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Hindawi
Mathematical Problems in Engineering
Volume 2021, Article ID 5558955, 18 pages
https://doi.org/10.1155/2021/5558955

Research Article

Finite-Time H_∞ State Estimation for Markovian Jump Neural Networks with Time-Varying Delays via an Extended Wirtinger's Integral Inequality

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Received 26 February 2021; Accepted 21 July 2021; Published 5 August 2021

Academic Editor: Xindong Peng

This study investigates the finite-time boundedness for Markovian jump neural networks (MJNNs) with time-varying delays. An MJNN consists of a limited number of jumping modes wherein it can jump starting with one mode then onto the next by following a Markovian process with known transition probabilities. By constructing new Lyapunov–Krasovskii functional (LKF) candidates, extended Wirtinger's, and Wirtinger's double inequality with multiple integral terms and using activation function conditions, several sufficient conditions for Markovian jumping neural networks are derived. Furthermore, delay-dependent adequate conditions on guaranteeing the closed-loop system which are stochastically finite-time bounded (SFTB) with the prescribed H_∞ performance level are proposed. Linear matrix inequalities are utilized to obtain analysis results. The purpose is to obtain less conservative conditions on finite-time H_∞ performance for Markovian jump neural networks with time-varying delay. Eventually, simulation examples are provided to illustrate the validity of the addressed method.

1. Introduction

Due to the great significance of neural networks (NNs) for both practical and theoretical purposes, their dynamics have been explored widely in recent years, such as pattern recognition, signal processing, solving optimization problems, static image processing, associative memories, target tracking, and automatic control. Therefore, many research subjects have been studied in a broad spectrum of stability analysis, passivity analysis, control, filtering design, and state estimation and synchronization, concerning to NNs [1–6]. In [4], passive filter design for fractional-order quaternion-valued neural networks with neutral delays and external disturbance has been studied. The authors in [6] investigated stability criteria of quaternion-valued neutral-type delayed neural networks. In many implementations of NNs, time delays are inevitable [7] and can lead NNs to instability and oscillation. Hence, the stability analysis with time delays in the NN models under consideration has attracted considerable attention [8–14].

Due to interconnection failures, sudden environment changes, components, and so on, plenty of structural parameters of neural networks may mutate. In general, there are finite modes in the neural network,s switching or jumping from one mode to another mode by a random form. A Markov chain can be used to describe jumping between different modes of neural networks, and the kinds of systems are called Markovian jump neural networks [15–18]. Many practical control systems can be modeled as Markovian jump neural networks, such as air intake systems and economic systems [19]. In an MJNN, hopping among operation modes is specified by a Markov process, so it is important to understand the impacts of its stochastic attributes on the stability analysis of delayed MJNNs. Some previous works [15–18] have discussed certain standard results in relation to MJNN stability analysis. In [20], the authors conducted an asymptotic stability analysis for stochastic and static NNs with time-varying delays that are mode-dependent. The use of linear matrix inequalities (LMIs) has led to important and interesting results concerning various types of NN with MJ parameters [21, 22]. The mode-dependent MJNNs with time-varying delays and incomplete transition rates can be found in [23], wherein some LMI-based conditions are proposed to obtain the required results.

In some cases, we are interested in knowing how the modeled system behaves within fixed- and finite-time intervals. In other words, given an initial bounded state, we require the system to remain in a state that is not superior to a particular threshold during a specified time interval. Since this type of stability ensures a faster convergence of the system, it has been widely used in various NNs, such as the MJNNs, and synchronizing neural networks [24]. An important example can be found in controlling the trajectory of a spacecraft between its initial and final locations within a specified time interval. However, because of the lack of other finite-time-bounded operational conditions, it is natural that research interest has shifted to Lyapunov stability in this paper. In addition, based on LMI results, the idea of finite-time boundedness (FTB) has been revisited here. We also studied that finite-time stability involves dynamical systems whose part of the trajectory converges to an equilibrium state in a finite time. Note that the finite-time stability with control frameworks has gained significant attention in recent years [25–28]. In [29], the authors discussed the finite-time L_2-gain performance of MJNNs. The design of a finite-time passive controller for uncertain MJ systems is optimized in [28], wherein a robust and fuzzy finite-time passive control is defined along with the finite-time stochastic stability of a nonlinear MJ system. However, finite-time H_∞ state estimation of MJ systems has not been studied much for NNs. This is a primary inspiration for this study. The main contributions of this study are listed as follows:

(1) The comprehensive Markovian jump neural networks with state and input constraints are studied.

(2) We have introduced a novel Lyapunov–Krasovskii functional (LKF), including time-varying delays.

(3) Wirtinger's double integral inequality, introduced by Park et al. [30], and Wirtinger's integral inequality, extended by Zhang et al. [31], are introduced into the time-derivative of LKF. This time-derivative forms the LMIs which are FTB. These LMIs deliver more effective outcomes in comparison to previous works. The numerical examples are also given.

(4) To show the real-life application, the four-tank water pumping system and network circuit are considered in this paper in terms of the NN model to show feasibility on a benchmark problem.

Notations are as follows:
$\mathbb{R}^n$: n-dimensional Euclidean space
$P>0$: the matrix P is a symmetric matrix
min (P): minimum eigenvalue of P
max(P): maximum eigenvalue of P
I: identity matrix
diag $\{\cdot\}$: diagonal matrix
$*$: symmetric matrices

2. Preliminaries and Problem Formulation

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\Omega, \mathcal{F}$, and $\mathcal{P}$ represent sample space, σ-algebra of events, and probability measure defined on $\mathcal{F}$, respectively. Let parameter $\{r_t, t \geq 0\}$ be a right continuous Markov chain taking values on $(\Omega, \mathcal{F}, \mathcal{P})$ a finite set $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Pi = (\pi_{ij})_{N\times N}$ given by

$$\Pr\{(r_t + \Delta_t) = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{1}$$

where $r_t \in \mathcal{S}$, $\Delta t > 0$, $\lim_{\Delta t \longrightarrow 0} (o(\Delta t)/\Delta) = 0$, and π_{ij} denotes the transition probability from modes i to j satisfying $\pi_{ij} \geq 0$, for $i \neq j$, with $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$, $i, j \in \mathcal{S}$.

Consider the MJNNs with time-varying delays are as follows:

$$\left.\begin{aligned} \dot{x}(t) &= -\mathbf{A}(r_t)x(t) + \mathbf{B}(r_t)h(x(t)) + \mathbf{B}_d(r_t)h(x(t-\delta(t))) + J + \mathbf{E}_1(r_t)w(t), \\ y(t) &= \mathbf{C}(r_t)x(t) + \mathbf{D}(r_t)x(t-\delta(t)) + \mathbf{E}_2(r_t)w(t), \\ z(t) &= \mathbf{G}(r_t)x(t), \\ x(t) &= \phi(t), \quad t \in [\delta, 0], \end{aligned}\right\}, \tag{2}$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^T \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output measurement, $z(t) \in \mathbb{R}^m$ denotes the estimated signal, $w(t) \in \mathbb{R}^q$ represents exogenous disturbance belonging to $L_2[0, \infty)$, $h(x(t)) = [h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t))]^T \in \mathbb{R}^n$ is a neuron activation function, and $J \in \mathbb{R}^n$ denotes an external input constant vector. $\mathbf{A}(r_t) > 0$ is a diagonal matrix, and $\mathbf{B}(r_t), \mathbf{B}_d(r_t), \mathbf{E}_1(r_t), \mathbf{E}_2(r_t), \mathbf{C}(r_t), \mathbf{D}(r_t)$, and $\mathbf{G}(r_t)$ are connection weight matrices. A time-varying delay is denoted as $\delta(t)$, where $0 \leq \delta(t) \leq \overline{\delta}$ and $\dot{\delta}(t) \leq \mu$, such that $\overline{\delta}$ and μ are known constants. For each possible value of $r(t) = i, i \in \mathcal{S}$, a matrix $\mathbf{A}(r_t)$ is denoted by $\mathbf{A}_i$ and all other matrices with appropriate dimensions are denoted by $\mathbf{B}_i$, $\mathbf{B}_{di}$, $\mathbf{E}_{1i}$, $\mathbf{E}_{2i}$, $\mathbf{C}_i$, $\mathbf{D}_i$, and $\mathbf{G}_i$.

Assumption 1. Each neuron activation function $h_k(t)$ $(k = 1, 2, \ldots, n)$ is continuous and bounded and satisfies the following condition:

$$\varrho_k^- \leq \frac{h_k(x_1) - h_k(x_2)}{x_1 - x_2} \leq \varrho_k^+, \quad \forall x_1, x_2 \in \mathbb{R}, x_1 \neq x_2, \tag{3}$$

where ϱ_k^- and ϱ_k^+ are constant. Then, we define the followings

$$\begin{aligned} L_1 &= \operatorname{diag}\{\varrho_1^-, \varrho_2^-, \ldots, \varrho_n^-\}, L_2 = \operatorname{diag}\{\varrho_1^+, \varrho_2^+, \ldots, \varrho_n^+\}, \\ M_t &= \operatorname{diag}\{\varrho_1^-\varrho_1^+, \varrho_2^-\varrho_2^+, \ldots, \varrho_n^-\varrho_n^+\}, \\ M_u &= \operatorname{diag}\left\{\frac{\varrho_1^- + \varrho_1^+}{2}, \frac{\varrho_2^- + \varrho_2^+}{2}, \ldots, \frac{\varrho_n^- + \varrho_n^+}{2}\right\}. \end{aligned} \tag{4}$$

Assumption 2. The external disturbance $w(t)$ fluctuates and satisfies the following inequality:

$$\int_0^T w^T(t)w(t)\mathrm{d}t \leq d, \quad d \geq 0. \tag{5}$$

For a MJNN defined as (2), a state estimator is constructed as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = -\mathbf{A}_i\tilde{x}(t) + \mathbf{B}_i h(\tilde{x}(t)) + \mathbf{B}_{di} h(\tilde{x}(t - \delta(t))) + J + K_i(y(t) - \tilde{y}(t)), \\ \tilde{y}(t) = \mathbf{C}_i\tilde{x}(t) + \mathbf{D}_i\tilde{x}(t - \delta(t)), \\ \tilde{z}(t) = \mathbf{G}_i\tilde{x}(t), \\ x(t) = 0, \quad t \in [\overline{\delta}, 0], \end{cases} \tag{6}$$

where $\tilde{x}(t) \in \mathbb{R}^n$ denotes the estimated state and $\tilde{z}(t) \in \mathbb{R}^q$ is the estimated measurement of $z(t)$. Then, estimator gain matrix K_i is to be constructed.

By defining the error $e(t) = x(t) - \tilde{x}(t)$, $f(e(t)) = h(x(t)) - h(\tilde{x}(t))$, and $\overline{z}(t) = z(t) - \tilde{z}(t)$, an error system can be obtained in the following form:

$$\begin{aligned} \dot{e}(t) &= -(\mathbf{A}_i + K_i\mathbf{C}_i)e(t) - K_i\mathbf{D}_i e(x(t - \delta(t))) + \mathbf{B}_i f(e(t)) + \mathbf{B}_{di} f(e(t - \delta(t))) + (\mathbf{E}_{1i} - K_i\mathbf{E}_{2i})w(t), \\ \overline{z}(t) &= \mathbf{G}_i e(t), \end{aligned} \tag{7}$$

where $e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T \in \mathbb{R}^n$ denotes the state vector of modeled system and $f(e(t)) = [f(e(t)) f_2(e(t)), \ldots, f_n(e(t))]^T$, and $f(e(t)) = g(x(t)) - g(\tilde{x}(t))$ is the transformed activation function. From Assumption 1, the neuron activation function satisfies

$$l_a^- \leq \frac{f_a(\rho)}{\rho} \leq l_a^+, \tag{8}$$

where $\rho \in \mathbb{R}$ and $\rho \neq 0$.

Definition 1 (stochastically finite-time stable (SFTS) [27]). Given time constant $T > 0$, an MJNN defined as (7) with $w(t) = 0$ is SFTS with respect to (c_1, c_2, T, R) if there exists a positive matrix $R > 0$ and scalars $c_1 > 0$ and $c_2 > 0$, such that the following inequality holds:

$$\mathbb{E}\left[x_0^T(t)Rx_0(t)\right] < c_1 \Rightarrow \mathbb{E}\left[x^T(t)Rx(t)\right], \quad t \in [0, T]. \tag{9}$$

Definition 2 (stochastically finite-time boundedness (SFTB) [27]). Given a time constant $T > 0$, an MJNN defined as (7) is said to be SFTB with respect to (c_1, c_2, T, R, d), where there exist $R > 0$ and scalars $c_1 > 0$ and $c_2 > 0$, such that the following inequality holds:

$$\mathbb{E}\left[x_0^T(t)Rx_0(t)\right] < c_1 \Longrightarrow \mathbb{E}\left[x^T(t)Rx(t)\right], \quad t \in [0, T]. \tag{10}$$

Definition 3 (see [32, 33]). For $T > 0$, an MJNN defined as (7) is said to be SFTB with respect to (c_1, c_2, T, R, d) and with a prescribed level of noise attenuation $\gamma > 0$ under a zero initial condition if it holds:

$$\mathbb{E}\left\{\int_0^T z^T(s)z(s)\mathrm{d}s\right\} \leq \gamma^2\mathbb{E}\left\{\int_0^T w^T(t)w(t)\mathrm{d}t\right\}. \tag{11}$$

Definition 4 (see [34]). A functional $V(x(t), r(t), t > 0) = V(x(t), r)$ is said to be a stochastic positive functional. Its weak infinitesimal operator can be defined as

$$\mathscr{L}(V(x(t), r(t))) = \lim_{\Delta t \longrightarrow 0} \frac{1}{\Delta t}[\mathbb{E}V(x(t+\Delta t), r(t+\Delta t), t+\Delta t)|x(t), r(t) = i - V(x(t), i, t)]. \quad (12)$$

Lemma 1 (see [30]). *Let a constant matrix $\mathbb{M} > 0$; the following condition can be defined for all differentiable function ϕ in $[a, b] \longrightarrow \mathbb{R}^n$ for scalars a and b with $a < b$:*

$$-\frac{b^2 - a^2}{2} \int_{-a}^{-b} \int_{t+\theta}^{t} \dot{\eta}^T(s)\mathbb{M}\dot{\eta}(s)\mathrm{d}s\mathrm{d}\theta \leq -\Omega_1^T \mathbb{M}\Omega_1 - 2\Omega_2^T \mathbb{M}\Omega_2, \quad (13)$$

where $\Omega_1 = (b-a)\eta(t) - \int_{t-a}^{t-b} \eta(s)ds$ and $\Omega_2 = -(b-a)/2\eta(t) - \int_{t-a}^{t-b} \eta(s)ds + 3/b - a\int_{-a}^{-b} \int_{t+\theta}^{t} \eta(s)dsd\theta$.

Lemma 2 (see [35]). *For any constant matrix $M > 0$, the following inequality holds for all continuously differentiable function φ on $[a, b] \longrightarrow \mathbb{R}^{n\times n}$:*

$$(b-a)\int_a^b \varphi^T(s)M\varphi(s)\mathrm{d}s \geq \left(\int_a^b \varphi(s)\mathrm{d}s\right)^T \times M\left(\int_a^b \varphi(s)\mathrm{d}s\right) + 3\Omega^T M\Omega, \quad (14)$$

where $\Omega = \int_a^b \varphi(s)ds - 2/b - a\int_a^b \int_a^s \varphi(\theta)d\theta ds$.

Lemma 3 (see [31]). *For a given symmetric matrix $W_2 = W_2^T > 0$, the following inequality holds for all continuously differential function x in $[\alpha, \beta] \longrightarrow \mathbb{R}^n$:*

$$(\beta - \alpha)\int_\alpha^\beta \dot{x}^T(s)W_2\dot{x}(s)\mathrm{d}s \geq \xi^T \Xi \xi, \quad (15)$$

where

$$\xi = \left[x^T(\beta)\, x^T(\alpha)\, \frac{2}{\beta - \alpha}\int_\alpha^\beta x^T(s)\mathrm{d}s\, \frac{12}{(b-a)^2}\int_\alpha^\beta \int_\alpha^s x^T(\theta)\mathrm{d}s\mathrm{d}\theta\right]^T,$$

$$\Xi = \begin{bmatrix} 9W_2 & -3W_2 & -36W_2 & 2W_2 \\ * & 9W_2 & 15W_2 & -5W_2 \\ * & * & 48W_2 & -15W_2 \\ * & * & * & 5W_2 \end{bmatrix}. \quad (16)$$

3. Methodologies and Theoretical Results

3.1. Finite-Time State Estimation. This section derives the SFTB of the error system in (7).

Theorem 1. *For scalars c_1, c_2, T, d, $\overline{\delta}$, μ, and α, an MJNN defined as (7) is SFTB in relation to (c_1, c_2, T, R, d) if there exist feasible matrices $P_i > 0$, $Q_s > 0$, $W_s > 0$ $(s = 1, 2, 3)$, $\mathscr{U}_t > 0$, $\mathscr{U}_u > 0$, $\mathscr{N}$, and X, where P_i, Q_s, and W_s are symmetric positive definite (PD), and $\mathscr{U}_t > 0$ and $\mathscr{U}_u > 0$ are diagonal, such that the following inequality holds:*

$$\Psi = [\psi_{ij}]_{9\times 9} < 0, \quad (17)$$

$$e^{\alpha T}[c_1\Lambda + \lambda_9 d] < \lambda_1 c_2, \quad (18)$$

where

$$\begin{aligned}
&\psi_{11} = \sum_{j=1}^{N}\pi_{ij}P_j + Q_1 + Q_2 + \bar{\delta}W_1 - 9W_2 - \bar{\delta}W_3 - \frac{\bar{\delta}^2}{4}w_3 - M_t\mathscr{U}_t - \mathscr{N}A_i - A_i^T\mathscr{N}^T - L_iC_i - C_i^T L_i^T,\\
&\psi_{12} = L_iD_i, \psi_{13} = 3W_2, \psi_{14} = 2P_i - \mathscr{N} - A_i^T\mathscr{N}^T - C_i^T L_i^T, \psi_{15} = 36W_2 + \bar{\delta}W_3 + \frac{\bar{\delta}}{2}W_3,\\
&\psi_{16} = -2W_2 - \frac{3}{2}W_3, \psi_{17} = M_u\mathscr{U}_t + \mathscr{N}B_i,\ \psi_{18}\\
&\psi_{22} = -(1-\mu)Q_1 - M_t\mathscr{U}_u, \psi_{23} = 0, \psi_{24} = D_i^T L_i^T, \psi_{25} = 0, \psi_{26} = 0, \psi_{27} = 0, \psi_{28} = M_u\mathscr{U}_u, \psi_{29} = 0,\\
&\psi_{33} = -Q_2 - 9W_2,\ \psi_{34} = 0,\ \psi_{35} = -30W_2,\\
&\psi_{36} = -5W_2, \psi_{37} = 0, \psi_{38} = 0, \psi_{39} = 0, \psi_{44} = \bar{\delta}W_2 + \frac{\bar{\delta}^4}{4}W_2 - \mathscr{N} - \mathscr{N}^T,\\
&\psi_{45} = 0, \psi_{46} = 0, \psi_{47} = \mathscr{N}B_i, \psi_{48} = \mathscr{N}B_{di}, \psi_{49} = \mathscr{N}E_{1i} - L_iE_{2i}, \psi_{55} = -\frac{1}{\bar{\delta}}W_1 - \frac{3}{\bar{\delta}}W_1 - 192W_2 - 2W_3,\\
&\psi_{56} = \frac{6}{\bar{\delta}^2}W_1 - 30W_2 + \frac{3}{\bar{\delta}}W_3, \psi_{57} = 0, \psi_{58} = 0, \psi_{59} = 0, \psi_{66} = -\frac{12}{\bar{\delta}^3}W_1 - 5W_2 - \frac{9}{\bar{\delta}^2}W_3, \psi_{67} = 0,\\
&\psi_{68} = 0, \psi_{69} = 0, \psi_{77} = Q_3 - \mathscr{U}_t, \psi_{78} = 0, \psi_{79} = 0, \psi_{88} = -Q_3 - \mathscr{U}_u, \psi_{89} = 0, \psi_{99} = -X.
\end{aligned} \tag{19}$$

In addition, the desired control gain matrices can be calculated by $K_i = \mathscr{N}^{-1}L_i$.

Proof. Construct LKF for an MJNN defined as (7):

$$V(t) = \sum_{i=1}^{5} V_i(t), \tag{20}$$

where

$$\begin{aligned}
&V_1(t) = e^T(t)P_ie(t),\\
&V_2(t) = \int_{t-\delta(t)}^{t} e^T(s)Q_1e(s)\mathrm{d}s + \int_{t-\bar{\delta}}^{t} e^T(s)Q_2e(s)\mathrm{d}s,\\
&V_3(t) = \int_{t-\delta(t)}^{t} f^T(e(s))Q_3f(e(s))\mathrm{d}s,\\
&V_4(t) = \int_{-\bar{\delta}}^{0}\int_{t+\theta}^{t} e^T(s)W_1e(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\bar{\delta}}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)W_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta,\\
&V_5(t) = \frac{\bar{\delta}^2}{2}\int_{-\bar{\delta}}^{0}\int_{\beta}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)W_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned} \tag{21}$$

By differentiating the above LKF to obtain its time derivatives along with the trajectory of the MJNN defined as (7), we obtain

$$\mathscr{L}V_1 = 2e^T(t)P\dot{e}(t) + e^T(t)\sum_{j=1}^{N}\pi_{ij}P_je(t), \tag{22}$$

$$\mathscr{L}V_2 = e^T(t)(Q_1 + Q_2)e(t) - (1-\delta_D)e^T(t-\delta(t)) \times Q_1e(t-\delta(t)) - e^T(t-\bar{\delta})Q_2e(t-\bar{\delta}), \tag{23}$$

$$\mathscr{L}V_3 = f^T(e(t))Q_3 f(e(t)) - f^T(e(t-\delta(t)))Q_3 f(e(t-\delta(t))), \tag{24}$$

$$\mathscr{L}V_4 = \overline{\delta}e^T(t)W_1 e(t) - \int_{t-\overline{\delta}}^{t} e^T(s)W_1 e(s)\mathrm{d}s + \overline{\delta}\dot{e}^T(t)W_2\dot{e}(t) - \int_{t-\overline{\delta}}^{t}\dot{e}^T(s)W_2\dot{e}(s)\mathrm{d}s, \tag{25}$$

$$\mathscr{L}V_5 = \left(\frac{\overline{\delta}^2}{2}\right)^2 \dot{e}^T(t)W_3\dot{e}(t) - \frac{\overline{\delta}^2}{2}\int_{\overline{\delta}}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)W_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta. \tag{26}$$

Utilizing Lemma 2, we obtain

$$-\int_{t-\overline{\delta}}^{t} e^T(s)W_1 e(s)\mathrm{d}s \le \frac{-1}{\overline{\delta}}\left(\int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s\right)^T \times W_1\left(\int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s\right) - \frac{3}{\overline{\delta}}\Phi_1^T W_1 \Phi_1, \tag{27}$$

where $\Phi_1 = \int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s - 2/\overline{\delta}\int_{-\overline{\delta}}^{0}\int_{t+\theta}^{t} e(s)\mathrm{d}s\mathrm{d}\theta$.

By applying Lemma 3, the following inequality can be written as

$$-\int_{t-\overline{\delta}}^{t}\dot{e}^T(s)W_2\dot{e}(s)\mathrm{d}s \le \begin{bmatrix} e(t) \\ e(t-\overline{\delta}) \\ \frac{1}{\overline{\delta}}\int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s \\ \frac{12}{\overline{\delta}^2}\int_{t-\overline{\delta}}^{t}\int_{t-\overline{\delta}}^{s} e(s)\mathrm{d}s\mathrm{d}\theta \end{bmatrix}^T \begin{bmatrix} -9W_2 & 3W_2 & 36W_2 & -2W_2 \\ * & -9W_2 & -30W_2 & 5W_2 \\ * & * & -192W_2 & 30W_2 \\ * & * & * & -5W_2 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\overline{\delta}) \\ \frac{1}{\overline{\delta}}\int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s \\ \frac{12}{\overline{\delta}^2}\int_{t-\overline{\delta}}^{t}\int_{t-\overline{\delta}}^{s} e(s)\mathrm{d}s\mathrm{d}\theta \end{bmatrix}. \tag{28}$$

By applying Lemma 1, we obtain

$$-\frac{\overline{\delta}^2}{2}\int_{\overline{\delta}}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)W_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \le -\begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix}^T \begin{bmatrix} W_3 & 0 \\ 0 & 2W_3 \end{bmatrix} \begin{bmatrix} \Phi_2 \\ \Phi_3 \end{bmatrix}, \tag{29}$$

where $\Phi_2 = \overline{\delta}e(t) - \int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s$ and $\Phi_3 = -\overline{\delta}/2e(t) - \int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s + 3/\overline{\delta}\int_{-\overline{\delta}}^{0}\int_{t+\theta}^{t} e(s)\mathrm{d}s\mathrm{d}\theta$.

From Assumption 1, we obtain

$$\begin{aligned} &[f_k(e_k(t)) - M_k^- e_k(t)][f_k(e_k(t)) - M_k^- e_k(t)] \le 0, \\ &[f_k(e_k(t-\delta(t))) - M_k^- e_k(t-\delta(t))] \times [f_k(e_k(t-\delta(t))) - M_k^- e_k(t-\delta(t))] \le 0, \\ &\text{where,} \quad k = 1, 2, \ldots, n. \end{aligned} \tag{30}$$

This can be written algebraically as

$$\begin{aligned} &\begin{bmatrix} e(t) \\ f(e(t)) \end{bmatrix}^T \begin{bmatrix} M_t & -M_u \\ * & I \end{bmatrix} \begin{bmatrix} e(t) \\ f(e(t)) \end{bmatrix} \le 0, \\ &\begin{bmatrix} e(t-\delta(t)) \\ f(e(t-\delta(t))) \end{bmatrix}^T \begin{bmatrix} M_t & -M_u \\ * & I \end{bmatrix} \begin{bmatrix} e(t-\delta(t)) \\ f(e(t-\delta(t))) \end{bmatrix} \le 0. \end{aligned} \tag{31}$$

Then, the following inequality holds for any positive matrices $\mathscr{U}_t = \mathrm{diag}\{u_1, u_2, \ldots, u_n\}$ and $\mathscr{U}_u = \mathrm{diag}\{\widehat{u}_1, \widehat{u}_2, \ldots, \widehat{u}_n\}$:

$$\begin{bmatrix} e(t) \\ f(e(t)) \end{bmatrix}^T \begin{bmatrix} M_t\mathscr{U}_t & -M_u\mathscr{U}_t \\ * & \mathscr{U}_t \end{bmatrix} \begin{bmatrix} e(t) \\ f(e(t)) \end{bmatrix} \leq 0, \tag{32}$$

$$\begin{bmatrix} e(t-\delta(t)) \\ f(e(t-\delta(t))) \end{bmatrix}^T \begin{bmatrix} M_t\mathscr{U}_u & -M_u\mathscr{U}_u \\ * & \mathscr{U}_u \end{bmatrix} \begin{bmatrix} e(t-\delta(t)) \\ f(e(t-\delta(t))) \end{bmatrix} \leq 0. \tag{33}$$

For convenience, consider a matrix $\mathscr{N}$ with appropriate dimension, and the following zero equality holds:

$$0 = 2\left[e^T(t) + \dot{e}^T(t)\right]\mathscr{N}\left[-\dot{e}^T(t) - (\mathbf{A}_i + K_i\mathbf{C}_i)e(t) + K_i\mathbf{D}_i e(t-\delta(t)) + \mathbf{B}_i f(e(t)) + \mathbf{B}_{di} f(e(t-\delta(t)) + (\mathbf{E}_{1i} - K_i\mathbf{E}_{2i})w(t)\right]. \tag{34}$$

Therefore, from (22)–(34), given that $\alpha > 0$, we obtain

$$\mathscr{L}V(e(t)) - \alpha V(x(t)) - \alpha w^T(t)Xw(t) \leq \vartheta^T(t)\mathbf{\Psi}\vartheta(t) < 0, \tag{35}$$

where

$$\vartheta^T(t) = \left[e^T(t)e^T(t-\delta(t))e^T(t-\overline{\delta})\dot{e}^T(t)\left(\int_{t-\overline{\delta}}^{t} e(s)\mathrm{d}s\right)^T\left(\int_{-\overline{\delta}}^{0}\int_{t+\theta}^{t} e(s)\mathrm{d}s\mathrm{d}\theta\right)^T f^T(e(t)f(e(t-\delta(t))w^T(t)\right]. \tag{36}$$

We can write this as

$$\mathscr{L}V(e(t)) \leq \alpha V(x(t)) + \alpha w^T(t)Xw(t). \tag{37}$$

Multiplying both sides of (37) by $e^{-\alpha t}$ and then integrating from 0 to t, where $t \in [0, T]$, we obtain

$$e^{-\alpha t}\mathbb{E}[V(e(t))] \leq \mathbb{E}[V(e(0))] + \alpha\int_0^t e^{\alpha s}w^T(s)Xw(s)\mathrm{d}s. \tag{38}$$

It can be simplified as

$$\mathbb{E}[V(e(t))] < e^{\alpha t}\left(\mathbb{E}[V(e(0))] + \alpha\int_0^t e^{\alpha s}w^T(s)Xw(s)\mathrm{d}s\right), \tag{39}$$

$$\mathbb{E}[V(e(t))] < e^{\alpha T}\left(\mathbb{E}[V(e(0))] + \lambda_9 d\right). \tag{40}$$

Let $\overline{P}_i = R^{-1/2}P_iR^{-1/2}$, $\overline{Q}_1 = R^{-1/2}Q_1R^{-1/2}$, $\overline{Q}_2 = R^{-1/2}Q_2R^{-1/2}$, $\overline{Q}_{3i} = R^{-1/2}Q_3R^{-1/2}$, $\overline{W}_1 = R^{-1/2}W_1R^{-1/2}$, $\overline{W}_2 = R^{-1/2}W_2R^{-1/2}$, and $\overline{W}_3 = R^{-1/2}W_3R^{-1/2}$.

Conversely,

$$\begin{aligned}
\mathbb{E}[V(e_0, 0)] &= \lambda_{\max}(\overline{P}_i)e^T(0)\mathrm{Re}(0) + \lambda_{\max}(\overline{Q}_1)\int_{-\delta(0)}^{0} e^T(s)\mathrm{Re}(s)\mathrm{d}s + \lambda_{\max}(\overline{Q}_2)\int_{-\overline{\delta}}^{0} e^T(s)\mathrm{Re}(s)\mathrm{d}s + \lambda_{\max}(\overline{Q}_3)\max\left|M_t^-, M_u^+\right|^2 \\
&\quad\times\int_{-\delta(0)}^{0} e^T(s)\mathrm{Re}(s)\mathrm{d}s + \lambda_{\max}(\overline{W}_1)\int_{-\overline{\delta}}^{0}\int_{\theta}^{0} e^T(s)\mathrm{Re}(s)\mathrm{d}s\mathrm{d}\theta + \lambda_{\max}(\overline{W}_2)\int_{-\overline{\delta}}^{0}\int_{\theta}^{0} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad+ \lambda_{\max}(\overline{W}_3)\int_{-\overline{\delta}}^{0}\int_{\beta}^{0}\int_{\theta}^{0} \dot{e}^T(s)R\dot{e}(s)\mathrm{d}s\mathrm{d}\theta \\
&\leq \left\{\lambda_{\max}(\overline{P}) + \overline{\delta}\lambda_{\max}(\overline{Q}_1) + \overline{\delta}\lambda_{\max}(\overline{Q}_2) + \overline{\delta}\max\left|M_t^-, M_u^+\right|^2\lambda_{\max}(\overline{Q}_3) + \frac{\overline{\delta}^2}{2}\lambda_{\max}(\overline{W}_1) + \frac{\overline{\delta}^2}{2}\lambda_{\max}(\overline{W}_2) + \frac{\overline{\delta}^5}{12}\lambda_{\max}(\overline{W}_3)\right\} \\
&\quad\sup_{-\overline{\delta}\leq s\leq 0}\left\{e^T(s)\mathrm{Re}(s), \dot{e}^T(s)R\dot{e}(s)\right\}, \\
V(x(t)) &\leq e^{\alpha T}\left(\Lambda c_1 + d\lambda_9\right),
\end{aligned} \tag{41}$$

where

$$\Lambda = \lambda_2 + \overline{\delta}\lambda_3 + \overline{\delta}\lambda_4 + \overline{\delta}\max\left|M_t^-, M_u^+\right|^2\lambda_5 + \frac{\overline{\delta}^2}{2}\lambda_6 + \frac{\overline{\delta}^2}{2}\lambda_7 + \frac{\overline{\delta}^5}{12}\lambda_8. \quad (42)$$

Note that

$$\mathbb{E}[V(x(t))] \geq \lambda_{\min}(\overline{P})\mathbb{E}\left[x^T(t)Rx(t)\right] = \lambda_1\mathbb{E}\left[x^T(t)Rx(t)\right]. \quad (43)$$

Then, from (18), we obtain

$$\mathbb{E}\left[x^T(t)Rx(t)\right] < c_2. \quad (44)$$

Based on Definition 2, an MJNN defined as (7) is SFTB. □

Corollary 1. *Given scalars c_1, c_2, T, $\overline{\delta}$, μ, and α, an MJNN defined as (7) with $w(t) = 0$ is SFTB in relation to (c_1, c_2, T, R) if there exist feasible matrices $P_i > 0$, $Q_s > 0$, $W_s > 0$ $(s = 1, 2, 3)$, $\mathscr{U}_t > 0$, $\mathscr{U}_u > 0$, and $\mathscr{N}$, where P_i, Q_s, and W_s are symmetric PD, and $\mathscr{U}_t > 0$ and $\mathscr{U}_u > 0$ are diagonal such that the following inequality holds:*

$$\Psi_1 = \left[\psi_{i,j}\right]_{8\times 8} < 0, \quad (45)$$

$$e^{\alpha T}\left[c_1\Lambda\right] < \lambda_1 c_2, \quad (46)$$

where ψ_{ij} is defined in Theorem 1.

Proof. It can be proved in a similar way as Theorem 1. The proof is omitted for brevity. □

3.2. Finite-Time H_∞ State Estimation

Theorem 2. *Given scalars c_1, c_2, T, $\overline{\delta}$, μ, d, and α, an MJNN defined as (7) is SFTB in relation to (c_1, c_2, T, R, d) with noise attenuation $\gamma > 0$ if there exist feasible matrices $P_i > 0$, $Q_s > 0$, $W_s > 0$ $(s = 1, 2, 3)$, $\mathscr{U}_t > 0$, $\mathscr{U}_u > 0$, $\mathscr{N}$, and X, where P_i, Q_s, and W_s are symmetric PD, and $\mathscr{U}_t > 0$ and $\mathscr{U}_u > 0$ are diagonal, such that the following inequality holds:*

$$\overline{\Psi} = \begin{bmatrix} \Psi_1 & \widehat{\Psi} & G_i \\ * & \overline{\psi}_{99} & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (47)$$

$$e^{\alpha T}\left[c_1\Lambda + \gamma^2 d\right] < \lambda_1 c_2, \quad (48)$$

where $\Psi_1 = [\psi_{i,j}]_{8\times 8}$, $\widehat{\Psi} = col[\psi_{i9}]$, $i = 1, 2, \ldots, 8$,

$$\begin{aligned}
&\psi_{11} = \sum_{j=1}^{N}\pi_{ij}P_j + Q_1 + Q_2 + \overline{\delta}W_1 - 9W_2 - \overline{\delta}W_3 - \frac{\overline{\delta}^2}{4}w_3 - M_t\mathscr{U}_t - \mathscr{N}\mathbf{A}_i - \mathbf{A}_i^T\mathscr{N}^T - L_i\mathbf{C}_i - \mathbf{C}_i^T L_i^T,\\
&\psi_{12} = L\mathbf{D}_i, \psi_{13} = 3W_2, \psi_{14} = 2P_i - \mathscr{N} - \mathbf{A}_i^T\mathscr{N}^T - \mathbf{C}_i^T L_i^T, \psi_{15} = 36W_2 + \overline{\delta}W_3 + \frac{\overline{\delta}}{2}W_3,\\
&\psi_{16} = -2W_2 - \frac{3}{2}W_3, \psi_{17} = M_u\mathscr{U}_t + \mathscr{N}\mathbf{B}_i, \psi_{18} = \mathscr{N}\mathbf{B}_{di}, \psi_{19} = \mathscr{N}E_{1i} - L_i\mathbf{E}_{2i},\\
&\psi_{22} = -(1-\mu)Q_1 - M_t\mathscr{U}_u, \psi_{23} = 0, \psi_{24} = \mathbf{D}_i^T L^T, \psi_{25} = 0, \psi_{26} = 0, \psi_{27} = 0, \psi_{28} = M_u\mathscr{U}_u, \psi_{29} = 0,\\
&\psi_{33} = -Q_2 - 9W_2, \psi_{34} = 0, \psi_{35} = -30W_2, \psi_{36} = -5W_2, \psi_{37} = 0, \psi_{38} = 0, \psi_{39} = 0,\\
&\psi_{44} = \overline{\delta}W_2 + \frac{\overline{\delta}^4}{4}W_2 - \mathscr{N} - \mathscr{N}^T, \psi_{45} = 0, \psi_{46} = 0, \psi_{47} = \mathscr{N}\mathbf{B}_i, \psi_{48} = \mathscr{N}\mathbf{B}_{di}, \psi_{49} = \mathscr{N}\mathbf{E}_{1i} - L_i\mathbf{E}_{2i},\\
&\psi_{55} = -\frac{1}{\overline{\delta}}W_1 - \frac{3}{\overline{\delta}}W_1 - 192W_2 - 2W_3, \psi_{56} = \frac{6}{\overline{\delta}^2}W_1 - 30W_2 + \frac{3}{\overline{\delta}}W_3, \psi_{57} = 0, \psi_{58} = 0, \psi_{59} = 0,\\
&\psi_{66} = -\frac{12}{\overline{\delta}^3}, W_1 - 5W_2 - \frac{9}{\overline{\delta}^2}W_3, \psi_{67} = 0, \psi_{68} = 0, \psi_{69} = 0, \psi_{77} = Q_3 - \mathscr{U}_t, \psi_{78} = 0, \psi_{79} = 0,\\
&\psi_{88} = -Q_3 - \mathscr{U}_u, \psi_{89} = 0, \overline{\psi}_{99} = -\gamma^2 I.
\end{aligned} \quad (49)$$

Proof. In a similar way to the proof in Theorem 1, we obtain

$$\mathbb{E}\{\mathscr{L}V(e(t))\} + \overline{z}^T(t)\overline{z}(t) - \gamma^2 w^T(t)w(t) < \vartheta^T(t)\overline{\Psi}\vartheta(t). \quad (50)$$

It can be deduced from (47) and (50) that

$$\mathbb{E}\{\mathscr{L}V(e(t))\} + \overline{z}^T(t)\overline{z}(t) - \gamma^2 w^T(t)w(t) < 0. \quad (51)$$

By integrating (51) from 0 to T, we obtain

$$\mathbb{E}\left\{V(x(t)) - V(x(o)) + \int_0^T \overline{z}^T(t)\overline{z}(t)\mathrm{d}t - \gamma^2 \int_0^T w^T(t)w(t)\mathrm{d}t\right\} < 0. \tag{52}$$

Subsequently, the following inequality is obtained:

$$\mathbb{E}\left\{\int_0^T \overline{z}^T(t)\overline{z}(t)\mathrm{d}t\right\} \leq \gamma^2 \mathbb{E}\left\{\int_0^T w^T(t)w(t)\mathrm{d}t\right\}. \tag{53}$$

Hence, we conclude that the MJNN defined as (7) is SFTB. □

Remark 1. Consider the following error system from the MJNN defined as (7), with $w(t) = 0$ and without MJ parameters:

$$\begin{aligned}\dot{e}(t) = &-(\mathbf{A} + K\mathbf{C})e(t) - K\mathbf{D}e(x(t - \delta(t)))\\ &+ \mathbf{B}f(e(t)) + \mathbf{B}_d f(e(t - \delta(t))).\end{aligned} \tag{54}$$

Corollary 2. *Given scalars $\overline{\delta}$ and μ, the error system (54) with $w(t) = 0$ is said to be stable if there exist feasible matrices $P_i > 0$, $Q_s > 0$, $W_s > 0$ $(s = 1, 2, 3)$, $\mathscr{U}_t > 0$, $\mathscr{U}_u > 0$, and $\mathscr{N}$, where P_i, Q_s, and W_s are symmetric PD, and $\mathscr{U}_t > 0$ and $\mathscr{U}_u > 0$ are diagonal, such that the following inequality holds:*

$$\overline{\Psi} = \left[\overline{\psi}_{i,j}\right]_{8\times 8} < 0, \tag{55}$$

where

$$\begin{aligned}
&\overline{\psi}_{11} = Q_1 + Q_2 + \overline{\delta}W_1 - 9W_2 - \overline{\delta}W_3 - \frac{\overline{\delta}^2}{4}w_3 - M_t\mathscr{U}_t - \mathscr{N}\mathbf{A} - \mathbf{A}^T\mathscr{N}^T - L\mathbf{C} - \mathbf{C}^T L^T, \overline{\psi}_{12} = \mathrm{LD},\\
&\overline{\psi}_{13} = 3W_2, \overline{\psi}_{14} = 2P - \mathscr{N} - \mathbf{A}^T\mathscr{N}^T - \mathbf{C}^T L^T, \overline{\psi}_{15} = 36W_2 + \overline{\delta}W_3 + \frac{\overline{\delta}}{2}W_3, \overline{\psi}_{16} = -2W_2 - \frac{3}{2}W_3,\\
&\overline{\psi}_{17} = M_u\mathscr{U}_t + \mathscr{N}\mathbf{B}, \overline{\psi}_{18} = \mathscr{N}\mathbf{B}_{di}, \overline{\psi}_{22} = -(1-\mu)Q_1 - M_t\mathscr{U}_u, \overline{\psi}_{23} = 0, \overline{\psi}_{24} = \mathbf{D}^T L^T, \overline{\psi}_{25} = 0,\\
&\overline{\psi}_{26} = 0, \overline{\psi}_{27} = 0, \overline{\psi}_{28} = M_u\mathscr{U}_u, \overline{\psi}_{33} = -Q_2 - 9W_2, \overline{\psi}_{34} = 0, \overline{\psi}_{35} = -30W_2, \overline{\psi}_{36} = -5W_2, \overline{\psi}_{37} = 0,\\
&\overline{\psi}_{38} = 0, \overline{\psi}_{44} = \overline{\delta}W_2 + \frac{\overline{\delta}^4}{4}W_2 - \mathscr{N} - \mathscr{N}^T, \overline{\psi}_{45} = 0, \overline{\psi}_{46} = 0, \overline{\psi}_{47} = \mathscr{N}\mathbf{B}, \overline{\psi}_{48} = \mathscr{N}\mathbf{B}_{di},\\
&\overline{\psi}_{55} = -\frac{1}{\overline{\delta}}W_1 - \frac{3}{\overline{\delta}}W_1 - 192W_2 - 2W_3, \overline{\psi}_{56} = \frac{6}{\overline{\delta}^2}W_1 - 30W_2 + \frac{3}{\overline{\delta}}W_3, \overline{\psi}_{57} = 0, \overline{\psi}_{58} = 0,\\
&\overline{\psi}_{66} = -\frac{12}{\overline{\delta}^3}W_1 - 5W_2 - \frac{9}{\overline{\delta}^2}W_3, \overline{\psi}_{67} = 0, \overline{\psi}_{68} = 0, \overline{\psi}_{77} = Q_3 - \mathscr{U}_t, \overline{\psi}_{78} = 0, \overline{\psi}_{88} = -Q_3 - \mathscr{U}_u.
\end{aligned} \tag{56}$$

Proof. Following similar ideas as in the proof of Theorem 1. The proof is omitted for brevity. □

Remark 2. Consider a NN from the MJNN defined as (7) with $C = 0$, $D = 0$, $w(t) = 0$, and no MJ parameters:

$$\dot{e}(t) = -\mathbf{A}e(t) + \mathbf{B}f(e(t)) + \mathbf{B}_d f(e(t - \delta(t))). \tag{57}$$

Corollary 3. *Given scalars $\overline{\delta}$ and μ, the error system (57) with $w(t) = 0$ is said to be stable if there exist feasible matrices $P_i > 0$, $Q_s > 0$, $W_s > 0$ $(s = 1, 2, 3)$, $\mathscr{U}_t > 0$, $\mathscr{U}_u > 0$, and $\mathscr{N}$, where P_i, Q_s, and W_s are symmetric PD, and $\mathscr{U}_t > 0$ and $\mathscr{U}_u > 0$ are diagonal, such that the following inequality holds:*

$$\widetilde{\Psi} = \left[\widetilde{\psi}_{i,j}\right]_{8\times 8} < 0, \tag{58}$$

where

$$\begin{aligned}
&\tilde{\psi}_{11} = Q_1 + Q_2 + \overline{\delta}W_1 - 9W_2 - \overline{\delta}W_3 - \frac{\overline{\delta}^2}{4}w_3 - M_t\mathcal{U}_t - \mathcal{N}\mathbf{A} - \mathbf{A}^T\mathcal{N}^T, \tilde{\psi}_{12} = 0, \tilde{\psi}_{13} = 3W_2,\\
&\tilde{\psi}_{14} = 2P - \mathcal{N} - \mathbf{A}^T\mathcal{N}^T, \tilde{\psi}_{15} = 36W_2 + \overline{\delta}W_3 + \frac{\overline{\delta}}{2}W_3, \tilde{\psi}_{16} = -2W_2 - \frac{3}{2}W_3, \tilde{\psi}_{17} = M_u\mathcal{U}_t + \mathcal{N}\mathbf{B},\\
&\tilde{\psi}_{18} = \mathcal{N}\mathbf{B}_{di}, \tilde{\psi}_{22} = -(1-\mu)Q_1 - M_t\mathcal{U}_u, \tilde{\psi}_{23} = 0, \tilde{\psi}_{24} = 0, \tilde{\psi}_{25} = 0, \tilde{\psi}_{26} = 0, \tilde{\psi}_{27} = 0, \tilde{\psi}_{28} = M_u\mathcal{U}_u,\\
&\tilde{\psi}_{33} = -Q_2 - 9W_2, \tilde{\psi}_{34} = 0, \tilde{\psi}_{35} = -30W_2, \tilde{\psi}_{36} = -5W_2, \tilde{\psi}_{37} = 0, \tilde{\psi}_{38} = 0,\\
&\tilde{\psi}_{44} = \overline{\delta}W_2 + \frac{\overline{\delta}^4}{4}W_2 - \mathcal{N} - \mathcal{N}^T, \tilde{\psi}_{45} = 0, \tilde{\psi}_{46} = 0, \tilde{\psi}_{47} = \mathcal{N}\mathbf{B}, \tilde{\psi}_{48} = \mathcal{N}\mathbf{B}_{di},\\
&\tilde{\psi}_{55} = -\frac{1}{\overline{\delta}}W_1 - \frac{3}{\overline{\delta}}W_1 - 192W_2 - 2W_3, \tilde{\psi}_{56} = \frac{6}{\overline{\delta}^2}W_1 - 30W_2 + \frac{3}{\overline{\delta}}W_3, \tilde{\psi}_{57} = 0, \tilde{\psi}_{58} = 0,\\
&\tilde{\psi}_{66} = -\frac{12}{\overline{\delta}^3}W_1 - 5W_2 - \frac{9}{\overline{\delta}^2}W_3, \tilde{\psi}_{67} = 0, \tilde{\psi}_{68} = 0, \tilde{\psi}_{77} = Q_3 - \mathcal{U}_t, \tilde{\psi}_{78} = 0, \tilde{\psi}_{88} = -Q_3 - \mathcal{U}_u.
\end{aligned} \tag{59}$$

Proof. It can be proved in a similar way to Theorem 1. The proof is omitted for brevity. □

Remark 3. The stability analysis of time-delay systems can be classified into two categories, i.e., delay-dependent stability criteria and delay-independent ones. Also, it is well known that delay-dependent stability criteria, which use the information on the size of time delays, are less conservative than delay-independent ones. Thus, more attention has been paid to the derivation of delay-dependent stability criteria for time-delay systems.

Remark 4. It is important to note that some pioneering works have been done on finite-time H_∞ state estimation for Markovian jump neural networks based on interval time-varying delay with simple LKF techniques. In [29], the authors studied finite-time boundedness for Markovian jump neural networks with L_2 gain analysis. Authors in [26] formulated finite-time stabilization of uncertain neural networks. Exponential state estimation problem has been designed for Markovian jumping neural networks in [18]. The model consider in this present study is more practical than that proposed by [18, 26, 29], whereas in this paper, we consider finite-time H_∞ state estimation problem with the combination of Markovian jump neural networks' interval time-varying delay model, which is another advantage. However, the authors in [18, 26, 29] used some simple techniques in LKFs to solve the stability problems to those articles. A new LKF with double and triple integral terms and utilizing extended Writnger's integral inequality (EWII) techniques has been proposed for the stochastically finite-time bounded analysis of Markovian jump system in this paper. Consider that some less conservative results can occur in our method and can be provided in the numerical example section with real-life examples. Hence, the results presented in this paper are essentially new.

Remark 5. Typically, finite time stability with H_∞ control, state estimation approach, and interval time-varying delay is not simply applied to Markovian jump neural networks. Some research publications have handled such issues [*17*, *18*, *26*, *29*]. As it is, the author utilized some elementary LKFs to deal with the stability problems in those articles. Novel LKF with EWII has been proposed; in addition, the developed stochastic stability criteria tested for feasibility of the benchmark problem to explore the real-world application in this paper. However, the desired control was completely studied for the considered neural network model with the real-world application problem (e.g., four-tank pumping system and network circuit), which is the principle commitment and inspiration of our work.

4. Numerical Examples

This section shows our results through some numerical examples on MJNNs with 2 operation modes to demonstrate the effectiveness of the proposed approach.

Example 1. Consider an MJNNs with MJ parameters ($i = 2$):

$$\begin{aligned}
&\mathbf{A}_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} -0.7 & 0.3 \\ 0.6 & -0.8 \end{bmatrix}, \mathbf{B}_{d1} = \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.4 \end{bmatrix}, \mathbf{E}_{11} = \begin{bmatrix} 0.1 & 0.9 \\ -0.6 & 0.2 \end{bmatrix},\\
&\mathbf{C}_1 = \begin{bmatrix} 0.4 & 0.8 \\ -0.7 & 0.2 \end{bmatrix}, \mathbf{D}_1 = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.2 \end{bmatrix}, \mathbf{E}_{21} = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 0.8 \end{bmatrix}, \mathbf{G}_1 = \begin{bmatrix} 0.3 & 0 \\ 0.2 & 0.4 \end{bmatrix},\\
&\mathbf{A}_2 = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -0.5 & 0.2 \\ 0.4 & -0.5 \end{bmatrix}, \mathbf{B}_{d2} = \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.4 \end{bmatrix}, \mathbf{E}_{12} = \begin{bmatrix} 0.07 & 0.8 \\ -0.5 & 0.1 \end{bmatrix},\\
&\mathbf{C}_2 = \begin{bmatrix} 0.5 & 0.7 \\ -0.8 & 0.1 \end{bmatrix}, \mathbf{D}_2 = \begin{bmatrix} -0.6 & 0 \\ 0 & -0.24 \end{bmatrix}, \mathbf{E}_{22} = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 0.8 \end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix},\\
&\pi = \begin{bmatrix} 4 & -4 \\ -3 & 3 \end{bmatrix}, \overline{\delta} = 2.5, \mu = 0.3, d = 0.02, T = 5, c_1 = 1, c_2 = 4, \alpha = 0.0002.
\end{aligned} \tag{60}$$

The activation functions are given as $M_t = \text{diag}\{0, 0\}$ and $M_u = \text{diag}\{1, 1\}$. By solving the LMIs in Theorem 2, we can obtain a feasible solution:

$$\begin{aligned}
&P_1 = \begin{bmatrix} 180.9128 & 184.5018 \\ 184.5018 & 198.2708 \end{bmatrix}, P_2 = \begin{bmatrix} 191.9509 & 182.5133 \\ 182.5133 & 179.4404 \end{bmatrix}, Q_1 = \begin{bmatrix} 70.8390 & 21.9994 \\ 21.9994 & 15.5734 \end{bmatrix},\\
&Q_2 = \begin{bmatrix} 2.4863 & 2.4205 \\ 2.4205 & 3.2684 \end{bmatrix}, Q_3 = \begin{bmatrix} 27.3140 & 19.0082 \\ 19.0082 & 14.7271 \end{bmatrix}, W_1 = \begin{bmatrix} 1.7407 & 1.5347 \\ 1.5347 & 1.9210 \end{bmatrix},\\
&W_2 = \begin{bmatrix} 0.1952 & 0.1575 \\ 0.1575 & 0.1754 \end{bmatrix}, W_3 = \begin{bmatrix} 0.0496 & 0.0350 \\ 0.0350 & 0.0324 \end{bmatrix}.
\end{aligned} \tag{61}$$

Then, we obtain state estimator gain matrices as

$$K_1 = \begin{bmatrix} 2.8449 & 1.1379 \\ -1.6262 & -0.6468 \end{bmatrix}, K_2 = \begin{bmatrix} 2.2627 & 0.9719 \\ -1.1607 & -0.4569 \end{bmatrix}. \tag{62}$$

Thus, the system is SFTB with the external disturbance $\gamma = 0.90$.

To demonstrate the capability of the proposed approach, we show the effectiveness of the theoretical results, as shown in Figures 1–4 . Figure 1 demonstrates the MJ mode r_t. Figures 2 and 3 show the behaviors of the error system and state estimation of the error system, respectively. Figure 4 illustrates that the state $x(t)$ of the system converges to zero. Furthermore, the superiority of our theoretical results is demonstrated through the simulation result of $x^T(t)Rx(t)$ in Figures 5 and 6. Therefore, the proposed MJNN (7) is STFB.

Example 2. Consider the NN (57) with the following parameters:

$$\begin{aligned}
&\mathbf{A} = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},\\
&\mathbf{B}_d = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},\\
&M_t = \text{diag}\{0, 0, 0, 0\}, M_u = \text{diag}\{0.1137, 0.1279, 0.7994, 0.2368\}.
\end{aligned} \tag{63}$$

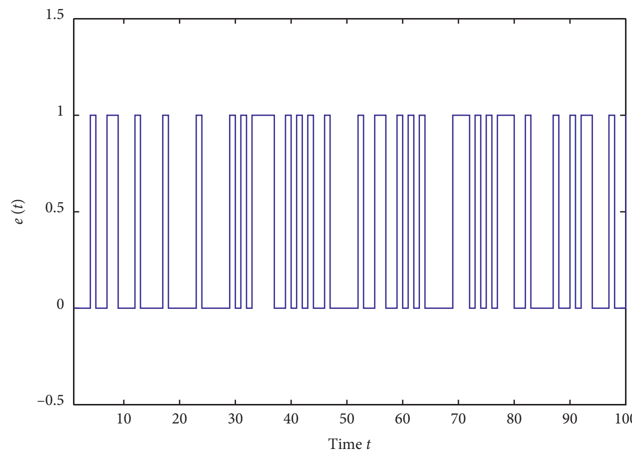

FIGURE 1: Markovian jumping mode.

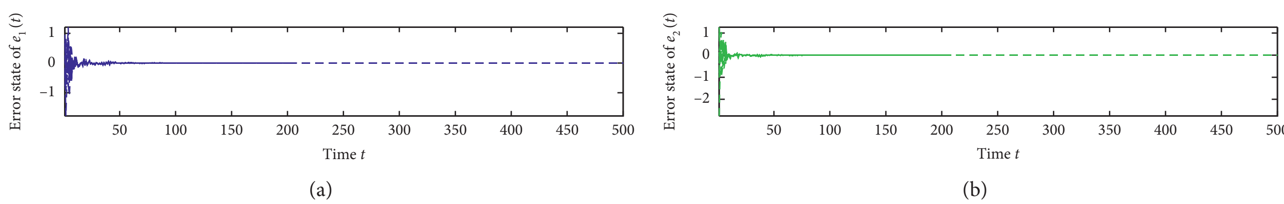

FIGURE 2: Estimation errors $e_1(t)$ and $e_2(t)$.

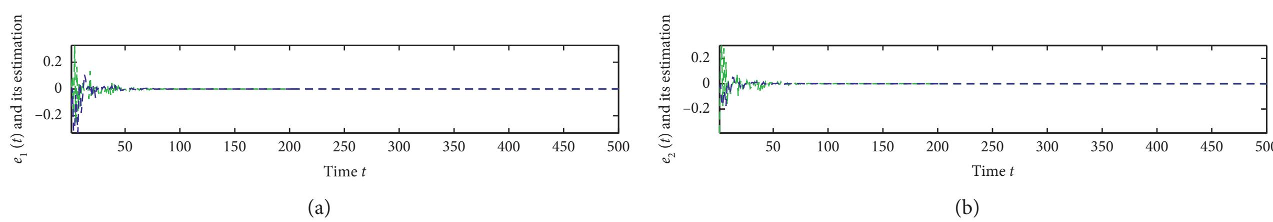

FIGURE 3: $e(t)$ and its estimation.

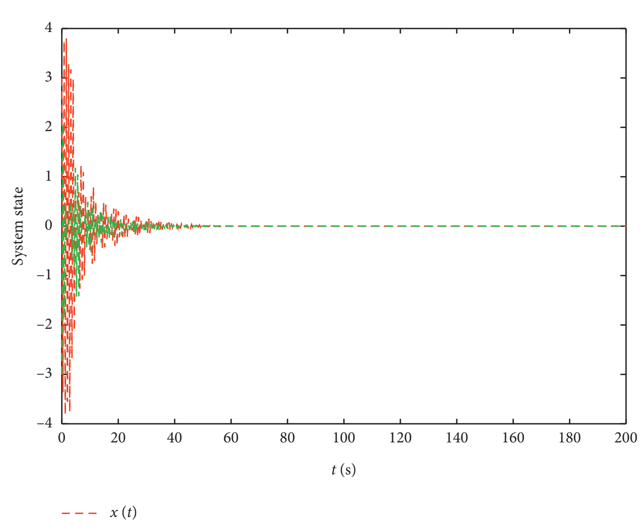

FIGURE 4: State trajectories of the system.

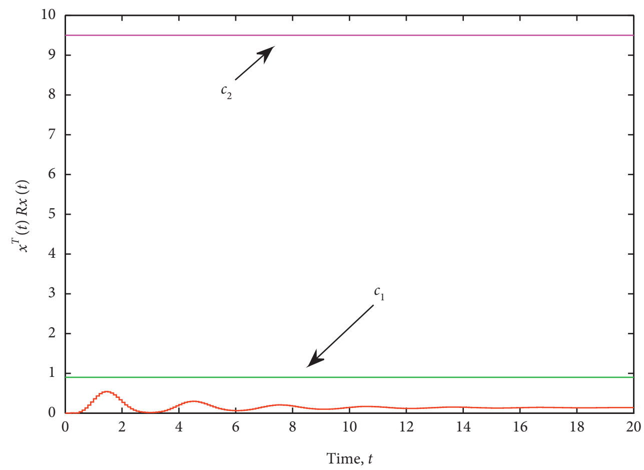

FIGURE 5: State trajectories of $x^T(T)Rx(t)$.

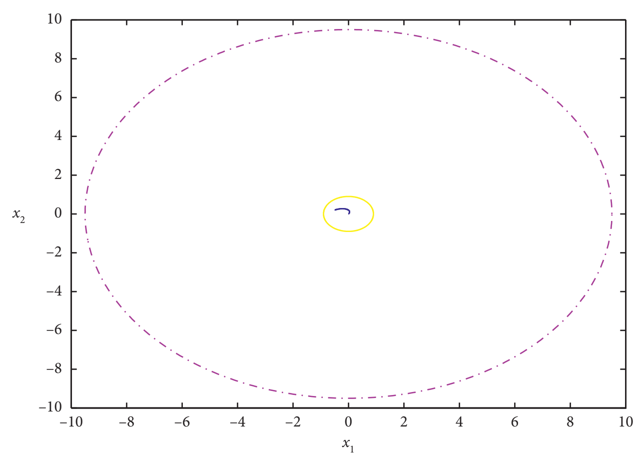

FIGURE 6: Evolution of $x^T(T)Rx(t)$.

This example shows a comparison of the conservativeness in the stability condition concerning the results in [36–38]. The maximum allowable delay bound (MADB) of $\overline{\delta}$ for various μ can be calculated using the MATLAB LMI toolbox. The MADBs of $\overline{\delta}$ for some values of μ in Example 2 are summarized in Table 1. We found that the outcomes of our proposed method produced better results than the previous research [36–38].

Example 3. Consider the NN (57) with the following parameters:

$$\mathbf{A}=\begin{bmatrix}2 & 0\\ 0 & 3.5\end{bmatrix},\mathbf{B}=\begin{bmatrix}-1 & 0.5\\ 0.5 & -1\end{bmatrix},\mathbf{B}_d=\begin{bmatrix}-0.5 & 0.5\\ 0.5 & 0.5\end{bmatrix},M_t=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},M_u=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{64}$$

For some values of μ, the MADBs of $\overline{\delta}$ are obtained and summarized in Table 2. We compare these results with those of previous studies [36, 39, 40]. As shown in Table 2, the MADB are larger than those obtained from [36, 39, 40]. It shows the superiority that the proposed stability criterion is less conservative than the previous works.

Remark 6. We calculated upper bounds with different delta, and they are listed in Tables 1 and 2. We provide comparisons with the results obtained in previous studies to show the improvements obtained by our proposed method.

Example 4. The NNs have similar characteristics to the neurons in a biological organism, leading to the nervous system. The NNs can represent not only the nervous systems with neurons but also the engineering systems such as the four-tank water pumping system, as shown in Figure 7. The four-tank water pumping system is equipped with 2 water

Table 1: The maximum delay upper bounds $\overline{\delta}$ for given μ in Example 2.

Method	0.5	0.8	0.9
[36]	3.6954	2.7711	2.5795
[37]	3.8709	3.3442	3.1291
[38]	4.2749	3.1993	2.9504
Corollary 3	4.5008	3.8621	3.5402

Table 2: The maximum allowable delay upper bounds $\overline{\delta}$ for given μ in Example 3.

Method	0.8	0.9
[36]	0.8784	0.8484
[39]	0.8841	0.8570
[40]	0.9631	0.9324
Corollary 3	1.3680	1.1035

pumps and 4 interconnected tanks with two valves. Voltage v_1 and v_2 are two input processes of two supplying pumps. The four-tank water pumping system can be modeled as a neural network model. Previous studies in [41–43] suggested the state-space equations of this four-tank system which is an application of the neural networks. State feedback controller modeled as follows:

$$\dot{\widehat{x}}(t) = \widehat{A}_0(\widehat{x}(t)) + \widehat{A}_1(\widehat{x}(t-\delta_1)) + \widehat{B}_0(\widehat{u}(t-\delta_2)) + \widehat{B}_1(\widehat{u}(t-\delta_3)), \tag{65}$$

where

$$\begin{aligned}\widehat{A}_0 &= \begin{bmatrix} -0.0021 & 0 & 0 & 0 \\ 0 & -0.0021 & 0 & 0 \\ 0 & 0 & -0.0424 & 0 \\ 0 & 0 & 0 & -0.0424 \end{bmatrix}, \widehat{A}_1 \\ \widehat{B}_0 &= \begin{bmatrix} 0.1113\gamma_1 & 0 & 0 & 0 \\ 0 & 0.1042(1-\gamma_2) & 0 & 0 \end{bmatrix}, \widehat{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0.1113(1-\gamma_1) \\ 0 & 0 & 0.1042(1-\gamma_2) & 0 \end{bmatrix}, \\ \gamma_1 &= 0.333, \gamma_2 = 0.307, \widehat{u} = \widehat{K}\widehat{x}(t), \widehat{K} = \begin{bmatrix} -0.1609 & -0.1765 & -0.0795 & -0.2073 \\ -0.1977 & -0.1579 & -0.2288 & -0.0772 \end{bmatrix}.\end{aligned} \tag{66}$$

Another control problem of our interests is obtained by adding transport delays $\delta(t)$ through delaying the inlet of incoming water into the tanks. Hence, the proposed approach has been used to study this problem here. Time-varying transport delays between valves and tanks have also been considered in the previous works, but they have not been considered the following aspects. For simplicity, it was assumed that $\delta_1 = 0, \delta_2 = 0$, and $\delta_3 = \delta(t)$ (since $\delta(t) \le \delta$). In this example, the control input $\widehat{u}(t)$ indicates the amount of water pumped. Therefore, it is naturally a nonlinear function and can be written as follows:

$$\begin{aligned}\widehat{u}(t) &= \widehat{K}\widehat{f}(\widehat{x}(t)), \\ \widehat{f}(\widehat{x}(t)) &= \left[\widehat{f}_1(\widehat{x}_1(t)), \ldots, \widehat{f}_4(\widehat{x}_4(t))\right]^T, \\ \widehat{f}_i(\widehat{x}_i(t)) &= 0.1\left(\left|\widehat{x}_i(t)+1\right| - \left|\widehat{x}_i(t)-1\right|\right), \\ i &= 1, 2, \ldots, 4.\end{aligned} \tag{67}$$

The four-tank system (65) can be rewritten to the form of system (57) with $\widehat{K} = 1$ as follows:

$$\dot{e}(t) = -\mathbf{A}e(t) + \mathbf{B}f(e(t)) + \mathbf{B}_d f(e(t-\delta(t))), \tag{68}$$

where $\mathbf{A} = \widehat{A}_0 - \widehat{A}_1$, $\mathbf{B} = \widehat{B}_0\widehat{K}$, $\mathbf{B}_d = \widehat{B}_1\widehat{K}$, and $f(\cdot) = \widehat{f}(\cdot)$. In addition, $M_t = \mathrm{diag}\{0,0,0,0\}$ and $M_u = \mathrm{diag}\{0.1, 0.1, 0.1, 0.1\}$ with $\delta = 6.5$ and $\mu = 0.5$. Using MATLAB LMI toolbox and solving the inequalities in Corollary 2, we are able to obtain feasible solution, which lead to a conclusion that FTPS (68) is stable.

Example 5. A continuous-time artificial NN containing n units can be described as the following well-known differential equations in [44]:

$$\left\{ \frac{\mathrm{d}e_i(t)}{\mathrm{d}t} = -\frac{e_i(t)}{R_iC_i} + \sum_{j=1}^{n} W_{ij}y_j(t) + u_i(t), y_i(t) = f_i(e_i(t)). \right. \tag{69}$$

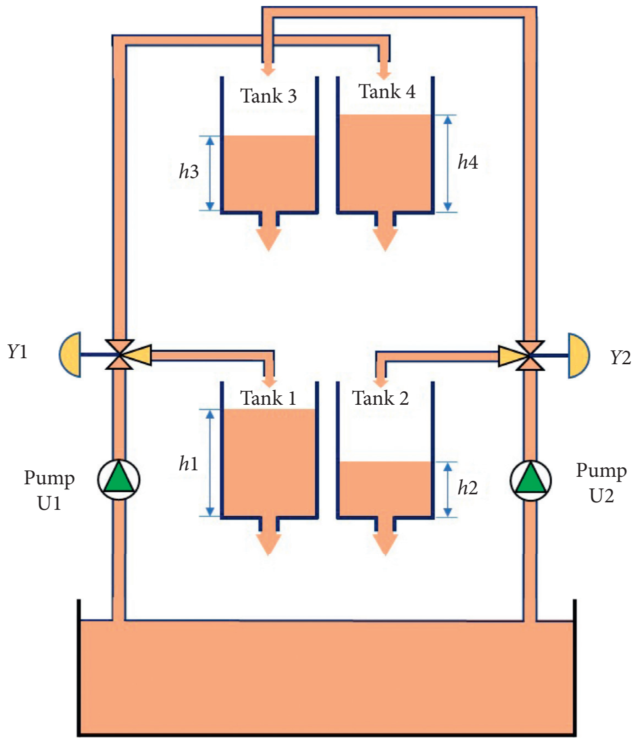

FIGURE 7: Schematic representation of the four-tank water pumping system.

This nonlinear system is implemented by a simple resistance-capacitance (RC) network circuit. It is shown in Figure 8, where $u_i = e_i$ and $V_i = f_i(e_i(t))$ are input and output voltage of the i^{th} amplifier, where V_i and $-V_i$ are two output terminals of the i^{th} amplifier, and the value R_i is defined as

$$\frac{1}{R_i} = \frac{1}{\sigma_i} + \sum_{j=1}^{n} \frac{1}{R_{ij}},$$
$$W_{ij} = \begin{cases} +\dfrac{1}{R_{ij}}, & R_{ij} \text{ is connected to } V_j, \\ -\dfrac{1}{R_{ij}}, & R_{ij} \text{ is connected to } -V_j. \end{cases} \tag{70}$$

Thus, nonlinear system (69) can be rewritten in the following form:

$$\dot{e}(t) = -\mathbf{A}e(t) + \mathbf{B}f(e(t)) + \mathbf{B_d}u, \tag{71}$$

with

$$\mathbf{A} = \begin{bmatrix} \dfrac{1}{R_1C_1} & 0 & \cdots & 0 \\ 0 & \dfrac{1}{R_2C_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{1}{R_nC_n} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \dfrac{W_{11}}{C_1} & \dfrac{W_{12}}{C_1} & \cdots & \dfrac{W_{1n}}{C_1} \\ \dfrac{W_{21}}{C_2} & \dfrac{1}{W_{21}/C_2} & \cdots & \dfrac{W_{2n}}{C_2} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{W_{n1}}{C_n} & 0 & \cdots & \dfrac{W_{nn}}{C_n} \end{bmatrix}, \mathbf{B_d} = \begin{bmatrix} \dfrac{1}{C_1} & 0 & \cdots & 0 \\ 0 & \dfrac{1}{C_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{1}{C_n} \end{bmatrix}. \tag{72}$$

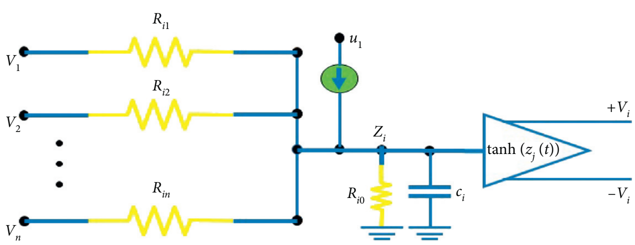

FIGURE 8: Circuit diagram for delayed NN.

The product $R_iC_i = \delta_i, i = 1, 2, \ldots, n$, is called as the time constant of the i^{th} neuron. An identical time constant for each neuron would require, that is, $C_i = C$ and $R_i = R$, for all i. In this case, every individual value for δ_i would have to be chosen in a way that compensates for C_i and R_i. It is important to note that the time constant δ_i describes the convergence of the neural state e_i of the i^{th} neuron. Because of the high-level gain of the transfer function, the output V_i might be saturated very fast. Thus, even if the state e_i is still far from reaching its equilibrium point, the output V_i might already be saturated, and by observing only V_i, it might appear as if the circuit had converged in merely a fraction of the time constant δ_i.

Consider the delayed neural networks (71) with the following parameters: $n = 2, C_1 = C_2 = R_1 = R_2 = w_{11} = 1$, $w_{22} = -1, w_{12} = 1.5, w_{21} = -1.5$, and $s_1 = s_2 = 0$. The neural network equations are, therefore, described as

$$\begin{cases} \dfrac{\mathrm{d}e_1(t)}{\mathrm{d}t} = -e_1(t) + f(e_1(t)) + 1.5 f(e_2(t)), \\ \\ \dfrac{\mathrm{d}e_2(t)}{\mathrm{d}t} = -e_2(t) - 1.5 f(e_2(t)) - f(e_2(t)), \end{cases} \tag{73}$$

and $f(e(t)) = [f_1(e_1(t)) \ldots f_n(e_n(t))]^T \in \mathbb{R}^n$, which satisfy

$$l_a^- \leq \frac{f_a(x_1) - f_a(x_2)}{x_1 - x_2} \leq l_a^+, \quad \forall x_1, x_2 \in \mathbb{R}, x_1 \neq x_2, \tag{74}$$

We can choose the value $f(e_k(t)) = \tanh e(t)$, which implies $l_1 = 0$ and $l_2 = 0.5I$, and we now apply Corollary 3 to system (71) by choosing $\mu = 0.5$ and $\delta = 0$ and $\mathbf{B_d} = 0$. Then, we can get (58) is feasible. Figure 9 shows the state responses of the system with the interval $[1, -0.5]^T$. Thus, the neural network (71) is asymptotically stable.

5. Conclusion

Herein, we studied the SFTB of MJNNs with time-varying delays. Using an LKF with Wirtinger's integral inequalities, a sufficient condition was derived such that the MJNNs were SFTB and satisfied a prescribed level of H_∞ disturbance attenuation in a finite-time interval. We illustrated the effectiveness of our main results with five numerical examples. We also compared to show that our results are less conservative than some existing ones. Future works focus on the discrete versions of these inequalities and their applications. [45–47].

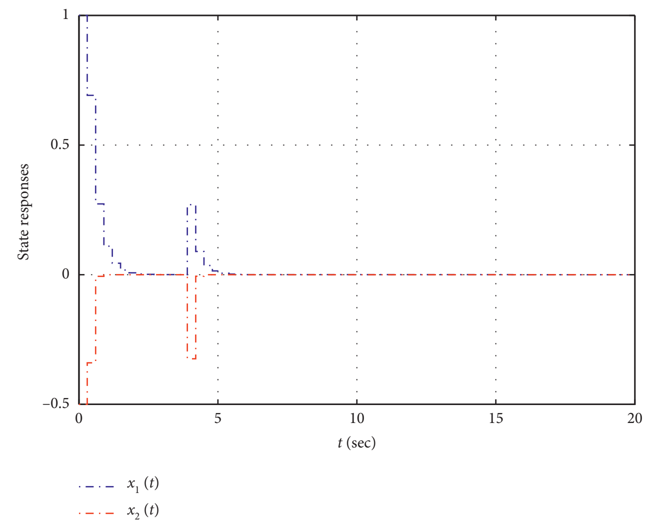

FIGURE 9: Circuit diagram for delayed NN.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (no. 2018R1A2B3008890).

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