

$$z_t \sim ST(\eta_t, \lambda_t) \quad (8)$$

$$z_t \sim T(\lambda_t) \quad (9)$$

$$z_t \sim N \quad (10)$$

where equation (6) is the GARCH specification, (7) is the APARCH model, and equations (8), (9) and (10) are three types of residual distribution, i.e. the Skewed t, t and Gaussian distribution, respectively.

After the initial estimation, we save the standard residual terms, z_t, which are to be plugged into the copula model in the next step for estimating parameters of the dependence structure.

Pair-copula construction for dependence structure

A brief introduction to the pair copula construction à la Bedford and Cooke (2002) is presented here. Consider a random vector $X = (X_1, \dots, X_n)$ with a joint density function of $f(x_1, \dots, x_n)$. The pair copula decomposition is a result of the combined application of conditional density equation and the density form of Sklar's theorem, as in the following:

$$f(a,b) = f(a|b) \cdot f(b) \tag{11}$$

$$f(a,b) = c(F(a), F(b)) \cdot f(a) \cdot f(b) \tag{12}$$

By applying the conditional density equation, the joint density function $f(x_1, \dots, x_n)$ can be expressed as:

$$f(x_1, \dots, x_n) = f(x_n) \cdot f(x_{n-1}|x_n) \cdot f(x_{n-2}|x_{n-1}, x_n) \cdots f(x_1|x_2, \dots, x_n) \tag{13}$$

The order of the variables is changeable. By applying the density form of Sklar's theorem, each factor on the right-hand side of the above equation can be decomposed into a product of several conditional pair-copulas and an unconditional marginal density function as shown below:

$$f(x_1|x_2, x_3) = \frac{c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2) \cdot f(x_3|x_2)}{f(x_3|x_2)}$$

$$= c_{13|2}[F(x_1|x_2), F(x_3|x_2)] \cdot f(x_1|x_2) \tag{14}$$

where $f(x_1|x_2)$ can be further decomposed using the same method, so:

$$f(x_1|x_2,x_3) = c_{13|2}[F(x_1|x_2),F(x_3|x_2)] \cdot c_{12}[F(x_1),F(x_2)] \cdot f(x_1) \quad (15)$$

[Insert Fig. 1 around here]

The choices of the pair variables of the copulas are also changeable. These various types are organised by the "vines" structure. Typical examples are the "C-vine" (canonical vine) and the "D-vine" (Kurowicka and Cooke, 2006). The main difference between them is that the C-vine places more emphasis on a pivotal variable as a root to connect other variables, whereas the D-vine states parallel relationship among variables. Fig. 1 demonstrates the comparison between the two structures in a 5-variables case. The n-dimensional density functions of the D-vine and C-vine decomposition are given by equations (16) and (17), respectively:

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\dots,i+j-1}\left\{F\left(x_i|x_{i+1},\dots,x_{i+j-1}\right),F\left(x_{i+j}|x_{i+1},\dots,x_{i+j-1}\right)\right\} \quad (16)$$

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\left\{F\left(x_j|x_1,\dots,x_{j-1}\right),F\left(x_{j+i}|x_1,\dots,x_{i+j-1}\right)\right\} \quad (17)$$

The likelihood function can be calculated using the same formulae as above, after the sample for x_k is decided, i.e. the standardised residuals from the GARCH estimation and the type of pair-copulas are determined.

In total, we have 12 currencies as candidates for the optimal currency portfolio. The sample time period spans for 9 years. To determine the best fit type of copula for each pair of variables on the vine nodes, we offer a range of 31 copulas which is wide enough to capture the complex dependence between the 12 currencies. For different layers of pair copula, we use 10 different copulas specifically the Gaussian, Student t, Clayton, Gumbel, Frank, Joe, Clayton-Gumbel, Joe-Gumbel, Joe-Clayton, and Joe-Frank copulas. Of these 10 copulas, 7 have their variants that are rotated 180 degrees, 90 degrees, and 270 degrees, making a total of 31 copulas. The copulas without variants are the Gaussian, Student-t and Frank. This setting allows the Archimedean copulas to capture any asymmetric dependence between upper and lower tails, and enables the rotated copulas to capture similar features in the second and third quarters of the dependence. This will be further illustrated later when analysing the currency returns data. The estimation is carried out by maximizing the pseudo-likelihood. The algorithms are based on modification of Aas *et al.* (2009) and the package ‘CDVine’ in R.[1]

The distribution building is finalised by combining the univariate returns and the copula dependence model. Monte Carlo simulations are conducted to generate each distribution containing 500,000 observations.[2] In generating the return distribution, GARCH forecasts for the portfolio management period, assumed in

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[1] The algorithm that the authors compiled can be obtained upon request.

[2] 1-million-sample-distribution is tried at some time points, showing no significant differences.

this study to be 1 year until next adjustment of compositions, are required and the average of these forecasts is incorporated in the return distribution.

To compare with the pair-copula model, a Gaussian copula model is also estimated using the same dataset from univariate currency returns. The estimation is straightforward, for only the covariance parameters are involved. It is found that the Gaussian copula cannot capture the asymmetric and complex dependence features in the data.

b. The investor's preference

In our study, the portfolio optimization problem can be summarised as maximization of appropriate expected utility while the utility function is based on the distributions from the above models:

$$\max_{w} E(U(W)) \tag{18}$$

$$W = 1 + w'R \tag{19}$$

where w is a vector representing the weights of currencies, R a vector of currency returns, and W is the wealth value of the portfolio.

The commonly used utility function is that of the power Constant Relative Risk Aversion (CRRA). Although this specification has preferences for higher moments, but the weights on them are rather small. Following Gul (1991), Ang *et al.* (2005) and Hong *et al.* (2007), we use the Disappointment Aversion (DA) preference for our optimization objective, on the ground that the commonly used

CRRA utility function is a local mean-variance preference. The DA utility is defined by the following equation:

$$DA(W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_w} u(W)dF(W) + A\int_{\mu_w}^{\infty} u(W)dF(W)\right) \qquad (20)$$

where $u(\cdot)$ is the felicity function in the form of CRRA utility:

$$u(W) = \begin{cases} (1-\gamma)^{-1} \cdot (W)^{1-\gamma} \ \ if\ \gamma \neq 1 \\ \ln(W)\ \ if\ \gamma = 1 \end{cases}, \qquad (21)$$

μ_w is the certainty equivalent according to the CRRA power utility; $F(\cdot)$ is the cumulative distribution function of the wealth; and K is a constant scalar given by:

$$K = P(W < \mu_w) + AP(W > \mu_w). \qquad (22)$$

The DA preference is a transformation based on the chosen $u(\cdot)$, or the CRRA power utility function in this case, in which the risk aversion parameter (RA) stands for the risk preference of the representative investor. The transformation puts different weights upon utility above and below the reference point, μ_w. Usually parameter A is set to be smaller than 1 so that the utility below average (the loss) gives larger impacts than the utility above the average (the profit). For example, if A is set to be 0.5, then the lower part of the utility is given twice the weight given to the upper part utility. This emphasis on the loss rather than profit is in accordance with the management nature of the central banks, whose primary goal is to avoid negative shocks to foreign assets rather than to increase wealth. Parameter A stands for the asymmetry preference of the representative investor. Therefore the optimization problem becomes:

$$\max_{w} DA(W) \tag{23}$$

$$W = 1 + w'R \tag{24}$$

In our analysis, we set up three levels of DA parameter, A, to be 0.25, 0.45 and 0.65, and four levels of relative risk aversion coefficient in the CRRA power utility function RA , to be 3, 7, 10 and 20. Similar range of risk aversion are used in Campbell and Viceira (1999), Ait-Sahalia and Brandt (2001) and Patton (2004).

3. DATA DESCRIPTION AND RESULTS ANALYSIS

a. Data description and the investment strategy

Unlike when calculating securities returns, to compute returns of each currency we need two types of datasets, i.e. the interest rate of the currency-issuing country and the exchange rate of the foreign currency to the currency of the home country, which is China in our case. To concentrate on the currency effect, we assume that international reserves are solely invested in government bonds. To comprehend the effects of diversification, a sufficient number of currency assets are to be included in a foreign currency portfolio. We select 12 currencies for the central bank of China. Therefore, we need 12 corresponding interest rates of these countries and 12 foreign exchange rates to the Chinese yuan. The horizon of the data sample is from 1 January 1999 to 31 December 2009 and the data are in daily frequency.

The interest rate dataset consists of 8 interbank rates and 4 money market rates. Of the 8 interbank rates, 7 are from the London market, i.e. the London Interbank Offered Rate (LIBOR) and the remaining one is the interbank rate for the country to which the home currency belongs, in this case Singapore Sibor. All 8 interbank rates are from Thomson Reuters DataStream. Due to data availability, the other four rates are money market rates from the IMF International Financial Statistics. Table 1 presents a summary of the interest rates.

[Insert Table 1 around here]

As to the exchange rates, 8 of the total 12 are from Thomson Reuters DataStream. Historic data on exchange rates of the Korean won and Russian rouble against the Chinese yuan are from a foreign exchange service company.[3] Table 2 gives a summary of the data sources.

[Insert Table 2 around here]

Currency returns are derived by combining the interest rate and exchange rate returns:

$$r_{i,t} = s_{i,t} + b_{i,t} \tag{25}$$

where $b_{i,t}$ is the interest rate of currency i and $s_{i,t}$ is the exchange rate return of currency i against the Chinese yuan.

For tractability, we assume that it is desirable for reserve managers to adopt a buy-and-hold investment strategy with yearly rebalancing. We take previous three years' daily returns as the base for estimating coefficients on model parameters and use one-year-ahead values from the conditional mean and volatility models as the corresponding expected values. Economic values are used as the performance measure, following Ang *et al.* (2005) and Hong *et al.* (2007). This measure is based on portfolio distributions, and indicates how

[3] OANDA Corporation. www.oanda.com.

much certainty equivalent wealth is needed for the worse model to have the same amount of utility as the better distribution model.

b. Empirical analysis of univariate currency returns

Descriptive analyses of the 12 currency returns during the sample period are carried out. Table 3 displays the results for the whole sample from 1999 to 2009. [4] The features of autocorrelation, heteroskedasticity and non-normal distributions are common among all currency returns. All currencies have big skewness and/or excess kurtosis. Normality of their returns is rejected by the Jarque-Bera tests. The prevalent non-normal distribution prompts us to add t distribution and skewed t distribution to modelling the residuals. With respect to the autocorrelation in conditional mean and volatility clustering, the Ljung-Box tests on raw data and squared returns are performed with 5 and 10 lag lengths. The LM ARCH test of Engle (1982) is also carried out. All 12 currencies have at least one test indicating autocorrelation or heteroskedastcity. This finding motivates us to apply the ARMA-GARCH/APARCH model.

In order to prove the consistence of the merits of our copula method, the empirical analysis covers 9 years from 2001 to 2009. To illustrate the empirical

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[4] In order to prove the consistency of the merits of our copula method, individual tests covering 2001 to 2009 are carried out. These empirical features in univariate returns as well as in dependence structure are universally presented in all years. Details on individual years can be obtained from the authors upon request.

motivations for applying the copula model and its effects after application, year 2005 is used as an example. These empirical features in univariate returns as well as in dependence structure are universally presented in all other years. Details on other years can be obtained from the authors upon request.

[Insert Table 3 around here]

The parameters for modelling each currency returns are presented in Table 4. The best model is determined by selecting the minimal AIC. The first two rows show the best fit type of conditional mean and conditional variance models. APARCH models explain asymmetries in some skewed currencies. The selection of residuals distribution type is also as expected from the descriptive statistics. Euro and pound sterling are fitted with normal distribution whereas the US dollar and the New Zealand dollar with high skewness are fitted with skewed Student-t distribution. Other currencies with high excess kurtosis are accounted for by t distributions. Most of the parameters are found to be significant, as indicated with bold typeface.

[Insert Table 4 around here]

Table 5 reveals the effectiveness of ARMA-GARCH/APARCH models in removing the time-dynamics in currency returns. The Ljung-Box and LM ARCH tests show all currency returns' residuals are now white noise. Kolmogorov-Smirnov tests are performed to compare residuals with their fitted distribution. The result shows that no currency can reject its best fit distribution. These results provide solid foundations for copula modelling.

[Insert Table 5 around here]

c. Analysis of dependence

Descriptive analyses of the dependence are also carried out. Table 6 reports the results for 2005 as an example. The lower triangular lists three dependence measures, i.e. the upper tail dependence, lower tail dependence and Kendall's tau. For example, in the 7th row and 2nd column of the table, the three numbers 0.6148, 0.3734 and 0.3630 indicate that the relation between the 7th currency AUD and the 2nd currency euro has a Kendall's tau of 0.3630, and its upper tail is greater than the lower tail. This implies that it has a fat-tail with tail dependence greater than zero. It also suggests the existence of asymmetric dependence, which indicates that extreme losses occur less often than do extreme earnings. The upper triangular of Table 6, further illustrates dependence between two variables. The empirical meta contour graphs are fitted in their corresponding positions. For example, the dependence between AUD and the euro, in the 2nd row and 7th column, is shown to be clearly asymmetric.

[Insert Table 6 around here]

Our vine copula structure allows a wide selection of copula functions. The flexibility of the approach manifests in two aspects. First, it can capture fat-tails and asymmetric dependence. Such dependence is complex, especially in high dimensional situations. As revealed in Table 6, many currency pairs have greater than zero tail dependence and uneven upper and lower tails. Conventional assumption of Gaussian and elliptical copulas are unable to capture these features, which may significantly affect portfolio optimization. See Figs. 2 and 3 for further illustration.

[Insert Figs. 2 and 3 around here]

Fig. 2 contains four graphs depicting the relation between the CHF and CAD in 2005. The scatter plot in the upper left, and the chi-plot in the upper right using the method of Fisher and Switzer (1985) are for the whole sample; the chi-plot in the lower left is for both variables increasing together above their averages (the upper tail dependence), and the one in the lower right is for their decreasing together (the lower tail dependence). The horizontal axis of a chi-plot is the distance between the data point (x, y) and the centre of the dataset, whereas the vertical axis is a correlation coefficient on dichotomised values of the two variables.

From the first chi-plot we can see that since the right half of this graph describes data moving in the same direction (rising or falling at the same time) and the left half describes data moving in different directions (one rises/falls, while the other falls/rises), the fact that dependence on the right is greater than that on the left means these two currencies are more correlated when increasing or decreasing simultaneously. Further, on reading the points towards the right of the plot (the furthest distance from the centre) the tail dependence is found to be above zero. This shows the fat-tail. Comparison between the second and third chi-plots shows that the upper tail has greater dependence than the lower tail, since the higher correlation points are from the upper tail in the lower left graph, rather than the lower tail in the lower right graph, and this reveals asymmetry.

Fig. 3 shows that the relation described in Fig. 2 can be captured exactly by a D-vine structure. The figure includes three meta-contour plots. The first is the empirical contour, the second is taken from the estimated best fit copula in the D-vine structure, whereas the third is a comparison with the Gaussian copula if no selection is permitted. It can be seen that the Clayton-Gumbel copula in the second plot better captures the essence of the empirical dependence.

To facilitate the demonstration of this point, Fig. 4 gives the same scatter plot and chi-plots as in Fig. 2 for the whole sample again from 1999 to 2009 for the purpose of showing such feature is universal. From the whole sample case in Fig. 4, it is also discovered from the chi-plots that the dependence is actually distributed unevenly. The non-zero dependence in the upper and lower ends means fat-tails, and the different patterns in the lower half two chi-plots indicate dependence asymmetry. Such features are typical and universal in all the individual years.

[Insert Fig. 4 around here]

The second aspect of our copula model's flexibility lies in the rotated copulas included in the fitting range, especially those Archimedean copulas being rotated 90 and 270 degrees. This makes it possible for our approach to capture dependence between variables that are correlated when moving in different directions. In the vine structures only part of the nodes are fed with the original residuals data. Many nodes need to be changed according to the conditional distribution functions. As such, there is a good chance that the dependence between changed variables is fit best by a rotated copula. Fig. 5 shows a meta-contour of the second copula in the sixth tier in the D-vine structure for the dependence of currency returns in 2005. It can be seen that the correlation in the upper left corner is greater than in the lower right corner. This best fit copula is a 270 degree rotated Clayton copula.

In Fig.6 similar discovery of rotated copulas capturing the relationship of currencies moving in different directions is shown again using the whole sample from 1999 to 2009. It is a plot of meta-contour of the second copula in the eighth tier in the D-vine structure, with the best fit copula to be a 90 degree rotated BB8 copula.

[Insert Fig. 5 and Fig. 6 around here]

To formally test the overall fit of the pair copula models, we conduct the Vuong ratio test (Vuong, 1989) by comparing the C-vine and D-vine copulas with a Gaussian copula and by comparing between the two vine structures. The Vuong test is a likelihood-ratio based test often used for comparing different non-nested models.

Table 7 presents the Vuong test statistics and p-values for three sets of comparisons. The test results are interpreted in terms of the p-values. If the p-value of a test is smaller than 5%, we prefer the first model at the 5% significance level. If it is greater than 95%, the second model is preferred. Thus we can see from the tests that both C-vine and D-vine copulas are to be preferred over the Gaussian copula. The flexibility provided by the vine-structures and inspected individually in above examples are highly effective in the overall 12-dimensional joint dependence in the sample years. However, the comparison between the C- and D-vines, is less conclusive. A winner can be selected if we raise the significance level from 5% to 10%. Below the 10% significance level, the D-vine is preferred for 2002 and 2008, whereas ethe C-vine is desired only for 2005. For all other the years the difference is hardly significant. The fact that the D-vine has a slight edge over the C-vine is probably due to the fact that in the first tiers of C-and D-vines, the latter contains more highly correlated pairs.

[Insert Table 7 around here]

d. Influences of risk aversion and disappointment aversion

Tables 8 and 9 show seven statistics that describe the optimal portfolio under different constructions. In addition to conventional measures such as portfolio mean, standard deviation, and the Sharpe ratio, we also look for skewness, kurtosis, VaR (value at risk) and CVaR (the conditional value at risk). Table 8 provides an overview of copula model estimates when the risk aversion variable (RA) takes different values; the disappointment avoidance variable, A, is set for 2005 at $A = 0.25$, which is the least of the three commonly adopted disappointment avoidance values.

Table 9 is a comparison under three values of A when R$A = 20$ for the same year of 2005. Generally speaking, for 2005, the average daily returns of the optimal portfolios across the models are all positive. The distinction between the three models of Gaussian copula, D-vine and C-vine methods is clear in terms of skewness and kurtosis. For the rest of the measures, the differences are not as apparent, which lends the support for our use of DA preference. With the DA utility function, the portfolio optimization can take into consideration the higher moments like skewness and kurtosis, which is the distinction between vine and Gaussian models.

[Insert Tables 8 and 9 around here]

In Table 8, one can see the effects of a change in risk aversion in any of the three models, especially in terms of the conventional risk measure, i.e. standard deviations. As the degree of risk aversion of the central bank increases, the portfolio with highest DA influence has less standard deviations and lower average returns. Table 9 shows the influence of the disappointment aversion effects. The smaller the value taken by A, the less tolerance of a negatively skewed distribution, implying that the possibility of negative extreme events is more stringently excluded. As expected, in all copula models skewness increases with the value of A. In what follows, we shall choose a pair of RA and A whose values are assumed to be the most likely representation of the central bank's preference. Given that the central bank is a very conservative institution in managing investment of its foreign reserves, we set A to take the smallest value from the range, i.e. 0.25, while RA equals to 20, the largest out of the four values to represent the behaviour of China's central bank.[5]

e. Expected Economic value of switching from mean-variance to pair-copula method

The notion of expected economic values can be traced back to Ang *et al.* (2005) and Hong *et al.* (2007). It calculates the certainty equivalent wealth gains based on the better fitted distribution model as compared to the coarser model. In this study, we use expected economic value to represent how much is earned by the pair-copula model compared to the mean-variance model. In so doing, we assume DA utility for the Chinese central bank and take into account the asymmetries, fat-tails and dependence complexities in the returns distribution. Hence, this performance measure is built on a comprehensive base that incorporates the conservative property of the central bank and the advantages offered by copula modelling.

Let us denote the certainty equivalent wealth of a mean-variance model as W^{nor} and the certainty equivalent wealth of the D-vine model as W^{copu}. The certainty equivalent wealth is a scalar which will give the same amount of DA utility if the distribution of the wealth is plugged into the utility function. The notion of the expected economic values is that if the D-vine distribution is

[5] Results of other values of A and RA can be obtained upon request.

believed to be true, how much percentage of returns that the investor needs giving up in order to have the same DA utility as can be obtained from the traditional mean-variance method. This can also be regarded as the economic value of switching from a mean-variance to a pair-copula model. Denoting this amount as CE, it can be solved through the following equations:

$$DA(W^{nor}) = \frac{1}{K}\Big(\int_{U(w^*)<E(\mathrm{U}(w^*))} U(w^*)p(R^{copu})\,dR^{copu} + A\int_{\mathrm{U}(w^*)>E(\mathrm{U}(w^*))} U(w^*)p(R^{copu})\,dR^{copu}\Big) \tag{26}$$

where

$$w^* = 1 + R^{copu} - CE \tag{27}$$

Table 10 displays the expected economic value of switching from mean-variance to the D-vine model when the disappointment avoidance parameter is taken to be 0.25 with five different risk aversion preferences. Across all risk preferences, Table 10 records that the annualised gain ranges from 0.563 basis points to 15.5%and the average is 0.962%. The annualised gains are calculated from the result from daily data assuming that there are 250 working days in a year. When the central bank of China takes the most conservative stance so that $\mathrm{RA} = 20$, the average annual gain is even higher, at 1.05% for the period from 2001 to 2009. It should be noted that the increases in economic value are calculated based on the simulated returns rather than the out-of-sample data. Hence the economic values are expected, not realised.

[Insert Table 10 around here]

f. Comparison with foreign debt and trade constraints

In this sub-section, we analyse influences of two *ad hoc* weight constraints on the choice of currency portfolio. These two sets of constraints are in correspondence to the currency shares of China's external debt and shares of bilateral trade between China and a particular partner in China's total foreign trade. We have shown that the pair-copula method is beneficial, but the gains are obtained when no constraints are imposed on currency weights.

Taking foreign trade and debt into consideration will make our model resemble the reality more closely. One major function of a country's foreign reserves is to fulfil the payment needs of international trade and debt. These two constraints of minimal weights are set up following Papaioannou *et al.* (2006). Further application of this set up can be found in Wu (2007).

Table 11 presents trade shares of Chinese partners according to the IMF's Direction of Trade. We take 50% of these shares as the minimal weight in the optimal currency structure for China's foreign reserves. For example, in China's total international trade in 2009, trade with the US accounts for 13.55% of China's total trade in value terms and so we assume that in China's currency structure of foreign reserves, at least 6.775% should be kept in the USD.

The second constraint involves China's international financial activity. The currency shares of China's external debt are obtained from the Global Development Finance Database of the World Bank, and are listed in Table 12. A threshold of 50% of these currency shares are taken for the minimal weight of the corresponding currency in China's currency composition of foreign reserves.

Table 13 shows annual gains of the expected economic value with foreign debt and international trade constraints. The average annualised expected economic value under the debt constraints is 4.12% and under the trade constraints it is 13.4%. These are greater than that in the case without weight constraints.

[Insert Tables 11, 12 and 13 around here]

g. Optimal currency composition for China's reserves

We report estimates of the optimal currency composition for China's foreign reserves in Tables 14 and 15. The estimation is based on the generally preferred D-vine copula construction for the sample period of 2001 to 2009. Results in Table 14 are those obtained under the trade constraints, while outcome in Table 15 are derived with the external debt constraints. Across the sample years, we see a clear pattern of currency distributions, i.e. the US dollar, euro and Japanese yen are the three main currencies that consistently dominate the currency structure of China's reserves. Of these first tier currencies, the US dollar maintains the leading position despite occasionally being challenged in the early 2000s by the Japanese yen (in 2001) and the euro (in 2003). However, although the dollar's primary standing is solid, its edge over other currencies is not as great as conventionally thought. Generally, in China's case, the optimal proportion for the dollar in the reserves is around 40-45%. The big-three currencies are followed by a large group of second-tier currencies. This research has derived optimal shares for each of these currencies in China's reserves. They provide ample rooms for China to diversify its reserve holdings into non-dollar assets.

Table 16 shows the optimal currency composition for China if the Gaussian copula model with international trade constraints is used. The results are generally similar to those of the previous exercises, in that if we attend to both the trade and debt constraints in the copula model we derive an average proportion of 41.75% for the USD, whereas the conventional estimate of China's USD reserves is above 60%. However in comparison with theD-vine copula results (in Table 14), allocations under the Gaussian copula show heavier concentration on several currencies. This means that the Gaussian copula approach may have squeezed the space for efficient currency diversification.

[Insert Tables 14, 15 and 16 around here]

4. CONCLUSIONS

An appropriate currency structure is an essential aspect of sound management of foreign reserves. In this paper, we set up a flexible framework based on pair-copula construction. This approach allows us to model critical features of currency returns, including the asymmetry, fat-tails and complex dependence structure. In the context of China, we apply the copula model to analyse how these features affect the currency returns and to derive an optimal currency structure for China's reserves management.

Each currency return is first modelled using a variety of ARMA-GARCH filters with different residual distributions to best suit dynamics in univariate returns series. The dependency structure to connect each currency returns are then modelled by pair-copula construction with two different vine structures. Based on the established distribution we use the preference under the disappointment aversion effect as the optimizing objective to obtain the optimal currency composition. Our comparison shows that the mean-variance method cannot reflect the skewness whereas the pair-copula method can capture the features of higher moments such as skewness and kurtosis. Our further comparison shows the expected economic value of switching to the pair-copula models from the mean-variance framework. Considering the enormous amount of the international reserves held by emerging economies such as China, the central bank in our model can achieve sizable gains.

To analyse the Chinese case, we mimic China's currency shares of external payments by imposing *ad hoc* weight restrictions according to China's foreign trade and debt relations. Evidence shows that the pair-copula model with the D-vine structure has advantages over other methods. In this approach, the US dollar consistently takes the largest share in China's reserve currency composition. However, incorporation of the features of asymmetry, fat tails and complex dependence structure would allow more rooms for other currencies to be chosen for currency diversification of China's reserves. It is therefore desirable and feasible for China to adopt the copula approach the currency composition of its reserves and diversification is important for countering dependence complexities to manage currency composition of its huge and growing reserves.

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TABLE 1
Summary of Interest Rates

Interest Rates Data								
Country	**US**	**EURO**	**JAPAN**	**UK**	**SWITZERLAND**	**CANADA**	**AUSTRALIA**	**SINGAPORE**
Type	Interbank rates (12 Month)							
Market	LIBOR							SIBOR
Frequency	Daily	Daily	Daily	Daily	Daily	Daily	Daily	Daily
Source	Thomson Reuters DataStream							
Mnemnic Code	BBUSD12	BBEUR1Y	BBJPY12	BBGBP12	BBCHF12	BBCAD12	BBAUD12	SNGIB1Y

Interest Rates Data				
Country	**NEWZEALAND**	**SOUTH KOREA**	**RUSSIA**	**THAILAND**
Type	Money Market Rate			
Frequency	Daily	Daily	Daily	Daily
Source	IMF International Financial Statistics			

Source: Authors' compilation

TABLE 2
Exchange Rate Data

Foreign Exchange Rate Data						
Currency	**USD**	**EURO**	**JPY**	**GBP**	**CHF**	**CAD**
Type	WM/Reuters Mid Price					
Frequency	Daily					
Source	Thomson Reuters DataStream					
Mnemnic Code	CHIYUA$	CHEURSP	CHJPYSP	CHIYUAN	CHCHFSP	CHCADSP

Foreign Exchange Rate Data Specification						
Currency	**AUD**	**SGD**	**NZD**	**THB**	**KRW**	**RUB**
Type	WM/Reuters Mid Price				Mid Price	
Frequency	Daily					
Source	Thomson Reuters DataStream				OANDA	
Mnemnic Code	CHAUDSP	CHSGDSP	CHNZDSP	CHTHBSP		

Source: Authors' compilation

TABLE 3

Descriptive Statistics for Currency Returns (Whole Sample)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
Skewness	-8.889	0.245	-0.116	-0.007	0.176	-0.163	-0.428	-0.014	-0.355	0.133	-2.339	-0.469
Excess Kurtosis	270.580	2.927	4.982	4.942	2.022	4.274	8.116	3.281	2.795	12.797	38.808	70.490
Jarque-Bera	**8.790E+06**	**1052.600**	**2973.900**	**2919.100**	**503.660**	**2196.400**	**7961.700**	**1286.600**	**993.710**	**19585.000**	**1.827E+05**	**5.941E+05**
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LM ARCH	0.030	**41.791**	**36.542**	**38.095**	**14.852**	**206.530**	**291.630**	**31.107**	**155.030**	**83.285**	**119.700**	**306.770**
p-value	0.970	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ljung-Box 5	**11.106**	3.577	**15.555**	**15.276**	**16.610**	**11.834**	**18.869**	**15.490**	3.170	**179.684**	**79.632**	**175.617**
p-value	0.049	0.612	0.008	0.009	0.005	0.037	0.002	0.008	0.674	0.000	0.000	0.000
LB 10	**33.744**	**23.590**	**22.798**	**26.513**	**21.893**	**38.255**	**38.029**	**24.541**	9.490	**235.190**	**101.496**	**264.868**
p-value	0.000	0.009	0.012	0.003	0.016	0.000	0.000	0.006	0.486	0.000	0.000	0.000
LB Square5	0.169	**144.383**	**125.566**	**442.611**	**52.145**	**1119.980**	**967.616**	**110.909**	**541.421**	**531.598**	**380.376**	**562.222**
p-value	0.999	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LB Square10	**0.385**	**246.545**	**185.378**	**861.929**	**85.085**	**2192.640**	**2004.390**	**226.359**	**931.561**	**1165.690**	**491.188**	**797.854**
p-value	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 4

Univariate Returns Model Estimation (2005)

	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
mean type	Arma (3, 3)	Arma (3, 2)	Arma (3, 1)	Arma (3, 3)	Arma (2, 1)	Arma (3, 1)	Arma (2, 2)	Arma (2, 3)	Arma (3, 3)	Arma (3, 2)	Arma (3, 1)	Arma (3, 3)
variance type	Aparch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Aparch (1, 1)	Garch (1, 1)	Garch (1, 1)	Garch (1, 1)	Aparch (1, 1)	Aparch (1, 1)
Distribution	sstd	norm	std	norm	std	Std	std	std	sstd	std	std	std
Mu	1.101E-07	5.080E-04	-2.470E-06	8.790E-04	-4.490E-06	**7.090E-06**	**-2.390E-05**	2.750E-04	1.740E-04	1.370E-04	5.610E-07	9.160E-06
p-value	4.536E-01	3.124E-01	5.979E-02	1.682E-01	9.860E-01	2.160E-06	2.000E-16	1.611E-01	1.247E-01	1.657E-01	NA	7.469E-01
ar1	**3.920E-01**	**-6.730E-01**	**8.940E-01**	**-9.580E-01**	-1.960E-01	**9.290E-01**	**4.840E-02**	-1.000E+00	**-4.530E-01**	-2.720E-01	9.620E-01	**2.030E-01**
p-value	2.000E-16	3.920E-04	2.000E-16	3.630E-05	4.770E-01	2.000E-16	2.000E-16	NA	1.660E-06	5.620E-02	NA	1.003E-02
ar2	**2.720E-01**	**-6.740E-01**	**1.080E-01**	**-6.960E-01**	-4.080E-03	**8.830E-02**	**9.510E-01**	**-5.120E-01**	**3.160E-01**	**5.580E-01**	7.940E-03	**-2.190E-01**
p-value	2.000E-16	6.450E-07	1.343E-02	4.660E-05	9.240E-01	3.960E-02	2.000E-16	1.970E-04	6.800E-04	2.180E-05	6.163E-01	3.620E-06
ar3	**3.400E-01**	-4.810E-02	-2.210E-03	**-6.850E-01**		-3.900E-02			**7.700E-01**	**2.170E-01**	1.760E-02	**6.890E-01**
p-value	2.000E-16	1.910E-01	9.522E-01	8.970E-04		2.322E-01			2.000E-16	2.790E-04	NA	2.000E-16
ma1	**-4.950E-01**	**6.840E-01**	**-1.000E+00**	**1.000E+00**	8.980E-02	**-1.000E+00**	**-9.090E-02**	9.390E-01	**4.320E-01**	-5.110E-02	-9.820E-01	-1.390E-01
p-value	2.000E-16	2.560E-04	2.000E-16	1.090E-04	7.450E-01	2.000E-16	2.000E-16	NA	5.770E-07	7.102E-01	NA	7.174E-02
ma2	**-1.890E-01**	**6.870E-01**		**7.340E-01**			**-9.410E-01**	**4.620E-01**	**-3.380E-01**	**-6.780E-01**		**2.660E-01**
p-value	2.000E-16	1.450E-07		6.560E-05			2.000E-16	9.600E-04	3.810E-05	1.880E-12		5.400E-08
ma3	**-1.890E-01**			**6.490E-01**				-4.200E-02	**-8.200E-01**			**-7.070E-01**
p-value	2.000E-16			5.820E-04				2.027E-01	2.000E-16			2.000E-16
Omega	**3.360E-05**	7.380E-07	7.300E-07	**9.050E-07**	1.090E-06	5.950E-07	8.140E-07	1.900E-07	7.440E-07	6.510E-07	5.680E-09	**1.290E-04**
p-value	2.430E-03	1.840E-01	1.236E-01	7.026E-02	2.120E-01	1.724E-01	1.840E-01	1.395E-01	1.276E-01	2.664E-01	1.000E+00	3.896E-02
alpha1	**2.500E-01**	1.350E-02	**5.000E-02**	**4.320E-02**	3.540E-03	**2.810E-02**	1.830E-02	**3.620E-02**	**1.620E-02**	**1.410E-01**	**1.000E+00**	**3.220E-01**
p-value	1.030E-07	1.169E-01	4.850E-03	5.818E-03	6.680E-01	2.120E-02	3.230E-01	1.358E-02	2.051E-02	1.487E-03	1.760E-02	3.170E-03

gamma1	8.880E-02						**3.310E-01**				1.070E-01	1.200E-01
p-value	5.160E-01						1.680E-05				3.909E-01	3.207E-01
beta1	**8.140E-01**	**9.660E-01**	**9.310E-01**	**9.260E-01**	**9.760E-01**	**9.540E-01**	**9.610E-01**	**9.380E-01**	**9.710E-01**	**8.710E-01**	**8.860E-01**	**6.380E-01**
p-value	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	2.000E-16	9.730E-09
Delta	**6.680E-01**						2.000E+00				**8.360E-01**	**1.240E+00**
p-value	2.240E-08						1.990E-01				7.850E-07	4.100E-03
Skew	**9.890E-01**								**8.740E-01**			
p-value	2.000E-16								2.000E-16			
Shape	**2.680E+00**		**5.030E+00**		**5.450E+00**	**6.040E+00**	**6.640E+00**	**6.210E+00**	**6.780E+00**	**4.320E+00**	**2.010E+00**	**2.870E+00**
p-value	4.440E-16		2.550E-06		4.090E-05	1.450E-05	4.140E-05	7.290E-06	8.640E-05	7.320E-09	2.000E-16	3.690E-14

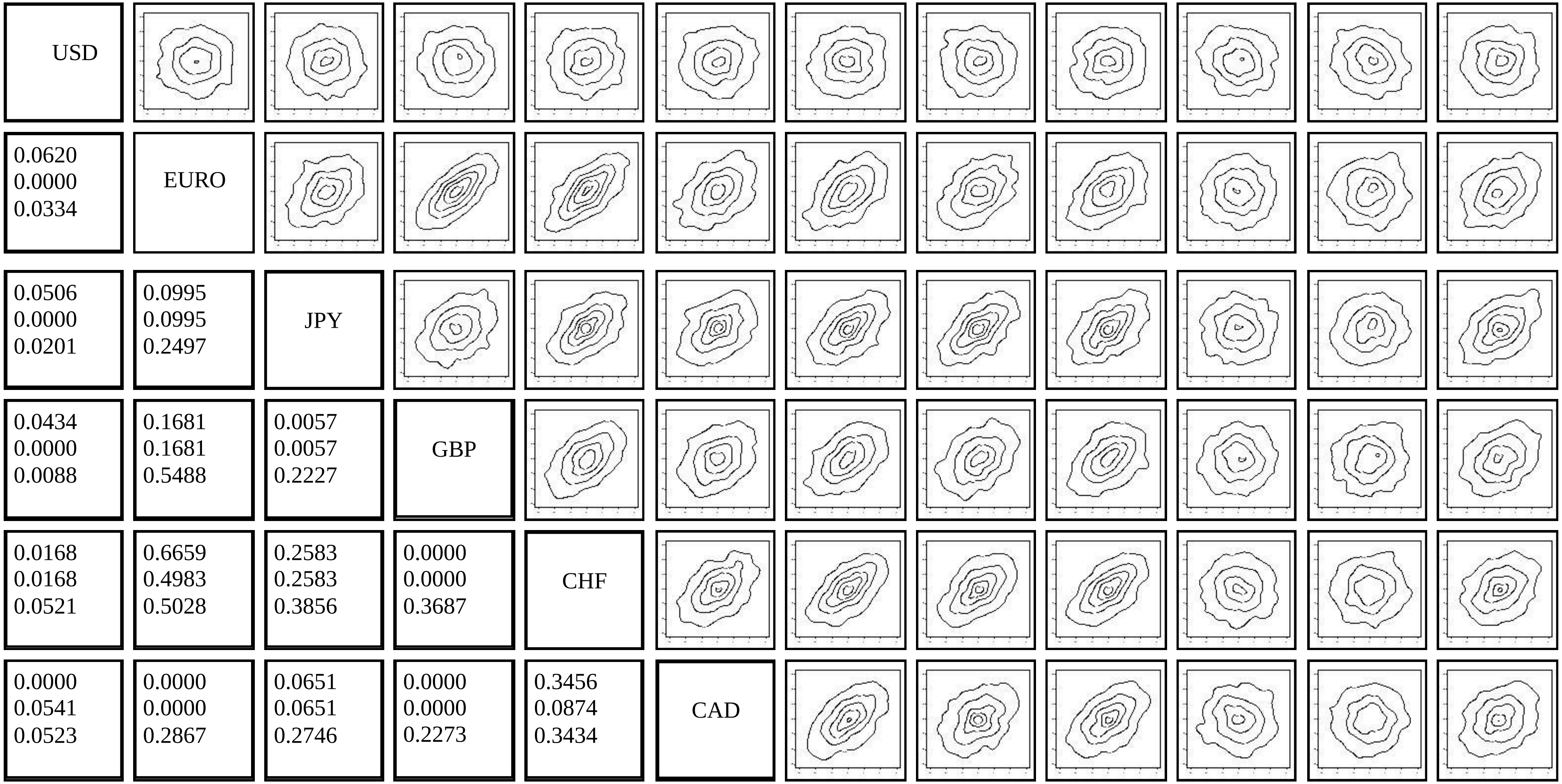
Notes: (i).The first two rows in the table indicate the type of mean and variance functions for each currency returns and their best fit lag lengths. The third row reports the best fit distribution forms for their residuals. Skewed Student-t, Student-t and Gaussian distributions are respectively denoted by ‘sstd’, ‘std’, and ‘norm’. (ii).The rest of the table lists coefficient values and their p-values to indicate significance for corresponding models in the first three rows. Significance is highlighted with the bold fonts.

TABLE 5
Statistical Tests for Effectiveness of Univariate Models (2005)

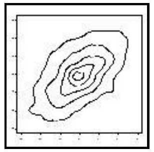
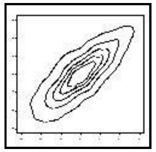
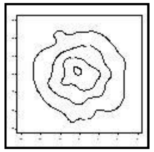
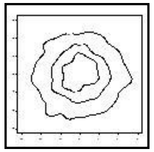
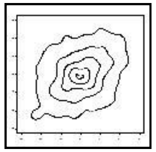
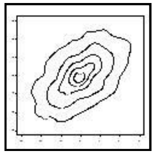
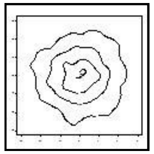
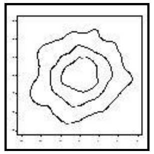
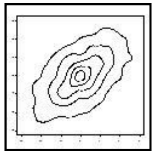
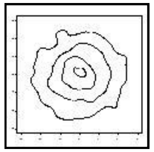
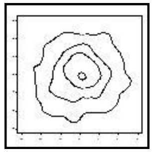
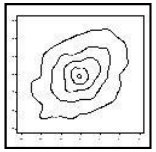
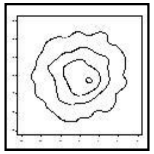
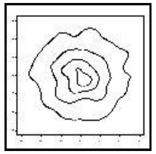
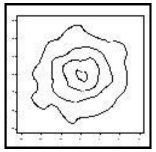
	USD	EURO	JPY	GBP	CHF	CAD	AUD	SND	NZD	KRW	RUB	THB
Ljung-Box 10	0.027	3.253	7.919	4.143	6.027	5.697	6.633	4.113	9.764	10.670	0.004	4.908
p-value	1.000	0.975	0.637	0.941	0.813	0.840	0.760	0.942	0.461	0.384	1.000	0.897
Ljung-Box 15	0.043	7.915	9.752	6.079	12.138	6.673	17.349	7.794	18.370	12.990	0.004	17.094
p-value	1.000	0.927	0.835	0.978	0.669	0.966	0.298	0.932	0.244	0.603	1.000	0.313
LB Square10	0.013	16.025	7.202	4.169	8.473	8.265	12.441	9.547	5.005	11.034	0.004	3.423
p-value	1.000	0.099	0.706	0.939	0.583	0.603	0.257	0.481	0.891	0.355	1.000	0.970
LB Square 15	0.020	19.544	9.240	7.160	9.754	11.013	20.304	16.794	18.920	15.732	0.004	4.065
p-value	1.000	0.190	0.865	0.953	0.835	0.752	0.161	0.331	0.217	0.400	1.000	0.998
LM ARCH	0.016	18.649	7.214	4.534	8.852	9.762	11.552	11.575	6.107	12.429	0.753	3.870
p-value	1.000	0.097	0.843	0.972	0.716	0.637	0.482	0.480	0.911	0.412	1.000	0.986
KS test	0.030	0.028	0.043	0.026	0.042	0.033	0.045	0.037	0.033	0.020	0.046	0.028
p-value	0.489	0.640	0.137	0.708	0.153	0.426	0.111	0.228	0.362	0.920	0.080	0.572

Notes: (i). LB stands for the Ljung-Box test and LB 10 means the Ljung-Box test on raw data with 10 lags. LB Squre15 means the Ljung-Box test on squared terms with a lag length of 15. (ii). All tests in the table are presented with both coefficient values and their probability values (p-values) to indicate the hypothesis rejection. None of the null hypothesis can be rejected.

TABLE 6
Descriptive Analysis of Dependence (2005)



USD											
0.0620 0.0000 0.0334	EURO										
0.0506 0.0000 0.0201	0.0995 0.0995 0.2497	JPY									
0.0434 0.0000 0.0088	0.1681 0.1681 0.5488	0.0057 0.0057 0.2227	GBP								
0.0168 0.0168 0.0521	0.6659 0.4983 0.5028	0.2583 0.2583 0.3856	0.0000 0.0000 0.3687	CHF							
0.0000 0.0541 0.0523	0.0000 0.0000 0.2867	0.0651 0.0651 0.2746	0.0000 0.0000 0.2273	0.3456 0.0874 0.3434	CAD						

0.0686 0.0000 0.0490	0.6148 0.3734 0.3630	0.2170 0.2170 0.3952	0.0293 0.0293 0.3378	0.2858 0.2858 0.4585	0.0000 0.0000 0.4193	AUD					
0.0000 0.0000 -0.0118	0.0000 0.0000 0.3160	0.2437 0.2437 0.4642	0.0000 0.0000 0.2713	0.1367 0.1367 0.3728	0.0986 0.0986 0.2533	0.1331 0.1331 0.3966	SND				
0.0041 0.0041 0.0535	0.0000 0.0000 0.3299	0.2302 0.2302 0.3659	0.2095 0.2632 0.3283	0.2833 0.2833 0.4411	0.0706 0.0706 0.3673	0.6757 0.4250 0.6572	0.0810 0.0810 0.3768	NZD			
0.0000 0.0000 -0.0565	0.0000 0.0000 0.0078	0.0000 0.0000 -0.0145	0.0000 0.0000 0.0091	0.0000 0.0000 -0.0048	0.0000 0.0000 -0.0528	0.0000 0.0082 0.0019	0.0000 0.0000 0.0104	0.0000 0.0000 0.0240	KRW		
0.0000 0.0000 -0.1062	0.0751 0.0000 0.0166	0.0000 0.0000 0.0697	0.0032 0.0000 0.0600	0.0000 0.0000 0.0168	0.0000 0.0000 0.0120	0.0000 0.0000 0.0256	0.0000 0.0000 0.0640	0.0000 0.0012 0.0400	0.0543 0.0000 0.0398	RUB	
0.0000 0.0000 -0.0062	0.0000 0.0000 0.2107	0.5830 0.2848 0.3222	0.0000 0.0000 0.1981	0.0158 0.0158 0.2168	0.1606 0.0124 0.1730	0.1242 0.1242 0.2254	0.6049 0.3469 0.3853	0.2257 0.0037 0.2123	0.0000 0.0000 -0.0205	0.0000 0.0000 0.0417	THB

Notes: The lower triangular lists three dependence measures: the upper and lower tail dependence and Kendall's tau, respectively. The upper triangular are empirical meta-contour graphs.

TABLE 7
Vuong Test for Three Pairs of Comparisons

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
C-Gaussian	5.975	5.811	6.446	5.573	4.283	5.209	5.446	6.252	4.893
p-value	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
D-Gaussian	5.634	5.964	6.321	5.332	4.528	4.995	6.205	6.253	6.400
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C-D	0.695	0.116	-0.394	0.739	-1.692	0.173	-1.208	-0.101	-1.491
p-value	0.487	0.908	0.693	0.460	0.091	0.863	0.227	0.920	0.136

Notes: (i). C-Gaussian means comparison between C-vine copula and Gaussian copula. (ii). The Vuong tests are interpreted by inspecting p-values. If it is smaller than the significance level, the former model in the comparing pair is preferred. If larger than one minus the significance level the latter is preferred. No decision can be made if in the middle.

TABLE 8

Descriptive Statistics for Different Degrees of Risk Aversion

when A=0.25 for 2005							
RA=3							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000451	0.005678	0.079404	-0.24963	4.856321	-0.00895	-0.24946
D-vine	0.000452	0.005992	0.075445	-0.23347	5.728709	-0.0094	-0.26427
C-vine	0.000449	0.005454	0.082337	-0.38095	14.861	-0.00844	-0.23346
RA=7							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000434	0.005085	0.085346	-0.10572	4.692941	-0.00785	-0.21691
D-vine	0.000433	0.005309	0.081606	-0.26512	11.21563	-0.00815	-0.227
C-vine	0.000439	0.00513	0.08565	-0.46894	23.37002	-0.00783	-0.2161
RA=10							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.00042	0.004794	0.087557	-0.08073	4.685982	-0.00736	-0.20315
D-vine	0.000424	0.005117	0.082878	-0.22989	14.21159	-0.00781	-0.21689
C-vine	0.000433	0.005005	0.086599	-0.45262	23.90282	-0.0076	-0.20958
RA=20							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
Gaussian	0.000307	0.002536	0.12104	-0.06807	5.009664	-0.0038	-0.10557
D-vine	0.000322	0.003789	0.085092	-0.01971	20.90247	-0.00575	-0.15901
C-vine	0.000333	0.003756	0.08871	-0.42971	23.38223	-0.00568	-0.15653

TABLE 9

Descriptive Statistics for Different Values of Asymmetry Preference

Portfolio Descriptive Statistics when RA=20 for 2005							
A=0.25							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002536	0.12104	-0.06807	5.009664	-0.0038	-0.10557
D-vine	0.000322	0.003789	0.085092	-0.01971	20.90247	-0.00575	-0.15901
C-vine	0.000333	0.003756	0.08871	-0.42971	23.38223	-0.00568	-0.15653
A=0.45							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002538	0.120982	-0.06858	5.007607	-0.00381	-0.10566
D-vine	0.000322	0.003788	0.085094	-0.01976	20.91866	-0.00575	-0.15898
C-vine	0.000333	0.003756	0.08871	-0.42983	23.38422	-0.00568	-0.15654
A=0.65							
Model	Mean	s.d.	Sharpe ratio	skewness	kurtosis	VaR	CVaR
MV	0.000307	0.002534	0.121074	-0.06889	5.011703	-0.0038	-0.10551
D-vine	0.000322	0.003789	0.085093	-0.01991	20.91404	-0.00575	-0.15899
C-vine	0.000333	0.003757	0.088708	-0.43131	23.37734	-0.00568	-0.15655

Notes: (i).A is the disappointment avoidance parameter with its values ranging in [0,1]. With the disappointment avoidance utility, the investor treats the earnings above the expectation only as A times of the losses below the expectation. The smaller the value of A, the more emphases the investor puts on losses below expectation than on earnings above. (ii).RA is the risk aversion parameter. The higher the value of RA, the more risk averse the investor is. (iii).s.d. is short for standard deviations. The Sharpe ratio is calculated as the ratio between mean and s.d. representing return per unit of risk. VaR is short for Value at Risk. CVaR is short for Conditional Value at Risk.

TABLE 10

Expected Economic Value of Switching from Gaussian Copula to D-Vine Copula Modelling

Economic value of Gaussian copula to D-vine when A=0.25				
	RA=3	RA=7	RA=10	RA=20
2001	8.68E-04	6.15E-04	2.04E-02	2.33E-02
2002	1.86E-04	3.13E-04	4.18E-04	4.63E-04
2003	5.63E-05	2.70E-04	2.93E-04	8.60E-03
2004	1.06E-02	6.53E-03	2.04E-03	3.05E-03
2005	2.53E-04	3.00E-04	4.55E-03	2.14E-02
2006	3.88E-03	1.11E-02	4.93E-03	7.95E-03
2007	1.92E-04	0.1515	4.78E-03	7.80E-03
2008	7.15E-03	2.70E-03	4.78E-03	1.41E-02
2009	4.75E-03	3.50E-03	4.78E-03	8.28E-03

Notes: (i).The table shows the annualised expected economic value for attending features of asymmetries and fat-tails by switching from the Gaussian copula to the D-vine copula modelling. The value is calculated as how much earnings can be deducted to lower the D-vine copula model's utility down to the same level as the mean-variance model's utility. (ii).A is the disappointment avoidance parameter with its values ranging in [0,1]. Under the disappointment avoidance utility, the investor treats the earnings above the expectation only as A times of the losses below the expectation. The smaller the value of A means that the more emphases the investor puts on losses below the expectation than earnings. (iii).RA is the risk aversion parameter. The higher the value of RA, the more risk averse the investor is.

TABLE 11
Trade Shares of China's Partners

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	15.80%	15.67%	14.87%	14.72%	14.92%	14.94%	13.94%	13.06%	13.55%
EURO	12.26%	11.61%	12.39%	12.24%	12.26%	12.35%	12.79%	12.93%	12.79%
JPY	17.22%	16.41%	15.69%	14.53%	12.97%	11.78%	10.85%	10.42%	10.37%
GBP	2.02%	1.83%	1.69%	1.71%	1.72%	1.74%	1.81%	1.78%	1.77%
CHF	0.47%	0.43%	0.42%	0.45%	0.41%	0.39%	0.44%	0.44%	0.44%
CAD	1.45%	1.28%	1.18%	1.34%	1.35%	1.32%	1.39%	1.35%	1.34%
AUD	1.76%	1.68%	1.59%	1.76%	1.91%	1.86%	2.01%	2.29%	2.71%
SND	2.14%	2.26%	2.27%	2.31%	2.34%	2.32%	2.17%	2.05%	2.17%
NZD	0.23%	0.23%	0.21%	0.22%	0.19%	0.17%	0.17%	0.17%	0.21%
KRW	7.04%	7.10%	7.43%	7.79%	7.87%	7.63%	7.36%	7.27%	7.07%
RUB	2.09%	1.92%	1.85%	1.83%	2.04%	1.89%	2.21%	2.22%	1.75%
THB	1.41%	1.38%	1.49%	1.50%	1.53%	1.57%	1.59%	1.61%	1.73%

Source: International Monetary Fund: Direction of Trade, various issues.

TABLE 12

Currency Shares of China's External Debt

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	74.08%	72.45%	71.27%	70.77%	74.69%	76.27%	80.62%	81.68%	83.83%
EURO	4.74%	5.69%	7.16%	9.02%	8.00%	8.39%	8.07%	6.62%	6.21%
JPY	14.54%	15.39%	16.73%	15.92%	13.47%	12.02%	8.38%	9.14%	7.86%
GBP	0.10%	0.11%	0.11%	0.10%	0.09%	0.08%	0.07%	0.04%	0.03%
CHF	0.10%	0.11%	0.11%	0.10%	0.07%	0.06%	0.04%	0.03%	0.01%

Source: World Bank: Global Development Finance Database

TABLE 13

Expected Economic Value of Switching from Mean-Variance to D-Vine Copula Modelling

Economic Values Constrained when A=0.25 and RA=20									
	2001	2002	2003	2004	2005	2006	2007	2008	2009
Debt Cons	6.33E-03	3.33E-04	2.47E-04	6.10E-03	0.142	1.11E-03	8.38E-03	0.1238	0.083
Trade Cons	0.552	4.70E-15	1.58E-15	0.0965	0.223	2.68E-03	0.23525	0.09075	1.51E-03

Notes: (i).The table shows the annualised expected economic value for attending features of asymmetries and fat-tails by switching from mean-variance to D-vine copula Modelling. The value is calculated as how much earnings can be deducted to lower the D-vine copula model's utility down to the same level as the mean-variance model's utility.(ii).The optimal currency compositions based on which the economic value is obtained are calculated with debt or trade constraints. These constraints are set as minimal weights of currencies for China's debt or transactions with its trading partners, and the weights are taken as 50% of each partner' share in China's debt or trade relation.

TABLE 14

Currency Composition by D-vine Copula with Trade Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	7.97%	38.07%	7.65%	35.45%	12.46%	19.92%	7.46%	50.15%	31.14%
EURO	6.21%	7.25%	21.10%	9.48%	11.13%	6.48%	6.77%	6.70%	6.80%
JPY	75.41%	8.29%	7.96%	7.37%	11.48%	5.97%	5.53%	22.78%	24.64%
GBP	1.39%	8.29%	18.58%	7.29%	5.89%	11.76%	1.30%	1.15%	1.00%
CHF	0.34%	13.58%	0.49%	0.25%	5.22%	0.33%	0.37%	1.23%	0.75%
CAD	0.72%	1.30%	0.90%	0.99%	12.03%	9.83%	16.19%	1.24%	2.28%
AUD	1.01%	1.00%	1.09%	0.98%	5.48%	2.24%	1.67%	2.06%	5.52%
SND	1.33%	1.45%	1.29%	1.30%	6.20%	28.94%	2.44%	5.41%	2.46%
NZD	0.22%	2.54%	34.78%	8.91%	4.59%	1.48%	0.64%	0.30%	0.63%
KRW	3.53%	4.50%	4.03%	5.46%	8.90%	8.22%	3.68%	3.84%	3.55%
RUB	1.05%	12.96%	1.12%	21.76%	10.90%	4.03%	46.93%	3.20%	1.25%
THB	0.82%	0.76%	1.02%	0.75%	5.73%	0.79%	7.02%	1.94%	20.00%

TABLE 15

Currency Composition by D-vine Copula with Debt Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	46.32%	45.99%	35.77%	35.52%	37.68%	38.48%	40.69%	49.46%	45.86%
EURO	2.48%	4.97%	19.05%	15.41%	4.21%	4.49%	4.41%	3.94%	7.91%
JPY	7.40%	7.78%	8.46%	8.08%	6.82%	6.09%	4.29%	14.36%	13.72%
GBP	0.99%	9.74%	3.41%	0.48%	0.33%	8.23%	0.43%	0.17%	2.58%
CHF	0.20%	14.75%	0.33%	0.25%	0.12%	0.17%	0.18%	2.08%	5.95%
CAD	41.11%	0.70%	0.23%	0.24%	27.24%	6.72%	17.37%	4.22%	0.02%
AUD	0.18%	0.17%	0.31%	0.17%	12.58%	0.93%	0.85%	4.73%	5.05%
SND	0.30%	0.39%	0.12%	0.13%	0.20%	22.78%	1.02%	7.56%	1.60%
NZD	0.13%	2.94%	31.51%	38.77%	6.52%	1.25%	0.63%	4.18%	6.27%
KRW	0.70%	2.49%	0.28%	0.89%	2.81%	7.48%	1.08%	2.82%	1.89%
RUB	0.07%	9.96%	0.28%	0.05%	0.88%	3.32%	23.53%	4.54%	5.14%
THB	0.12%	0.12%	0.24%	0.02%	0.60%	0.05%	5.51%	1.94%	4.02%

TABLE 16

Currency Composition by Gaussian Copula with Trade Constraints

	2001	2002	2003	2004	2005	2006	2007	2008	2009
USD	32.97%	32.71%	7.67%	7.60%	7.72%	9.12%	7.46%	43.90%	35.22%
EURO	6.25%	7.17%	21.46%	27.59%	6.34%	6.47%	6.77%	6.51%	6.87%
JPY	8.75%	8.32%	7.98%	7.47%	6.59%	5.97%	5.52%	21.84%	25.15%
GBP	1.57%	7.18%	13.51%	1.48%	1.11%	8.22%	1.30%	0.94%	1.01%
CHF	0.39%	16.89%	0.58%	0.55%	0.31%	0.33%	0.37%	0.29%	0.73%
CAD	39.95%	1.68%	0.99%	1.11%	26.96%	13.11%	19.56%	0.85%	2.88%
AUD	1.09%	1.05%	1.38%	1.10%	5.99%	1.61%	1.67%	1.31%	5.98%
SND	1.42%	1.70%	1.32%	1.36%	1.37%	42.38%	2.45%	1.57%	2.63%
NZD	0.27%	3.20%	37.85%	43.74%	5.76%	0.78%	0.63%	0.24%	0.54%
KRW	5.24%	4.91%	4.10%	4.16%	4.32%	6.60%	4.16%	3.69%	3.62%
RUB	1.23%	14.30%	2.06%	2.87%	32.50%	4.56%	42.60%	1.78%	1.24%
THB	0.86%	0.88%	1.10%	0.96%	1.02%	0.85%	7.49%	17.07%	14.11%