Identifying the Cognitive Predictors of Early Counting and Calculation Skills: Evidence from a Longitudinal Study

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Abstract

The extent that phonological, visual-spatial STM and non-symbolic quantitative skills support the development of counting and calculation skills was examined in this 14-month longitudinal study of 125 children. Initial assessments were made when the children were 4:8. Phonological awareness, visual-spatial STM and non-symbolic approximate discrimination predicted growth in early calculation skills. These results suggest that both the approximate number system and domain-general phonological and visual-spatial skills support early calculation. In contrast, only performance on a small non-symbolic quantity discrimination task (where the presented quantities were always within the subitising range) predicted growth in cardinal counting skills. These results suggest that the development of counting and calculation are supported by different cognitive abilities.

Keywords: counting; calculation; approximate number system; phonological awareness; visual-spatial short-term memory

Identifying the Cognitive Predictors of Early Counting and Calculation Skills: Evidence from a Longitudinal Study

Learning to count and calculate are vital first steps towards mathematical competence. Theoretical models (Krajewski & Schneider, 2009; LeFevre et al., 2010) have identified verbal, visual-spatial and quantitative abilities as important independent predictors of mathematical development. The current study examines the influence of these abilities on the development of early number skills. The extent to which phonological awareness, visual-spatial short-term-memory (STM) and non-symbolic quantitative skills predict *growth* in counting and calculation is examined.

The Development of Counting and Calculation

Young children develop their counting and calculation skills through informal everyday experiences and later through formal school-based instruction. *Sequential counting* refers to the ability to recite the number-word sequence and acknowledge the position of a number word in this sequence without necessarily understanding its cardinal meaning (Fuson, 1992; Gelman & Gallistel, 1978). The initial stages of sequential counting often develop before children enter formal schooling (Case & Griffin, 1990; Gelman & Gallistel, 1978; Mix, Sandhofer, & Baroody, 2005; Siegler, 1991; Spelke, 2000; Wynn, 1992). Gradually children develop the ability to apply their knowledge of the number-word sequence to enumerate sets (Gelman, Meck, & Merkin, 1986; Wynn, 1992). This serial quantification process is referred to as *cardinal counting* and requires mapping each number-word onto each item in a set in one-to-one correspondence to acknowledge the exact number of items in a collection (Fuson, 1988, 1992; Gelman & Gallistel, 1978). Many pre-school children can also complete non-verbal calculations where the quantities are represented by objects (Barth, La Mont, Lipton, & Spelke, 2005; Huttenlocher, Jordan, &

Levine, 1994; Jordan, Huttenlocher, & Levine, 1992; Levine, Jordan, & Huttenlocher, 1992; Rasmussen & Bisanz, 2005; Starkey & Gelman, 1982; Zur & Gelman, 2004). However, fewer pre-school children are able to perform *formal calculations* (involving number words or symbols). Proficiency with formal calculations increases dramatically during the first years of schooling (Jordan et al., 1992; Levine et al., 1992; Rasmussen & Bisanz, 2005). The importance of these number skills is supported by empirical studies indicating that both early counting (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Johansson, 2005) and calculation (Krajewski & Schneider, 2009; LeFevre et al., 2010; Aunio & Niemivirta, 2010) are effective predictors of later mathematical attainment. Furthermore, the establishment of secure counting and calculation skills is a core aim of early Primary curricula (e.g. United States National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; England, Department of Education, 2014; Singapore, Ministry of Education, Singapore, 2012). It is important to understand the cognitive basis of counting and calculation as both are fundamental building blocks of early mathematics and key topics in the first years of children's education.

Theoretical Models of Number Processing and Mathematical Development

Dehaene's triple-code model of number processing (Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Stanescu, Pinel, & Tsivkin, 1999) identifies three different types of representations utilised during mathematical tasks. It is proposed that all numerical tasks involve the processing of abstract numerical representations which are associated with neural activity in the intraparietal sulcus. Visual-spatial or phonological representations may also be recruited depending on the nature of the task. For example, phonological representations are utilised in tasks such as arithmetic fact retrieval, whilst visual-spatial representations are utilised in tasks such as number comparison where reference to an internal number line is required. Dehaene's model has influenced developmental models of early mathematics with both LeFevre et al.'s (2010) pathways model and Krajewski and Schneider's (2009) arithmetical development model proposing that phonological processing, visual-spatial STM and quantitative skills differentially influence the development of different number skills. These cognitive skills therefore form the focus of the current study.

Cognitive Determinants of Early Counting and Calculation

Phonological awareness.

Phonological awareness refers to the ability to encode, access and manipulate speech sounds within words (Alloway, Gathercole, Willis, & Adams, 2004). It is one of a family of phonological processing skills that also includes phonological memory and rate of access to phonological codes (often assessed using rapid naming tasks) (Hecht, Torgeson, Wagner, & Rashotte, 2001; Snowling, 2000). Whilst memory skills may influence children's performance on phonological awareness tasks (Hecht et al., 2001), the strength of children's underlying long-term phonological representations significantly influences their phonological awareness (Fowler, 1991; Snowling, 2000).

Krajewski and Schneider (2009) and Koponen, Salmi, Eklund and Aro (2013) report associations between phonological processing abilities and young children's sequential counting skills. This is consistent with the proposition that phonological awareness facilitates the acquisition of number words in the same way that it supports the acquisition of other types of vocabulary (see Bowey, 1996; Gathercole, 2006 for discussions of the role of phonological processes in vocabulary development). Alongside this proposal, it is also argued that the strength of the phonological representations of arithmetic facts in long-term memory impacts on the efficiency with which they can be retrieved (see De Smedt, Verschaffel, & Ghesquière, 2009;

Simmons & Singleton, 2008). However, whilst phonological representations may influence arithmetic fact retrieval in older children and adults, it may not influence formal calculation skills in younger children who have not yet established a store of arithmetic facts in long-term memory (Siegler, 1996; Siegler & Shrager, 1984).

The current study aims to determine whether phonological awareness predicts growth in sequential counting, as predicted by Krajewski and Schneider (2009) and Koponen et al. (2013), as well as to clarify whether phonological awareness predicts growth in early formal calculation.

Visual-spatial STM.

Visual-spatial STM supports the generation, retention and manipulation of visual-spatial information (Logie, 1995) and is a predictor of mathematical attainment in early childhood (Bull, Espy, & Wiebe, 2008; Krajewski & Schneider, 2009; LeFevre et al., 2010; Simmons, Singleton, & Horne, 2008). However, the specific early number skills that visual-spatial STM underpins remain ambiguous. Krajewski and Schneider (2009) propose that visual-spatial STM supports cardinal, but not sequential counting. This is consistent with a concurrent relationship between three-year-olds' visual-spatial abilities and a task assessing their cardinality understanding (Ansari et al., 2003).

Associations between visual-spatial STM and children's performance on non-verbally presented arithmetic have also been reported (Krajewski & Schneider, 2009; LeFevre et al., 2010; Rasmussen & Bisanz, 2005). It is suggested that visual-spatial STM is associated with performance on non-verbal arithmetic problems because young children use non-verbal mental models to solve them (Huttenlocher et al., 1994; Rasmussen & Bisanz, 2005). However, the evidence that visual-spatial STM supports formal arithmetic (where the problems are presented using number words or symbols) is more ambiguous. Although a dual-task study supported the role of visual-spatial STM in early formal calculation (McKenzie, Bull, & Gray, 2003), some correlational studies have failed to identify a specific relationship between visual-spatial STM and formal arithmetic (Rasmussen & Bisanz, 2005; Simmons, Willis, & Adams, 2012). It may be that formal arithmetic is less influenced by visual-spatial STM than non-verbal arithmetic because children abandon non-verbal mental models in favour of counting strategies when the problems are presented aurally using number words.

The current study aims to determine whether visual-spatial STM predicts growth in cardinal counting as proposed by Krajewski and Schneider (2009) as well as to clarify whether visual-spatial STM predicts growth in early formal calculation skills.

Non-symbolic quantitative skills.

Two distinct systems for quantity processing have been proposed; one for small and precise magnitude representations and one for approximate magnitude representations (Feigenson, Dehaene, & Spelke, 2004). Such a distinction is based on the argument that these two representational systems function under different principles (Feigenson et al., 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008) and recruit different neural networks (Dehaene & Cohen, 1997; Dehaene et al., 1999; Lemer, Dehaene, Spelke, & Cohen, 2003).

The approximate number system (ANS).

The ANS is a cognitive system that generates 'noisy' abstract representations of quantities enabling approximate numerical discriminations to be made (Feigenson et al., 2004). Children's ability to discriminate between two arrays of items in conditions that prohibit verbal counting has typically been used to assess their ANS precision (see Halberda & Feigenson, 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011, 2013 for examples of typical tasks). Individual differences in approximate quantity discrimination tasks have been found to relate concurrently, retrospectively and longitudinally to mathematical attainment (e.g. Bonny & Lourenco, 2013; Halberda, Mazzocco, & Feigenson, 2008; Inglis et al., 2011; Libertus et al., 2011, 2013; Mussolin, Nys, Leybaert, & Content, 2012). However, other studies have failed to identify this relationship (Göbel, Watson, Lervåg, & Hulme, 2014; Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Lonnemann, Linkersdorfer, Hasselhorn, & Lindberg, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, van den Bussche, & Reynvoet, 2012; Soltész, Szucs, & Szucs, 2010) or suggested that this relationship is inconsistent over development (Bonny & Lourenco, 2013). Moreover two recent studies (Fuhs & McNeil, 2013; Gilmore et al., 2013) propose that positive findings could be due to non-symbolic approximate discrimination tasks unintentionally tapping inhibition skills in children, rather than the precision of their quantity representations. In both studies performance was weaker on anticorrelated trials (where surface area is inversely correlated with the number of items in the set), potentially because children had to inhibit responses based on the sets' total surface area to respond accurately. Both studies found that only performance on anticorrelated trials predicted children's mathematical attainment. Consequently the extent to which the ANS supports the development of formal mathematics is contentious.

The current study uses an approximate quantity discrimination task to evaluate the influence of the ANS on early counting and calculation. Because theoretical models propose the involvement of abstract analogue representations in calculation (e.g. Dehaene et al., 2003; Krajewski & Schneider, 2009; LeFevre et al., 2010), we anticipated that the approximate discrimination task would predict growth in calculation skills.

The precise number system (PNS) and subitising.

The extent to which the PNS supports the development of formal mathematics has been subject to less empirical scrutiny than the ANS. Whilst studies of mathematical development do not often refer explicitly to the PNS, some investigations have examined the relationship between subitising skills and mathematics in children (e.g. LeFevre et al., 2010). Subitising is the ability to enumerate small sets (of up to three or four discrete items) precisely and quickly without recourse to verbal counting (Kaufman, Lord, Reese, & Volkmann, 1949). It has been argued that subitising is a function of the PNS (Feigenson et al., 2004), with weak subitising skills sometimes being attributed to an impaired 'number module' (a modular cognitive system that processes quantity) (Butterworth, 1999, 2005, 2010). Alternative explanations of subitising do not situate it within a domain-specific module, but rather view it as reliant on domain-general cognitive skills such as working memory (see Cowan, 2001; Trick & Pylyshyn, 1994 for domain-general accounts of subitising abilities).

Weak subitising has been associated with mathematical difficulties (Fischer, Gebhardt, & Hartnegg, 2008; Landerl, Bevan, & Butterworth, 2004; Schleifer & Landerl, 2011). Furthermore, experimental evidence suggests that subitising skills support children's early understanding of the cardinal value of small number words (Benoit, Lehalle, & Jouen, 2004) and subitising efficiency has been shown to have a concurrent relationship with non-verbal calculation (LeFevre et al., 2010). An association between performance on the 'number sets test' and formal calculation has also been reported (Fuchs et al., 2010). The 'number sets test' assesses the efficiency with which children manipulate small quantities within and just outside the subitising range. However, this test requires knowledge of number symbols and therefore performance may be influenced by the associations between abstract quantity representations and the symbols rather than speed of subitising *per se*.

The current study utilises a novel small discrimination task that assessed children's ability to quickly and precisely discriminate among non-symbolic quantities within the subitising range. Consistent with Benoit et al.'s (2004) association between cardinal understanding and subitising, we expected performance on the small discrimination task to predict growth in children's cardinal counting skills.

Aim of the study

The core aim of the study is to determine the extent to which phonological awareness, visual-spatial STM and non-symbolic quantitative skills support growth in sequential counting, cardinal counting and calculation during children's first year of schooling. Previous longitudinal studies that have examined the cognitive influences on early counting and calculation have either not assessed quantitative skills directly (e. g. Koponen et al., 2013; Krajewski & Schneider, 2009), used a subitising measure alone (LeFevre et al., 2010) or used quantitative skills measures that require direct knowledge of the formal number system (Fuchs et al., 2010). We extended previous studies by including a typical non-symbolic approximate discrimination task similar to those used to index children's ANS (e.g. Halberda & Feigenson, 2008; Inglis et al., 2011; Libertus et al., 2011, 2013). The current study enabled the relative influence of the ANS to be evaluated alongside that of phonological and visual-spatial STM skills. The use of a longitudinal design, with initial individual differences in the outcome measures controlled, enabled stronger conclusions about the skills that predict *growth* in early counting and calculation to be drawn.

In addition to specific measures of sequential counting, cardinal counting and formal calculation, standardised measures of mathematics and reading attainment were administered. These standardised measures enabled us to examine the extent that the attainment of the sample was representative of the general population, to replicate previous studies that have examined the relationship between ANS functioning and general mathematics attainment and to explore whether any relationships between the cognitive predictors and the number skills measures were specific to the number domain.

Method

Participants

Children were recruited from the Reception classes of five primary schools in England. Data was collected during the Reception Year (T_1) when children were 4 years, 8 months (*SD* = 4.05) and 14 months later in Year 1 (T_2) when children were 5 years, 10 months (*SD* = 3.96). At the start of the study 131 children were recruited. However, two children did not complete the assessments at T_1 (one due to prolonged absence and one because she expressed a desire to leave the study). Three children did not complete the assessments at T_2 (one due to prolonged absence and two because their families moved abroad). One further child was excluded from the analyses because she did not follow the task instructions at T_1. This resulted in a final sample of 125 children (68 males) who are included in all the analyses presented. When compared with the national average, two of the participating schools had a below average proportion of children eligible for free school meals, two had an average proportion and one had an above average proportion (free school meals eligibility levels were obtained from the schools' most recent inspection report http://www.ofsted.gov.uk).

Measures

Phonological awareness. Two oral subtests from the Preschool and Primary Inventory of Phonological Awareness (PIPA) (Dodd, Crosbie, MacIntosh, Teitzel, & Ozanne, 2000) were administered. In the *Syllable Segmentation* subtest the child is asked to reproduce a word said by the experimenter while tapping a drum picture to indicate each syllable in the word. This test consists of four practice items and 12 experimental items. In the *Rhyme Awareness* subtest the child is asked to identify the word (from a choice of four) that does not rhyme with the others. This test consists of two practice items and 12 experimental items. For these two subtests internal consistency is .84 and .83 respectively (PIPA manual, Dodd et al., 2000).

Visual-spatial STM. Two subtests from the Automatic Working Memory Assessment (Alloway, 2007) were administered. Both tests consist of three practice items and seven levels of increasing difficulty presenting up to six different items per level. In the *Block Recall* subtest the child is asked to use a finger to tap the same blocks (in the same order as) a video of a finger tapping blocks did previously. In the *Mazes Memory* subtest the child is asked to use a finger to trace the route inside a black maze that a red line previously showed for three seconds. For both subtests the standardised scoring and stopping rules were applied. For these two subtests test-retest reliability is .83 and .81 respectively (Alloway, Gathercole, & Pickering, 2006).

Computerised discrimination tasks. Three computerised tasks programmed with E-prime were used to assess children's speed of processing, approximate discrimination and small discrimination skills. For these three tasks children viewed two stimuli simultaneously presented and spatially separated (one on each side) on a computer screen. They were asked to press as fast and accurately as they could the key that matched the side (left or right) where they believed the correct answer appeared. Responses were made via a two-key response box. The correct answer appeared the same number of times on each side of the screen in a random order.

Speed of processing. A baseline measure of children´s Response Times (RTs) on a two-choice non-numerical task was obtained to index general speed of processing. In each trial, two triangles were presented until the participant responded. One triangle had eyes and a smile and the other was empty. Children had to indicate which of the two triangles contained the happy face. There were three practice trials and 12 test trials. A feedback message was displayed for 2,000 ms after each response. Then the next trial automatically started. Accuracy and RTs were recorded.

Approximate discrimination. We used an adaptation of Nordman, Bull, Davidson and Church's (2009, September) non-symbolic approximate discrimination task. Two arrays of circles that contained the sterling pound symbol were presented for 2,000 ms. Circles in one set were green and in the other set were purple. Children had to indicate the more numerous array.

This task consisted of four practice trials and 100 test trials. The number of circles in each set varied from five to 35, with each pair depicting a numerical ratio difference ranging from 1.1 to 1.5. Ten pairs were presented for each ratio bin used in the task (see Table 1). The circles varied in size inter and intra set and ranged in diameter from 1 cm to 1.9 cm. Half of the test trials were surface area correlated (i.e. the more numerous set had the larger total surface area) and the other half were surface area anticorrelated (i.e. the more numerous set had the smaller total surface area). The number of correlated and anticorrelated trials was balanced within each ratio bin and the task as a whole. Feedback was given after each response and consisted of the presentation of either a chest full of coins (after a correct response) or an empty chest (after an incorrect response) for 2,000 ms. If no response was given within 2,000 ms an incorrect response was automatically recorded and no feedback was provided. The following trial automatically started after feedback or time limit elapsed. Correct responses given within the time limit were recorded. As this was a lengthy task, test trials were presented in four blocks to prevent fatigue. Encouraging feedback was given after each block.

Table 1: Stimulus Pairings in the Approximate Discrimination Task for Each Numerical Ratio Bin

Ratio Bin	Left answer required					Right answer required				
1.5	9:6	15:10	18:12	27:18	30:20	6:9	10:15	12:18	18:27	20:30
1.4	7:5	14:10	21:15	28:20	35:25	5:7	10:14	15:21	20:28	25:35
1.3	9:7	13:10	18:14	26:20	30:23	7:9	10:13	14:18	20:26	23:30
1.2	6:5	12:10	18:15	24:20	30:35	5:6	10:12	15:18	20:24	25:30
1.1	11:10	12:11	22:20	32:29	33:30	10:11	11:12	20:22	29:32	30:33

Note. Adapted from Nordman, Bull, Davidson and Church (2009, September). Ratio Bin = n_2 / n_1, n_2 being the larger set.

Small discrimination. Two arrays separated by a black vertical line were presented, each array consisting of one to three red circles. Children had to indicate the more numerous array. The stimuli remained visible until they responded. This task consisted of four practice trials and 36 experimental trials balanced across three numerical comparisons (1:2, 2:3 and 1:3). The circles varied in size inter and intra set and ranged in diameter from 0.2 cm to 4.6 cm. A feedback message was displayed for 2,000 ms after each response. Then the following trial automatically started. Accuracy and RTs were recorded.

Contour length and surface area were manipulated so that these continuous variables could not be consistently associated with the correct response. We manipulated the total surface area of the sets in 18 of the trials. Six trials were created in which the total surface area of the sets was equated, six trials in which the correct set had the larger total surface area and six trials in which the incorrect set had the larger total surface area. We manipulated contour length in the other 18 trials. Six trials were created where the total contour length of both sets was equated, six trials in which the correct set had the longer total contour length and six trials on which the incorrect set had the longer contour length. The trials appeared in a pseudo-random order.

In all three computerised discrimination tasks one point was awarded for each correct response.

Number skills.

Sequential counting. Children were asked to recite the number-word sequence starting from one and were stopped at 20. They were then asked to recite the next six number words starting from 25, 65, 75, 95 and 155. Children were stopped after four consecutive incorrect answers.

Cardinal counting. Children were asked how many animals there were on a sheet of card (e.g. 'How many bears are there?'). Printed on each card were randomly distributed pictures of animals (each animal was approximately 2.5 cm^2). There were 20 items with the number of animals increasing on each trial. The numbers of animals in the items were 3, 5, 7, 8, 10, 11, 13, 15, 16, 18, 20, 23, 25, 27, 30, 35, 42, 51, 66 and 97. Children were told they could touch the pictures if they wished. Children were stopped after four consecutive incorrect answers.

Story problems. Arithmetical problems presented in story format were used to assess children's formal calculation skills. The experimenter read the story problem to the child. A picture that related to the story context but provided no concrete support for the calculation was shown during each problem. For example, *'Four people live in a house. Three people move in with them. How many people live in the house now?'* was read, while the child saw a picture of a house. The problems were presented in increasing difficulty in two sections: additions and subtractions. The addition section consisted of five problems where the addends were both single-digit numbers followed by five problems where a single-digit number had to be added to a two-digit number. The subtraction section consisted of five problems involving two single-digit numbers followed by five problems where a single-digit number had to be subtracted from a two-digit number. Children were stopped after four consecutive incorrect answers on each section. In all three number skills tasks one point was awarded for each correct answer.

Standardised attainment measures.

Mathematics. Two subtests of the Wechsler Individual Achievement Test Second UK Edition (WIAT-II^{UK}) (Wechsler, 2002) were administered. The *Mathematical Reasoning* subtest includes counting, identifying shapes and problem solving. The *Numerical Operations* subtest includes numeral identification, numeral writing and solving written arithmetic problems. For both subtests the inter-item reliability for the standardisation sample is <.80 (WIAT-II^{UK} manual, Wechsler, 2002).

***Reading*.** The *Early Word Recognition* subtest of the York Assessment of Reading for Comprehension (YARC) (Hulme et al., 2009) was administered. The child is asked to read aloud 15 regular and 15 irregular words presented alternately. Internal consistency is .98 (YARC manual, Hulme et al., 2009).

For all the standardised tests the standardised scoring and stopping rules were applied.

General Conceptual Ability.

Two subtests of the Early Years Core Scales of the British Ability Scales Second UK Edition (BAS II^{UK}) (Elliott, Smith, & McCulloch, 1996) were administered to assess children's general conceptual ability. The *Picture Similarities* subtest assesses non-verbal reasoning. The child sees a set of four different pictures and is given a card containing an additional picture that needs to be matched with the picture in the set it best relates to. The *Naming Vocabulary* subtest assesses expressive vocabulary. The child is asked to name individual pictures. The internal consistency for the two subtests is .81 and .75 respectively (BAS II^{UK} manual, Elliott et al., 1996). For both ability tests the standardised scoring and stopping rules were applied.

Procedure

Ethical approval was granted by the university research ethics panel. Written consent was gained from the schools' head-teachers and the parents or guardians of the children. Prior to assessment, verbal assent was requested from each child. Children were assessed individually in a quiet area of their school.

At T_1 participants had their speed of processing, phonological awareness, visual-spatial STM, quantity discrimination skills, number skills and mathematical attainment assessed. At T_2

participants' had their speed of processing, quantity discrimination skills, number skills, general conceptual ability and mathematical and reading attainment assessed.

Computer tasks were combined with verbal and/or pencil-and-paper tasks in each session. For both time points, each child completed three sessions lasting approximately 20 minutes each (the sessions were slightly longer at T_2 because the children completed more items in some tests). The order of the presentation of the assessments was randomised across children.

Results

Phonological Awareness and Visual-spatial STM

Principal Component Analysis (PCA) was conducted to produce composite measures of Phonological Awareness and Visual-spatial STM. Composite measures are preferable to using single tests because they reduce the influence of task-specific variance (see Bowey, 2005 for a discussion of this issue). The raw scores for Syllable Segmentation (M = 4.56, SD = 2.39), Rhyme Awareness (M = 5.07, SD = 2.51), Block Recall (M = 10.80, SD = 3.20) and Mazes Memory (M = 6.12, SD = 4.54) were entered into a two fixed-factor PCA. The Kaiser-Meyer-Olkin value was acceptable (KMO = .60) according to Kaiser (1974). Bartlett's test of sphericity was significant ($\chi^2(6) = 25.61$, $p < .001$), indicating that it is appropriate to conduct PCA on these variables. Results from the PCA were a first Component consisting of Block Recall (factor loading = .74) and Mazes Memory (factor loading = .85) and a second Component consisting of the Syllable Segmentation (factor loading = .78) and Rhyme Awareness (factor loading = .76).

All factor loadings were greater than .36 and therefore represent substantive significant values (Stevens, 1992). The resulting factor scores from the PCA were used as the Visual-spatial STM (first Component) and Phonological Awareness scores (second Component) for further analyses.

Table 2: Descriptive Statistics for Performance on the Speed of Processing, Approximate Discrimination and Small Discrimination Tasks at T_1 and T_2

	Task	*M*	*SD*	ZSkewness	ZKurtosis	Min.-Max.
T_1	Speed of Processing					
	RT	887.66	291.45	7.18	8.09	309.66-2111.25
	Accuracy	10.61 (88.42)	1.67	-7.14	7.14	3-12
	Approximate Discrimination					
	Accuracy (max. 100)	59.05	10.87	-1.09	-1.58	29-79
	Accuracy correlated trials (max.50)	30.02 (60.04)	5.83	-0.45	-1.02	14-44
	Accuracy anticorrelated trials (max.50)	29.03 (58.06)	5.76	-1.41	-0.91	15-41
	Accuracy with 1.1 ratio excluded (max.80)	49.19 (61.49)	9.96	-1.14	1.32	21-69
	Accuracy correlated trials with 1.1 ratio excluded (max.40)	24.72 (61.8)	5.22	0.14	-1.09	11-37
	Accuracy anticorrelated trials with 1.1 ratio excluded (max.40)	24.47 (61.17)	5.44	-1.59	-1.35	10-35
	Small Discrimination					
	Total RT	1549.81	803.94	11.91	18.51	521.06-4953.54
	Total accuracy	32.62 (90.61)	4.43	-8.36	6.65	18-36
T_2	Speed of Processing					
	RT	675.69	148.11	8.23	14.53	453.55-1465.10
	Accuracy (max.12)	11.44 (95.33)	.91	-11.59	19.91	7-12
	Approximate Discrimination					
	Accuracy (max. 100)	70.56	9.97	-3.23	1.60	40-88
	Accuracy correlated trials (max.50)	35.47 (70.94)	5.56	-2.95	1.84	17-47
	Accuracy anticorrelated trials (max.50)	35.09 (70.18)	5.23	-2.91	-0.05	20-43
	Accuracy with 1.1 ratio excluded (max.80)	59.56 (74.45)	9.05	-3.54	1.49	30-75
	Accuracy correlated trials with 1.1 ratio excluded (max.40)	29.94 (74.85)	5.09	-3.45	1.21	14-40
	Accuracy anticorrelated trials with 1.1 ratio excluded (max.40)	29.62 (74.05)	4.64	-3.18	0.77	15-37
	Small Discrimination					
	Total RT	932.42	172.78	5.95	3.00	577.59-1587.16
	Total accuracy	33.59 (93.33)	2.82	-8.54	9.72	22-36

Note. RT = Response Time. Percentage accuracy shown in brackets. Correlated trials = Trials where surface area directly correlates with the number of items in the set. Anticorrelated trials = Trials where surface area inversely correlates with the number of items in the set.

Approximate Discrimination

The descriptive statistics for accuracy on the approximate discrimination task are shown in Table 2. We chose to report accuracy as the index of children's ANS precision because it has been shown to be a stronger predictor of mathematics attainment than Weber fractions or RTs (Libertus et al., 2013), because Weber fractions have been shown to be highly correlated with percentage accuracy (Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013) and because the fit of the psychophysical model can be volatile in children (Mazzocco, Feigenson, & Halberda, 2011; Mussolin et al., 2012).

[INSERT FIGURE 1 ABOUT HERE]

The impact of ratio. Figure 1 shows the proportion of correct responses for the different ratio bins at T_1 and T_2. To verify that this task engaged children's ANS we examined whether accuracy rates showed evidence of the standard ratio effect (see Halberda et al., 2008). The overall pattern had the typical ANS ratio signature with larger ratios being associated with more accurate responses at both time points. The only inconsistent finding was a slightly higher accuracy rate for ratio bin 1.2 than for ratio bin 1.3 at both time points. We tested the effect of ratio difficulty and time on performance using a 5 X 2, ratio bin (1.5, 1.4, 1.3, 1.2, 1.1) and time (T_1, T_2) repeated-measures ANOVA, with percentage correct as the dependent measure. There was a significant main effect of ratio bin, $F(4, 121) = 77.60$, $p < .001$, $\eta_p^2 = .72$. Consistent with previous findings (e.g. Fuhs & McNeil, 2013; Halberda & Feigenson, 2008), children's average percentage correct decreased as the numeric ratio decreased, $F_{linear}(1, 124) = 267.38$, $p < .001$, $\eta_p^2 = .68$. Furthermore, pairwise comparisons indicated that accuracy rates on larger numerical ratio bins compared to accuracy rates on smaller numerical ratio bins were always significantly higher ($d < .10$, $p < .01$ for all comparisons) with the exception of the comparison between 1.2 and 1.3 which did not differ significantly ($d = .01$, $p = .40$). Consistent with previous crosssectional and longitudinal findings (Halberda & Feigenson, 2008; Libertus et al., 2013) performance improved with age, $F(1, 124) = 117.37$, $p < .001$, $\eta_p^2 = .49$. There was a significant interaction between ratio bin and time, $F(4, 121) = 5.97$, $p < .001$, $\eta_p^2 = .17$. Examination of Figure 1 suggests this reflects a steeper fall in performance as numerical ratios decreased at T_2 compared to T_1.

As children found smaller ratio bins difficult (particularly at T_1) we conducted one-sample t-tests in order to determine whether they were performing significantly above chance on each ratio bin. At T_1 performance was significantly above chance except for the lowest ratio bin, ratio 1.5 $t(124) = 9.46$, $p < .001$, ratio 1.4 $t(124) = 9.10$, $p < .001$, ratio 1.3 $t(124) = 8.35$, $p < .001$, ratio 1.2 $t(124) = 7.17$, $p < .001$, ratio 1.1 $t(124) = -0.64$, $p = .52$. At T_2 children's accuracy increased so they performed above chance on all ratio bins; ratio 1.5 $t(124) = 22.32$, $p < .001$, ratio 1.4 $t(124) = 21.79$, $p < .001$, ratio 1.3 $t(124) = 15.43$, $p < .001$, ratio 1.2 $t(124) = 19.55$, $p < .001$, ratio 1.1 $t(124) = 5.31$, $p < .001$. When analysing the relationships between approximate discrimination at T_1 and the other variables we calculated accuracy based on the four ratio bins where the children scored above chance. This meant that the relationships between approximate discrimination and the other variables would not be diluted by the inclusion of the 1.1 ratio bin where children were performing at chance.

The impact of surface area congruency. Performance on correlated trials was not significantly better than on anti-correlated trials at either T_1 (when the 1.1 was excluded), $F(1, 124) = 0.99$, $p > .05$, $\eta_p^2 < .01$ or T_2, $F(1, 124) = 0.99$, $p > .05$, $\eta_p^2 = .01$. This indicates that surface area congruency did not have a significant influence on children's performance on this task.

Small Discrimination and Speed of Processing

The descriptive statistics for accuracy and RTs on the speed of processing task and on the small discrimination task are shown in Table 2. Mean accuracy for these tasks were approximately 90%. This would be expected on these relatively simple tasks. To more sensitively index individual differences on these tasks we used RTs in all further analyses. The mean RTs for all correct responses taking less than two times the interquartile range from the median for each child were used. This trimming aimed to eliminate the influence of outliers due to children's distractibility or very fast responses that are unlikely for this age. Similar data trimming techniques have been used with similar data in child samples (e.g. Butterworth, 2003; Libertus et al., 2011).

Number Skills, Attainment and Ability Measures

The descriptive statistics for the number skills, attainment and ability measures are shown in Table 3. A good spread of scores was obtained for all number skills and attainment measures. The mean standardised scores for the attainment measures are close to 100. These indicate that the attainment levels of the sample were broadly representative of the performance of children in the United Kingdom. The raw scores for the number skills and attainment measures and the ability scores for the ability measures were used for further analyses.

The concurrent criterion validity of the number skills measures was demonstrated by their relationships with the standardised mathematics measures. Single step multiple regressions with the three number skills at T_1 entered simultaneously indicated that sequential counting, cardinal counting and story problems were unique predictors of Mathematical Reasoning at T_1 (β = .21, p =.03; β =.32, p <.001; β =.19, p =.03 respectively, R^2 = .29, p <.001, df = 3, 121) and sequential and cardinal counting were unique predictors of Numerical Operations at T_1 (β = .30, p <.01; β =.30, p =.001, respectively, R^2 = .29, p <.001, df = 3, 121). Single step multiple regressions with the three number skills at T_2 entered simultaneously indicated that cardinal counting and story problems were unique predictors of Mathematical Reasoning at T_2 ($\beta = .19$, $p = .01$; $\beta =.59$, $p <.001$ respectively, $R^2 = .41$, $p <.001$, $df = 3, 121$) and of Numerical Operations at T_2 ($\beta = .21$, $p <.01$; $\beta =.57$, $p <.001$ respectively, $R^2 = .41$, $p <.001$, $df = 3, 121$). This pattern of findings suggests that sequential counting explains less variance and calculation explains more variance in mathematical attainment as children grow older.

Table 3: Raw Scores, and Standardised and Ability Scores (where applicable) for the Number Skills, Mathematical and Reading Attainment, and General Conceptual Ability Measures at T_1 and T_2

	M	*SD*	ZSkewness	ZKurtosis	Min - Max
Sequential counting (max.50)					
Sequential counting T_1	24.38	9.31	4.09	0.53	9 - 48
Sequential counting T_2	39.72	10.12	-4.59	-0.98	15 - 50
Cardinal counting (max. 20)					
Cardinal counting T_1	8.15	3.54	0.18	-2.05	1 - 17
Cardinal counting T_2	13.14	3.60	-3.86	0.88	2 - 19
Story Problems (max. 20)					
Story Problems T_1	3.26	1.83	1.32	4.58	0 - 11
Story Problems T_2	8.38	4.11	0.32	-2.21	0 - 18
Mathematical Reasoning (WIAT-II^{UK})					
Raw Scores T_1	10.89	3.34	1.04	0.09	4 - 21
Standard scores T_1	101.18	8.59	-2.03	2.59	79 - 123
Raw Scores T_2	18.63	4.41	0.50	1.12	6 - 32
Standard scores T_2	102.95	12.72	1.18	-0.28	70 - 132
Numerical Operations (WIAT-II^{UK})					
Raw Scores T_1	4.89	2.09	-2.77	-1.32	0 - 8
Standard scores T_1	97.08	7.97	-2.29	0.57	78 - 109
Raw Scores T_2	8.34	1.95	-1.45	-1.32	4 - 13
Standard scores T_2	97.61	10.14	-1.95	0.49	68 - 120
Reading (YARC)					
Raw Scores T_2	19.65	7.28	-1.41	-2.23	4 - 30
Standard scores T_2	104.64	11.81	-0.68	-0.95	74 - 127
Ability Measures (BAS II)					
Picture Similarities (ability scores) T_2	87.33	8.60	-4.14	14.46	41 - 111
Naming Vocabulary (ability scores) T_2	116.72	14.11	0.27	1.88	78 - 161

Note. Descriptive statistics for the standard scores in both mathematical attainment subtests were based on a sub-sample of 50 children at T_1, as the rest of the children in the sample were too young for the test norms to be applied at that time. BAS II ability scores are comparable to raw scores in that they represent children's relative performance on the test and are not derived from age-standardised norms. See the BAS II manual (Elliott et al., 1996) for full details.

Correlation Analyses

The concurrent correlations between the variables assessed at T_1 are presented in Table 4. After controlling for age, the only significant relationship between the four cognitive predictors was a weak relationship between visual-spatial STM and approximate discrimination. The longitudinal correlations between the cognitive variables measured at T_1 and the outcome and ability variables measured at T_2 are shown in Table 5. Predictors at T_1 have marked differences in their relationships with the outcome and ability measures at T_2. Approximate discrimination correlated with all three number skills, both mathematical attainment tests, and with reading and naming vocabulary after controlling for children's age. Small discrimination correlated significantly with cardinal counting skills and calculation skills (story problems) and with reading after controlling for children's age. Visual-spatial STM skills correlated with children's calculation skills (story problems), with both mathematical attainment tests and with picture similarities after controlling for children's age. Phonological awareness correlated with children's sequential counting skills, calculation skills (story problems), reading attainment and with naming vocabulary after controlling for children's age.

Table 4: Zero-order Concurrent Correlations (above the diagonal) and Partial Correlations Controlling for Age (below the diagonal) Between the Variables Assessed at T_1

	2	3	4	5	6	7	8	9	10	11	12	13
1. Age	-.30**	.31***	.23*	.35***	-.33***	.35***	.25**	.42***	.23*	.18	.31***	.37***
2. Speed of processing	-	-.13	-.12	-.12	-.41***	-.18*	-.22*	-.13	-.20*	-.02	-.11	-.28**
3. Approximate discrimination	-.04	-	.93***	.94**	-.18*	.30**	.20*	.34***	.29**	.17	.34***	.39***
4. Approximate discrimination (correlated trials)	-.05	.93***	-	.75***	.37***	.24**	.21*	.32***	.29**	.20*	.37***	.39***
5. Approximate discrimination (anti-correlated trials)	-.02	. 93***	.73***	-	-.20*	.26**	.16	.32***	.24**	.12	.26**	.34***
6. Small discrimination	.34***	-.09	-.07	-.09	-	-26**	-.19*	-.25**	-.32***	-.14	-.18*	-.33***
7. Visual-spatial STM	-.08	.18*	.18	.17	-.16	-	< .01	.14	.21*	.11	.29**	.40***
8. Phonological awareness	-.16	.13	.16	.08	-.12	-.09	-	.40***	.27**	.26**	.30**	.33***
9. Sequential counting	< .01	.25**	.26**	.20*	-.13	-.01	.33***	-	.47***	.34***	.42***	.45***
10. Cardinal counting	-.14	.23*	.26**	.18*	-.27**	.15	.23*	.42***	-	.18*	.45***	.45***
11. Story problems	-.04	.13	.16	.12	-.09	.05	.22*	.30**	.14	-	.31***	.28**

12.	Mathematical reasoning	-.02	.27**	.33***	.17*	-.09	.21*	.25**	.34**	.41***	.28**	-	.61***
13.	Numerical operations	-.20*	.32***	.34***	.25**	-.24**	.31***	.26**	.36***	.40***	.24**	.56***	-

Note. *p < .05, two-tailed. **p < .01, two-tailed. ***p < .001, two-tailed.

Table 5: Longitudinal Correlations between the Speed of Processing and Cognitive Predictors Assessed at T_1 and the Number Skills, Attainment and General Conceptual Ability Measures Assessed at T_2

		T_2							
		Number skills			Attainment			Ability	
		Sequential counting	Cardinal counting	Story problems	Mathematical reasoning	Numerical operations	Reading	Picture similarities	Naming vocabulary
T_1	Speed of processing	-.08	-.05	-.20*	-.20*	-.08	-.12	-.14	-.04
	Approximate discrimination	.28**	.24**	.37***	.33***	.31***	.31***	.04	.36***
	Approximate discrimination (correlated trials)	.29**	.24**	.41***	.31***	.32***	.35***	.07	.36***
	Approximate discrimination (anti-correlated trials)	.24**	.22*	.28**	.30**	.27**	.24**	.01	.32***
	Small discrimination	-.18*	-.29**	-.27**	-.21*	-.27**	-.28***	.01	.00
	Visual-spatial STM	.18*	.18*	.36***	.34***	.30**	.22*	.24**	.13
	Phonological awareness	.28**	.12	.32***	.24**	.09	.28**	.08	.24**
	Speed of processing	-.03	-.02	-.11	-.12	.04	-.05	-.12	-.01
	Approximate discrimination	.24**	.22*	.29**	.26**	.23*	.26**	.02	.35***
	Approximate discrimination (correlated trials)	.26**	.21*	.36***	.27**	.26**	.31***	.05	.35***
	Approximate discrimination (anti-correlated trials)	.19*	.19*	.19*	.22*	.16	.17	-.01	.30***
	Small discrimination	-.13	-.27**	-.18*	-.13	-.17	-.22*	.01	.04
	Visual-spatial STM	.13	.15	.27**	.26**	.20*	.15	.23*	.10
	Phonological awareness	.24**	.10	.26**	.18	< .01	.23*	.06	.22*

Note. Zero-order longitudinal correlations are presented above the horizontal division and longitudinal partial correlations controlling for age are presented below the horizontal division.

**p* < .05, two-tailed. ***p* < .01, two-tailed. ****p* < .001, two-tailed.

Regression Analyses

We present two sets of regression analyses, in the first we control for the background variables, in the second for the background variables and the autoregressive effect of the criterion variable at T_1. If a cognitive predictor remains significant when autoregressor effects are controlled, then it can be concluded that it predicts *growth* in the outcome measure and stronger evidence for a causal relationship is demonstrated. However, we also present the regressions without controlling for autoregressive effects for three key reasons. First, it enables comparison of the number skills and mathematics attainment results with those for reading attainment (which was not assessed at T_1). Second, it enables comparison between the results of the present study and others that do not control for autoregressive effects (e. g. Bull et al., 2008; De Smedt et al., 2009; LeFevre et al., 2010). Finally, it enables identification of variables that predict when the autoregressor is not controlled, but do not predict when it is. If a cognitive predictor predicts an outcome variable only when the autoregressor is not controlled, it is consistent with the predictor variable having a stronger relationship with the outcome variable earlier in development (i. e. at T_1 rather than T_2) (see Bowey, 2005 for a discussion of the issues relating to the interpretation of results when controlling for autoregressor effects).

The regression models shown in Table 6 examine the relationships between the cognitive variables at T_1 and the outcome variables T_2, without controlling for the autoregressor. The control variables (age, speed of processing and the two ability measures) were entered as a first step. By controlling for speed of processing we were confident that any associations between small discrimination and the outcome measures are due to the efficiency with which the children can discriminate small quantities rather than due to individual differences in their processing speed *per se*. The cognitive predictors were entered as a second step. The cognitive predictors explained a significant proportion of the variance in all the outcome measures over and above the control variables.

The outcome measures had very different relationships with the predictors. Sequential counting was predicted by phonological awareness, cardinal counting was predicted by small discrimination, and calculation (story problems) was predicted by approximate discrimination, visual-spatial STM and phonological awareness. Mathematical Reasoning was predicted by visual-spatial STM, whereas reading was predicted by phonological awareness. Although together the variables in the final step of the regression model significantly increased the variance explained in the Numerical Operations measure, none of the cognitive predictors explained unique variance.

Table 6: Hierarchical Regressions Analysing the Relations between the Cognitive Predictors Assessed at T_1 and the Number Skills, Mathematical and Reading Attainment Measures Assessed at T_2

		Sequential Counting T_2		Cardinal Counting T_2		Story Problems T_2		Mathematical Reasoning T_2		Numerical Operations T_2		Reading T_2	
		β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2	β	ΔR^2
Model 1	Step 1. Control Variables												
	Age T_1 (months)	.15		.09		.30**		.23**		.37***		.23*	
	Speed of Processing	-.03		-.02		-.09		-.11		.04		-.06	
	Picture Similarities	.00		-.03		.09		.10		-.01		-.13	
	Naming Vocabulary	.12		.19*		.20*		.33***		.15		.21*	
			.04		.05		.17***		.22***		.16***		.13**
Model 2	Step 1. Control Variables												
	Age T1 (months)	.00		-.05		.10		.10		.25**		.07	
	Speed of Processing	.06		.10		-.01		-.05		.10		.04	
	Picture Similarities	-.02		-.03		.05		.06		-.02		-.15	
	Naming Vocabulary	-.00		.14		.07		.26**		.10		.11	
	Step 2. Cognitive Predictors												
	Approximate Discrimination	.19		.13		.18*		.11		.15		.16	
	Small Discrimination	-.09		-.29**		-.10		-.07		-.17		-.18	
	Visual-spatial STM	.12		.09		.22*		.20*		.14		.14	
	Phonological Awareness	.23*		.05		.22*		.10		-.03		.19*	
			.11**		.11**		.13***		.06*		.08*		.11**

Note. Values reported are the standardised regression coefficients. Model 1 df = 4, 120; Model 2 df = 8, 116.

*$p < .05$. **$p < .01$. ***$p < .001$.

The regression models shown in Table 7 determine whether the cognitive predictors predicted *growth* in the number skills and the mathematical attainment measures. Score at T_1 on the outcome measure being predicted (criterion variable) was entered as a first step. The control variables were entered as a second step. The cognitive predictors were entered together as a third step.

When the autoregressor was controlled, phonological awareness no longer predicted sequential counting at T_2. Cardinal counting continued to be predicted by small discrimination and calculation (story problems) continued to be predicted by approximate discrimination, visual-spatial STM and phonological awareness. None of the individual variables explained unique variance in the mathematical attainment measures and the additional variance explained by the final steps of these regression models was not statistically significant.

Table 7: Hierarchical Regressions Analysing the Relations Between the Cognitive Predictors Assessed at T_1 and the Number Skills and Mathematics Measures at T_2 (with the autoregressor controlled)

		Sequential Counting T_2		Cardinal Counting T_2		Story Problems T_2		Mathematical reasoning T_2		Numerical reasoning T_2	
		β	Δ R^2	β	Δ R^2	β	Δ R^2	β	Δ R^2	β	Δ R^2
Model 1	Step 1. Autoregressor										
	Criterion variable at T_1	.37***		.54***		.25**		.55***		.47***	
			.14***		.29***		.06**		.30***		.22***
Model 2	Step 1. Autoregressor										
	Number skill at T_1	.36***		.53***		.15		.43***		.39***	
	Step 2. Control Variables										
	Age T_1 (months)	.01		-.00		.26**		.11		.26**	
	Speed of Processing	-.02		.06		-.10		-.10		.11	
	Picture Similarities	.01		-.03		.08		.10		-.03	
	Naming Vocabulary	.09		.09		.15		.15		.03	
			.01		.01		.13**		.06*		.06[b]
Model 3	Step 1. Autoregressor										
	Criterion variable at T_1	.27**		.48***		.09		.39***		.35**	
	Step 2. Control Variables										
	Age T1 (months)	-.08		-.06		.09		.05		.22*	
	Speed of Processing	.03		.11		-.02		-.07		.13	
	Picture Similarities	-.02		-.03		.04		.08		-.03	
	Naming Vocabulary	.01		.09		.05		.12		.03	
	Step 3. Cognitive Predictors										
	Approximate Discrimination	.14		.07		.18*		.08		.10	
	Small Discrimination	-.06		-.18*		-.09		-.05		-.13	
	Visual-spatial STM	.13		.05		.22*		.13		.06	
	Phonological Awareness	.16		-.03		.21*		.04		-.10	
			.05		.04[a]		.12**		.02		.03

Note. Values reported are the standardised regression coefficients; Model 1 df = 1, 123; Model 2 df = 5,119; Model 3 df = 9,115. [a]Whilst Δ R^2 for the final step predicting cardinal counting does not reach statistical significance when all the cognitive predictors are entered simultaneously, it does when small discrimination is entered in the final step as a single variable.. [b]p = .051

*$p < .05$. **$p < .01$. ***$p < .001$.

Discussion

In the present study we determined that phonological awareness, visual-spatial STM and non-symbolic quantitative skills make unique contributions to the development of early counting and calculation. Small quantity discrimination predicted growth in cardinal counting skills, whereas phonological awareness, visual-spatial STM and approximate quantity discrimination predicted growth in calculation skills.

Phonological Awareness

Consistent with previous studies identifying an association between phonological awareness and sequential counting (Koponen et al., 2013; Krajewski & Schneider, 2009), phonological awareness predicted sequential counting at T_2 (see Table 6). However, phonological awareness was no longer a significant predictor when sequential counting at T_1 was controlled for (see Table 7). This pattern of findings is more consistent with phonological awareness influencing the number sequence knowledge of school entrants rather than the *growth* in number sequence during the first year of schooling (see Bowey, 2005 for a discussion of the interpretation of effects that are eliminated when the autoregressor is controlled). A succinct explanation of these findings would be that phonological awareness influences the rate at which children acquire the first few number words in the same way that phonological awareness influences all vocabulary acquisition (see Bowey, 1996; Gathercole, 2006), but later extension of the number sequence depends to a greater extent on children's understanding of the base 10 system of number (i. e. understanding where a decade transition should occur, understanding the logical nature of decade-unit word structures). Logical application of the base system may be less reliant on phonological awareness than establishing the sequence of the first few number words.

Nevertheless, phonological awareness predicted growth in calculation skills. As children at this age have typically not yet established a comprehensive store of arithmetic facts (Siegler, 1996; Siegler & Shrager, 1984), it is likely that phonological sensitivity supports formal calculations via the use of other strategies. Although phonological awareness does not predict growth in the length of children's number-word sequence (see Table 7), it may predict growth in their counting *speed*, which in turn influences their ability to use counting strategies effectively.

Visual-spatial STM

Visual-spatial STM did not predict growth in cardinal counting. Whilst it has previously been shown that visual-spatial ability is associated with children's understanding of cardinality principles (Ansari et al., 2003), our findings suggest that visual-spatial STM does not account for individual differences in children's ability to apply them. However, visual-spatial STM did predict growth in children's calculation skills. This aligns with Rasmussen and Bisanz's (2005) and Krajewski and Schneider's (2009) proposal that young children make use of visual-spatial mental models and therefore visual-spatial STM when calculating. It also aligns with McKenzie et al.'s (2003) study that indicated that young children utilise visual-spatial STM during formal calculations. As both phonological awareness and visual-spatial STM were shown to predict calculation, it is likely that in this age group children are employing a range of strategies (both verbal counting and non-verbal mental models). The choice of strategy may be influenced by the difficulty of the problem and by the child's individual preferences and abilities.

Approximate Discrimination

Approximate discrimination skills predicted growth in children's calculation skills. Furthermore, the analyses of correlated and anti-correlated trials suggested that this is more likely to be due to the task indexing the precision of children's ANS rather than their inhibitory skills. In the current study, the children did *not* perform significantly worse on the anticorrelated trials, suggesting that inhibition was not a strong influence on task performance[1]. Although inhibition may underpin the relationship between children's approximate discrimination skills and mathematical attainment in some studies (e. g. Fuhs & McNeil, 2013; Gilmore et al., 2013), it does not appear to underpin the relationship between approximate discrimination and formal calculation skills in the current study. The extent that inhibition influences approximate discrimination task performance and underpins its relationship with mathematical attainment may be influenced by sample characteristics (e.g. age, level of inhibition skills) and task characteristics (e.g. the *degree* of surface area incongruency, the ratios presented). Investigating the influence of these factors is an important area for further research.

The results suggest the ANS has a role supporting early calculation. This is consistent with neuropsychological models that identify the activation of abstract analogue representations of number during calculation (Dehaene et al., 2003) and theoretical models that have propose a role for quantitative skills in calculation development (Krajewski & Schneider, 2009; LeFevre et al., 2010). However, the approximate discrimination task does not predict growth in either cardinal or sequential counting skills. Future studies need to examine how the ANS supports early calculation skills. One possibility would be that a more precise ANS allows *estimates* of the answers to arithmetic problems to be more precisely generated and thus provides an alternative route for checking an answer generated via verbal counting or non-verbal mental model strategy. The relationship between ANS functioning and estimation skills in children needs to be explored to examine this hypothesis.

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[1] When supplementary regression analyses were conducted with correlated and anticorrelated trials entered as separate variables, anticorrelated trials were *not* a unique predictor of story problems, but correlated trials were. These analyses further support the argument that inhibition did not underpin the relationship between approximate discrimination and the outcome measures in this sample.

Approximate discrimination was not a significant predictor of mathematical attainment as indexed by either the Mathematical Reasoning or the Numerical Operations tests. Although inconsistent with some previous findings where ANS accuracy predicted mathematics attainment (Bonny & Lourenco, 2013; Halberda et al., 2008; Inglis et al., 2011; Libertus et al., 2011, 2013; Mussolin et al., 2012), the present results are consistent with a number of findings that have *not* found significant relations between tasks indexing the ANS and mathematics attainment (Göbel et al., 2014; Holloway & Ansari, 2009; Sasanguie, De Smedt et al., 2012; Sasanguie, van den Bussche et al., 2012). The pattern of our results (with the ANS supporting early calculation but not counting skills) suggests that the ANS has a differential influence on different number skills. As different mathematics attainment measures include a diverse range of number skills and the skills tested can differ at different ages, it is unsurprising that contrasting findings regarding the relationship between the ANS and mathematics attainment are reported.

Small Discrimination

Consistent with our prediction performance on the small discrimination task predicted growth in cardinal counting. The small discrimination task was designed to assess the efficiency with which children could discriminate between numerical sets within the subitising range. Children who subitise the items quickly would perform the task more efficiently than children who subitise more slowly or who rely on slower and more effortful counting strategies. Consequently, small discrimination may predict growth in cardinal counting skills because efficient subitising supports the development of cardinal counting. This explanation is consistent with previous findings reporting an association between subitising skills and young children’s understanding of cardinality (Benoit et al., 2004). Moreover, we are confident that this relationship is not underpinned by general processing speed because small discrimination predicts growth in cardinal counting when speed of processing is controlled (see Table 7).

Although subitising efficiency is a credible explanation of the relationship between small discrimination and cardinal counting, other possible accounts of this relationship should be considered. For instance, it is possible that quantity judgement speed *per se* (rather than specifically speed of subitising) underpins the relationship. In the current study we cannot exclude this possibility because we did not include a latency measure for the approximate discrimination task. It could also be that children who learn a rule-based strategy across the course of task (e.g. always choose the side that is not one circle and/or always choose three circles when presented) are more efficient and it is the ability to acquire such rules rather than subitising efficiency that underpins the task's ability to predict cardinal counting. Future studies can untangle these possibilities by examining the small discrimination task's relationships with more traditional measures of subitising (such as RT's slopes using enumeration paradigms), with latency measures of large approximate discrimination tasks and with other domain-general cognitive tests such as rapid automated naming and working memory.

Limitations, Future Directions and Implications

In the current study we included phonological awareness, visual-spatial STM and non-symbolic quantitative skills as predictors because these variables are identified in theoretical models (Krajewski & Schneider, 2009; LeFevre et al., 2010) describing the cognitive factors that influence early counting and calculation. Whilst these variables predicted a statistically significant proportion of the variance in counting and calculation skills, it is important to recognise that they only accounted for a small proportion of the total variance. Consequently other cognitive and non-cognitive variables (not currently included in these theoretical models) must be important in accounting for individual differences in these number skills. Future studies of early counting and calculation need to build on the current findings by considering the role of other cognitive (e.g. the central executive of working memory) and non-cognitive factors (e.g. the teaching and home environment), alongside the phonological, visual-spatial and quantitative skills explored in the present study.

The present study has methodological strengths in terms of its substantive sample, longitudinal nature, control of autoregressive effects and varied outcome measures. However, we also recognise that it has limitations. First, whilst we reduced the influence of task-specific variance on the results for phonological awareness and visual-spatial STM by using composite measures of these constructs, we only used single measures of small and approximate discrimination skills. At the time of the data collection there were no published studies demonstrating the validity of multiple quantitative processing measures. However recent studies have found promising relationships between multiple non-symbolic quantity processing tasks (e.g. Gilmore, Attridge, De Smedt, & Inglis, 2014; Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013). Replicating the present findings using composite measures for both approximate and small discrimination would further strengthen the conclusions. Second, while the cognitive precursors were assessed at T_1, the ability measures that were used as control variables were assessed at T_2. We recognise that it would have been ideal to include the ability measures at T_1. However, we did not believe longer assessment sessions would have been appropriate at that age (4 years). We were able to include these measures at T_2 because the children were older so the assessment sessions could be extended. The ability measures were included as a control in the analyses (despite their administration at T_2) to ensure a very conservative test of the impact of the cognitive predictors on the outcome measures. This was deemed preferable to a less conservative test where general conceptual ability was not controlled. Finally, we acknowledge that our findings may not be applicable at earlier or later points in children's development. In particular, although both counting and calculation skills developed between T_1 and T_2, all children had grasped the fundamentals of counting at T_1 (see the minimum scores for sequential and cardinal counting in Table 3). As discussed earlier, different cognitive skills may be involved in the *establishment* of counting skills than in the *extension* of these skills.

The pattern of findings reported in the current study suggests that during the first years of formal schooling children who have different cognitive profiles will experience difficulties with different number skills. In particular children with circumscribed deficits in visual-spatial STM and/or the ANS are unlikely to experience difficulties in extending their counting skills, but are more likely to experience difficulties with developing formal calculation skills. If future studies extend our findings by examining the extent that a wider range of cognitive abilities predict early number skills, it may be possible to assess children's cognitive profiles at the start of schooling and provide targeted support in the early number skills where they are most likely to experience difficulties. This type of targeted support would enable potential difficulties to be addressed before they become entrenched. Given that early number skill proficiency predicts later mathematical attainment (Aunio & Niemivirta, 2010; Aunola et al., 2004; Johansson, 2005; Krajewski & Schneider, 2009; LeFevre et al., 2010), this type of targeted intervention should enhance children's mathematical attainment as they progress thorough education.

Conclusion

The present study indicates that phonological awareness, visual-spatial STM and the ANS all support the growth of early calculation skills. It also demonstrates that fast discrimination between quantities within the subitising range predicts growth in cardinal counting.

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