

Arbitrary Modulation of Average Dwell Time in Discrete-Time Markov Chains based on Tunneling Magnetoresistance Effect

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Abstract—Stochastic processes (SPs) are widely used in many real-world fields, especially AI algorithms and models. A discrete-time Markov chain (DTMC) is a fundamental SP where the probability of each event depends only on the state attained in the previous event. DTMC is extensively used in signal processing and information theory, but the hardware generation of DTMC remains hindered by the difficulty in arbitrarily modulating the averaged dwell times (ADTs), i.e. the average time that the DTMC stays at one state. In this paper, we propose a two-step procedure to modulate the ADTs of a DTMC generated from single magnetic tunnel junction (MTJ), without being limited by additional restrictions such as a fixed ratio between the ADTs, widening the SP applications scope of the MTJ-based DTMC. This method has been verified via mathematical derivation and electrical characterization. The generation throughput and power consumption can also be conveniently modulated. This procedure provides a new hardware solution for the generation of stochastic signal in semiconductor IC chips.

Index Terms—Markov chain, magnetic tunnel junction, probabilistic switching, spintronic

I. INTRODUCTION

Markov process is a stochastic process with a sequence of possible events, in which the probability of the next event depends only on the state attained in the present event, and is independent of the past events [1] [2]. It is different from the generic random bit stream generation, in which the random

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variables are totally independent. Markov processes with discrete time are known as discrete-time Markov chains (DTMC), and are widely used in wireless communications [3] [4], Internet traffic modeling [5], information encryption [6] [7] and financial engineering [8], helping to provide stochasticity for the systems and meanwhile being mathematically trackable [9].

A DTMC is formed by segments of alternating consecutive appearing states, and the lengths of the segments, often called dwell times (DT), are randomly but exponentially distributed [2]. The average dwell time (ADT) represents DTs' distribution and it is desirable that the ADTs could be adjusted to any target sets, for many practical applications such as the stabilization of stochastic systems in robust control theory [10] and fuzzy systems [11].

Most of the existing DTMCs are generated by software algorithms. However, for the Internet-of-Things (IoT) applications, it is desirable that DTMCs could be directly generated by hardware devices, for the performance, area, and power considerations. Recently, noise and variability of scaling/emerging devices [12] has been utilized for DTMC generation. Random telegraph noise induced by defect carrier trapping/de-trapping [13] [14] is a natural Markov chain, but its ADTs cannot be arbitrarily modulated for a determined defect. The variability in a resistive-switching memory (RRAM) has been used to generate DTMC, but with non-modulable transition matrix [12] [15], limited speed (\sim ms), and limited endurance (10^5).

Spintronic devices have been used to generate telegraphic switching signal [16] [17] similar to the random telegraph noise [13] [14] and also belongs to the Markov chain family [18], but it requires both a magnetic field and electrical pulses, the former of which is not easy to implement in on-chip circuitry. Our recent work proposed a DTMC generator with excellent endurance and uniformity [19], based on the probabilistic switching [20] of single magnetic tunnel junction (MTJ) [21] [22] and a specially designed waveform, with longer endurance, lower energy consumption and pure electrical operation.

In this work, we proposed a two-step procedure to control the ADTs of DTMC, using only one MTJ device and electrical operations. The arbitrary ADTs pairs of the two states can be electrically modulated. The correctness of this procedure has been mathematically derived and experimentally verified. High-quality DTMCs have been generated in a wide range. The method also shows promising potential in scaling the energy consumption and boosting the throughput, and provides a

hardware solution for the practical hardware generation of DTMC in the IoT era.

II. DEVICE AND EXPERIMENT

The device is a bottom pinned perpendicular magnetization anisotropy (PMA) MTJ with a diameter of 78 nm. **Fig. 1a** demonstrates its magneto-resistive switching between the low-resistance parallel (P) state, and the high-resistance anti-parallel (AP) state with the cross section scanning electron microscopy (SEM) image (**inset of Fig. 1a**). **Fig. 1b** shows the sigmoid-like dependence of switching probability on pulse amplitudes with fixed pulse width of 500 ns, which can be explained by the Néel-Brown equation [23]:

$$P(t) = 1 - \exp(-t/\tau) \quad (1)$$

where $P(t)$ is the thermal switching probability, and τ is the relaxation time:

$$\tau = f_0^{-1} \exp(E_b/k_B T) \quad (2)$$

where f_0 is the attempt frequency, E_b is the energy barrier, and T is the temperature.

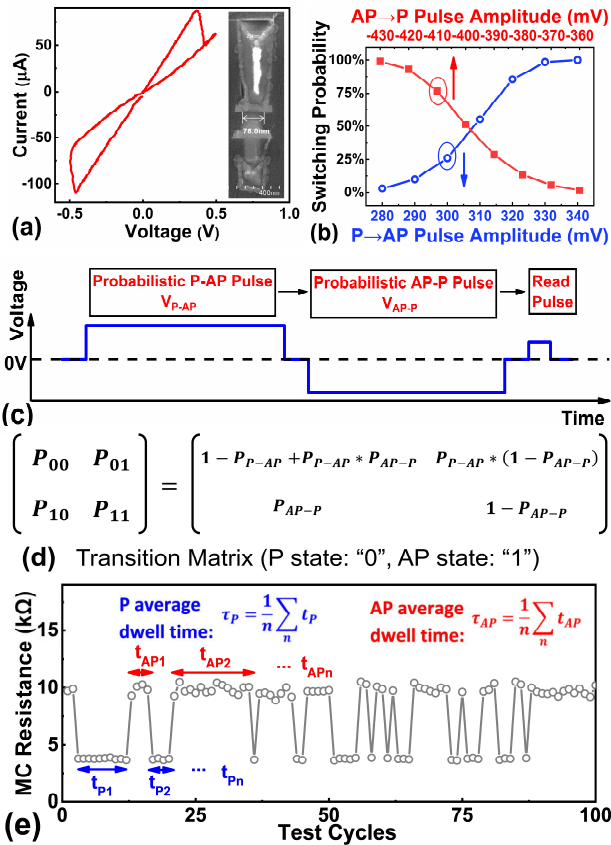


Fig.1. (a) The I-V curve of magneto-resistive switching. Inset: cross-section SEM image of the MTJ. (b) The sigmoid-like dependence of switching probability on pulse amplitudes with pulse width of 500 ns. (c) The three-pulse waveform. (d) The transition matrix. (e) Partial demonstration of the generated DTMC, with the dwell time (t_p and t_{AP}) and their average values (τ_p and τ_{AP}).

The DTMC is generated by consecutively applying the three-pulse waveform onto the MTJ (**Fig. 1c**). The details of DTMC generation setup and the transition matrix including transition probabilities P_{00} , P_{01} , P_{10} , P_{11} (**Fig. 1d**) can be found in [19]. Herein, the “time” in ADT refers to the number of consecutive identical bits, each of which is generated from one cycle. All

electrical measurements were done using the pulse measurement units in a Keithley 4200 semiconductor analyzer.

III. RESULTS AND DISCUSSIONS

In a DTMC, t_i denotes the dwell time, i.e. the time that the DTMC remains in one state i (in this work, $i = 0$ or 1 , which corresponds to the state P or AP, respectively) until the next transition, and n_i denotes the number of segments in state i , while τ_i denotes the averaged value of t_i . **Fig. 1e** partially demonstrates the generated DTMC, together with the t_i and τ_i . Mathematically, the dependence of τ_i on the transition probabilities can be derived in the following:

In the DTMC, t_i is a random variable with probability distribution

$$\Pr[t_i = m] = P_{ii}^{m-1}(1 - P_{ii}). \quad (3)$$

Then its expectation can be calculated as follows:

$$\begin{aligned} E[t_i] &= \sum_{m=1}^{\infty} m P_{ii}^{m-1}(1 - P_{ii}) \\ &= \frac{1 - P_{ii}}{P_{ii}} \sum_{m=1}^{\infty} m P_{ii}^m = \frac{1}{1 - P_{ii}}. \end{aligned} \quad (4)$$

In a DTMC of infinite length, it follows from the law of large numbers that τ_i converges to $E[t_i]$ (which is equal to $\frac{1}{1 - P_{ii}}$ when the DTMC length is infinite). According to the transition probabilities definition shown in **Fig. 1d**, the ADT of AP and P states, i.e. τ_{AP} and τ_P , converge to $\frac{1}{P_{AP-P}}$ and $\frac{1}{P_{P-AP}(1 - P_{AP-P})}$, respectively.

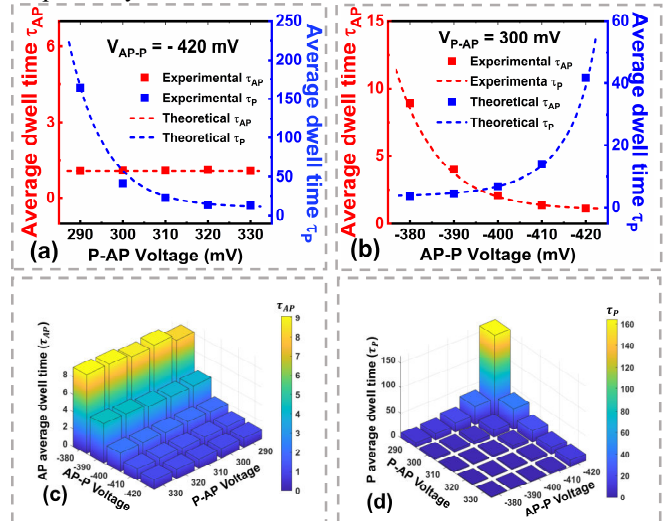


Fig.2. The dependence of τ_{AP} and τ_P on (a) V_{P-AP} and (b) V_{AP-P} is similar to the theoretical counterpart, with fixed V_{AP-P} and V_{P-AP} , respectively. The (c) τ_{AP} and (d) τ_P across a range of V_{P-AP} and V_{AP-P} .

Note that the P-AP (or AP-P) means that the device is intended to be switched from P (or AP) state to AP (or P) state with a specific probability. As show above, τ_{AP} is only determined by P_{AP-P} , while τ_P is determined by both P_{AP-P} and P_{P-AP} . Such dependency has been experimentally verified in **Fig. 2**: with a fixed V_{AP-P} of -420mV , when V_{P-AP} increases, τ_P gradually decreases, but τ_{AP} remains constant (**Fig. 2a**); if V_{P-AP} is fixed at 300mV and V_{AP-P} increases, τ_P increases but τ_{AP} decreases (**Fig. 2b**), both in agreement with the trend of

theoretical values after calibration. Such dependency is further supported by the distribution of τ_P and τ_{AP} with V_{AP-P} and V_{P-AP} covering a wider range, as visualized in the 3D plots in **Fig. 2c** and **2d** based on experimental data collected from a range of V_{P-AP} and V_{AP-P} . ADTs cover a wide range as the switching probability varies from 0 to 1. The different dependences of τ_P and τ_{AP} on the pulse conditions provides the inspiration that a target set of τ_{AP} and τ_P combination can be reached following a two-step procedure (**Fig. 3**): (i) Since τ_{AP} is not affected by P_{P-AP} , for a target τ_{AP} , a corresponding P_{AP-P} can be obtained, which is given by a suitable V_{AP-P} from the sigmoid-like dependence in **Fig. 1b**. (ii) Next, for the target τ_P , since P_{AP-P} is already determined, a suitable P_{P-AP} can be calculated given by the V_{P-AP} .

In this way, the two-step procedure facilitates the electrical modulation of any arbitrary ADT pairs of DTMC generated from a single MTJ entirely under electrical operation. Note that this methodology also works, if the polarities of the two probabilistic pulses in **Fig. 1c** are reversed: τ_P can be realized first, then τ_{AP} .

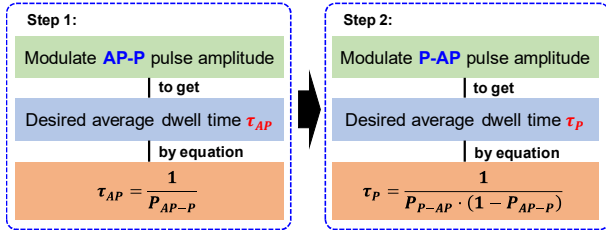


Fig. 3. The two-step procedure to modulate ADT in a DTMC: (i) the target τ_{AP} is achieved by setting V_{AP-P} , regardless of V_{P-AP} . (ii) with fixed V_{AP-P} and determined τ_{AP} , the target τ_P is achieved by setting V_{P-AP} .

Fig. 4 demonstrates the DTMCs with an example target set of ADTs ($\tau_{AP} = 2.5$ and $\tau_P = 1.6$), generated with four different pulse widths. As shown in **Fig. 4a**, the bits in a DTMC can be generated using probabilistic pulses with width scaling from $100 \mu\text{s}$ to 100ns , limited by the instrument's minimum pulse width level and the measurement time [24], while maintaining the same set of ADTs stably. Note that the method is several orders faster than the existing MTJ-based superparamagnetic oscillation [25] [26] which are normally with millisecond time scale [27]. This indicates the potential energy efficiency scaling to lower than 10 pJ/bit , as the energy consumption scales linearly with pulse width in a log-log plot (**Fig. 4b**). Furthermore, the dwell time, τ_P and τ_{AP} of the generated DTMCs all follow the exponential distribution, as evidenced in the cumulative density function (CDF) in **Fig. 4c** and **4d**, supporting their Markovian property [1].

The randomness of the DTMC has been tested by a series of methods [19]: in **Fig. 6a**, the absence of any discernible pattern in the bitmap confirms the balance and randomness of the DTMC bits with pulse width of 100 ns . The Hamming distance (HD) and entropy of this DTMC are 0.49 (close to ideal value 0.5) and 0.96 (close to ideal value 1.0), respectively. The autocorrelation function (ACF) of the DTMC with various pulse width (**Fig. 6b**) shows the average 95% confidence boundary being only ~ 0.03 , comparable to the established benchmarks [28].

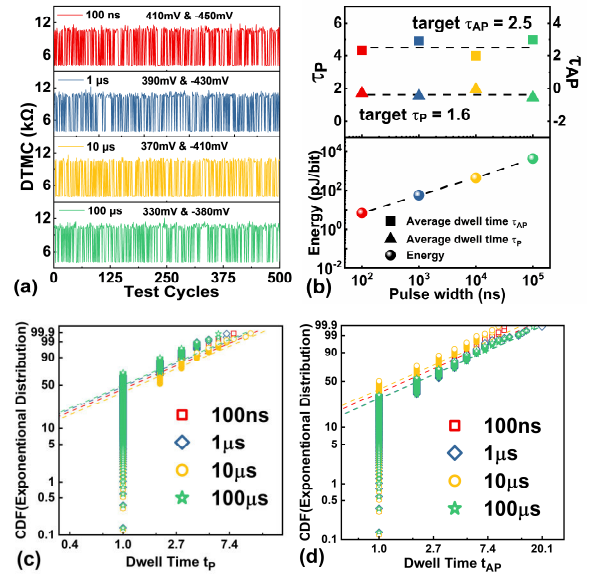


Fig. 4. (a) The generation speed shows scaling capability by reducing the probabilistic pulse width from $100 \mu\text{s}$ to 100ns . (b) With pulse amplitude adjustment, the generated DTMCs show the same ADT, while the energy per cycle can be scaled below 10 pJ . (c-d) The exponentially distributed cumulative density function (CDF) of τ_P and τ_{AP} in the DTMC, supporting their MC property.

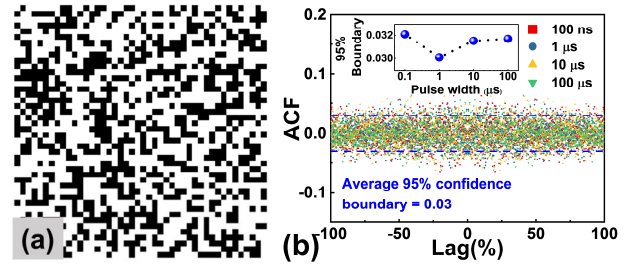


Fig. 5. (a) The bitmap (ap: black; p: white) with the pulse width of 100 ns shows the superior randomness performance of DTMC signal. (b) The autocorrelation function (ACFs) of the obtained DTMC signals show average 95% confidence boundary as low as only ~ 0.03 .

Recently, the concept of persistent dwell time (PDT) proves to be useful in investigating the Markov jump systems (MJSs) based on MCs. Compared with DT or ADT, PDT is more general because it can represent the intermittent occurrence of fast and slow switching existing in the practical and complex MJSs [11]. We will continue to work along this line and analyze the PDT in our DTMC which is a two-level switching signal generated from practical MTJ devices and could be affected by complex spintronic conditions.

IV. CONCLUSIONS

In this work, we proposed a two-step procedure to control the ADT of DTMC, using only one MTJ device with electrical operations. The arbitrary ADT pairs of the two states can be electrically modulated. The correctness of this procedure has been mathematically derived and experimentally verified. High-quality DTMCs have been generated with a wide range of ADTs. The proposed method also shows promising potential in scaling the energy consumption and boosting the throughput and provides a hardware solution for the practical hardware generation of DTMC in the IoT era.

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