A novel collaborative collision avoidance decision-making methodology based on potential collision areas for intelligent navigation

Jiongjiong Liu[1,2,3], Jinfen Zhang[1,2,3]*, Mingyang Zhang[4], Xuri Xin[5], Zaili Yang[5]

[1] *State Key Laboratory of Maritime Technology and Safety, Wuhan University of Technology, Wuhan, China*

[2] *Intelligent Transportation System Research Center, Wuhan University of Technology, Wuhan, China*

[3] *National Engineering Research Center for Water Transport Safety, Wuhan University of Technology, Wuhan, China*

[4] *Department of Mechanical Engineering, Aalto University, Espoo, Finland*

[5] *Liverpool Logistics, Offshore and Marine (LOOM) Research Institute, Liverpool John Moores University, Liverpool, UK*

*Corresponding author Email: jinfen.zhang@whut.edu.cn, Tel/Fax: 86 27 86589905

Abstract: Ship navigation safety is a crucial component of intelligent navigation, and collaborative collision avoidance plays a vital role in ensuring safety and conflict resolution by enhancing decision-making efficiency in multi-ship encounters. In this paper, a collaborative collision avoidance model is proposed to connect encounter risk and decision-making in a more intuitive way. It pioneers the quantification of collision risk based on potential collision areas and differentiates encountered ships, and a collaborative mechanism is further constructed to formulate a collision avoidance decision-making model. International Regulations for Preventing Collisions at Sea (COLREGs) are comprehensively considered by differentiating risk relationships among ships and specify their roles and responsibilities. A ship collaboration mechanism under action intention games is constructed based on rational thinking to form a decision-making model with game-decision cycles. The results demonstrate that the model can meet the safety requirements in case studies, providing a rational reflection, and accurately determines encounter stages. The results also indicate that ship differentiation and role assignment can adapt to abnormal actions. This research makes significant contributions to ship decision support in term of better collaboration among ships while reducing action conflicts, promoting the development of intelligent navigation technologies.

Keywords: Collision avoidance; collaborative decision-making; risk evaluation; potential collision area; intelligent navigation

1. Introduction

Shipping is critical to global economic development and sustainability, which accounts for 80% of the world's goods trade (Feng et al., 2024c; Li et al., 2022). Ship collisions, a significant hazard in maritime traffic, sometimes lead to devastating outcomes including fires, explosions, and sinking, which result in severe casualties, property damage, and environmental pollution (Feng et al., 2024b; Marino & Hajnsek, 2015). Statistics reveal that human factors, including poor assessment of ship encounters, inadequate communication and coordination between ships, and insufficient decision-making skills of officers on watch (OOWs), contribute to 60.6% of these accidents (Fan & Yang, 2024; Feng et al., 2024a). Consequently, several advancements have been proposed in intelligent navigation, decision support system (Wu et al., 2018), autonomous collision avoidance (Merwe et al., 2024) and intelligent navigation (Liu et al., 2024c), all aimed at minimizing human error-related accidents. Furthermore, International Maritime Organization (IMO) has established a classification for autonomous ships based on the degree of autonomy (IMO, 2023): L1) ship with automated processes and decision support; L2) remotely controlled ship with seafarers on board; L3) remotely controlled ship without seafarers on board; L4) fully autonomous ship. Regardless of L1~L4, a comprehensive collision risk assessment model is essential for effective conflict resolution (Xin et al., 2024), and collaboration among ships can make collision avoidance decisions executing more efficiently, thereby ensuring navigation safety (Akdag et al., 2022; Chaal et al., 2023).

Collision risk quantifies the likelihood or urgency of a collision, whose precision significantly influences decisions on collision avoidance (Yuan et al., 2023). The higher the risk, the more urgent actions are required. However, there is a gap in research directly linking collision risk or situational urgency to specific collision avoidance actions. Velocity obstacle (VO) is utilized to evaluate collision risk in the ship velocity field, obtaining feasible decisions derived from the complement set (Huang et al., 2018). However, VO fails in reflecting the urgency of different velocities within the set, potentially leading to dangerous scenarios during collision avoidance processes. Meanwhile, effective collision avoidance aims to ensure that ships do not appear at the same location at the same time, necessitating adequate safety spacing with economical actions (Zhao & Roh, 2019). Following this idea, this study initially identifies all potential collision areas—the spatial positions where own ship (OS) and target ships (TSs) might collide—and evaluates risk levels based on micro-decision amplitudes and macro-distribution of regional risk. Moreover, a collaborative mechanism is introduced through action intention games based on rational thinking. Finally, collaborative collision avoidance decisions are formulated by considering the encounter evolutions and action intention games in multi-ship encounters. This systematic approach provides with a framework for enhancing navigation safety and development of intelligent navigation, see Fig. 1.

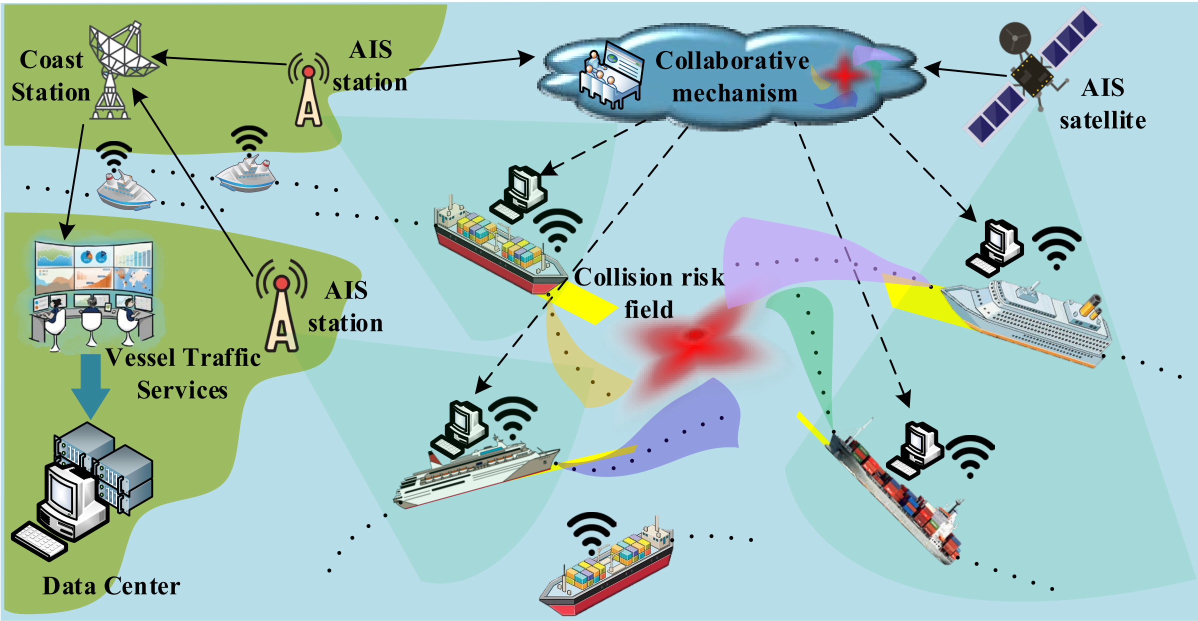


Fig.1 Collaborative collision avoidance in intelligent navigation

The remainder of this article is structured as follows: Section 2 provides a literature review of ship collision risk assessment and collision avoidance decision-making, aiming to identify gaps in existing studies and to underscore novel contributions. A collaborative collision avoidance decision-making model is proposed in section 3, involving collision risk based on potential collision areas and multi-ship collaborative collision avoidance based on action intention games. Case studies including a real collision accident and a simulation scenario in Section 4, are carried out to verify the decision-making model. In Section 5, conclusions are outlined.

2. Literature review

To understand the progress and trend of ship collision avoidance in more detail, this section summarizes existing research on ship collision risk assessment and collision avoidance decision-making, and identify the research gaps and contributions.

2.1 Ship collision risk assessment

Collision risk assessment is crucial for navigation safety and managing risks effectively (Sawada et al., 2024; Shi et al., 2022). The research method in this topic is categorized into two types: comprehensive collision risk indexes and ship domain (Xin et al., 2023a). The former constructs a collision risk model by selecting and integrating various parameters that can measure encounter situations (Liu et al., 2024a). Distance/Time to Closest Point of Approach (DCPA/TCPA) are the two most commonly used parameters (Li et al., 2019; Zhao & Fu, 2022), representing the spatial level and urgency of collision risk, respectively. Moreover, distance, relative orientation, ship speed ratio and others are also incorporated into collision risk modeling (Cai et al., 2021). To assign appropriate weights to each parameter, methodologies like evidence theory (Wu et al., 2018), analytic hierarchy process (Shi et al., 2022) and support vector machine (Song et al., 2023) are utilized.

Ship domain refers to an area around OS that is not allowed to be invaded by TSs to maintain a safe space (Szlapczynski et al., 2018). Based on actual traffic flow statistics, ship domain is first proposed by integrating International Regulations for Preventing Collisions at Sea (COLREGs) and ship collision avoidance responsibilities (Fujii, 1971). Its scale is influenced by ship speed, ship size, water conditions, traffic density and meteorological conditions (Silveira et al., 2022; Wang, 2013). The shape of ship domain also includes circular, rectangular, ellipse and their composites (Pietrzykowski & Wielgosz, 2021). Meanwhile, compared with DCPA and TCPA, Degree/Time of Domain Violation (d_{dv}/t_{dv}) are more suitable for elliptic ship domains (Szlapczynski & Szlapczynska, 2016; 2021). It combines these two methods to enhance the accuracy and relevance of risk evaluations. To enhance objectivity, data-driven collision risk models are developed (Yang et al., 2024). Automatic Identification System (AIS) data is utilized to obtain parameters and their weights from statistical perspectives (Ma et al., 2024; Zhao & Fu, 2022), and ship domain boundaries are also fitted based on spatiotemporal relationships, while the shape and scale of ship domains are determined (Silveira et al., 2022; Zhang & Meng, 2019), thereby reducing subjectivity comparing to traditional methods.

However, these two types of collision risk models only indicate the presence or value of a risk but do not directly facilitate collision avoidance decisions. Therefore, to improve practicability, a collision risk model should be constructed to directly reflect feasible decisions. VO makes decisions by analyzing the complement set of velocity states associated with risks, offering a qualitative description within velocity field (Huang et al., 2018). However, it cannot quantify the difficulty of ship collision avoidance actions, the evolving trends and urgency of encounter situations, making the description of ship collision risk less intuitive. Obstacle Zone by Target (OZT) (Sawada et al., 2021) and Dangerous Area of Collision (DAC) (Hashimoto et al., 2022) are used to identify geographical areas where ships may collide, and a collision risk distribution is formed by quantifying state risk in Yoshioka et al. (2023) and Zhang et al. (2023a). However, it is crucial to accurately evaluate the risk level to clarify the encounter stages, evaluating COLREGs compliance and the decision-making effectiveness. Therefore, it is essential to quantify ship risk based on geographical risk.

In summary, the quantitative results of collision risk in the previous studies are not directly applicable to collision avoidance decision-making, and existing research on ship risk distribution can neither directly indicate the collision risk level, nor facilitate easy evaluation of the decision-making effectiveness. Therefore, further research is needed to explore how to integrate these two studies to develop a quantitative model based on collision risk distribution.

2.2 Collision avoidance decision-making

Collision avoidance decision-making is a vital area of maritime traffic research (Huang et al., 2020; Li et al., 2024a), and is also one of the key technologies for the practical application of intelligent/autonomous ships (Liu et al., 2024c). Decision-making for collision avoidance can be categorized into centralized and distributed modes (Wang et al., 2023). Taking into account navigation characteristics, the distributed decision-making mode, in which ships take collision avoidance actions by independently assessing encounter situations, is more applicable (Zhang et al., 2015). As a legal document to guide ships decision-making at sea, COLREGs divides encountered ships into give-way and stand-on ships (Zhao & Roh, 2019). Stand-on ships may take coordinated actions according to the actions of give-way ships, especially if a give-way ship fails to take appropriate actions in accordance with COLREGs (Szlapczynski et al., 2018; Zhao & Roh, 2019). Meanwhile, in multi-ship encounter scenarios, safe navigable ships should not be affected by avoidance actions to bring new collision risks or more urgent dangers. Collision avoidance actions should also consider the special encounter scenario, such as involving the ship restricted in its ability to manoeuvre or not under command, with differentiated assessments for encountered ships (Cockcroft & Lameijer, 2012). However, in most existing research, all ships are set to take actions according to COLREGs and decision models (Lyu et al., 2023). There is a research gap in the adaptive capacities for handing abnormal actions of TSs.

With the advancement of collision avoidance decision-making research, it has gradually expanded from complete information scenarios with navigation states and action intentions known to part or all information sealed (Akdag et al., 2022), making it necessary to predict action intentions. For example, by discretizing steering angles, ships trajectories under different collision avoidance actions can be predicted, and the most effective collision avoidance scheme is found based on collision risk assessments (Li et al., 2019). Moreover, considering that ships can exchange action intentions through navigation aids, such as Very High Frequency (VHF), collaborative decision-making can be achieved more safely and economically (Akdag et al., 2022). Therefore, to ensure the rational and economical of decision-making, a collaboration mechanism should be constructed between encountered ships. Collision avoidance models are developed based on the exchange and negotiation of action intentions and navigation trajectories between ships, which greatly improves efficiency and ship actions effectiveness (Akdag et al., 2022; Hornauer et al., 2015; Liu et al., 2022). Liu et al. (2022) introduce a coordination collision avoidance model with intention notifications, and the model selects the most favorable strategy by predicting ships' navigation states based on these action intentions. Hornauer et al. (2015) develop a model to find feasible trajectories for all ships by exchanging and negotiating planning trajectories generated by an improved A* algorithm. Akdag et al. (2022) propose a collaborative decision-making model by exchanging their planned trajectories and improving the model predictive control algorithm. However, when the action intentions or trajectories of TSs cannot be obtained, such as inter-ship communication failure or delay, the effectiveness of decision-making is reduced significantly. So these methods rely heavily on the high reliability of information interaction between ships. Wang et al. (2023) construct a collaborative decision-making model based on cooperative game, and formulate a global responsibility allocation mode. This provides with a new idea for ship collision avoidance decision-making.

In general, to enhance the adaptive capabilities of collision avoidance decision-making, it should further address the collaborative actions among ships, and permit stand-on ships taking appropriate actions when give-way ships fail to fulfill their responsibilities, as mandated by COLREGs. Meanwhile, building a collaborative mechanism to enable appropriate anti-collision operations between ships without information interaction and making decisions in real time mode based on the predicted situation evolution would greatly enhance applications in practice.

2.3 Research contributions

By analyzing ship collision risk and collision avoidance decision-making, this study proposes a new potential collision areas concept to connect encounter risk and decision-making more intuitively. The research contributions can be summarized as follows:

- Using an off-centered elliptic ship domain to determine potential collision areas and quantify regional risk distributions, collision risk is calculated by incorporating both dimensions of micro and macro, which makes collision avoidance decision-making more intuitive.
- Encountered ships are differentiated based on risk relationships between neighbor ships, and their roles are assigned with collision avoidance responsibilities, thus providing a more comprehensive consideration of COLREGs.
- Based on rational thinking, a collaborative mechanism is formulated to obtain ship action intentions through a strategy game, and a collaborative collision avoidance decision-making model within a game-decision cycle is formed by taking into account action intentions to predict encounter evolution.

3. Methodology and modeling

This paper adopts the intention game equilibrium between ships as collaborative decision strategies, and then forms avoidance schemes by real-time decision making. In this section, the logical framework of collaborative collision avoidance decision research based on potential collision areas is outlined, as depicted in Fig. 2. It encompasses two parts: collision risk evaluation and collaborative collision avoidance decision-making. The results of the collision risk evaluation are used as inputs of the collaborative collision avoidance decision model. The details are as follows:

Part 1: Collision risk evaluation based on potential collision areas. Under multi-ship encounter scenarios, the potential collision areas are identified based on ship domain with off-centered elliptic (Section 3.1.1). The micro-level collision risk is constructed by defining the feasible decision space, which corresponds to the minimum avoidance amplitudes required to avoid the potential collision areas (Section 3.1.2). A ship collision risk field is developed through quantitative evaluation of the state risk at any position, and the macro-level collision risk is defined by taking into account the ship collision risk coverage ratio and the average state risk (Section 3.1.3). Finally, the collision risk evaluation based on potential collision areas is achieved by integrating the collision risk across both dimensions (Section 3.1.4).

Part 2: Multi-ship collaborative collision avoidance based on action intention games. The encountering ships are divided into a game group and a keeping group based on whether there is a collision risk, with further differentiation in the game group based on ship avoidance ability (Section 3.2.1). The multi-ship encounter situation is modeled as a collision avoidance intention game, and the game equilibrium is determined as the collaborative decision strategies (Section 3.2.2). Among them, a hierarchical game reward function is constructed, taking into account ship safety, COLREGs compliance, and economic factors (Section 3.2.3). Finally, collaborative collision avoidance decisions are achieved through real-time situation evaluation and avoidance stage judgment in multi-ship encounters (Section 3.2.4).

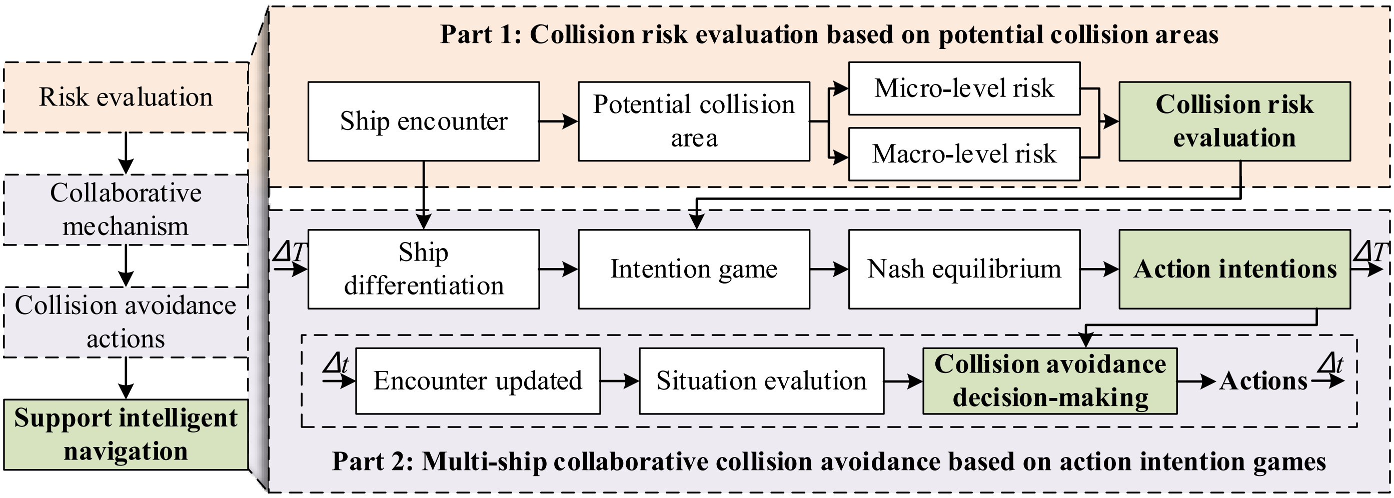


Fig. 2 The modeling logic of collaborative collision avoidance

3.1 Collision risk evaluation based on potential collision areas

3.1.1 Ship domain and potential collision area

To ensure navigation safety, a safety region should be kept for the clearance from TSs throughout the entire encounter (Szlapczynski et al., 2018). The concept of the ship domain has been proposed and widely utilized to assess and quantify collision risk (Pietrzykowski & Wielgosz, 2021; Yim et al., 2018). Based on AIS trajectories analysis (Horteborn et al., 2019; Silveira et al., 2022), with areas toward the bow and starboard being wider than those toward the stern and port (Zhang & Meng, 2019). Therefore, an off-centered ellipse, depicted in Fig. 3, is selected in this study, where L is ship length, and its major and minor semi-axes are $10L$ and $5L$. OS is situated to the rear and left of the ellipse's center, with backward and port offsets of $2.5L$ and $1.25L$ (Szlapczynski & Szlapczynska, 2016; Szlapczynski et al., 2024).

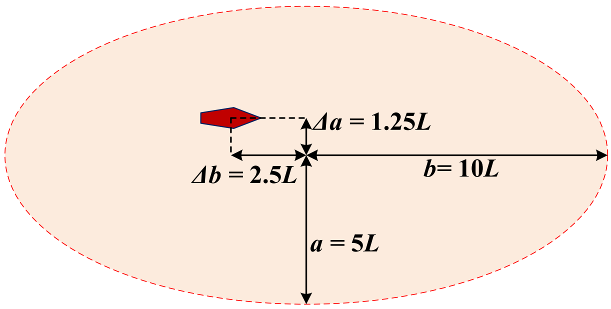


Fig. 3 An off-centered elliptic ship domain

There are four types of safety standards when using ship domain to determine collision risk (Szlapczynski & Szlapczynska, 2017): 1) OS's domain should not be violated; 2) TS's domain should not be violated; 3) Both domain of OS and TS remain inviolate; 4) ship domains should not overlap. Given the asymmetrical nature of off-centered elliptic, ships may interpret collision risks differently even in identical encounter scenarios (Szlapczynski & Szlapczynska, 2016). Meanwhile, since the actual decision-making is in distributive mode (Zhang et al., 2015), this study adopts the first type of collision risk from OS's perspective, focusing on whether TSs infringe upon OS's domain. This perspective is more aligned with the subjective feelings, risk cognitions and navigation habits of OOWs (Wu et al., 2021). Therefore, potential collision areas are defined as areas where TSs may pose a collision risk to OS, that is, a set of OS's positions in scenarios where TSs violate OS's domain.

The ship status can be expressed as $\boldsymbol{S} = (\boldsymbol{P}, \boldsymbol{V}, L)$, where $\boldsymbol{P} = [x, y]$ represents the ship position in X ($W\rightarrow E$) and Y ($S\rightarrow N$) coordinates; $\boldsymbol{V} = [v, c]$ is the ship velocity in terms of magnitude and heading; L is the ship length. At t_0, OS's status is $\boldsymbol{S}_o(t_0)$, TS's status is $\boldsymbol{S}_t(t_0)$. At t, the collision risk between OS and TS can be expressed in Eq. 1.

$$\begin{cases} \boldsymbol{P}_t(t) \in \boldsymbol{P}_o(t) \oplus Conf\boldsymbol{P}(\boldsymbol{P}_o(t),\ \boldsymbol{V}_o(t)) \\ \boldsymbol{P}_o(t) = \boldsymbol{P}_o(t_0) + \int_{t_0}^{t} \boldsymbol{V}_o(t)dt \\ \boldsymbol{P}_t(t) = \boldsymbol{P}_t(t_0) + \int_{t_0}^{t} \boldsymbol{V}_t(t)dt \end{cases} \tag{1}$$

where, $Conf\boldsymbol{P}(\boldsymbol{P}_o(t), \boldsymbol{V}_o(t))$ represents the ship domain at $\boldsymbol{P}_o(t)$ and $\boldsymbol{V}_o(t)$ at t, and $\oplus$ denotes the Minkowski addition.

Potential collision areas can be obtained by the velocity (including magnitude and heading) and t under satisfactory conditions in Eq. 1. Based on the definitions of d_{dv} and t_{dv}, when states of OS and TS are available, and if d_{dv} >0 and $t \in [\min(t_{dv}), \max(t_{dv})]$, then TS invades OS's domain. The corresponding points of potential collision areas $\boldsymbol{sP}$ can be obtained in Eq. 2.

$$\boldsymbol{sP}(\boldsymbol{V}_o) = \left\{ \boldsymbol{P}_o(t) = \boldsymbol{P}_o(t_0) + \int_{t_0}^{t} \boldsymbol{V}_o(t)dt \in \boldsymbol{R} \mid d_{dv}>0,\ t \in [\min(t_{dv}),\ \max(t_{dv})] \right\} \quad (2)$$

Ship velocity $\boldsymbol{V}_o$ is expressed as magnitude v_o and heading c_o respectively, and potential collision areas are expressed in x-y form. Eq. 2 can be rewritten in Eq. 3.

$$\boldsymbol{sP}(v_o, c_o) = \left\{ \begin{bmatrix} x_o(t) \\ y_o(t) \end{bmatrix} = \begin{bmatrix} x_o(t_0) \\ y_o(t_0) \end{bmatrix} + \int_{t_0}^{t} \begin{bmatrix} v_o(t)\sin c_o(t) \\ v_o(t)\cos c_o(t) \end{bmatrix} dt \in R \mid d_{dv}>0,\ t \in [\min(t_{dv}),\ \max(t_{dv})] \right\} \quad (3)$$

Based on Rule 6 of COLREGs, safe speed, in actual navigation situations, ships do not accelerate to avoid collisions (He et al., 2017), and the amplitude of turning is less than 90° (Neatby & Thornhill, 2024). Therefore, the potential collision area $\boldsymbol{P}$ can be expressed in Eq. 4.

$$\boldsymbol{P} = \bigcup_{c=c_o-90^\circ}^{c_o+90^\circ} \left(\bigcup_{v=0}^{v_o} \boldsymbol{sP}(v, c) \right) \quad (4)$$

Meanwhile, there are n (n>1) TSs in multi-ship encounters, so the potential collision area $\boldsymbol{MP}$ is the union of $\boldsymbol{P}_i$ (i = 1, 2, …, n) for each TS, which can be expressed in Eq. 5.

$$\boldsymbol{MP} = \bigcup_{i=1}^{n} \boldsymbol{P}_i \quad (5)$$

3.1.2 Collision risk model based micro-level situation

The feasible decision space refers to all maneuvering actions that can ensure safety, including turning heading and changing speed. The minimum feasible amplitude, which is the minimum action amplitude to keep clearance from all TSs, corresponds to different avoidance difficulty and reflects varying levels of collision risk from a micro perspective.

Since COLREGs requires ships to navigate at a safe speed without acceleration to avoid collision, three possible actions to avoid collision are turning starboard, turning port and deceleration (Huang et al., 2020; Zhou et al., 2018). In Section 3.1.1, potential collision areas are determined, and the critical value of TSs invading OS domain can also be obtained, including the minimum amplitude for starboard c_s, port c_p and deceleration ratio d_v, respectively. Meanwhile, the maximum of these minimum amplitudes Mc_s, Mc_p and Md_v are selected to quantify collision risk under multi-ship encounter in Eq. 6.

$$\begin{cases} Mc_s = \max\limits_{i=1}^{n} c_s(i) \\ Mc_p = \min\limits_{i=1}^{n} c_p(i) \\ Md_v = \min\limits_{i=1}^{n} d_v(i) \end{cases} \quad (6)$$

where Mc_s, Mc_p and Md_v represent the minimum amplitudes of turning starboard, port and deceleration ratio that can ensure safety in multi-ship encounter scenarios, respectively. Taking into account difficulty and navigation habits, corresponding risk evolution characteristics should change slowly in the early stage, then increase in a faster pace, and finally stabilize at a maximum value of 1. Therefore, collision risk models, r_{cs}, r_{cp}, r_v, are constructed for three possible actions, turning to starboard, turning to port and deceleration, as shown in Eqs. 7~9, respectively.

$$r_{cs} = \begin{cases} 1, & Mc_s > C_s \\ \left(\frac{\sin(Mc_s)}{\sin(C_s)}\right)^2 + e^{1-\frac{\sin(C_s)}{\sin(Mc_s)}} - \left(\frac{\sin(Mc_s)}{\sin(C_s)}\right)^2 e^{1-\frac{\sin(C_s)}{\sin(Mc_s)}}, & Mc_s \le C_s \end{cases} \quad (7)$$

$$r_{cp} = \begin{cases} 1, & Mc_p < C_p \\ \left(\frac{\sin(Mc_p)}{\sin(C_p)}\right)^2 + e^{1-\frac{\sin(C_p)}{\sin(Mc_p)}} - \left(\frac{\sin(Mc_p)}{\sin(C_p)}\right)^2 e^{1-\frac{\sin(C_p)}{\sin(Mc_p)}}, & Mc_p \ge C_p \end{cases} \quad (8)$$

$$r_v = \begin{cases} 1, & 1 - Md_v < C_v \\ 4(1-Md_v)^2 + e^{\frac{C_v}{1-Md_v}-1} - 4(1-Md_v)^2 e^{\frac{C_v}{1-Md_v}-1}, & 1 - Md_v \ge C_v \end{cases} \quad (9)$$

where C_s, C_p and C_v are model parameters, representing the maximum tolerable amplitudes. The larger the amplitudes required for avoidance, the higher the corresponding risks, with all values ranging from 0 to 1. When the minimum amplitudes exceed these thresholds, the corresponding risk value is 1. Based on statistical analysis of AIS data (Liu et al., 2024b), C_s, C_p and C_v are set as 53.9, 63.7 and 0.5.

Since there are two turning modes, the smaller one is taken as the turning result. Additionally, turning is easier to manipulate, and also is more apparent, making it more commonly used than deceleration (He et al., 2017; Zhou et al., 2018). Thus, in risk assessment, the weight of turning should be much larger than that of deceleration. To construct a comprehensive micro-level collision risk model R_a, which takes into account both turning and deceleration, the risk models r_{cs}, r_{cp}, r_v, are fused using Eq. 10, by incorporating different weights for turning and deceleration. With reference to Cho et al. (2022) and Liu et al. (2024b), their weights are set as 0.8 and 0.2, respectively.

$$R_a = 0.8\min(r_{cs}, r_{cp}) + 0.2 r_v \quad (10)$$

Since all ranges of r_{cs}, r_{cp}, r_v are from 0 to 1, the range for R_a is also [0, 1].

3.1.3 Collision risk model based on macro-level situation

In Section 3.1.2, a collision risk model is constructed based on feasible decisions. It provides with micro-level modeling by considering the minimum feasible decision amplitude. To fully understand ship encounter situations, this section outlines a collision risk model from the macro-level situation.

In Zhang et al. (2021b), the time for ship encounter identification is set to 30min to the closest point. Therefore, taking into account the actual navigation and avoidance characteristics, an evaluation area, as shown in Fig. 4, is created to quantify collision risk from a macro perspective, whose left and right boundaries are respectively current heading C_h turning port/starboard with 30° (C_h ± 30°), and ship forward heading is partitioned at 0.1h (6 min) intervals until 0.5h (30 min).

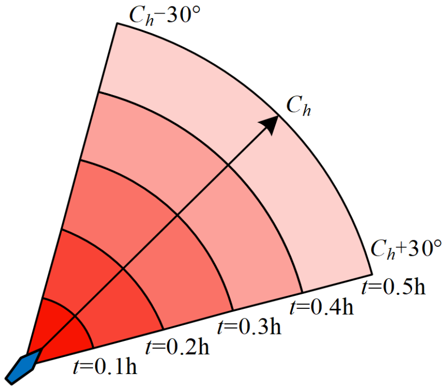


Fig. 4 An evaluation area for ship collision risk at the macro-level

To calculate the coverage ratio, statistics and calculations are carried out for each of the five partitions. In the statistical process, the potential collision area is discretized to determine whether the direction and distance of points are located within the area and in which interval, to determine the coverage ratio s_{co} in Eq. 11.

$$s_{co} = 0.2\sum_{i=1}^{5}\frac{s_i}{S_i} \qquad (11)$$

where s_i represents the number of points in potential collision areas located in the i^{th} interval; S_i represents the total number of points of the i^{th} interval after discretization. Considering that ship collision risk increases with the increase of s_{co}, an exponential model r_c is constructed based on s_{co} in Eq. 12.

$$r_c = \begin{cases} 1 & s_{so} \geq 0.2 \\ s_{co} e^{\frac{s_{co}}{0.2} - 1 + \ln 5} & s_{so} < 0.2 \end{cases} \qquad (12)$$

According to Eq. 12, r_c increases exponentially when s_{co} < 0.2, and reaches the maximum value 1 when s_{co} ⩾ 0.2, and the range of r_c is [0, 1].

Since the collision risk varies at different positions within the potential collision area, it is necessary to evaluate the collision risk for each state. DCPA and TCPA are widely used to evaluate collision risk (Xie et al., 2019). However, as shown in Fig. 3, the safety distance required in various directions is different. Therefore, DCPA and TCPA cannot accurately characterize ship relations or determine collision risks in an off-centered ellipse domain, and the conventional metrics of DCPA and TCPA are substituted with d_{dv} and t_{dv} (Szlapczynski & Szlapczynska, 2016). The state collision risk model r_s is constructed according to Eqs. 13~15.

$$r_s = k_1 r_d + k_2 r_t \qquad (13)$$

$$r_d = \begin{cases} 0, & d_2 \geq d_{dv} \\ 0.5 - 0.5\sin\left[\dfrac{180^\circ}{d_2 - d_1}\left(\dfrac{d_1 + d_2}{2} - d_{dv}\right)\right], & d_2 < d_{dv} < d_1 \\ 1, & d_{dv} \geq d_1 \end{cases} \qquad (14)$$

$$r_t = \begin{cases} 0, & t_2 < \min(t_{dv}) \\ \left(\dfrac{t_2 - t_{dv}}{t_2 - t_1}\right)^2, & t_1 < \min(t_{dv}) < t_2 \\ 1, & t_1 > \min(t_{dv}) \end{cases} \qquad (15)$$

where k_1 and k_2 are the weights of spatial risk r_d and temporal risk r_t respectively, and are set as 0.5215 and 0.4785 based on Xie et al. (2019) and Zhang et al. (2021b). In the spatial risk r_d, d_1 and d_2 are ship collision occurrence and ship domain boundary corresponding to d_{dv}, with values of 0.9 and 0 respectively. In the temporal risk r_t, t_1 and t_2 represent the time when risk becomes intolerable and ship collision avoidance begins to be considered, with values of 6min and 30min, respectively.

Since different intervals have varying importance on collision risk, to further quantify the urgency of collision risk, the evaluation area is divided into five partitions with different weights, i.e. k_i (i=1, 2,…, 5) set as 0.2, 0.4, 0.6, 0.8 and 1, as along with the distance to the OS being decreased. The weighted average state risk s_{av} for the entire region is calculated in Eq. 16:

$$s_{av} = \sum_{i=1}^{5} k_i \frac{\sum_{s_i} r_s}{S_i} \qquad (16)$$

Considering that ship collision risk increases with the increase of s_{av}, a model r_a is constructed with an exponential function of s_{av} according to Eq. 17.

$$r_a = \begin{cases} 1 & s_{av} \geq 0.25 \\ s_{av} e^{\frac{s_{av}}{0.25} - 1 + \ln 4} & s_{av} < 0.25 \end{cases} \qquad (17)$$

According to Eq. 17, r_a increases exponentially when $s_{av} < 0.25$, and reaches the maximum value 1 when $s_{av} \geqslant 0.25$, and the range of r_a is [0, 1].

To comprehensively assess ship collision risk at the macro-level, it is necessary to combine the two risk models which are built based on the coverage ratio s_{co} and the average state risk s_{av}. To satisfy the monotonically increasing relationship (Liu et al., 2024b), Eq. 18 is used to fuse the two risk models to form a macro-level collision risk model R_m.

$$R_m = r_c + r_a - r_c r_a \qquad (18)$$

In Eq. 18, R_m is defined as a monotonically increasing function within the range of r_c and r_a. To ensure the reliability of the risk model, the fused value is no smaller than any sub-risk model. This guarantees that the model adequately reflects the potential risks identified, and its value range is [0, 1].

3.1.4 Collision risk model based on potential collision areas

The micro-level indexes (minimum feasible decision amplitudes) and the macro-level indexes (the coverage ratio and the average of state risk) are calculated based on potential collision areas, and the corresponding collision risk models are constructed respectively. To involve both micro-level and macro-level collision risk, Eq. 19 is used to fuse the risk models into a final collision risk model R.

$$R = R_a + R_m - R_a R_m \qquad (19)$$

To enhance the reliability of the collision risk evaluation process, the fusion method is similar to Eq. 18. Both the micro-level risk R_a and macro-level risk R_m have value ranges between [0, 1]. Meanwhile, the final risk model R is defined as a monotonically increasing function within the range of R_a and R_m, ensuring that the overall risk value is not smaller than any individual component risk model. This structure guarantees that the final collision risk model R captures the complete spectrum of potential risks, with its value also ranging from 0 to 1.

3.2 Collaborative collision avoidance based on intention games

Accurate acquisition of TSs' action intentions can greatly reduce the difficulty of decision-making, mitigate losses of taking actions, and improve efficiency of collision avoidance. Therefore, based on rational thinking, this section constructs a game model that considers ship safety, COLREGs compliance and economic factors, and determines action intentions based on Nash equilibria. This forms a collaborative collision avoidance decision-making framework to ensure safety and efficiency in multi-ship encounters.

3.2.1 Differentiated ships in encounter situations

In multi-ship encounters, the roles of ships may vary, such as give-way and stand-on, and some ships may pass other ships at a safe distance without taking actions. However, due to the coupling relationship between encountered ships, this compresses the ship's decision space (Chauvin & Lardjane, 2008). Ships should avoid increasing collision risk while also preventing the formation of another close-quarters situation or new collision risks. Thus, it is necessary to differentiate ships based on risk relationships between neighbor ships. To avoid a new collision risk between give-way ships and safe navigable ships, encountered ships are divided into game group (involving give-way and stand-on ships) and keeping the group according to the criterion as to whether there is a collision risk between OS and other ships. Since ships in the keeping group are not at risk of collision, they can ensure navigation safety by keeping their heading and speed. Ships in the game group should take actions in accordance with COLREGs and common practice of seafarers (Li et al., 2024c), and avoid to form new collision risks in the process of decision-making considering ships states of keeping group simultaneously. This method reduces complexity and improves decision-making efficiency by limiting the number of ships involved in the game.

Meanwhile, COLREGs also requires taking appropriate actions by stand-on ships (Rule 17) to avoid close-quarters situation or immediate danger. Additionally, when a give-way ship has not taken actions complying with COLREGs, stand-on ships may take actions independently. Therefore, when give-way ships cannot take actions according to game results, such as ships restricted in their ability to manoeuvre or not under command (Wu et al., 2018), these ships are marked as GNA (Give-way Not taking appropriate Actions) to be recognized and must be accounted for in both the game and decision-making process. Moreover, stand-on ships should take appropriate actions as soon as possible to ensure navigation safety. Therefore, there are three types of ships in this study: the ship in the game group without GNA, the ship in the game group with GNA and the ship in the keeping group. Tab. 1 shows the differentiated ships algorithm.

Tab. 1 The algorithm of differentiated ships in encounter situations

```
Algorithm: Differentiated ships in encounter situations
Input: Encountered ship status S_i(t) = [P_i(t), V_i(t), L_i(t)], i = 1, 2,…, n;
Output: The game group G_g (including GNA) and keeping group G_k
Calculate encounter parameters between any ship i and j,
  including distance d_ij ,d_dvij, and t_dvij
For   i = 1, 2,…, n in S
    For   j = 1, 2,…, n in S
        If   i ≠ j & d_ij <6n mile & d_dvij >0 & t_dvij >0
            Add S_i to game group G_g;
            If   S_i is a give-way ship & cannot take actions
                Mark S_i with GNA
            End if
            break;
        End if
    End for
    If   S_i is not added to G_g
        Add S_i to keeping group G_k
    End if
End for
```

The algorithm input is ship status, and encountered ships are differentiated by evaluating their encounter parameters with neighbors. A ship is placed in the game group if a ship within 6n mile violates its ship domain. Then, this ship would be further evaluated to determine whether it should be marked as GNA. Otherwise the ship joins the keeping group. Ships in the keeping group do not need to make decisions. For ships in the game group without GNA, decisions need to be made according the surrounding ships, including ships in the keeping group and in the game group with GNA. Since the number of ships in the game being less than than the number of encountered ships, and the ship behaviors in keeping group is available, ship differentiation reduces the complexity of the game and simplifies encounter situations.

3.2.2 Ship action intention game model and Nash equilibrium

Due to the strong coupling of spatiotemporal relationships between encountered ships, the uncertainty of encounter evolution and the difficulty of decision-making increase sharply with the number of ships (Cho et al., 2022; Wang et al., 2023). To reduce the difficulty of collision avoidance and achieve coordinated actions, OOWs can communicate with encountered ships using VHF during actual navigation, clarifying their action intentions, and thus formulating reasonable decision-making plans (Akdag et al., 2022). However, for fully autonomous ships or autonomous/manned ships, obtaining the action intentions of TSs requires a new identification or prediction model based on the characteristics of encounter relationships (Li et al., 2024b).

In this study, a game model G (N, S, u) is established for multi-ship encounters to obtain action intentions of TSs by solving Nash equilibria based on rational thinking. N, S, u represent the players, strategy and reward function of game G. S = {go straight, turn starboard, turn port, decelerate, accelerate}; u is the player's reward function, and is also the common knowledge of the game G.

For any multi-ship encounter scenario, N is the ships in the game group, with $n \geqslant 1$. There are five strategies in S, and the ship speed should not exceed its initial speed. In the calculation of game rewards, the heading changes ±5° and the speed changes ±1kn for each game period ΔT=60s. Only one collision avoidance action is selected for each game period. Reward function u is an evaluation model that takes into account ship safety, COLREGs compliance, and economic factors. See Section 3.2.3 for details.

Since there are 5^n strategy combinations per game period with n ships in the game group without GNA and the number of ships is limited, according to the existence theorem, Nash equilibrium in this study can be expressed as follows:

For the game G (N, S, u), if there is a strategy combination (s_1*, s_2*, ..., s_n*), where $s_i^* \in S$ (i = 1,2,... ,n) is the player i's strategy, such that, u_i (s_1*, s_2*,... s_n*)$\geqslant u_i$ (s_i, s_{-i}*). Then, the strategy combination, (s_1*, s_2*,..., s_n*), is a Nash equilibrium for G.

The model is a pure strategy game, and Nash equilibriums can be solved using a lineation method (Fu, 2021). When multiple Nash equilibria exists and ships still face collision risk, the equilibrium with the lowest risk level is selected. Otherwise, the strategy combination that most stabilizes ship behavior is chosen.

3.2.3 Reward function under multi-ship game scenario

In this part, game evaluation models considering ship safety, COLREGs compliance, and economic factors are constructed respectively, and the game reward function is formed by assigning different weights to fuse these evaluation models.

(a) Safety evaluation model

For each strategy combination of encountered ships, potential collision areas of ships in the game group are calculated. Additionally, under any strategy combination, whether there is a collision risk with ships in the keeping group should be determined. If collision risk exists, a new potential collision area should be superimposed according to Section 3.1.1, followed by a collision risk assessment.

Ship collision risk model measures the risk level of ship encounters. In game reward calculations, the smaller the risk, the greater the reward. So, a safety evaluation model *ms* is constructed based on the risk evaluation result *R* in Eq. 20.

$$ms = 1 - R \qquad (20)$$

(b) COLREGs compliance model

COLREGs is usually used in ship collision accidents analysis and responsibility identification. The requirements of COLREGs are in ample time and with good seamanship, substantial and resulting in keeping clearance (Liu et al., 2022). COLREGs divides ship encounter into three situations (Cockcroft & Lameijer, 2012): overtaking, head-on and crossing, and provides decision guidance for different situations and stages. The analysis based on the COLREGs is as follows:

It is critical to clarify the criteria of classification encounter situations based on the COLREGs. Kim & Park (2024) investigated navigators and verified that the ambiguity of COLREGs has a negative impact on collision avoidance decisions. This result was also mentioned in Park et al. (2023). Therefore, based on statistical analysis of AIS data (Zhang et al., 2023b), encountered ships is divided into 6 categories (Head-on, Overtaking, Overtaken, Crossing with small angle, Crossing with large angle, Crossing with stand-on) taking into account ship avoidance actions and position relationships. The classification criteria for these categories refers to Liu et al. (2022) in this paper.

Since ship collision avoidance decision-making involves determining when and what actions to take, to ensure that ships have sufficient time and distance to take appropriate actions, it is essential to study the latest time or minimum distance required for safe avoidance (Yim & Park, 2022). Especially in restricted waters, ship's maneuverability and the environmental conditions affecting its trajectory must also be considered (Lee & Park, 2024). However, different encounter situations and ship types have different requirements for the latest time/minimum distance. In open waters, due to the wider water, 6n mile begins to avoid TSs and is a widely accepted distance in the navigation industry (Cockcroft & Lameijer, 2012). Meanwhile, the close-quarters and immediate danger situations are determined according to collision risk values with regard to Liu et al. (2024b). Based on Rule 13 of COLREGs (Overtaking), an overtaking ship is required to adopt a give-way, allowing for the collision avoidance mode to be chosen according to real-time encounter scenarios (Longo et al., 2024), and the deceleration of stand-on ship is considered a coordinated action (Zhang et al., 2024). Rule 14 clearly requires that all ships in a head-on situation turn starboard. According to Rule 15, turning port makes it easier to pass from the bow of TSs in crossing situation with small angle, and the description of COLREGs is avoiding crossing other ships' ahead. However, since turning starboard is not apparent in crossing situation with large angle, turning port should also be considered. Moreover, altering course alone may be the most effective action according to Rule 8, so speed change should be given lower priority than turning course.

Moreover, stand-on ships shall take appropriate actions to avoid forming close-quarters or immediate danger situations (Huang et al., 2020). There is no limit on collision avoidance modes for overtaken ship; stand-on ships in crossing situations, if conditions permit, do not turning to port. Additionally, when a stand-on ship encounters a give-way ship marked with GNA, the stand-on ship should take actions as soon as possible.

Based on the above analysis, COLREGs compliance model is established with continuous values according to two indexes, collision avoidance mode and amplitude. The reward of collision avoidance modes mr_m is shown in Tab. 2.

Tab. 2 The COLREGs compliance reward of collision avoidance modes mr_m

Situation	Stage	Action			
		Straight	Port	Starboard	Speed
Head-on	-	0	0	0.5	0
Overtaking	-	0	0.5	0.5	0
Overtaken	B	0.5	0	0	0.25
	A/GNA	0	0.5	0.5	0.25
Crossing with small angle	B	0	0	0.5	0.25
	A/GNA	0	0.25	0.5	0.25
Crossing with large angle	B	0	0.25	0.5	0.5
	A/GNA	0	0.5	0.5	0.5
Crossing with stand-on	B	0.5	0	0	0
	A/GNA	0	0.25	0.5	0.5

where A and B represent after/before the close-quarters situation.

To meet the substantial requirement of COLREGs, the reward of collision avoidance amplitudes, heading mr_c and speed mr_v, is in Eqs. 21~22.

$$mr_c = 0.5\min\left(\frac{d_h}{k_h},\ 1\right) \qquad (21)$$

$$mr_v = 0.5\min\left(\frac{d_v}{k_v},\ 1\right) \qquad (22)$$

where d_h and d_v represent the collision avoidance amplitudes of heading and speed, k_h and k_v are model parameters representing amplitudes sufficient to be easily observed or detected by TSs, with k_h and k_v set as 15° and 0.5v respectively, where v is the initial speed. By combining two rewards, collision avoidance modes and amplitudes, the final COLREGs compliance model mr is in Eq. 23.

$$mr = mr_m + \max(mr_c,\ mr_v) \qquad (23)$$

Since there are n (n>1) ships around OS in multi-ship encounter scenarios, the average is taken as the evaluation result of COLREGs compliance, which is expressed in Eq. 24.

$$mr = \frac{1}{n}\sum_{i=1}^{n} mr_i \qquad (24)$$

where mr_i is the COLREGs compliance of OS with ship i.

(c) Economic evaluation model

Collision avoidance actions taken by ships decrease navigation efficiency and alter navigation states (Zhang et al., 2023b), as shown in Fig. 5, resulting in increased navigation distance and time, while the original states are often more reasonable and economical. To prevent excessive deviating from their original route, economic indexes are set to constrain ship actions. In this study, economic evaluation is modeled from two aspects: navigation efficiency and state recovery.

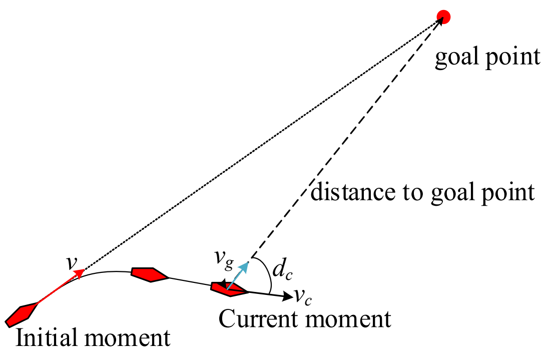


Fig. 5 The diagram of economic evaluation and its parameters

Since ship's heading or speed changes with different actions, the distance to its destination is decreasing. Therefore, the ratio of the velocity component v_g in the direction from the current position towards the destination and the ship initial speed v is used to characterize navigation efficiency n_e according to Eq. 25.

$$n_e = \frac{v_g}{v} = \frac{v_c \sin(d_c)}{v} \qquad (25)$$

Since ships often sail at an economic speed (Zhou et al., 2018), they need to return to their initial states. Therefore, the economic evaluation based on navigation states is assessed according to the cost of returning to initial state, including change amplitudes in heading and speed. Eqs. 26~27 can be used to calculate the time of turning to goal point t_g and the time of recovering to the initial speed t_s.

$$t_g = d_g / k_g \qquad (26)$$

$$t_s = d_s / k_s = \frac{v - v_c}{k_s} \qquad (27)$$

where d_g and d_s are the required heading and speed change amplitudes to resume navigation towards the goal point and initial speed; k_g and k_s are the parameters representing the heading and speed changes per unit time. To keep consistent with strategy changes, k_g and k_s are set as 5°/min and 1kn/min, respectively.

The higher the navigation efficiency and the shorter the state recovery time, the better the economic performance. Therefore, Eq. 28 is used for economic evaluation me taking into account navigation efficiency and state recovery.

$$me = n_e \cos\left(\frac{t \cdot \pi}{12}\right) \qquad (28)$$

where, $t = \min(\max(t_g, t_s), 6)$, and Eq. 28 takes into account the efficiency of the current velocity, represented by the component of the velocity approaching the goal point in Eq. 25, and the time required to resume navigation towards the goal point, and multiplies to get the final result, with a value range [0, 1].

(d) Hierarchical game reward model

Game evaluation models are constructed for safety, COLREGs compliance and economic indexes of encounter scenarios. To evaluate collision avoidance effect and facilitate game decision-making, a hierarchical game reward model m is constructed by incorporating the results of three-dimensional evaluation in Eq. 29.

$$m = \begin{cases} 0.6ms + 0.4mr & d_{dv} > 0 \\ 1 + 0.7mr + 0.3me & d_{dv} = 0 \end{cases} \qquad (29)$$

Eq. 29 is a hierarchical model based on navigation safety, with safety and COLREGS compliance as its main objective (Hu et al., 2020; Zhao & Roh, 2019). When there is still collision risk, the model only considers safety and COLREGs compliance, with weights of 0.6 and 0.4 respectively, and a value range [0, 1]. When safety is ensured, the safety evaluation value is set as 1, and the weights of COLREGs compliance and economic evaluation are set as 0.7 and 0.3 respectively, with a value range [1, 2]. Through hierarchical calculations under different encounter strategies, the game is more inclined to choose a strategy to ensure safety.

3.2.4 Collaborative decision-making in multi-ship encounters

By calculating the game rewards for each strategy combination and calculating Nash equilibrium, the action intentions of ships in the game group can be obtained based on rational thinking, and applied to ship state prediction, allowing evaluation the evolution of encounter situations. If a ship is marked with GNA due to failing to take actions, the stand-on ship would take actions immediately.

The game interval period ΔT is set as 60s, taking into account the calculation cost and stability of action intentions. Ships evaluate encounter situations and determine the best action based on intentions, carrying out a real-time decision-making process (with a decision-making period Δt = 1s) in which the maximum changes in rudder and speed are set as 2°/s and 0.04kn/s respectively. The collaborative collision avoidance decision-making algorithm under game-decision cycles is shown in Tab. 3.

Tab. 3 Collaborative collision avoidance decision-making algorithm

Algorithm: Collaborative collision avoidance decision-making
Input: Encountering ship status $\boldsymbol{S}_i(t) = [\boldsymbol{P}_i(t), \boldsymbol{V}_i(t), L_i(t)]$, $i = 1, 2,\ldots, n$;
Output: Encountering ship status from t+1 to end: $\boldsymbol{S}_i\|_{t+1\to\text{end}}$
1. Obtain/update all encountering ship status $\boldsymbol{S}_i$;
2. Obtain the results of game group $\boldsymbol{G}_g$ and keeping group $\boldsymbol{G}_k$; (Section 3.2.1)
3. If ~ isempty $\boldsymbol{G}_g$
4. Determine all game strategy combinations $(a_1, a_2,..., a_n)$;
5. If $\boldsymbol{S}_i$ with GNA
6. The game strategy combinations is $(a_1, a_2, ...,a_{i-1}, a_{i+1}..., a_n)$ //The number of combinations is 5^m (m is the number of ships in $\boldsymbol{G}_g$ without GNA)
7. End if
8. For $c = 1, 2,\ldots, 5^m$
9. For $i = 1, 2,\ldots, m$
10. Calculate the potential collision area of ship i under the c^{th} combination; (Section 3.1.1) //All encountering ships including ships in $\boldsymbol{G}_k$ and $\boldsymbol{G}_g$ with GNA
11. Calculate game reward under the c^{th} combination; (Section 3.2.3)
12. End for
13. End for
14. Solve the Nash equilibrium and determine action intentions of each ship in $\boldsymbol{G}_g$ without GNA;
15. For t=1, 2, …, ΔT // Game step ΔT = 60s
16. For $i = 1, 2,\ldots, m$
17. Predicate the ship status in $\boldsymbol{G}_k$ and $\boldsymbol{G}_g$ with GNA //Keep heading and speed;
18. Predicate the ship status in $\boldsymbol{G}_g$ without GNA based on action intentions in Line 12;
19. Evaluate encounter situation and determine the best action based on Eq. 29; // Keep the current collision avoidance action consistent with the game result
20. Update status of ship i at t in $\boldsymbol{G}_g$ without GNA;
21. End for
22. End for
23. End if
24. If ~ isempty $\boldsymbol{G}_g$ with GNA
25. For $i = 1, 2,\ldots, k$ // k is the number of ships in $\boldsymbol{G}_g$ with GNA
26. Update status of ship i at t in $\boldsymbol{G}_g$ with GNA //Keep heading and speed;
27. End for
28. End if
29. If ~ isempty $\boldsymbol{G}_k$

30. For $i = 1, 2, \ldots, l$ // l is the number of ships in $\boldsymbol{G}_k$
31. Update status of ship i at t in $\boldsymbol{G}_k$ //Keep heading and speed;
32. End for
33. End if
34. If Any ship with collision risk or not heading towards their goal point with initial speed
35. Go to Line 1;
36. Else
37. Finish the avoidance decision process;
38. End if

4. Case studies and discussions

To evaluate the effectiveness of proposed model, a collision accident between CF CRYSTAL and SANCHI and a four-ship simulation scenario are used to verify the proposed model. In the first case, collision avoidance actions are taken by ships at different distances, compared to the collision accident data, and the influence of actions on encounter parameters is analyzed. Under the four-ship simulation scenario, all ships are set to take actions according to game intentions, and then the decision-making model is analyzed by setting a ship to violate the game intentions/COLREGs due to ships restricted in their ability to manoeuvre or not under command with GNA, to comprehensively verify the effectiveness of the collaborative decision-making.

4.1 Case study I: Collision accident between CF CRYSTAL and SANCHI

When the collision accident between CF CRYSTAL and SANCHI occurred, the visibility and conditions were good, and there was sufficient space to take actions (Li et al., 2021). The collision trajectories are shown in Fig. 6. The two ships did not take collision avoidance actions during the whole collision process, resulting in a collision accident.

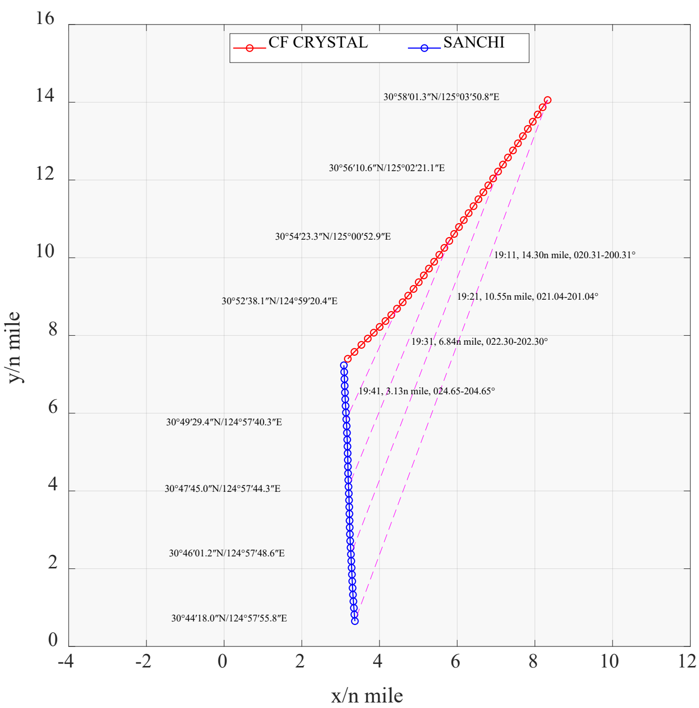


Fig. 6 Collision process between CF CRYSTAL and SANCHI

Taking 19:11 (local time) as the initial moment, collision risk curves of two ships are obtained, as shown in Fig. 7. Generally, SANCHI risk is higher than CF CRYSTAL, indicating that collision risk from give-way ship's perspective is larger than from stand-on ship's perspective, especially at close distances. The collision risk increases significantly after 18 min and reaches 0.5 at 22 minutes (about 6n mile away). According to encounter situations, the range of 0.2~0.7 is risk of collision (Liu et al., 2024b), corresponding to the duration in which give-way ships take actions. Moreover, 6n mile is widely accepted at the distance that collision avoidance actions should be activated. Therefore, this is consistent with Cockcroft & Lameijer (2012), Li et al. (2021), Liu et al. (2022) and Yuan et al. (2021).

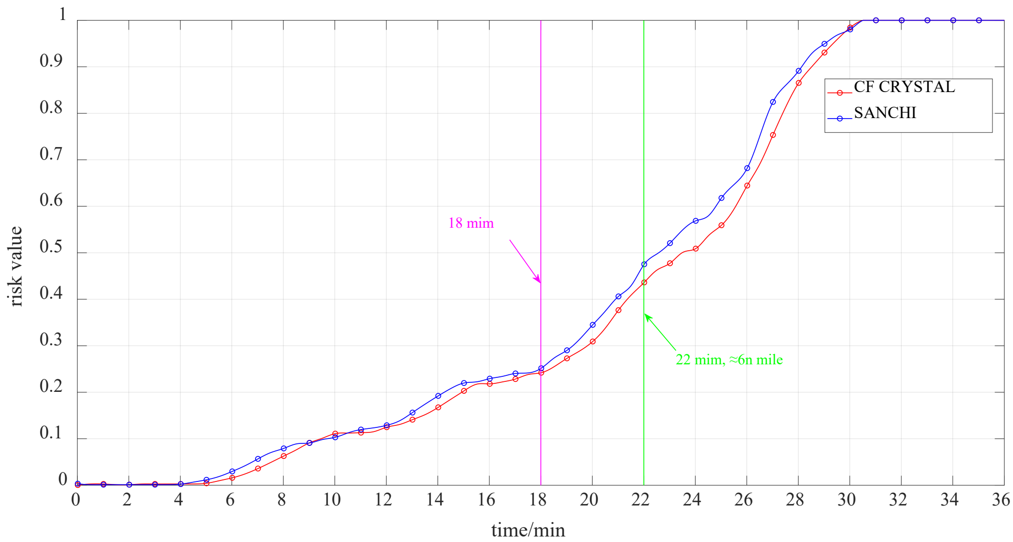


Fig. 7 Collision risk curves of CF CRYSTAL and SANCHI

Further, to analyze collision risk fields around ships, samples are taken with 1n mile intervals at distance less than 6n mile to obtain potential collision areas, as shown in Fig. 8, where a sector area surrounded by solid yellow lines represents the evaluation area defined in Section 3.1.3, the left and right boundaries being the current heading C_h±30°, and the interval of each dotted line in this area is 6min. The dotted yellow lines on the left and right of the feasible decision area represent the results by turning port -63.7° and starboard 53.9°of C_h respectively. As distances between ships decrease, potential collision areas become brighter (indicating higher state risk) and closer to ships. Meanwhile, its shape gradually expands to the left, limiting the possibility of ships turning port to keep clearance.

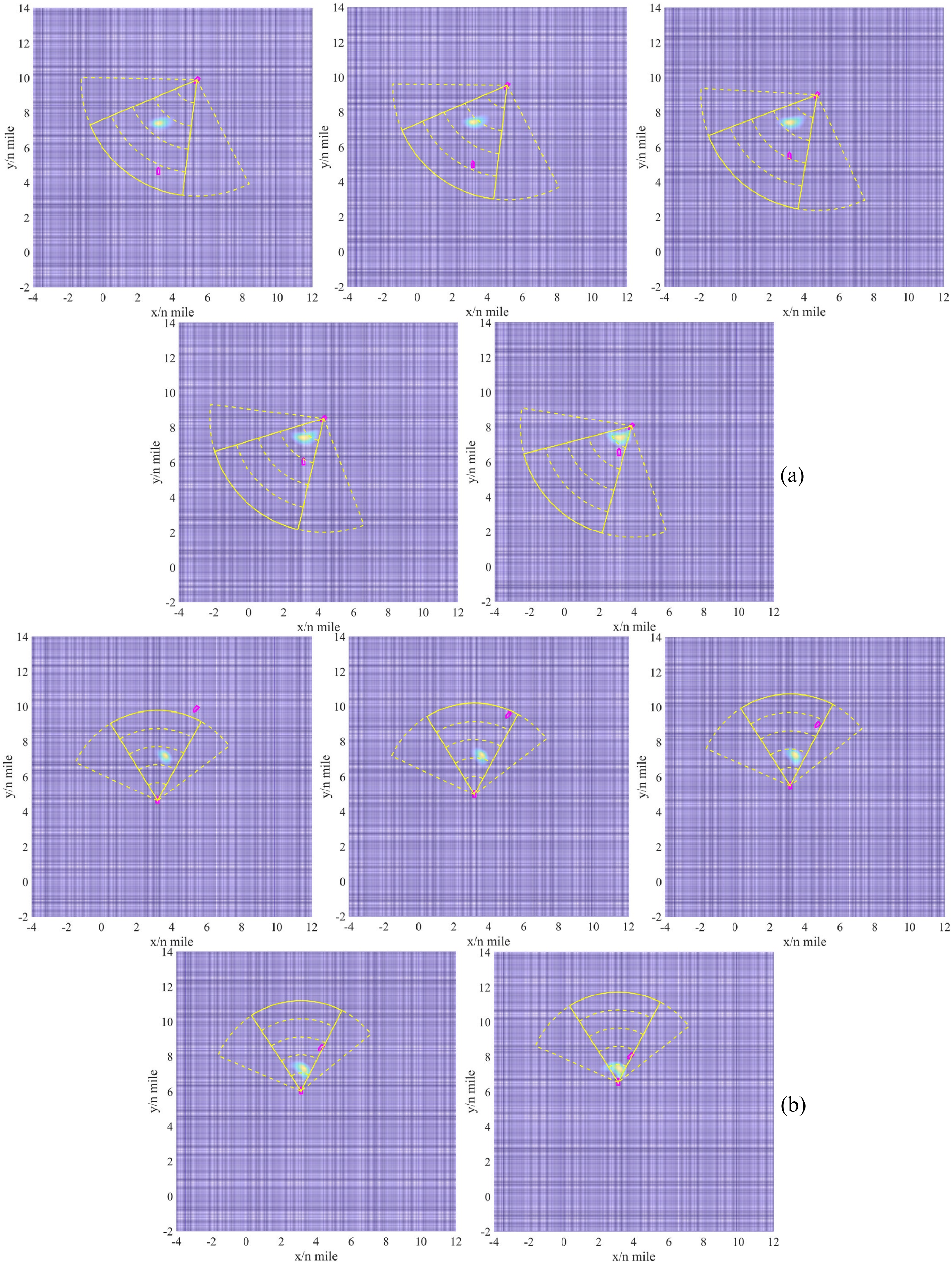
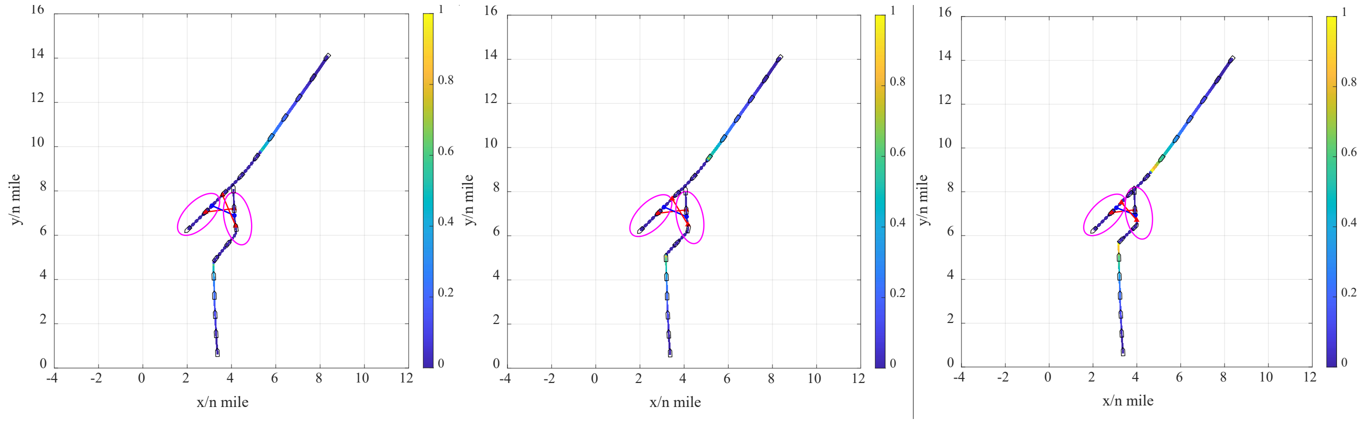
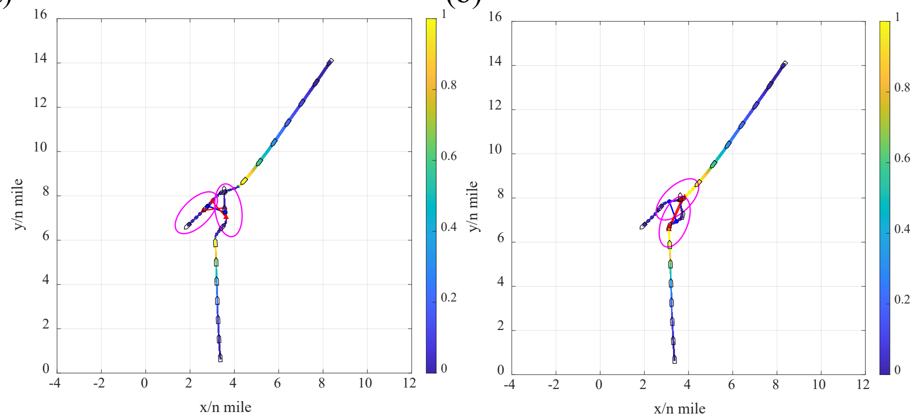


Fig. 8 Collision risk fields around CF CRYSTAL (a) and SANCHI (b) (6 ~ 2n mile)

The proposed decision-making model is applied to the encounter between CF CRYSTAL and SANCHI, and collision avoidance actions are analyzed by setting different decision-making distances, with ships trajectories shown in Fig. 9. In Fig. 9 (a) ~ (c), the trajectories correspond to decisions starting at 6, 5, and 4n mile, with only the give-way ship (SANCHI) takes actions; Fig. 9 (d) and (e) show decisions starting at 3 and 2n mile, with both ships taking actions simultaneously to avoid a close-quarters situation/imminent danger. Fig. 10 and 11 show the comparison of encounter spatial parameters and velocity curves. Since there is a long distance in the early encounter without actions, time starts from 20 min.

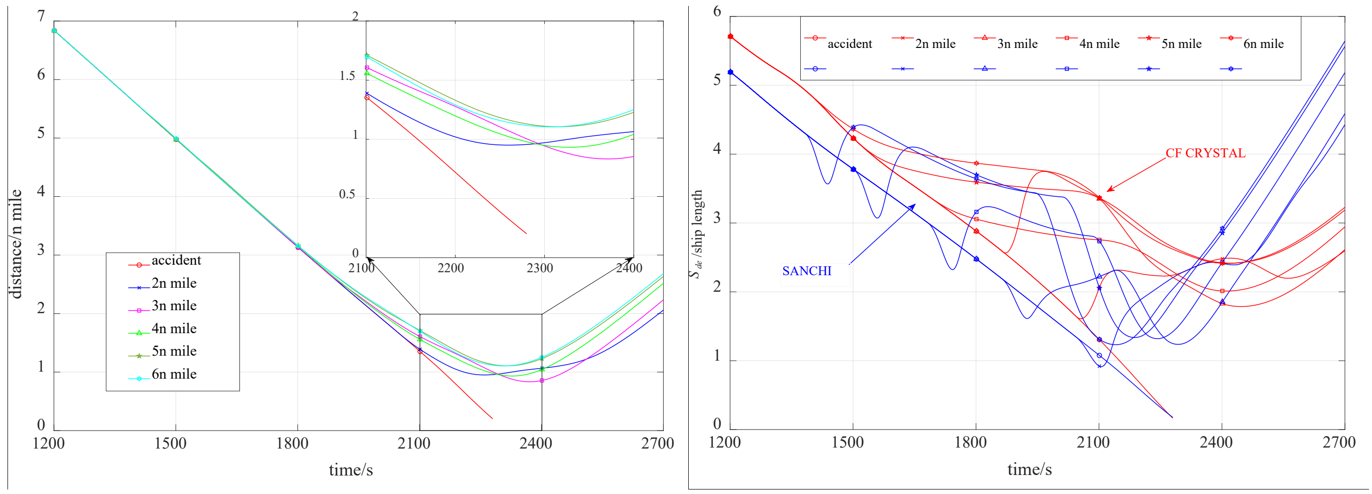


(a) (b) (c)



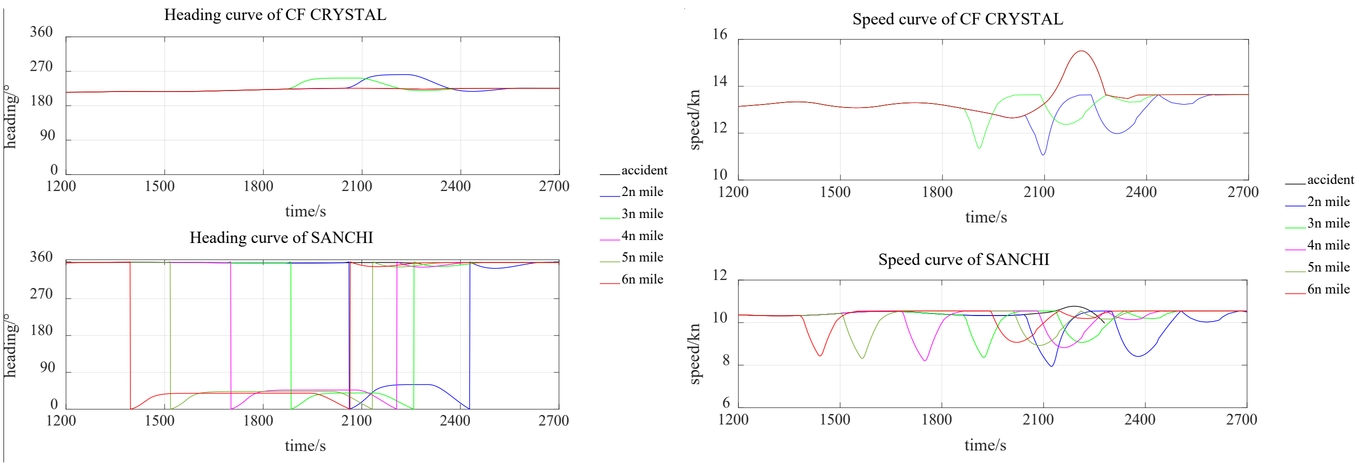
(d) (e)

Fig. 9 Collision avoidance trajectories with different decision-making distances



(a) (b)

Fig. 10 The comparison results of encounter spatial parameters (Left: distance; Right: S_{de})



(a) (b)

Fig. 11 The velocity curves under different decision-making distances (Left: heading; Right: speed)

According to Fig. 9, the motion trajectories of two ships are navigable according to actual AIS data before making decisions, and ships take a right action in all encounters based on game results. The color of trajectories represents the risk value at current position. The magenta segment shows the line connecting two ships positions corresponding to the minimum S_{de}, which is a spatial parameter, ship domain extension parameter, defined and calculated in Liu et al. (2024b). The triangles on both segment sides represent ship positions, and the magenta ellipse represents the conflict area surrounded by a ship domain. Due to the asymmetry of ship domains with ellipse, the moment corresponding to the minimum S_{de} of two ships is asynchronous, so there are two segments. The blue segment represents the smallest distance between ships, with circles at both ends representing ship positions. According to Fig. 10 (a), the minimum distance is at 3n mile starting to take actions, and two ships keep clear of each other. Although the minimum distance at 2n mile starting to take actions is larger than that at 3n mile, CF CRYSTAL infringes on the ship domain of SANCHI near 2100s (Fig. 9 (e)), and S_{de} of SANCHI is less than 1 (Fig. 10). Fig. 11 shows the curves of heading and speed. Under avoidance actions taken by SANCHI alone, the later it takes action, the larger the required amplitude. When both ships are taking actions, the turning amplitude of SANCHI at 3n mile is smaller than that at 5n mile, due to CF CRYSTAL taking actions together, which is a common result of game equilibrium and collaborative decision-making. At 2n mile to starting taking actions, both ships take urgent and substantial actions, but still cannot avoid ship domain of SANCHI infringed. When decisions start at 6, 5 and 4n mile, only one curve (6n mile) is shown in heading and speed curves of CF CRYSTAL.

4.2 Case study II: A simulation scenario in multi-ship encounter

In this section, a 4-ship encounter scenario is designed to further verify the effectiveness of the risk and collaborative decision-making models. The initial ships parameters are shown in Tab. 4 and collision risk fields of potential collision areas are shown in Fig. 12, where each ship is 6n mile from its nearest target ships (t = 675s).

Tab. 4 The initial ships parameters of 4-ship encounter scenario

NO.	Position/*n mile*	Heading/°	Speed/*kn*	Ship length/*m*	Goal point//*n mile*
S1	(0, -7.5)	000	15	222	(0, 67.5)
S2	(0, 7.5)	180	15	222	(0, -67.5)
S3	(6, 0)	270	12	185.2	(-54, 0)
S4	(-6, 0)	090	12	185.2	(54, 0)

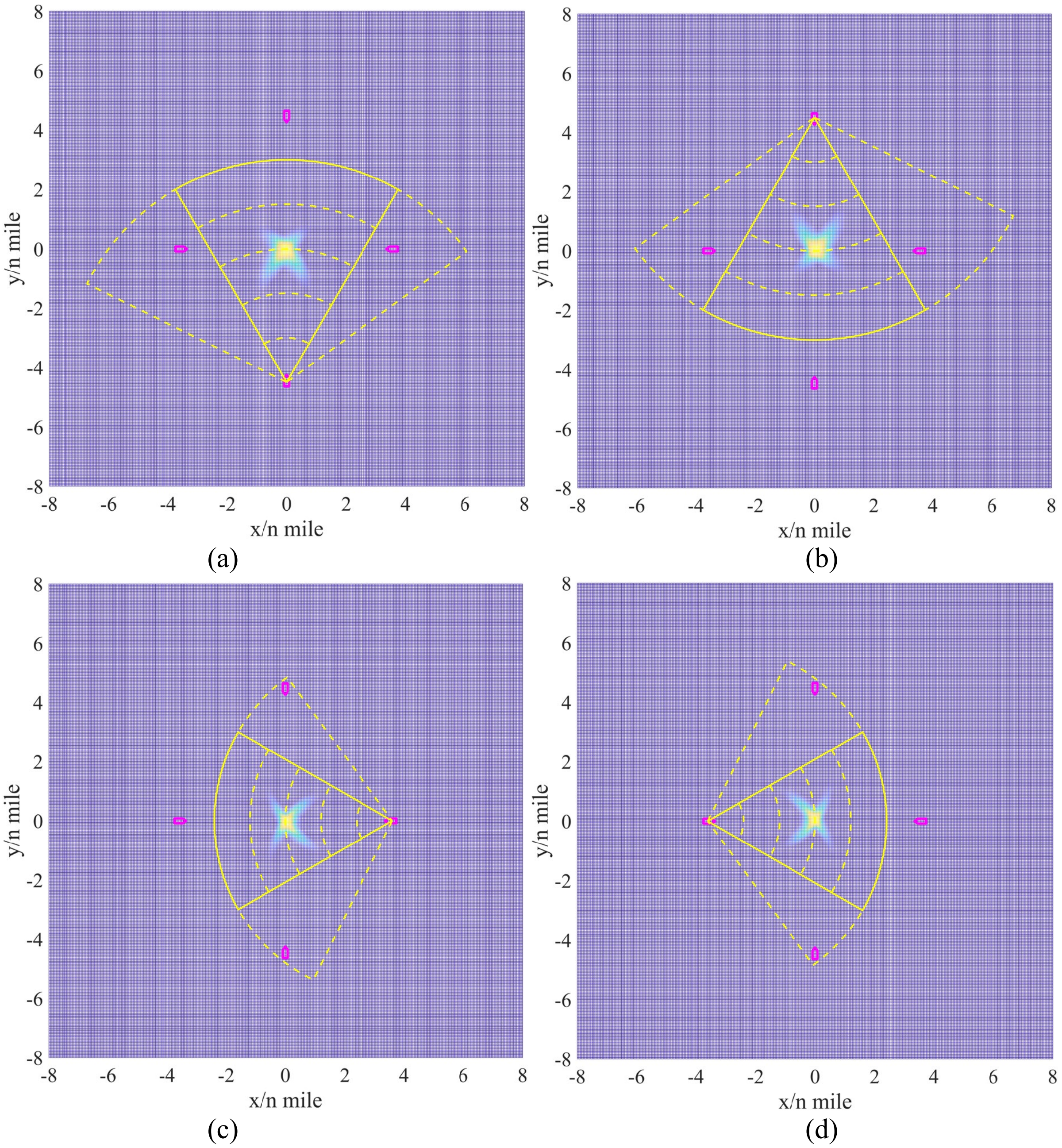
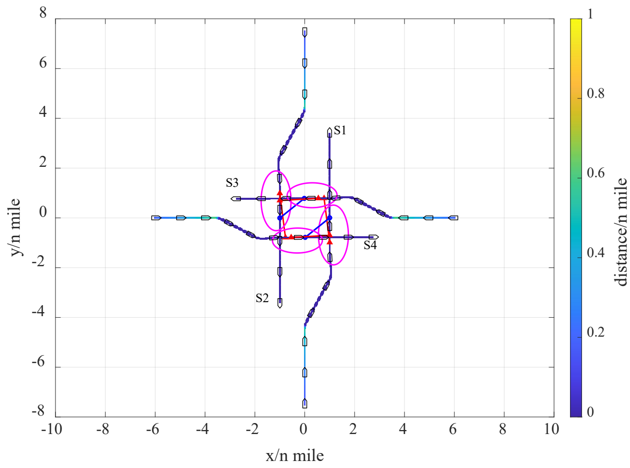


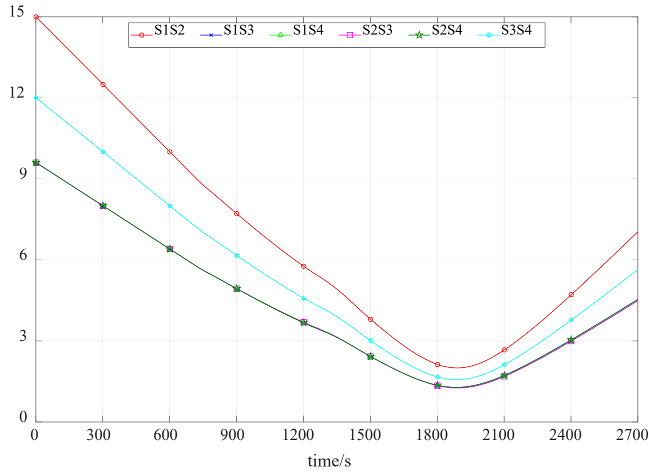
Fig. 12 Collision risk fields of potential collision areas for each ship

4.2.1 Simulation 1

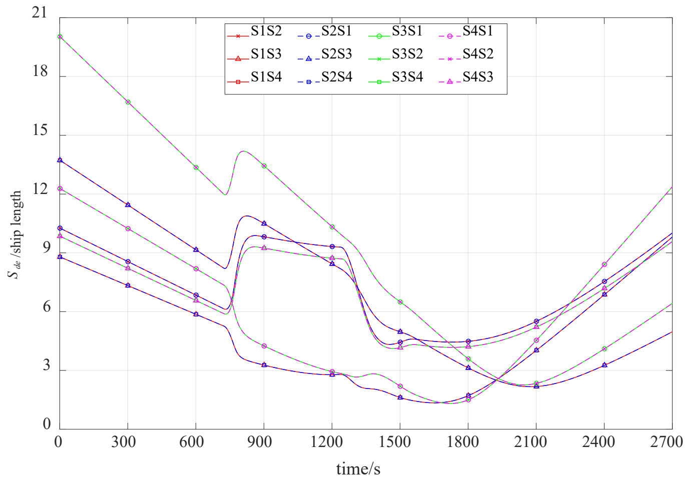
Fig. 13 shows the collision avoidance trajectories and related parameter results of the four ships taking collision avoidance actions according to their game action intentions. Since the encounter is a symmetric scenario, the game results of S1 and S2, S3 and S4 are consistent. Meanwhile, parameters of some ships are also consistent. In encounter trajectories (Fig. 13 (a)), the magenta segments and blue segments are marked under the minimum S_{de} and distance, respectively. All ships avoid violating other ship domains and pass at a safe distance. Fig. 13 (b) shows the variation curve of distances between ships, except for distance between S1 and S2, S3 and S4, and other four curves are coincident before taking actions and further split into two bunches (S1S3 and S2S4, S1S4 and S2S3) as a result of collision avoidance actions. Fig. 13 (c) shows the curve of S_{de}, in which all appear in pairs and meet the safety requirements (the minimum S_{de} > 1). Fig. 13 (d)~(f) shows the curves of rudder angle, heading and speed respectively. At 721s, all ships begin to take actions of turning starboard, with turning amplitudes of S1/S2 at 29.06°, heading stabilizing at 359.99° and 179.99° at 1237s, respectively, with the minimum speed 12.48kn, and the whole process is 856s. The amplitudes of S3/S4 are 28.20°, starting to return to destination direction at 1243s, and finally heading to 269.71° and 89.71°. The entire process is 862s with the minimum speed 10.02kn.



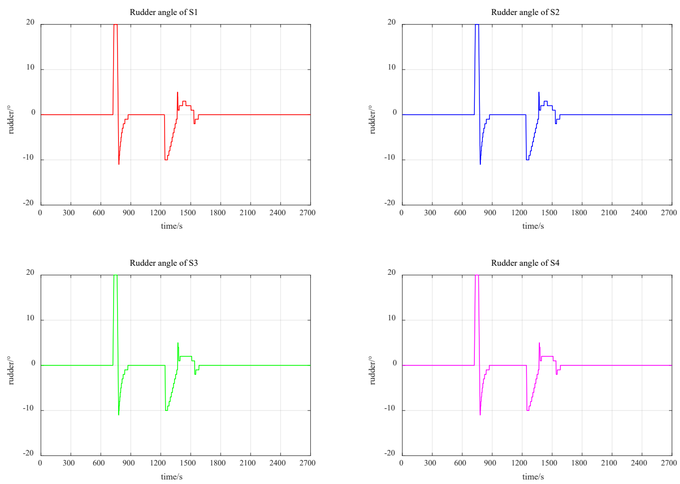
(a) encounter trajectories of each ship



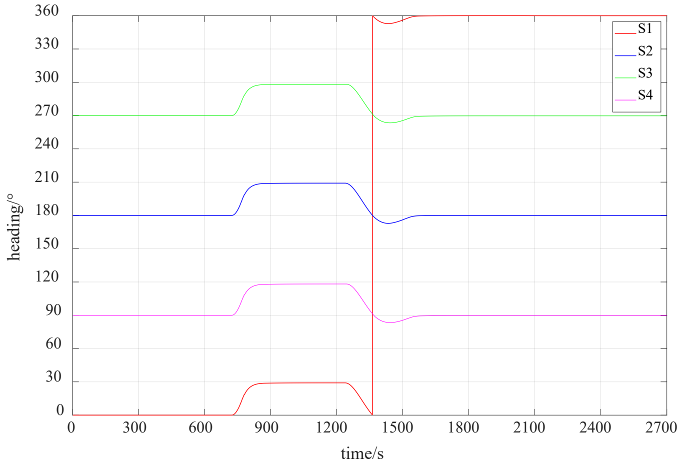
(b) distance curves between ships



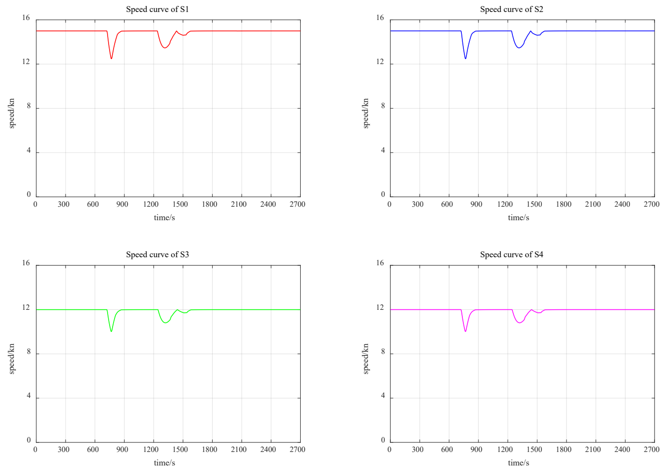
(c) S_{de} curves between ships



(d) rudder angle curves of each ship



(e) heading curves of each ship

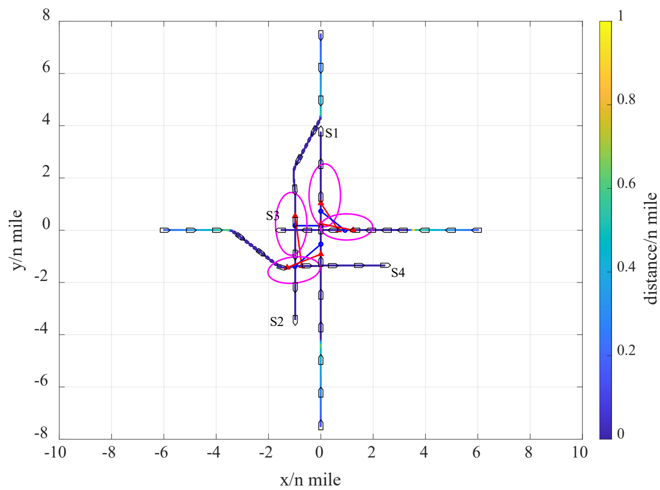


(f) speed curves of each ship

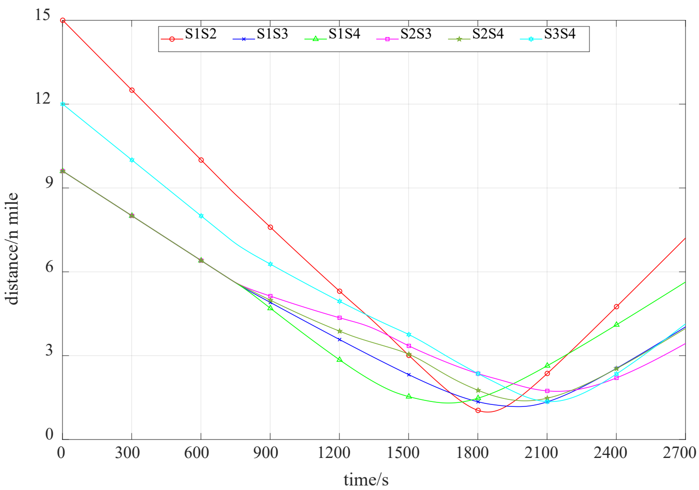
Fig. 13 The results of all the four ships taking actions based on the game action intentions

4.2.2 Simulation 2

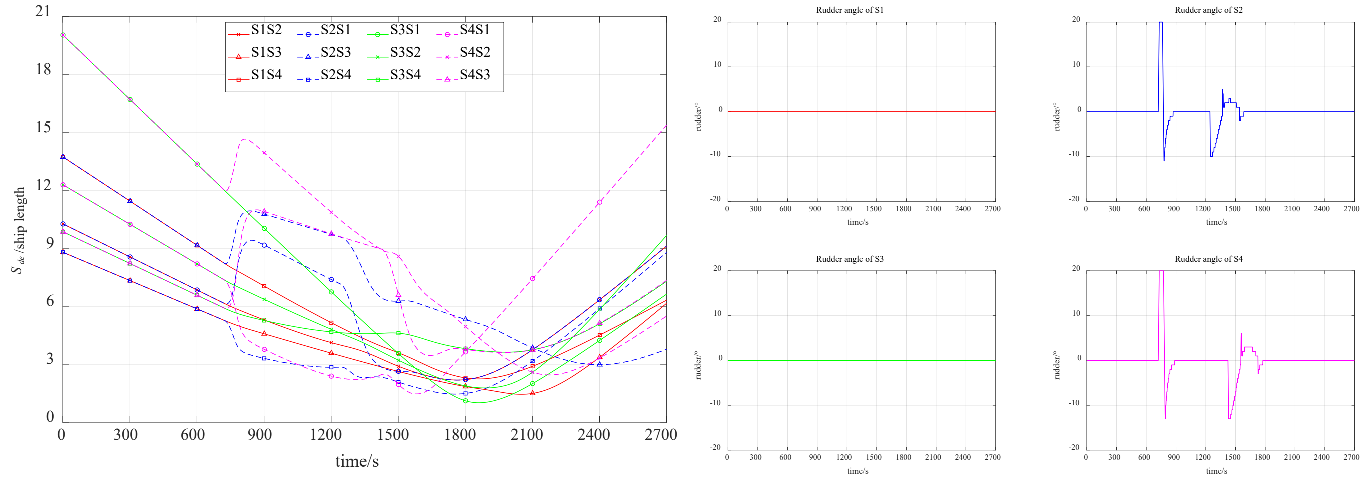
In this simulation, S1 is set to not take actions according to game intentions and marked with GNA. The simulation results are shown in Fig. 14. Since S1 does not participate in the game decision, the situation is no longer symmetric. According to the results, all ships avoid infringing on ship domains of TSs and keep clearance. Ship positions corresponding to the minimum S_{de} and distance are shown in Fig. 14 (a). Fig. 14 (b) shows the distance curve. During whole encounter process, the minimum distance is between S1 and S2 with 0.98n mile. According to Fig. 14 (c), the minimum S_{de} occurs between S3 and S1 with 1.01. Comparing Fig. 14 (b) and (c), S_{de} is more sensitive than distance when taking actions. Due to S1 keeping heading and speed without taking action, S2 and S4 are required to take actions by turning starboard, but S3 is required to reduce speed. S2 turns starboard with 28.20° and finally stabilizes at 179.94° towards to goal point with the process duration 863s and the minimum speed 12.52kn. S3 decelerates to avoid collisions, reducing its speed from 12kn to 7.24kn, with the whole process lasting 1258s. The amplitude of S4 is 37.52° and it approaches to the goal point at 89.59° after taking actions at 1055s, with the minimum speed 9.67kn during entire process.



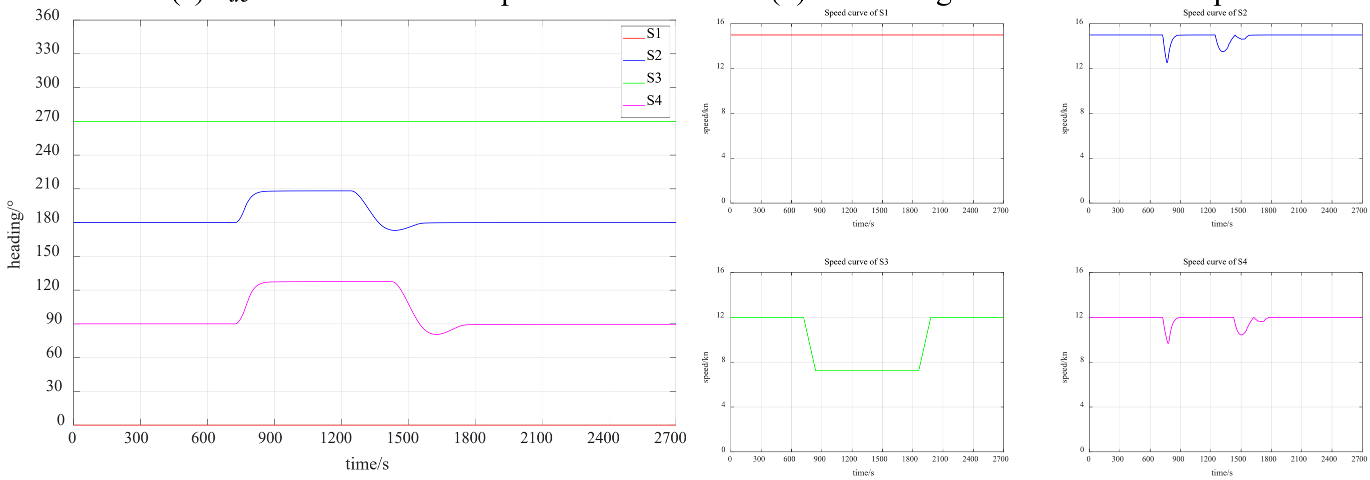
(a) encounter trajectories of each ship



(b) distance curves between ships



(c) S_{de} curves between ships (d) rudder angle curves of each ship



(e) heading curves of each ship (f) speed curves of each ship

Fig. 14 The results of S1without taking actions based on the game action intentions

4.3 Results analysis and discussions

In the CF CRYSTAL and SANCHI case, collision risk increases as the distance decrease, especially after 18 min. At 22 min (about 6n mile), the risk value is 0.5 and it is consistent with the division of risk of collision stage, 0.2~0.7, at the moment when give-way ships should take actions and in the middle of division as defined in (Liu et al., 2024b). The widely-accepted node of 6n mile also proves the rationality of these research results. Moreover, compared with Li et al. (2021) and Yuan et al. (2021), the trend and range of the proposed model are similar, conforming to COLREGs and navigation risk cognition (Wu et al., 2021). In the collision avoidance simulations, the distance curves of 6, 5, 4n mile are set respectively to start taking actions, by analyzing spatial attributes, distance and S_{de}, safety can be ensured with give-way ships taking actions alone. At closer distances, simulations are carried out with 3 and 2n mile starting to actions, and the COLREGs and game collaborative mechanism are considered to complete decision-making between two ships. Although the two ships take evasive actions simultaneously, the ship domain of SANCHI is violated with 2n mile starting to actions, as its ship domain violation could not be avoided at such a close distance.

In the multi-ship case, a symmetric scenario is set up, and the decision-making is made to verify the validity and rationality of collision avoidance. According to the results, the actions of corresponding ships (S1 and S2, S3 and S4) are consistent, and the decision-making model can ensure safe navigation. To further verify the economy of the proposed model, a result comparison with Liu et al. (2022) is shown in Tab. 5. Compared with the amplitude, the actions taken by the proposed model are larger due to considering the substantial requirements of COLREGs; meanwhile, while ensuring safety, the duration of the entire avoidance process is significantly shortened. Compared to Liu et al. (2022), the proposed model achieves a smaller drift distance and higher economic efficiency. Further, by setting S1 to keep heading and speed, other ships make decisions considering departing from the COLREGs. By marking GNA in the decision-making algorithm, collision avoidance actions are taken early to ensure navigation safety.

Tab. 5 A result comparison with Liu et al. (2022) in all four ships taking actions

	Ship No.	Amplitude/°	Duration/*s*	Drift distance/*n mile*
Liu et al. (2022)	S1/S2	13.78	1571	1.14
	S3/S4	17.95	1600	1.20
Proposed model	S1/S2	29.06	856	1.06
	S3/S4	28.20	862	0.84

In multi-ship simulations, the combination strategies are coded into quinary numbers, 0~4 correspond to go straight, turn starboard, turn port, decelerate, accelerate respectively, and are sequenced from left to right. Therefore, game rewards changes periodically by 5^m, where m represents the number corresponding to ships, sequenced from right to left within the combination strategies, with the large period fluctuating periodically under the influence of the smaller period. The result of the first multi-ship simulation is shown in Fig. 15. Four ships participate in the game, and game result is '1111', indicating that all ships are turning starboard. This result corresponds to game strategy number 157, as indicated by the red line. In the second multi-ship simulation, only three ships (S2~S4) participate in the game, resulting in the outcome '141', which corresponds to game strategy number 47, and the result is shown in Fig. 16. Although the selected game results may not offer the largest rewards for any particular ship, overall, all ships achieve a higher reward. Moreover, according to the definition, no ship can increase its rewards by unilaterally changing its strategy, thus achieving an equilibrium state where any rational ship is willing to accept this decision.

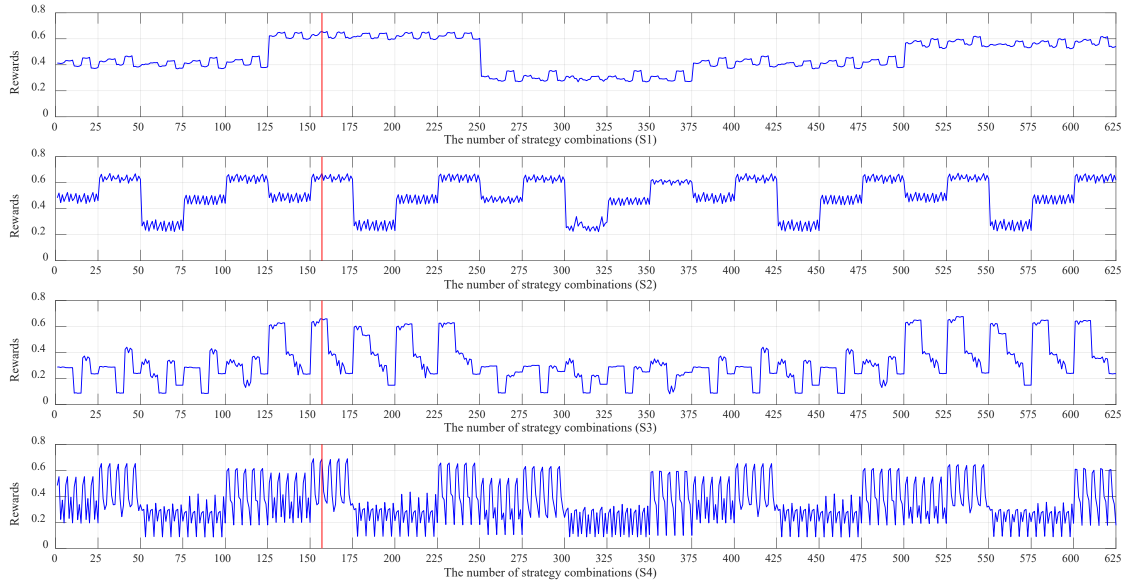


Fig. 15 The rewards for all combination strategies with all four ships participation

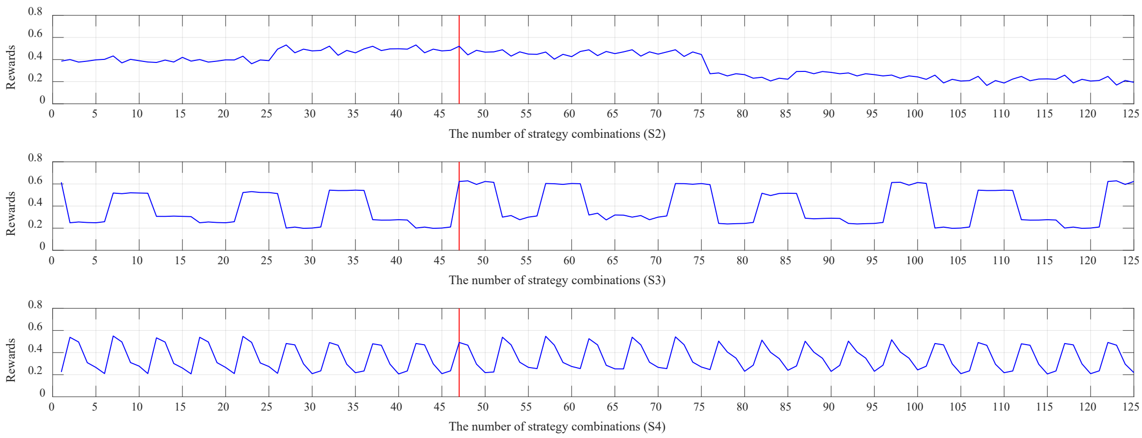


Fig. 16 The rewards for all combination strategies without S1's participation

Comparing two simulations in the second case, to address the change in S1's actions, S3 changes from turning to deceleration, and causing changes in the amplitudes of other ships directly or indirectly. The amplitude of S2 reduces slightly with 0.86°, and the amplitude of S4 increases significantly by 9.32°. This indicates that the change is not only reflected in collision avoidance modes, but also the collision avoidance amplitudes are not all increased, which reflects the complex coupling relationship between ships in multi-ship encounters (Cho et al., 2022; Wang et al., 2023). Ship collaborative decision-making based on intention games can deal with the coupling relationship accurately, and ensure navigation safety and economy. Comparing the curve of distance and S_{de}, it is less sensitive to collision avoidance actions than S_{de}. Therefore, when using an elliptical ship domain or other shapes with different safety standards required for various azimuths as conflict detection and collision risk assessment, more appropriate spatial attribute parameters should be selected to better evaluate encounters (Szlapczynski et al., 2018; Szlapczynski & Szlapczynska, 2016). This paper proposes a distributed decision-making model with a game collaborative mechanism based on rational thinking, which can better adapt to dynamic encounter scenarios and more effectively ensure navigation safety.

4.4 Limitations and further developments

In terms of the practical application of the model, collision avoidance decision-making algorithms and the model will be integrated into ship navigation software as a critical component of the intelligent navigation system to ensure safety during ship encounters. After addressing a few collision avoidance problems as follows, the applications could be broader and of greater contributions to the development of intelligent navigation.

- Differences in risk attitudes and ship motions. Since OOWs' navigation experience and subjective perceptions varies, their attitudes are also different (Wu et al., 2021). Meanwhile, how to control and track trajectories, predict TSs' trajectory also limits the evolution and assessment of encounter situations (Li & Yang, 2023). In the follow-up research, risk attitudes can be reasoned using Bayesian theory (Zhang et al., 2020) and other methods based on AIS data (Li et al., 2023), with construction and integration ship action control (Gil et al., 2019), trajectory tracking (Guo et al., 2024) and online maneuverability identification (Liu et al., 2024d) to further improve the performance of the decision-making model.
- Collision avoidance in restricted waters. This study primarily focuses on ship collision avoidance in open waters. Further, the model can be extended to adjust to restricted waters with static obstacles. It is necessary to obtain the position of static obstacle conflict areas (Zhang et al., 2021a), and navigable waters through navigation aids, and quantify and evaluate them together with potential collision areas. In this context, the parameters in the model should be adjusted to different waters.
- Multi-ship encounters division issues in actual navigation scenarios. Ships may join and leave a specific multi-ship encounter situations with a dynamic group and strong coupling among encountered ships (Xin et al., 2023b). To maintain a stable ship group, it is necessary to consider the spatiotemporal dynamics and the interference relationship between encountered ships, dividing the ship cluster or dynamic group and involving into collision avoidance decision-making models (Liu et al., 2023).
- Other aspects. In real navigation environments, unexpected situations often occur. Data transmission error and loss happens occasionally (Zhang et al., 2023b); the effectiveness of decision-making is also influenced by wind, waves and current (Zhang et al., 2021a). In addition, before the decision-making model being used in practice, multi-round reliability evaluations need to be undertaken and implemented by real ship testing (Liu et al., 2024c).

5. Conclusion

A collaborative decision-making model is proposed to realize anti-collision of multiple ships without information interaction among the encountered ships. The results demonstrate that the decision-making can meet the safety requirements in case studies. This research can be utilized to ship decision support and promote the development of intelligent navigation. Some new findings are summarized as follows:

- From the OS's perspective of risk cognitions, potential collision areas with an off-centered elliptic ship domain are determined by combining ship heading, speed and time of domain violation (Section 3.1.1), which can directly reflect the amplitude of collision avoidance decisions.
- By quantifying the state risk of potential collision areas, the collision risk field and risk value are formed based on feasible decisions and macro-level situations (Section 3.1.2~3.1.4), which can guide decision-making and evaluate its effectiveness.
- Encountered ships are differentiated based on risk relationships between neighbor ships, and their roles are assigned with collision avoidance responsibilities (Section 3.2.1), providing a more comprehensive consideration of COLREGs.
- Based on rational thinking, a collaborative mechanism is established to obtain ship action intentions through a strategy game (Section 3.2.2~3.2.3), and a collaborative collision avoidance decision-making model within game-decision cycles (Section 3.2.4) is developed to reduce calculation time and meet real-time requirements.
- The dynamic ship encounter is transformed into a static collision risk field by determining and quantifying potential collision areas, and the collaborative decision-making is developed by differentiating encountered ships to enhance the collision avoidance effectiveness (Tab. 5) and ensure navigation safety (Fig. 13, 14).

Acknowledgments

The research was supported by National Key Technologies Research & Development Program (2023YFB4302303), National Natural Science Foundation of China (52071247; 51920105014) and European Research Council project under the European Union’s Horizon 2020 research and innovation programme (TRUST CoG 2019 864724).

Appendix A. Lists of abbreviations and symbols

Tab.A.1 List of abbreviations

OOWs	Officers On Watch
IMO	International Maritime Organization
VO	Velocity Obstacle
OS	Own Ship
TS	Target Ships
DCPA	Distance to Closest Point of Approach
TCPA	Time to Closest Point of Approach
COLREGs	International Regulations for Preventing Collisions at Sea
d_{dv}	Degree of Domain Violation
t_{dv}	Time of Domain Violation
AIS	Automatic Identification System
OZT	Obstacle Zone by Target
DAC	Dangerous Area of Collision

VHF	Very High Frequency
GNA	Give-way Not taking appropriate Actions
A/B	After/Before the close-quarters situation

Tab. A.2 List of symbols

$\boldsymbol{S}_{o/t}(t)$	OS($_o$)/TS($_t$) ship statu at t
$\boldsymbol{P}_{o/t}(t)$	OS($_o$)/TS($_t$) ship position at t
$\boldsymbol{V}_{o/t}\ (t)$	OS($_o$)/TS($_t$) ship velocity at t
$[x, y]$	Coordinates of ship position $\boldsymbol{P}$ in X ($W \to E$) and Y ($S \to N$)
$[v, c]$	Magnitude and heading of velocity
L	Ship length
$Conf\boldsymbol{P}(\boldsymbol{P}, \boldsymbol{V})$	Ship domain of OS at $\boldsymbol{P}$ and $\boldsymbol{V}$
$\oplus$	Minkowski addition
$\boldsymbol{sP}$	Positions of potential collision area
$\boldsymbol{P}$	Potential collision area of a ship
$\boldsymbol{MP}$	Union of potential collision area of multiple ships
Mc_s, Mc_p, Md_v	Minimum amplitudes of turning starboard, port and deceleration ratio
r_{cs}, r_{cp}, r_v	Collision risk models based on Mc_s, Mc_p, Md_v
C_s, C_p, C_v	Tolerable maximum amplitudes of turning starboard, port and deceleration ratio
R_a	Collision risk model based on feasible decision space
s_{co}, s_{av}	Coverage ratio and average of state risk of potential collision area
C_h	Current heading
s_i	Number of points in potential collision areas located in the i^{th} interval
S_i	Total number of points of the i^{th} interval after discretization
r_c	Collision risk model based on s_{co}
r_s	State collision risk model
r_d, r_t	Spatial and temporal risk of state collision risk model
k_1, k_2	Weights of r_d and r_t of r_s
r_a	Collision risk model based on s_{av}
R_m	Collision risk model based on a macro-level situation
R	Collision risk model based on potential collision areas
G	Game model
N, S, u	Players, strategy and reward function of game G
ΔT	Game period /60s
ms	Reward function of safety evaluation model
mr_m	Reward function of collision avoidance modes
mr_c, mr_v	Reward fuction of collision avoidance amplitudes, heading and speed
d_h, d_v	Collision avoidance amplitudes of heading and speed
k_h, k_v	Amplitudes sufficient to be easily observed or detected by TSs
mr	Reward function of COLREGs compliance
v_g	ratio of the velocity component towards goal point
n_e	Navigation efficiency
t_g, t_s	Time of turning heading to resume towards goal point and initial speed
d_g, d_s	Heading and speed amplitudes to resume towards goal point and initial speed
me	Reward function of economic
m	Hierarchical game reward model
Δt	Decision-making period /1s
S_{de}	Ship domain extension parameter

Appendix B. The relationship of ship rudder and angular

The relationship of ship rudder and angular is assumed to follow the *Nomoto* model (Nomoto, 1956) in case studies and shown in Eq. B.1. Considering that ship engine mode is fixed power at open sea, steering generates torque to change the heading, causing deceleration. The larger the angular velocity, the more obvious the deceleration. The relation between ship deceleration V_{loss} and angular velocity r is shown in Eq. B.2.

$$T \cdot dr / dt + r = K\delta \qquad \text{(B.1)}$$

$$V_{loss} = 0.2 \cdot r / r_{max} \cdot v \qquad \text{(B.2)}$$

where, T and K represent ship's time constant and rudder gain respectively, and are set as 0.0579 and 69.9784 (Zhang et al., 2015); r is the angular velocity, δ represents the rudder angle, v is initial speed, and r_{max} represents the maximum angular velocity.

References

Akdag, M., Solnor, P., & Johansen, T. A. (2022). Collaborative collision avoidance for Maritime Autonomous Surface Ships: A review. *Ocean Engineering*, *250*, 17, 110920.

Cai, M. Y., Zhang, J. F., Zhang, D., Yuan, X. L., & Soares, C. G. (2021). Collision risk analysis on ferry ships in Jiangsu Section of the Yangtze River based on AIS data. *Reliability Engineering & System Safety*, *215*, 13, 107901.

Chaal, M., Ren, X., BahooToroody, A., Basnet, S., Bolbot, V., Banda, O. A. V., & Van Gelder, P. (2023). Research on risk, safety, and reliability of autonomous ships: A bibliometric review. *Safety Science*, *167*, 106256.

Chauvin, C., & Lardjane, S. (2008). Decision making and strategies in an interaction situation: Collision avoidance at sea. *Transportation Research Part F-Traffic Psychology and Behaviour*, *11*(4), 259-269.

Cho, Y., Han, J., & Kim, J. (2022). Efficient COLREG-Compliant Collision Avoidance in Multi-Ship Encounter Situations. *Ieee Transactions on Intelligent Transportation Systems*, *23*(3), 1899-1911.

Cockcroft, A. N., & Lameijer, J. N. F. (2012). *A Guide to the Collision Avoidance Rules* (seventh ed.).

Fan, S., & Yang, Z. (2024). Accident data-driven human fatigue analysis in maritime transport using machine learning. *Reliability Engineering & System Safety*, *241*, 109675.

Feng, Y. W., Wang, H. X., Xia, G. Q., Cao, W. J., Li, T. Y., Wang, X. J., & Liu, Z. J. (2024a). A machine learning-based data-driven method for risk analysis of marine accidents. *Journal of Marine Engineering and Technology*, 12.

Feng, Y. W., Wang, X. J., Chen, Q. L., Yang, Z. L., Wang, J., Li, H. H., Xia, G. Q., & Liu, Z. J. (2024b). Prediction of the severity of marine accidents using improved machine learning. *Transportation Research Part E-Logistics and Transportation Review*, *188*, 31, 103647.

Feng, Y. W., Wang, X. J., Luan, J. L., Wang, H., Li, H. J., Li, H. H., Liu, Z. J., & Yang, Z. L. (2024c). A novel method for ship carbon emissions prediction under the influence of emergency events. *Transportation Research Part C-Emerging Technologies*, *165*, 33, 104749.

Fu, H. F. (2021). On the existence of Pareto undominated mixed-strategy Nash equilibrium in normal-form games with infinite actions. *Economics Letters*, *201*, 3, 109771.

Fujii, Y. (1971). Traffic Capacity. *Journal of Navigation*, *24*(4), 543-552.

Gil, M., Wrobel, K., & Montewka, J. (2019). Toward a Method Evaluating Control Actions in STPA-Based Model of Ship-Ship Collision Avoidance Process. *Journal of Offshore Mechanics and Arctic Engineering-Transactions of the Asme*, *141*(5), 051105.

Guo, Q., Zhang, X., & Wang, X. (2024). Adaptive dynamic surface control for trajectory tracking of autonomous surface vehicles with input and output constraints. *Journal of Marine Engineering and Technology*, *23*(2), 113-121.

Hashimoto, H., Makino, H., Yoshioka, H., & Matsuda, A. (2022). Ship-stopping algorithm utilizing VecTwin rudder system for automatic collision prevention. *Ocean Engineering*, *251*, 111098.

He, Y. X., Jin, Y., Huang, L. W., Xiong, Y., Chen, P. F., & Mou, J. M. (2017). Quantitative analysis of COLREG rules and seamanship for autonomous collision avoidance at open sea. *Ocean Engineering*, *140*, 281-291.

Hornauer, S., Hahn, A., Blaich, M., & Reuter, J. (2015). Trajectory Planning with Negotiation for Maritime Collision Avoidance. *Transnav-International Journal on Marine Navigation and Safety of Sea Transportation*, *9*(3), 335-341.

Horteborn, A., Ringsberg, J. W., Svanberg, M., & Holm, H. (2019). A Revisit of the Definition of the Ship Domain based on AIS Analysis. *Journal of Navigation*, *72*(3), 777-794.

Hu, L., Naeem, W., Rajabally, E., Watson, G., Mills, T., Bhuiyan, Z., Raeburn, C., Salter, I., & Pekcan, C. (2020). A Multiobjective Optimization Approach for COLREGs-Compliant Path Planning of Autonomous Surface Vehicles Verified on Networked Bridge Simulators. *Ieee Transactions on Intelligent Transportation Systems*, *21*(3), 1167-1179.

Huang, Y., van Gelder, P. H. A. J. M., & Wen, Y. (2018). Velocity obstacle algorithms for collision prevention at sea. *Ocean Engineering*, *151*, 308-321.

Huang, Y. M., Chen, L. Y., Chen, P. F., Negenborn, R. R., & van Gelder, P. (2020). Ship collision avoidance methods: State-of-the-art. *Safety Science*, *121*, 451-473.

IMO. (2023). *Autonomous shipping*. https://www.imo.org/en/MediaCentre/HotTopics/Pages/Autonomous-shipping.aspx

Kim, J. K., & Park, D. J. (2024). Understanding of sailing rule based on COLREGs: Comparison of navigator survey and automated collision-avoidance algorithm. *Marine Policy*, *159*, 14, 105894.

Lee, H. J., & Park, D. J. (2024). Collision evasive action timing for MASS using CNN-LSTM-based ship trajectory prediction in restricted area. *Ocean Engineering*, *294*, 28, 116766.

Li, H. B., Wang, X., Soares, C. G., & Ni, S. K. (2024a). A collision-avoidance decision-making scheme based on artificial potential fields and event-triggered control. *Ocean Engineering*, *306*, 11, 118101.

Li, H. H., Jiao, H., & Yang, Z. L. (2023). AIS data-driven ship trajectory prediction modelling and analysis based on machine learning and deep learning methods. *Transportation Research Part E-Logistics and Transportation Review*, *175*, 39, 103152.

Li, H. H., Lam, J. S. L., Yang, Z. L., Liu, J. X., Liu, R. W., Liang, M. H., & Li, Y. (2022). Unsupervised hierarchical methodology of maritime traffic pattern extraction for knowledge discovery. *Transportation Research Part C-Emerging Technologies*, *143*, 24, 103856.

Li, H. H., Xing, W. B., Jiao, H., Yang, Z. L., & Li, Y. (2024b). Deep bi-directional information-empowered ship trajectory prediction for maritime autonomous surface ships. *Transportation Research Part E-Logistics and Transportation Review*, *181*, 31, 103367.

Li, H. H., & Yang, Z. L. (2023). Incorporation of AIS data-based machine learning into unsupervised route planning for maritime autonomous surface ships. *Transportation Research Part E-Logistics and Transportation Review*, *176*, 32, 103171.

Li, M. X., Mou, J. M., Chen, L. Y., He, Y. X., & Huang, Y. M. (2021). A rule-aware time-varying conflict risk measure for MASS considering maritime practice. *Reliability Engineering & System Safety*, *215*, 13, 107816.

Li, M. X., Mou, J. M., Chen, P. F., Rong, H., Chen, L. Y., Zhao, X. Y., & Wu, Y. (2024c). Ship autonomous collision avoidance decision from the perspective of navigation practice. *Ocean Engineering*, *310*, 22, 118713.

Li, S. J., Liu, J. L., & Negenborn, R. R. (2019). Distributed coordination for collision avoidance of multiple ships considering ship maneuverability. *Ocean Engineering*, *181*, 212-226.

Liu, C., Kulkarni, K., Suominen, M., Kujala, P., & Musharraf, M. (2024a). On the data-driven investigation of factors affecting the need for icebreaker assistance in ice-covered waters. *Cold Regions Science and Technology*, *221*, 104173.

Liu, J., Zhang, J., Yang, Z., Wan, C., & Zhang, M. (2024b). A novel data-driven method of ship collision risk evolution evaluation during real encounter situations. *Reliability Engineering & System Safety*, *249*, 110228.

Liu, J. J., Zhang, J. F., Yan, X. P., & Soares, C. G. (2022). Multi-ship collision avoidance decision-making and coordination mechanism in Mixed Navigation Scenarios. *Ocean Engineering*, *257*, 20, 111666.

Liu, J. L., Yang, F., Li, S. J., Lv, Y. Q., & Hu, X. J. (2024c). Testing and evaluation for intelligent navigation of ships: Current status, possible solutions, and challenges☆. *Ocean Engineering*, *295*, 14, 116969.

Liu, K., Wu, X., Zhou, Y., Yuan, Z., Yang, X., Xin, X., & Zhuang, S. (2023). A conflict cluster-based method for collision avoidance decision-making in multi-ship encounter situations. *Ocean Engineering*, *288*, 116038.

Liu, Y., Zhang, Q., Wang, L., An, S., He, Y., Fan, Z., & Deng, F. (2024d). Identification of Multi-Innovation Stochastic Gradients with Maximum Likelihood Algorithm Based on Ship Maneuverability and Wave Peak Models. *Journal of Marine Science and Engineering*, *12*(1), 142.

Longo, G., Martelli, M., Russo, E., Merlo, A., & Zaccone, R. (2024). Adversarial waypoint injection attacks on Maritime Autonomous Surface Ships (MASS) collision avoidance systems. Journal of Marine Engineering and Technology, 23(3), 184-195.

Lyu, H., Hao, Z., Li, J., Li, G., Sun, X., Zhang, G., Yin, Y., Zhao, Y., & Zhang, L. (2023). Ship Autonomous Collision-Avoidance Strategies-A Comprehensive Review. *Journal of Marine Science and Engineering*, *11*(4), 830.

Ma, Q., Tang, H., Liu, C., Zhang, M., Zhang, D., Liu, Z., & Zhang, L. (2024). A big data analytics method for the evaluation of maritime traffic safety using automatic identification system data. *Ocean & Coastal Management*, *251*, 107077.

Marino, A., & Hajnsek, I. (2015). Statistical Tests for a Ship Detector Based on the Polarimetric Notch Filter. *Ieee Transactions on Geoscience and Remote Sensing*, *53*(8), 4578-4595.

Merwe, K. v. d., Mallam, S., Nazir, S., & Engelhardtsen, O. (2024). Supporting human supervision in autonomous collision avoidance through agent transparency. *Safety Science*, *169*, 106329.

Neatby, H. C., & Thornhill, E. (2024). Turning and stopping of a ship with twin Z-drive thrusters. *Ocean Engineering*, *293*, 116641.

Nomoto, K. (1956). On the steering qualities of ships. *International shipbuilding progress*, *4*, 354-370.

Park, S., Kim, C. H., & Cheon, M. (2023). Analysis of marine incidents and policy implications: Case study of the South Korea. *Marine Policy*, *155*, 12, 105736.

Pietrzykowski, Z., & Wielgosz, M. (2021). Effective ship domain - Impact of ship size and speed. *Ocean Engineering*, *219*, 108423.

Sawada, R., Sato, K., & Majima, T. (2021). Automatic ship collision avoidance using deep reinforcement learning with LSTM in continuous action spaces. *Journal of Marine Science and Technology*, *26*(2), 509-524.

Sawada, R., Sato, K., & Minami, M. (2024). Framework of safety evaluation and scenarios for automatic collision avoidance algorithm. *Ocean Engineering*, *300*, 14, 117506.

Shi, Z., Zhen, R., & Liu, J. (2022). Fuzzy logic-based modeling method for regional multi-ship collision risk assessment considering impacts of ship crossing angle and navigational environment. *Ocean Engineering*, *259*, 111847.

Silveira, P., Teixeira, A. P., & Soares, C. G. (2022). A method to extract the Quaternion Ship Domain parameters from AIS data. *Ocean Engineering*, *257*, 20, 111568.

Song, J., Shoji, R., Tamaru, H., & Kayano, J. (2023). Modeling Human Encounter Situation Awareness Results Using Support Vector Machine Models. *Applied Sciences-Basel*, *13*(13), 7521.

Szlapczynski, R., Krata, P., & Szlapczynska, J. (2018). A Ship Domain-Based Method of Determining Action Distances for Evasive Manoeuvres in Stand-On Situations. *Journal of Advanced Transportation*, 3984962.

Szlapczynski, R., & Szlapczynska, J. (2016). An analysis of domain-based ship collision risk parameters. *Ocean Engineering*, *126*, 47-56.

Szlapczynski, R., & Szlapczynska, J. (2017). Review of ship safety domains: Models and applications. *Ocean Engineering*, *145*, 277-289.

Szlapczynski, R., & Szlapczynska, J. (2021). A ship domain-based model of collision risk for near-miss detection and Collision Alert Systems. *Reliability Engineering & System Safety*, *214*, 21, 107766.

Szlapczynski, R., Szlapczynska, J., Gil, M., Zyczkowski, M., & Montewka, J. (2024). Holistic collision avoidance decision support system for watchkeeping deck officers. *Reliability Engineering & System Safety*, *250*, 23, 110232.

Wang, N. (2013). A Novel Analytical Framework for Dynamic Quaternion Ship Domains. *Journal of Navigation*, *66*(2), 265-281.

Wang, S. B., Zhang, Y. J., Song, F. F., & Mao, W. A. (2023). A collaborative collision avoidance strategy for autonomous ships under mixed scenarios. *Journal of Navigation*, 25, Pii s0373463323000012.

Wu, B., Zong, L., Yan, X., & Soares, C. G. (2018). Incorporating evidential reasoning and TOPSIS into group decision-making under uncertainty for handling ship without command. *Ocean Engineering*, *164*, 590-603.

Wu, X. L., Liu, K. Z., Zhang, J. F., Yuan, Z. T., Liu, J. J., & Yu, Q. (2021). An Optimized Collision Avoidance Decision-Making System for Autonomous Ships under Human-Machine Cooperation Situations. *Journal of Advanced Transportation*, *2021*, 17, 7537825.

Xie, S., Chu, X. M., Zheng, M., & Liu, C. G. (2019). Ship predictive collision avoidance method based on an improved beetle antennae search algorithm. *Ocean Engineering*, *192*, 19, 106542.

Xin, X., Liu, K., Li, H., & Yang, Z. (2024). Maritime traffic partitioning: An adaptive semi-supervised spectral regularization approach for leveraging multi-graph evolutionary traffic interactions. *Transportation Research Part C: Emerging Technologies*, *164*, 104670.

Xin, X. R., Liu, K. Z., Loughney, S., Wang, J., Li, H. H., Ekere, N., & Yang, Z. L. (2023a). Multi-scale collision risk estimation for maritime traffic in complex port waters. *Reliability Engineering & System Safety*, *240*, 109554.

Xin, X. R., Yang, Z. L., Liu, K. Z., Zhang, J. F., & Wu, X. L. (2023b). Multi-stage and multi-topology analysis of ship traffic complexity for probabilistic collision detection. *Expert Systems with Applications*, *213*, 18, 118890.

Yang, Y., Liu, Y., Li, G. R., Zhang, Z. K., & Liu, Y. B. (2024). Harnessing the power of Machine learning for AIS Data-Driven maritime Research: A comprehensive review. *Transportation Research Part E-Logistics and Transportation Review*, *183*, 17, 103426.

Yim, J. B., Kim, D. S., & Park, D. J. (2018). Modeling perceived collision risk in vessel encounter situations. *Ocean Engineering*, *166*, 64-75.

Yim, J. B., & Park, D. J. (2022). Modeling evasive action to be implemented at the minimum distance for collision avoidance in a give-way situation. *Ocean Engineering*, *263*, 13, 112210.

Yoshioka, H., Hashimoto, H., & Makino, H. (2023). Decision-making algorithm for ship collision avoidance with collision risk map. *Ocean Engineering*, *286*, 115705.

Yuan, X., Zhang, D., Zhang, J., Wan, C., & Fan, L. (2023). A two-stage collision avoidance path planning approach for inland ferries under dynamic channel crossing risk conditions. *Ocean & Coastal Management*, *242*, 106692.

Yuan, X. L., Zhang, D., Zhang, J. F., Zhang, M. Y., & Soares, C. G. (2021). A novel real-time collision risk awareness method based on velocity obstacle considering uncertainties in ship dynamics. *Ocean Engineering*, *220*, 20, 108436.

Zhang, C., Zhang, D., Zhang, M. Y., Lang, X., & Mao, W. G. (2020). An integrated risk assessment model for safe Arctic navigation. *Transportation Research Part a-Policy and Practice*, *142*, 101-114.

Zhang, D., Han, J. R., Wu, D., & Mao, W. G. (2023a). The Model of Ship Navigation Risk Field for Risk Assessment of Icebreaker Convoy Operations. *Ieee Transactions on Intelligent Transportation Systems*, 15.

Zhang, J. F., Liu, J. J., Hirdaris, S., Zhang, M. Y., & Tian, W. L. (2023b). An interpretable knowledge-based decision support method for ship collision avoidance using AIS data. *Reliability Engineering & System Safety*, *230*, 20, 108919.

Zhang, J. F., Zhang, D., Yan, X. P., Haugen, S., & Soares, C. G. (2015). A distributed anti-collision decision support formulation in multi-ship encounter situations under COLREGs. *Ocean Engineering*, *105*, 336-348.

Zhang, L. Y., Ma, Z. C., Liu, J. G., Liu, S. J., & Song, J. (2024). Selfish or cooperative? Understanding the behavior of ships during an overtaking process from the spatiotemporal perspective-A case study of Qingdao Port, China. *Ocean Engineering*, *299*, 17, 117352.

Zhang, L. Y., & Meng, Q. (2019). Probabilistic ship domain with applications to ship collision risk assessment. *Ocean Engineering*, *186*, 11, 106130.

Zhang, M. Y., Conti, F., Le Sourne, H., Vassalos, D., Kujala, P., Lindroth, D., & Hirdaris, S. (2021a). A method for the direct assessment of ship collision damage and flooding risk in real conditions. *Ocean Engineering*, *237*, 109605.

Zhang, M. Y., Montewka, J., Manderbacka, T., Kujala, P., & Hirdaris, S. (2021b). A Big Data Analytics Method for the Evaluation of Ship - Ship Collision Risk reflecting Hydrometeorological Conditions. *Reliability Engineering & System Safety*, *213*, 21, 107674.

Zhao, L. B., & Fu, X. J. (2022). A Method for Correcting the Closest Point of Approach Index During Vessel Encounters Based on Dimension Data From AIS. *Ieee Transactions on Intelligent Transportation Systems*, *23*(8), 13745-13757.

Zhao, L. M., & Roh, M. I. (2019). COLREGs-compliant multiship collision avoidance based on deep reinforcement learning. *Ocean Engineering*, *191*, 15, 106436.

Zhou, K., Chen, J. H., & Liu, X. (2018). Optimal Collision-Avoidance Manoeuvres to Minimise Bunker Consumption under the Two-Ship Crossing Situation. *Journal of Navigation*, *71*(1), 151-168.