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Theory and application of possibility and evidence in reliability analysis and design optimization

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Abstract
Numerous design optimization methodologies and reliability analysis techniques have been developed to address aleatory and epistemic uncertainties in engineering system design. Aleatory uncertainty is modeled by statistical distributions, while epistemic uncertainty becomes an alternative in cases where data is sparse and cannot be fully captured statistically. Possibility and evidence theories are computationally efficient and robust for quantifying epistemic uncertainty in reliability analysis and design optimization. This paper provides a comprehensive analysis of existing methodologies, challenges, and opportunities in managing uncertainty in engineering systems. Additionally, the concepts and practical applications of possibility and evidence theories are reviewed. Potential future research directions are outlined ultimately. This paper provides the sector with a clear understanding of possibility theory and evidence theory and their developments.

Keywords: possibility theory, evidence theory, uncertainty, reliability, design optimization

1. Introduction

Uncertainty is one of concerns in engineering design [1]. Engineering practices are subject to multiple uncertainties across spatial scales, temporal scales, and design stages [2]. Addressing these uncertainties is crucial for the effectiveness of engineering design activities [3]. Aiming at enhancing the understanding (classifications, theories, and design considerations) related to uncertainty, a comprehensive review of approaches for addressing epistemic uncertainty is performed in this paper, with an emphasis on their application in reliability analysis and design practices [4].

Several terms have been defined to describe uncertainty, including indeterminacy, unpredictability, variability, irregularity, arbitrariness, vagueness, randomness, variability, and chance. Uncertainty is always associated with phenomena that is doubtful, problematic, and indefinite [5]. It is also connected to the confidence degree when a particular proposition or dataset is valid [6, 7]. Based on the existing definitions, Zimmermann provided a comprehensive definition

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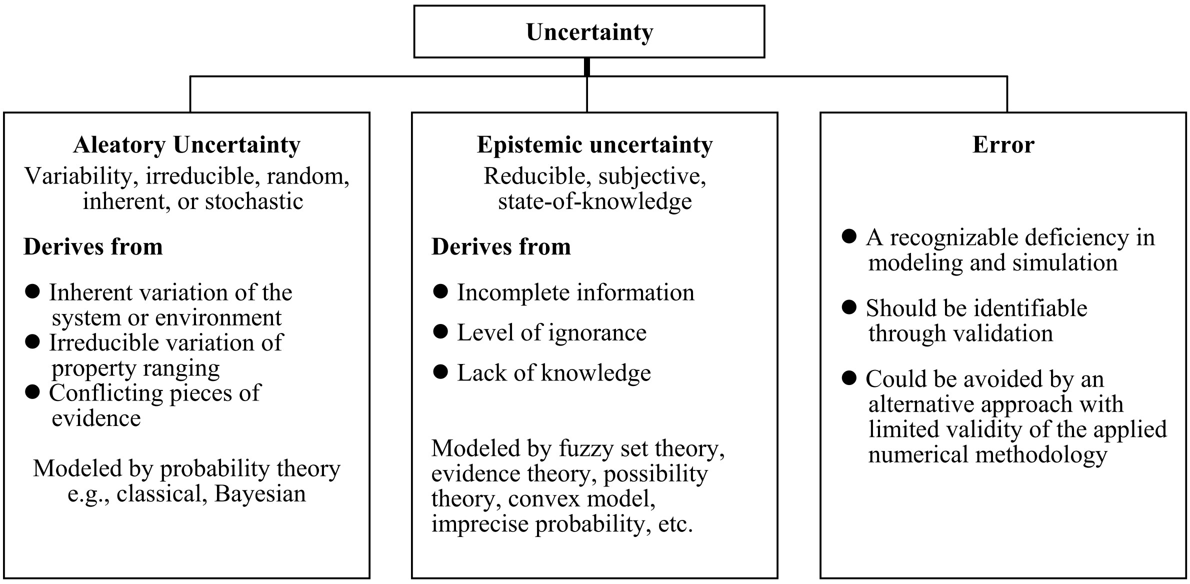

Figure 1. A classification of uncertainty.

'*Uncertainty refers to a situation in which an individual lacks sufficient information, both quantitatively and qualitatively, to accurately describe, prescribe, or predict a system, its behavior, or other characteristics deterministically and numerically*' [8].

Uncertainty can be classified into irreducible uncertainty (known as random uncertainty or aleatory uncertainty) and reducible uncertainty (known as epistemic uncertainty). The former arises from fundamental physical laws and cannot be completely removed. For instance, even under strict manufacturing conditions, the same material may show variations in their lifetime under identical environmental conditions and stresses. In contrast, reducible uncertainty arises from limited knowledge and can be mitigated through additional data collection, refined modeling methods, and enhanced measurement techniques. For example, Kong *et al* [9] employed an expert scoring method to address data scarcity and therefore improve the accuracy of failure analysis in offshore systems. Gan *et al* [10] constructed a knowledge graph to mitigate the uncertainty of maritime traffic accident causation to provide effective solution for accident analysis.

The distinct discrepancy between real system and simulation system is defined as error and uncertainty quantification models have been developed extensively to reduce the mentioned error [10, 11]. Probabilistic and possibilistic methods have been successfully applied in uncertainty quantification In engineering systems design [12, 13]. Random variables are used to describe uncertainties involved in probabilistic models. Probabilistic reliability methods are typically applied to systems with low to moderate complexity, whereas approaches such as fuzzy set theory or probability theory are suited for managing complex systems. On the other hand, according to the nature of available information, uncertainty is classified into conflict, non-specificity, and fuzziness. Conflict occurs when there exists contradictory information that provided by multiple data source. These data source provide different evidence, making it struggles to make a correct decision. Non-specificity arises when the available evidence supports multiple possible possibilities rather than a single definitive answer. Fuzziness occurs when the boundaries of a classification are not clearly defined, leading to subjective interpretations. Another framework categorizes uncertainty into: (i) metric uncertainty relating to variability and measurement uncertainty in observed data; (ii) structural uncertainty resulting from system complexity; (iii) temporal uncertainty involving uncertainty about past and future states, and (iv) translational uncertainty stemming from the interpretation of uncertain results [14]. Besides, a two-state classification framework defines uncertainty as ignorance and variability [15]. Variability represents differences among individuals, as well as spatial and temporal variations. It is inherent to a system and cannot be reduced. Ignorance stems from a lack of knowledge and can be reduced through improved measurements. An extension of this model is the three-state classification, which includes variability, uncertainty, and error [16]. Notably, while 'uncertainty' is often used interchangeably with random uncertainty, there is a clear distinction between random uncertainty, epistemic uncertainty, and error [6], as shown in figure 1.

Uncertainty mainly arises from insufficient and complex information, contradictory evidence, ambiguity, measurement errors and subjective beliefs [17, 18]. Specifically, including (i) external parameters of systems (temperature, radiation, etc); (ii) internal parameters of systems (material properties, etc); (iii) physical system modeling (conceptual or mathematical method); (iv) observational uncertainty; (v) uncertainty in solving mathematical models, including numerical or algorithmic errors; (vi) representation of digital solutions, (vii) measurement data, and; (viii) human errors.

A wide range of theories have been proposed to model uncertainties. Probability theory has been an effective tool for modeling stochastic uncertainty when sufficient data are

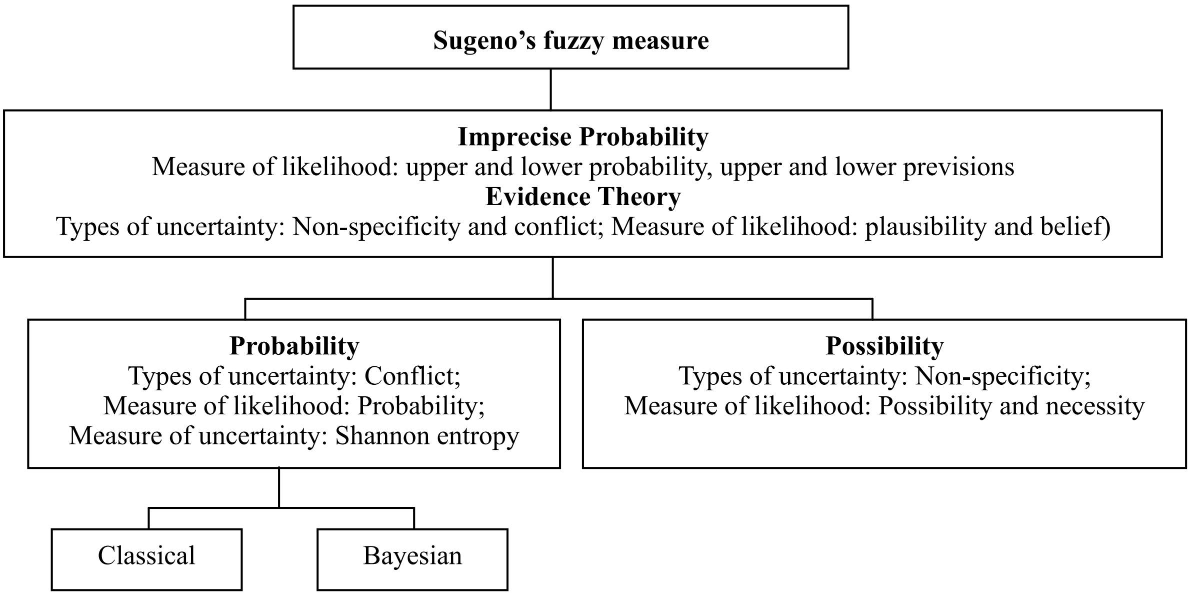

Figure 2. Families of uncertainty theories.

Table 1. Uncertainty measures [5].

Type	Name	Formula	Notation
Classical	Hartley information	$I(N) = \log_2 N$	N: cardinality of a crisp set; p_i: occurrence probability of the ith ($i = 1, 2, \ldots, n$) event; $U(\Pi)$: U-uncertainty of a ranking Π; A: fuzzy set; X: total number of items; C: fuzzy complement; $\mu_A(x)$: membership function of the fuzzy set A for element x; m: basic assignment; $\mathbb{F}$: set of focal elements; $Pl(A)$: plausibility (Pl) of subset A; Bel(A): belief (Bel) of subset A;
	Shannon entropy	$H(\mathrm{P}) = -\sum_{i=1}^{n} p_i \log_2 p_i$	
General	U-uncertainty	$U(\Pi) = \sum_{i=1}^{n} (\pi_i - \pi_{i+1}) \log_2 i$	
Vagueness	Measure of fuzziness	$f_C(A) = \lvert X \rvert - \sum_{x \in X} \lvert \mu_A(x) - C(\mu_A(x)) \rvert$	
Ambiguity	Measure of nonspecificity	$V(m) = \sum_{A \in \mathbb{F}} m(A) \log_2 \lvert A \rvert$	
	Measure of dissonance	$E(m) = -\sum_{A \in \mathbb{F}} m(A) \log_2 Pl(A)$	
	Measure of confusion	$C(m) = -\sum_{A \in \mathbb{F}} m(A) \log_2 \mathrm{Bel}(A)$	

available. However, its reliance on additivity axioms makes it less suitable for addressing epistemic uncertainty and reducible error, particularly in scenarios with sparse data [19, 20]. Numerical approaches, including imprecise probability and evidence, have been applied to address the above limitations [21, 22], see figure 2. These approaches offer significant flexibility to deal with the model-interdependency and uncertainty [23].

Investigating specific measures for uncertainties is one of the key aspects for uncertainty-related research [24]. These measures correspond to different types of uncertainty, see table 1. Neglecting uncertainty in design solutions can render systems sensitive to input variations, resulting in performance degradation or violation of critical design constraints [25–27]. Consequently, practical design under uncertainty has become increasingly prevalent in engineering. To enable design optimization, uncertainty properties and formalisms must be quantified. Real-world design or decision problems including uncertainty can be modeled by various uncertainty frameworks. For example, probability theory represents decision situations by conflicting and exclusive belief degrees, whereas opportunity theory describes them with nonspecific [28]. Each uncertainty management method emphasizes a distinct paradigm: (i) robust design aims to enhance product quality by reducing the influence of input variations; and (ii) reliability-based design aims to ensure the feasibility of a design under specified reliability levels [29–32]. However, traditional probabilistic analyses, which often overlook epistemic uncertainty, are criticized. Non-deterministic approaches provide powerful tools for uncertainty modeling, they also pose challenges for designers in selecting the most appropriate method.

This work aims to provide a comprehensive overview of reliability and design optimization methods under uncertainty. The rest of this paper is organized as follows. Section 2 presents the basic interpretations, advantages, and characteristics of the possibility theory and the evidence theory. Section 3 summarizes the applications of possibility theory and evidence theory in engineering design. Section 4 describes trends of these theories in engineering design, followed by conclusions in section 5.

2. Theoretical foundations of possibility theory and evidence theory

2.1. Possibility theory

Possibility theory, proposed by Zadeh in 1978, is one of the three key components of fuzzy theory, alongside fuzzy set theory and fuzzy logic [32]. In the case of ambiguous information, possibility theory can be applied to model the potential ambiguity and uncertainty from information [33]. Early contributions preferred to transform the natural language into probability distribution based on fuzzy relationship. Mathematical tools to model the fuzzy language and approximate reasoning using possibility theory are proposed aims to extend and formalize a framework for expressing knowledge through fuzzy propositions by natural language [34].

Notably, the possibility theory in fuzzy set shares the similar mathematical expectation with probability theory, which provides a subjective assessment based on the available information. Probability theory is well-suited for modeling random uncertainty by using precise probability distributions. In contrast, Possibility theory is more appropriate for handling epistemic uncertainty, where only partial or vague information is available. Possibility theory provides a more flexible and efficient framework for modeling such uncertainties [33]. For instance, Tang *et al* [27] integrated the possibility theory into design optimization framework to develop a proposed a possibility-based solution framework. Cai *et al* [35] developed a possibility generalized labeled multi-Bernoulli filter for multitarget tracking under epistemic uncertainty, in which the possibility theory was used to deal with the ignorance and partial knowledge about the system. Possibility shift scalars were integrated to guide constraint satisfaction and objective function improvements. Notable examples include the possibilistic interpretation, the modal logic interpretation [36], the evidence theory interpretation examined [37], and the fuzzy set interpretations [32].

2.1.1. Definition of possibility theory.

Possibility is a subjective measure that reflects the degree to which an individual believes that an event is likely to occur, or conversely, how the available evidence suggests that an event will occur [38]. Fuzzy theory extends classical set theory by allowing partial membership in a set rather than a strict binary classification. Probability shares the similar definition with possibility, the possibility is favored over probability in the context of decision-making involving unrepeated uncertainty [1]. The term 'probability' first appeared in 1975 research on probabilistic automata [39]. Another interpretation, rooted in evidence theory, describes probability as the confidence limit when evidence overlaps. Furthermore, possibility is considered an upper bound on probability. Another definition of probability states that the probability of an event is the minimum value within the interval. Although probability theory, possibility theory, and fuzzy theory all address uncertainty, they show differences:

(i) Both probability and possibility measure likelihood, but probability is based on additive measures, whereas possibility follows maxitive measures. Therefore, probability is more suitable for random uncertainty quantification and possibility is used in epistemic uncertainty where knowledge is incomplete or imprecise.
(ii) Possibility theory and fuzzy theory both handle imprecision, but possibility theory quantifies the plausibility of an event, while fuzzy theory describes gradual transitions using membership functions. Possibility distributions are derived from fuzzy membership functions, linking these two concepts.
(iii) Probability theory assumes crisp event definitions, while fuzzy theory allows gradual uncertainty representation. In cases where both randomness and vagueness coexist, a hybrid fuzzy-probability approach can be applied, where fuzzy variables define uncertain probability distributions.

2.1.2. Standard fuzzy-set interpretation of possibility theory.

Possibility distribution [33]: U denotes the discourse space. X refers to an input. F represents a fuzzy set. Notably, the value of all variables from U. $\mu_F(u)$ represents the affiliation function that defines the degree of compatibility between any u and F. If F serves as an elastic constraint on the possible values that can be assigned to X, then F is a fuzzy constraint on X (or connected to X), called $R(X)$. A fuzzy proposition 'X is F' is defined as:

$$R(X) = F. \tag{1}$$

According to the possibility assumption, no additional information can be used for the F beyond X. Consequently, the possibility distribution Π_X can be defined as:

$$\Pi_X = R(X). \tag{2}$$

The degree of possibility for X belongs to u is equal to the degree of affiliation in the case of X belongs to u. Specifically:

$$\forall u \in U,\ \pi_X(u) = \mu_F(u) \tag{3}$$

where, $\pi_X(u)$ represents the possibility measure of X taking the value of u. $\mu_F(u)$ describes the degree of membership of

u in fuzzy set F. Notably, the membership function $\mu_F(u)$ in possibility theory assigns a value between 0 and 1 to indicate the degree of membership of a value u belongs to a fuzzy set F.

The membership function $\mu_F(u)$ in possibility theory assigns a value between 0 and 1 to indicate the degree to which a value u belongs to a fuzzy set F. Unlike a probability density function, the membership function does not require normalization and does not imply additivity. Instead, it describes the plausibility of an event under vagueness and incomplete information. Possibility distributions are particularly useful in applications involving fuzzy decision-making, linguistic uncertainty, and expert-driven reasoning.

Possibility measure [32]: Π_X denotes the possibility distribution associated with a variable X that takes value in U, then the possibility measure $\pi(A)$ is defined as interval value of [0, 1]. When A is a non-fuzzy (crisp) subset of U, there is:

$$P_{\text{oss}}\{X \in A\} \equiv \pi(A) \equiv \sup_{u\in A} \pi_X(u). \tag{4}$$

When A is a fuzzy subset of U, it can be rewritten as:

$$P_{\text{oss}}\{X \text{ is } A\} \equiv \pi(A) \equiv \sup_{u\in U}(\mu_A(u) \wedge \pi_X(u)) \tag{5}$$

Properties of possibility measure [32]: According to equations (4) and (5), the properties can be defined as:

$$\pi(A \cup B) = \pi(A) \vee \pi(B). \tag{6}$$

Obviously, $\max(\pi(A), \pi(\bar{A})) = 1$, and the properties can be written as:

$$\pi(A \cap B) \leqslant \pi(A) \wedge \pi(B). \tag{7}$$

The early advancements in this field link fuzzy propositions and probability measures, which has been successfully applied as the standard interpretation of the possibilities in fuzzy set theory. Through the concept of probability distributions, the expression of propositions is transformed from natural language into a formal representation. This process involves controlling probability distributions based on rules for combining fuzzy sets, particularly fuzzy constraints [40]. It can be concluded that the initial interpretation of fuzzy sets was largely influenced by the mathematical similarities. In fuzzy set theory, α-cuts correspond to the fundamental structure of nested sets, which in probability theory are composed of focal elements [33].

2.1.3. Extensions on the standard interpretation. A pair of fuzzy measures are applied to describe the uncertainty of a proposition in possibility theory [41], namely the possibility measure $P_{\text{oss}}(A)$ and the necessity measure $N_{\text{ec}}(A)$. The subscript 'oss' represents the abbreviation of possibility and 'ec' represents the abbreviation of necessity. Possibility theory allows these measures to be defined as $(P_{\text{oss}}(A), P_{\text{oss}}(\bar{A}))$, or alternatively represented by the pair $(P_{\text{oss}}(A), N_{\text{ec}}(A))$. The necessity measure can be described through two dual formulations

$$N_{\text{ec}}(A) = 1 - P_{\text{oss}}(\bar{A}). \tag{8}$$

Similarly, equation (9) is used to limit the possibility measure:

$$P_{\text{oss}}(A \cup B) = \max(P_{\text{oss}}(A), P_{\text{oss}}(B)). \tag{9}$$

According to the properties of fuzzy measures, their features are expressed as follows [41]:

$$N_{\text{ec}}(A \cap B) = \min(N_{\text{ec}}(A), N_{\text{ec}}(B)) \tag{10}$$

$$\max(P_{\text{oss}}(A), P_{\text{oss}}(\bar{A})) = 1 \tag{11}$$

$$\min(N_{\text{ec}}(A), N_{\text{ec}}(\bar{A})) = 0 \tag{12}$$

$$P_{\text{oss}}(A) \geqslant N_{\text{ec}}(A). \tag{13}$$

2.1.4. Revised fuzzy-set interpretation of possibility theory. Klir [33] proposed a revised interpretation of the possibility theory of fuzzy sets to address the standard interpretation in the context of subnormal fuzzy sets, that is, the height of a fuzzy set F, $h_F = \sup_{u\in U} \mu_F(u) \neq 1$. Consider F is subnormal, the standard interpretation will be unstable. Such a point is supported by Liang [42]. Liang [42] also highlighted that one of the main properties of the possibility theory (equation (13)), fails to hold in the case of $h_F < 1$. To address this limitation, a security measure was proposed to replace the necessity function [43]:

$$C_{\text{ert}}(A) = \min(P_{\text{oss}}(A), N_{\text{ec}}(A)). \tag{14}$$

Huang *et al* [44] argued that this substitution breaks the basic demands of the possibility theory. As an alternative, they retained the necessity function while replacing equation (8) with the generalized equation:

$$N_{\text{ec}}(A) = h_F - P_{\text{oss}}(\bar{A}). \tag{15}$$

Considering F is normal, equation (15) will be transformed into equation (8). When all subsets belong to U, such transformation is recognized as a reasonable formulation. When $A = U$ or $\bigcup_{i\in I} A_i = U$, the possibility theory can be modified by substituting equations (4)–(9), respectively [45], where I represents a set of arbitrary indices. It is evident that the works primarily represent interpretations of other formal systems based on specific modifications of possibility theory rather than foundational interpretations of possibility theory for fuzzy sets. Returning to the interpretation for fuzzy sets, the properties of probability distribution function $\pi(u)$ is shown as follows

$$\sup_{u\in U} \pi(u) = 1. \tag{16}$$

The explanation of modified fuzzy sets of possibility theory are presented in table 2. The objectives of the explanation of possibility theory include: (i) to ensure consistency between all fuzzy sets, whether normal or not, so that no property of

Table 2. The explanation of modified fuzzy-set of possibility theory [33].

Standard interpretation	Revised interpretation
$\pi_X(u) = \mu_F(u)$	$\pi_X(u) = \mu_F(u) + c_F = \mu_F(u) + 1 - h_F$
$m_F(u) = \inf_{u \in U} \mu_F(u)$	$m_F(u) = \inf_{u \in U} \mu_F(u) + 1 - h_F$
$\sum_{A \in P(x)} m_F(A) = h_F < 1$	$\sum_{A \in P(x)} m_F(A) = 1$
$P_{oss}(A) \equiv \sup_{u \in U} \min(\mu_F(u), \mu_A(u))$	$P_{oss}(A) \equiv \sup_{u \in U} \min(\mu_F(u) + 1 - h_F, \mu_A(u))$

where, $\pi_X(u)$ is the possibility assigned to the element u in the fuzzy set X; $\mu_F(\cdot)$ refers to the membership function; h_F denotes the height of the fuzzy set F; c_F is a constant value given a fuzzy set F; m denotes the fundamental probability assignment function of evidence theory.

possibility theory is violated, (ii) to quantify the evidence from each fuzzy proposition m_F and incorporate this knowledge into the evidence theory framework, and (iii) to retain its intuitive meaning.

2.2. Evidence theory

Evidence theory, also referred to as DempsterShafer theory (DST), is founded on the concepts of upper and lower bounds of probabilities, which do not conform to the traditional additivity property. Both possibility and evidence theories generalize classical probability by allowing uncertainty beyond strict probability distributions. Notably, possibility theory can be derived from evidence theory by assigning mass functions in a way that maximizes plausibility. Every possibility distribution can be interpreted as a plausibility function in evidence theory, meaning possibility theory is a simplified, upper-bound representation of evidence theory. However, evidence theory is better for conflict resolution, while possibility theory is more efficient for handling vagueness and ranking uncertain outcomes. Evidence theory provides a comprehensive framework for representing belief functions, initially marking its importance in fields such as artificial intelligence [44]. Viewed as a mathematical model, evidence theory integrates empirical data to construct a consistent and logical representation of real-world uncertainties [46]. By accumulating evidence and explicitly addressing both uncertainty and ignorance, it offers a systematic approach for hypothesis development [47]. When the element of ignorance is removed entirely, the Dempster–Shafer framework reduces to the classical Bayesian model. This demonstrates how DST extends conventional probability theory by incorporating rules to combine multiple independent sources of evidence [48, 49].

There are multiple interpretations of the DST including probabilistic and non-probabilistic approaches [50–58]. In addition, several closely related advancements have been made in recent years [59–61]. Shafer's formulation is the most popular presentation of evidence theory, which is demonstrated in the book 'The Mathematical Theory of Evidence' [1]. It serves as the cornerstone of this field. Evidence theory begins with the concept of a frame of discernment (FD), which is a collection of mutually exclusive and exhaustive propositions, comparable to a finite sample space in probability theory. To measure uncertainty, it utilizes belief functions (Bel) and plausibility functions (Pl) as key tools. Within the DST framework, basic probability assignments (BPAs) are introduced to model evidence. Specific rules are then applied to aggregate and combine multiple evidence sources for reasoning under uncertainty.

2.2.1. Basic concepts of evidence theory.

Let U be a finite, non-empty universal set representing a collection of elements with shared characteristics, commonly referred to as the FD in DST. $\wp(U)$ is the power set of U. A is the subset of U. The available evidence is then represented by equations (17) and (18):

$$m: \wp(U) \to [0,1] \tag{17}$$

$$\sum_{A \in \wp(U)} m(A) = 1 \tag{18}$$

where m referred to as the BPA or the mass function, is used to quantify the degree of evidence that supports the assertion that an element of U belongs to a subset $A \in \wp(U)$. $m(A)$ reflects the degree of belief in the validity of such an assertion [4].

In a BPA, any subset $A \in \wp(U)$ where $m(A) \neq 0$ is defined as a focal element, and $m(A)$ is known as the weight of A. The set $\mathbb{F}$, which includes all focal elements of m represents the subsets within the FD that are supported by the available evidence supports. The pair $(\mathbb{F}, m)$ is termed the body of evidence or the belief structure.

2.2.2. Belief and plausibility measures.

Confidence and reliability measures are the fundamental to the mathematical theory of testing. According to an assignment of the basis probability m, these measures are defined as follows:

$$\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B) \tag{19}$$

$$\mathrm{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{20}$$

where, Bel and Pl refer to belief and plausibility function. Bel(A) and Pl(A) serve as the lower and upper bounds of event A, respectively. $m(B)$ denotes the BPA associated with subset

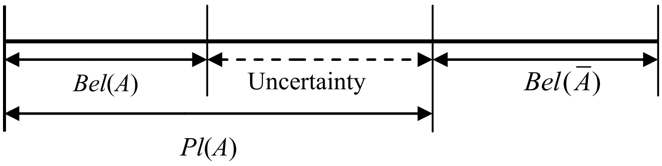

Figure 3. Relation of belief measure and plausibility measure.

B. These two measures are dual to each other, meaning that each can be uniquely determined from the other. This relationship is expressed mathematically by the following equation:

$$Pl(A) = 1 - \text{Bel}(\bar{A}). \tag{21}$$

Here, $\bar{A}$ denotes the complement of A in the classical sense. This concept emphasizes that the sum of all BPAs must be equal to one, as demonstrated in equation (15). Furthermore, a reverse approach can also be applied where necessary [4]

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \text{Bel}(B). \tag{22}$$

The important inequality properties of the Belief and Plausibility are written as [24]:

$$\text{Bel}(A_1 \cup A_2) \geqslant \text{Bel}(A_1) + \text{Bel}(A_2) - \text{Bel}(A_1 \cap A_2) \tag{23}$$

$$\text{Pl}(A_1 \cap A_2) \leqslant \text{Pl}(A_1) + \text{Pl}(A_2) - \text{Pl}(A_1 \cup A_2) \tag{24}$$

and there is:

$$\text{Bel}(A) + \text{Bel}(\bar{A}) \leqslant 1 \tag{25}$$

$$\text{Pl}(A) + \text{Pl}(\bar{A}) \geqslant 1 \tag{26}$$

where, $A_I \cap A_2$ meaning the event where either event $A_I$or A_2 both occur. $A_I \cup A_2$ meaning the event where both A_I and A_2 occur simultaneously. Bel(A) is total belief or evidence that input falls in A. In contrast, Pl(A) accounts for the additional belief or evidence resulting from the overlap of A with other focal elements [62]. The relationship between these two dual measures is expressed as follows:

$$\text{Pl}(A) \geqslant \text{Bel}(A). \tag{27}$$

Equations (18) and (27) are visualized by figure 3 [63].

2.2.3. Bodies of evidence. Evidence theory is used to quantify the uncertainty from missing and ambiguous information [51]. Within an abstract framework, it defines two sets: H representing hypotheses, and A representing arguments [64]. The triple (H, A, s) is known as the body of the argument where s denotes a support distribution. This algebraic structure underpins the foundation of evidence theory. Within the argument set (H, A, s), the elements of A do not necessarily have equal probability, as certain arguments may carry more weights than others. Consequently, the credibility of different hypotheses depends on the strength of their supporting arguments. While such arguments can be assessed using probabilities, the Boolean algebra A may become excessively large for assigning probabilities to every element. To resolve this, a sub-σ-algebra A_0 is introduced within A, where a probability $P(\alpha)$ is associated with each element α in A_0. As a result, $P(\alpha)$ functions as a probability measure on A_0. The structure represented by the quintuple (H, A, A_0, P, s) is collectively referred to as the body of evidence.

Suppose empirical evidence can be quantified and is represented as a set of value $\{m(A_1), m(A_2), \cdots, m(U)\}$, representing the *amounts of evidence* supporting the subsets $\{A_1, A_2, \cdots\}$ of a FD in U, respectively. Then each set of numbers $\{m_1, m_2, \cdots, m_U\}$ is called a *body of evidence*, where number m is usually normalized to satisfy,

$$\sum_i m(A_i) + m(U) = 1, \text{ where } m(U) > 0 \tag{28}$$

where, $\{m_1, m_2, \cdots\}$ represent the amount of empirical evidence supporting alternative possibilities $\{A_1, A_2, \cdots\}$. $m(U)$ is the belief assigned to the entire frame U. Its meaning will be clear with an example of belief formation in the biotech industry and the ensuing discussion to be presented in [46].

2.2.4. Combinations of evidence. There are four distinct categories of evidence derived from multiple sources that affect the process of combining information [65]: consonant evidence, consistent evidence, arbitrary evidence, and separate evidence. Evidence theory provides reliable framework for managing these various types by integrating probabilistic reasoning with classical set theory concepts. When evidence is derived from multiple sources, these sets of evidence can be combined using well-established combination rules [6]. Specifically, these rules serve as specialized methods for aggregating data from different sources. The most used combination rules are summarized in literatures [54, 65, 66].

Dempster's combination rule is a widely used method and serves as the basis for the Dempster–Shafer fusion approach. The joint evidence, denoted as m_{12}, is calculated by:

$$(m_1 \oplus m_2)(A) = m_{12}(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B) m_2(C)}, A \neq \emptyset \tag{29}$$

where B and C represent the propositions from each source (m_1 and m_2).

Dempster's rule may become unsuitable when a significant discrepancy exists in the available evidence, because of the normalization factor of the denominator. However, it is effective in cases where exists a sufficient consistency degree or agreement degree among views from different samples [6]. Alternative methods such as the mixture or average rule [65] and the extended combination rule [54] can be applied in scenarios where evidence from various sources lacks consistency or shows minimal agreement. Recent research on combination rules can be broadly categorized into two types: (i) methods for modifying Dempster's rule and (ii) approaches for correcting the original sources of evidence before applying the rule [67].

The advantages of evidence theory are summarized as follows [68]:

(1) The reducible uncertainty and irreducible uncertainty are solved using evidence theory, even with limited information. For example, it allows the expression of ignorance about the likelihood of an event by assigning high plausibility and low belief, offering a framework than the Bayesian approach.
(2) A measurement tool is established by the evidence theory which quantifies the uncertainty in risk assessment. For instance, the larger the gap between belief and plausibility, the greater the uncertainty in estimating event risk.
(3) Evidence theory applies more than just to single or simple classes; it also applies to joint classes.

Despite its advantages, Dempster–Shafer evidence theory faces some criticisms, primarily based on [69]:

(1) Dempster's combination rule yields unreasonable outcomes when inconsistent evidence is combined.
(2) It struggles to reconcile the perspectives of individuals with overlapping experiences, particularly in applications related to security analysis [70].
(3) Probability theory introduces inconsistencies in certainty estimates under specific conditions.

To realize the applicable of Dempster–Shafer and possibility theories to risk and reliability analysis, several challenges should be addressed [69]: (i) combining homogeneous bodies of evidence effectively; (ii) managing inconsistent information elements from multiple sources; (iii) ensuring that accepted judgments are reliable during the analysis; and (iv) accounting for the dependence of inaccuracies on the amount of available information.

3. Applications of possibility theory and evidence theory in reliability and design optimization

Probability theory faces limitations in addressing epistemic uncertainty and handling imprecise data or incomplete knowledge. In contrast, possibility theory and evidence theory provide robust alternatives, offering mathematical frameworks to represent uncertainty in complex systems, particularly when information about random variations is insufficient. Both theories have gained increasing attention across various scientific fields such as reliability analysis and uncertainty management. Similarly, evidence theory has evolved along a comparable trajectory.

3.1. General topics of applications

Possibility theory has found extensive applications in multiple domains, especially when it comes to measuring epistemic uncertainty in situations where expert opinions do not conflict [71]. For example, Rebane *et al* [72] designed a possibility theory framework to deal with uncertain spatial relationships, optimizing designs, and improving decision-making under uncertain conditions. Kühne and Edler [73] applied possibility distributions to handle measurement errors, imprecise experimental data, and process variability, ensuring more reliable chemical analysis and industrial applications. Significant advancements have been made in assessing reliability under uncertainty, as discussed in [74–78]. Notable developments include, but are not limited to: the data fusion rule for reliability modeling [79], possibility-based design optimization (PBDO) [80–85], and fuzzy reliability theory [58, 74, 86–88]. Applications of these approaches have extended beyond reliability engineering to fields such as civil and structural engineering [75], computational mechanics, military, energy, forestry [7], aerospace, and automotive engineering [89].

As a broader tool for analyzing uncertainty, possibility theory has also been applied across diverse domains. These include artificial intelligence (especially in the development of expert systems) [90], object detection and approximate reasoning [91–94], design optimization [85] (which includes multidisciplinary optimization [6]), uncertainty quantification [71, 90], risk assessment and reliability, as well as remote sensing classification [95], pattern recognition and image analysis and decision making [96, 97], data fusion [98], and fault diagnosis [17]. Despite its versatility, the adoption of evidence theory has been limited due to its reliance on epistemological assumptions that differ from classical and Bayesian probability theories [46].

Evidence theory develops numerical methods that integrate the moment concept and finite element methods to compute linear elastic static and dynamic responses of structures under epistemic uncertainty [99]. In real-world conditions, parameter correlations influence the reliability analysis. To address this, new evidence theory-based models were developed, incorporating copula functions and ellipsoid models to account for parameter correlations in structural reliability analysis [100]. Similar to the most probable point (MPP) [101] in probabilistic reliability analysis, the most probable focal element was introduced for reliability analysis using evidence theory [102]. Based on this idea, the first-order reliability method (FORM) and the second-order reliability method were developed to enhance the efficiency of reliability analysis under the framework of evidence theory [103].

A growing trend suggested to integrate multiple frameworks to address complex and dynamic environments, including: combining probabilistic and probabilistic approaches [104], probabilistic and probabilistic approaches [95], integrating probabilistic design optimization with robust design [82], probabilistic optimization with robust design [105], and integrating random and epistemic uncertainty into various design optimization processes [85]. Recent developments in uncertainty and reliability analysis using possibility and evidence theories can be categorized into: (i) theoretical advancements focusing on the reliability foundations of probability theory, such as imprecise reliability [106, 107] and fuzzy reliability [69], reliability [86], and (ii) computational or algorithmic advancements, including data fusion techniques in reliability assessment [108] and optimal design methods [109–112].

3.2. Development of reliability theory

Reliability refers to a system's capability to perform its intended function effectively and without failure. It is formally determined as the ability of a component or system to meet functional requirements under specified conditions for a defined period. Reliability is often quantified through measures such as probability, success rates, or feasibility, inherently linking it to failure mechanisms and quality degradation. Reliability can be categorized from multiple dimensions: (i) key areas: this includes topics such as reliability engineering, management, product warranty, and maintenance strategies; (ii) research focus: it encompasses activities like evaluation, forecasting, modeling, data analysis, implementation, and validation processes (e.g. testing); (iii) phases: design, manufacturing, and operational reliability, and (iv) objects: hardware, software, human, and structural reliability. Currently, prominent reliability design methodologies include fault tree analysis (FTA), failure mode and effects analysis (FMEA), and reliability optimization. Liu *et al* [113] combined FMEA and fuzzy FAT approach to evaluate the reliability of a subsea control system. It provided a comprehensive understanding of the subsea control system's reliability, enabling the development of effective risk mitigation strategies. Lin *et al* [114] presented an enhanced FTA method to evaluate the reliability and risk factors of a subsea pipeline system. This study enhanced risk identification, reduced subjectivity in node discovery, and simplified the mathematical calculations required for quantitative reliability assessments. Traditional engineering reliability analysis primarily employs probabilistic approaches, where system state variables are modeled using precise probability distributions. This approach provides accurate failure estimates when sufficient input data is available. However, in complex engineering decision-making scenarios, particularly in the early design stages and throughout the product's manufacturing and usage phases, numerous indeterminable factors arise due to limited knowledge. Consequently, as a result, decisions based solely on precise probabilistic analyses often fail to adequately represent real-world conditions. To address these limitations, recent fundamental and theoretical advancements in safety and reliability analysis have been developed to manage situations.

3.2.1. Imprecise reliability. **Origin and motivation.** Engineering design is a decision-making process where engineers often operate with insufficient information, leading to uncertainty. To represent this uncertainty clearly and quantitatively, its imprecise nature must be appropriately accounted for. Ambiguity may arise from the intrinsic indeterminacy of available evidence or the incomplete representation of evidence and beliefs [109]. Imprecise probabilities provide an effective approach for representing uncertainty in reliability and risk analysis by using probability intervals to capture uncertain knowledge. The primary motivation for employing imprecise probabilities lies in the fact that the confidence a decision-maker places in a probability estimate heavily depends on the quality and completeness of the underlying evidence. Therefore, it is crucial to explicitly express the uncertainty associated with probabilities to accurately convey the corresponding level of confidence [109]. Various frameworks have been developed to model inaccurate probabilities. A unified fuzzy probability theory can be found in [86, 102].

The imprecise probability theory. The imprecise probability theory refers to a collection of mathematical frameworks that include upper and lower bounds for probabilities, as well as predictions or expectations. It also incorporates possibility and necessity measures, confidence metrics, reliability functions, and other qualitative approaches [69]. Theoretical foundations of imprecise probability [106] emphasize a behavioral perspective and are based on three core principles: loss aversion, robustness, and natural extension. At the core of the behavioral interpretation lies the concept of a gamble, which is defined as a real-valued, bounded function over a specified domain. A gamble represents a reward dependent on the uncertain state of the system and each element of which belongs to the domain. Consistent theories of imprecise probability also rely on lower predictions (or expectations) and upper predictions probabilistic models. Reliability analysis focus on binary gambles where the reward is either zero or one. In this context, the bottom and top predictions are referred to as lower probabilities and upper probabilities, reflecting their probabilistic nature. When integrating information from various sources, it is essential to identify and differentiate between aligned (consistent) and conflicting (inconsistent) judgments or models. For consistent judgments, the tie rule is used to merge the lower and upper predictions. For inconsistent judgments, the unanimity rule, derived from the concepts of desirability and preference, is applied in literature [106].

Advances of imprecise probability theories. Fuzzy probability theories have proven valid and effective for reliability and risk analysis, despite certain challenges, such as difficulties in combining evidence or managing divergent judgments during derivation [69]. Significant advancements in this field include: a fuzzy probability theory consistent with behavioral interpretations based on decision theory and utility theory [106], and methods for constructing fuzzy limit state functions from sparse data [115]. The advantages, disadvantages, and applications of fuzzy probability in reliability can be found in [116] and [88], which introduces nonparametric predictive inference as a consistent framework for reliability when data are sparse. Additional research focuses on replacement and maintenance decisions [117], design decisions [44], imprecise probability theory and robust statistics [11], imprecise probability for sensitivity analysis [94], uncertainty

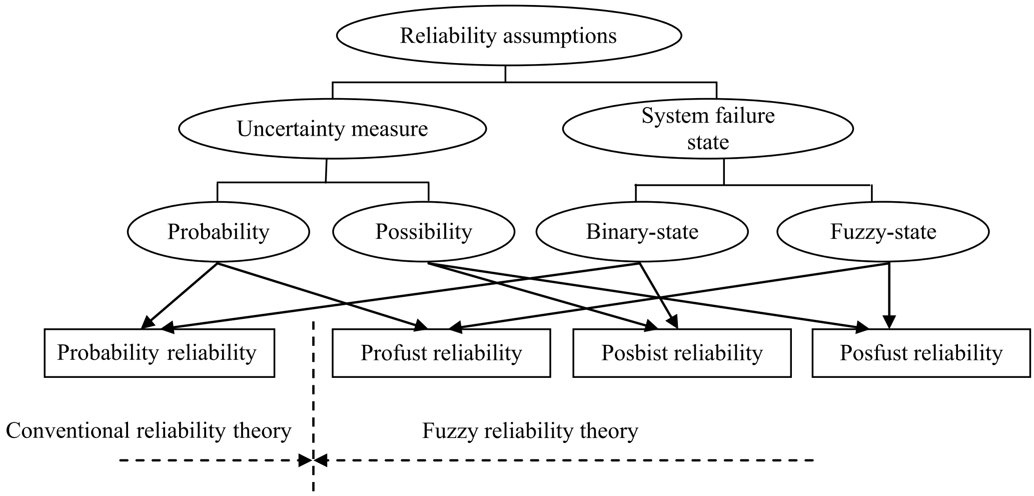

Figure 4. Reliability theories based on various fundamental assumptions.

propagation [118], polynomial expansion based on test theory, and imprecise probabilistic non-intrusive integration [119]. Bayesian optimization [120–122], multi-fidelity Kriging surrogate model [123]. These advancements have shown that, in certain design problems, explicitly representing uncertainty imprecision through imprecise probabilities can be particularly beneficial. While imprecise probability methods have opened new research opportunities, they also pose challenges, particularly in terms of computational complexity, highlighting the importance of diversity of research horizons for future developments in this field.

3.2.2. Fuzzy reliability. Fuzzy theory serves as a powerful tool for uncertainty analysis. The origins of fuzzy reliability theory can be traced to the need to handle reliability considerations in degradable computer systems in an elegant way, where the health states of system cannot be simply divided into failed or functional. Performance degradation, influenced by complex and uncertain factors, often leads to failures that do not occur randomly. Moreover, the determination of safety criteria frequently relies on technical judgment, introducing additional sources of uncertainty. Utilizing advanced uncertainty quantification tools, the concept of fuzzy reliability was introduced, grounded in fuzzy set theory [86]. Kabir [121] combined expert judgments and fuzzy set theory to evaluate the reliability of fuel distribution system. Reliability grounded in probability theory is viewed as a subset of fuzzy reliability frameworks and is also consistent with non-probabilistic and imprecise probability approaches. The former stems from its theoretical basis in fuzzy set theory, whereas the latter relates to the non-statistical characteristics of the information being analyzed.

Several types of fuzzy reliability approaches have been introduced, including profust reliability, posbist reliability, and posfust reliability theories, to address the limitations of traditional binary-state assumptions and probabilistic models. These approaches rely on diverse reliability criteria [43]. Among these, possibility theory has found broader applications, especially in fields like artificial intelligence and data fusion, surpassing probability theory and evidence theory in certain contexts. Existing studies mainly focus on theoretical construction and modification [124–127], or practical applications in engineering [128]. Several respective advancements include posbist reliability theory with a series, parallel, or k-out-of-n configurations [86], cold and warm redundant systems [57], repairable systems [129], posbist reliability behavior [88], FTA model aligned with posbist reliability theory [130], statistical fuzzy reliability evaluation, fuzzy truncated probabilistic distribution method [131], two-parameter Pareto lifetime distribution with vague shape and scale parameters [132], and dual hesitant fuzzy sets in conjunction with the inverse Weibull distribution [133]. Despite significant advancements, current fuzzy reliability models face practical limitations. One major issue is the inability of existing theories to fully capture a wide range of judgments regarding reliability. Even evidence-theory-based approaches to reliability analysis under incomplete information encounter similar challenges. In many real-world scenarios, a suitable and accurate possibility distribution consistent with statistical data may not exist, underscoring the need for more robust and interpretable frameworks. The difficulty in constructing and interpreting possibility distributions remains a critical barrier for practitioners. Figure 4 illustrates reliability theories based on various fundamental assumptions.

3.3. Computational developments related to design optimization under uncertainty

3.3.1. Design optimization. Design optimization has become a critical section in the development of high-tech products, evolving as a natural extension of advancements in computer-aided engineering [134]. The increasing complexity of modern systems, dynamic business requirements, and the integration of diverse technologies highlight the necessity of adopting a system-level approach to design optimization, moving beyond the traditional focus on individual components. Incorporating uncertainty into design optimization is essential as various uncertainties arise during both the design and operational phases.

A decision-making paradigm is applied for the design optimization under uncertainty, which can be expressed as follows:

$$\begin{aligned}
&\min_{\mathbf{d},\widetilde{\mathbf{d}}(\mathbf{X})} : f(\mathbf{d},\mathbf{X},\mathbf{P}) \\
&st.\ P\{G_i(\mathbf{d},\mathbf{X},\mathbf{P}) > 0\} \leqslant p_{f_i}\ i = 1,2,\ldots,np \\
&\mathbf{d}^L \leqslant \mathbf{d} \leqslant \mathbf{d}^U \\
&\widetilde{\mathbf{d}}(\mathbf{X})^L \leqslant \widetilde{\mathbf{d}}(\mathbf{X}) \leqslant \widetilde{\mathbf{d}}(\mathbf{X})^U \\
&\mathbf{d} \in \mathbb{R}^n, \mathbf{X} \in \mathbb{R}^{nr}, \mathbf{P} \in \mathbb{R}^q
\end{aligned} \tag{30}$$

where, $f(\mathbf{d},\mathbf{X},\mathbf{P})$ is the objective function, $\mathbf{d}$ is the vector of deterministic design variables in the n-dimensional real space $\mathbb{R}^n$. $\mathbf{X}$ is the uncertain variables corresponding to $\mathbf{d}$, $\widetilde{\mathbf{d}}(\mathbf{X})$ is the vector of design variables for uncertain variables, which is dependent on the uncertainty characteristics, such as, the mean values of random variables, and the midpoints of the upper and lower bounds of interval variables are taken as design variables respectively. $\mathbf{P}$ is the vector of uncertain parameters. The $\mathbf{d}^L$ and $\mathbf{d}^U$ define the lower and upper bounds of $\mathbf{d}$, while $\widetilde{\mathbf{d}}(\mathbf{X})^L$ and $\widetilde{\mathbf{d}}(\mathbf{X})^U$ represent the lower and upper bounds of $\widetilde{\mathbf{d}}(\mathbf{X})$ respectively. $G_i(\mathbf{d},\mathbf{X},\mathbf{P})$ is the constraint function which determines whether a design is feasible (reliable) or not. In a practical engineering optimization problem, the main criteria used to measure the effectiveness are usually cost and performance. $P\{\cdot\}$ represent failure probability, p_{f_i} is the threshold failure probability. $\mathbb{R}^{nr}$ is the real number space for uncertain variables, $\mathbb{R}^q$ is the real number space for uncertain parameters.

Traditional design optimization offers a valuable tool for analysis and design but faces significant limitations in addressing inherent uncertainties. These include variations in design variables and parameters, as well as model uncertainties, such as numerical errors in analysis tools [135]. To overcome these challenges, researchers [136] have developed specialized optimization methodologies aimed at reducing the computational costs associated with design optimization problems. Notable advancements include the exploration of decomposition strategies and approximation methods, particularly in aerospace and automotive engineering applications [137]. The diversity of system architectures, resource constraints, and types of uncertainties has driven the development of a wide range of design optimization models and methods for managing uncertainty [97].

3.3.2. PSDO. In practical engineering, aleatory and epistemic uncertainties are inevitable in the design stage but less considered. Among them, aleatory uncertainty is featured by the objectivity and unavoidability. It is typically modeled using probability theory when sufficient data is available. Epistemic uncertainty is featured by subjectivity and reducibility, stemming from data scarcity. In the case where precise statistical data is unavailable because of constraints such as budget limitations, inadequate facilities, and time restrictions, probabilistic methods may not be suitable for structural analysis and related applications. Alternative design methodologies are required to manage epistemic uncertainty and model physical uncertainty under limited information [109]. The mean performance of the possibility-based methods is optimized, subject to possibilistic constraints [76]. The general PBDO can be formulated as [76]:

$$\begin{aligned}
&\min_{\mathbf{d},\widetilde{\mathbf{d}}(\mathbf{X})} : f(\mathbf{d},\mathbf{X},\mathbf{P}) \\
&st.\ \Pi\{G_i(\mathbf{d},\mathbf{X},\mathbf{P}) > 0\} \leqslant \alpha_{t_i}\ i = 1,2,\ldots,np \\
&\mathbf{d}^L \leqslant \mathbf{d} \leqslant \mathbf{d}^U \\
&\widetilde{\mathbf{d}}(\mathbf{X})^L \leqslant \widetilde{\mathbf{d}}(\mathbf{X}) \leqslant \widetilde{\mathbf{d}}(\mathbf{X})^U \\
&\mathbf{d} \in \mathbb{R}^n, \mathbf{X} \in \mathbb{R}^{nr}, \mathbf{P} \in \mathbb{R}^q
\end{aligned} \tag{31}$$

where $f(\mathbf{d},\mathbf{X},\mathbf{P})$ is the objective function, $\mathbf{d}$ is the vector of deterministic design variables, $\mathbf{X}$ is the uncertain variables corresponding to $\mathbf{d}$, represented by fuzzy-random variables, $\widetilde{\mathbf{d}}(\mathbf{X})$ is the vector of design variables for uncertain variables $G_i(\mathbf{d},\mathbf{X},\mathbf{P})$ is the constraint condition, and $\mathbf{P}$ is the vector of uncertain parameters. $\widetilde{\mathbf{d}}(\mathbf{X})$ is the vector of the uncertain design variables. The bounds $\mathbf{d}^L$ and $\mathbf{d}^U$ define the lower and upper bounds of $\mathbf{d}$, while $\widetilde{\mathbf{d}}(\mathbf{X})^L$ and $\widetilde{\mathbf{d}}(\mathbf{X})^U$ represent the lower and upper bounds of $\widetilde{\mathbf{d}}(\mathbf{X})$ respectively. α_{t_i} represents a target failure possibility. np is the number of possibility constrains. n is the number of deterministic design variables in vector $\mathbf{d}$. nr is the number of fuzzy-random variables in vector $\mathbf{X}$. $\mathbb{R}^{\mathrm{n}}$ is the real number space for determine design variables. $\mathbb{R}^{\mathrm{nr}}$ is the real number space for fuzzy-random variables, $\mathbb{R}^q$ is the real number space for uncertain parameters.

Possibility analysis, also known as fuzzy analysis, has proven to be an effective tool for addressing uncertainty in design optimization. It offers several advantages: (i) retains the inherent randomness of physical variables through their membership functions, (ii) requires less computational complexity compared to probabilistic methods, particularly in extended fuzzy operations [24], (iii) provides more conservative design solutions than probabilistic methods, especially with respect to confidence levels [33], and (iv) delivers possibility assessments at the system level, extending beyond the capabilities of traditional reliability analysis. Several methods have been developed for numerical fuzzy analysis, including: vertex method, discretization method, level-cuts (α-cuts) method, multilevel-cut method, possibility index approach, performance measure approach (PMA), MPP search, and maximal possibility search.

Among these, the vertex methods have been successfully applied in practical. However, it may result in expensive computation cost for large-scale system and inaccurate outcomes in the case of maximum or minimum output response. To address challenges posed by nonlinear problems, the level-cutting method has been employed at different design stages. The multilevel-cut method is introduced to enhance the precision of peak response analysis, particularly for nonlinear structural designs. However, this approach remains computationally demanding when applied to PBDO. The PMA, however, has demonstrated significant numerical efficiency and stability in PBDO applications [137]. Unlike probabilistic reliability analysis, where the MPP is based on FORM, fuzzy analysis computes the MPP directly and exactly. Furthermore, while reliability analysis involves operations in an n-dimensional sphere, fuzzy analysis operates in an n-dimensional hypercube, simplifying the calculations [84].

One of the main research focuses of PBDO is to improve the numerical efficiency, accuracy and stability during the optimization process. During the optimization process, PMA addresses these challenges by replacing the probabilistic constraint (equation (31)) with a performance measure assessed at a defined confidence threshold [137]. For example, PMA improves the numerical efficiency, stability and accuracy [84] and integrated design platform have been developed to address physical uncertainty [109]. Reliability analysis under incomplete information has been incorporated into structural analysis and design [74]. While PMA is more cost-effective when the reliability index is high, additional calculations may be required when the reliability index is below the required threshold [138]. Therefore, the PMA+, an extension of PMA, has been developed to enhance its performance [82]. These studies indicated that PBDO generally produces more conservative results compared to probability-based RBDO, especially when reliability estimation is based on sparse information [139]. Recent developments in PBDO demonstrate its potential to address complex design problems under uncertainty. These include the opportunity-based framework for solving optimization problems under interval uncertainty [140], the adaptive Kriging method combined with active opportunity constraints (I-AK-AC) for PBDO [27], the fuzzy safety index (FSI) assuming that the fuzzy inputs are independently and identically distributed, and the constraints based on failure probabilities are reformulated into FSI-based constraints, presenting a FSI approach for solving PBDO problems [141].

3.3.3. Evidence-based design optimization (EBDO). As a general framework to quantify uncertainty, evidence theory has demonstrated significant qualitative value and computational efficiency across various applications. One of the main advantages of evidence theory lies in its flexibility to assign probability measures to groups or intervals without requiring precise assumptions about the probabilities of individual elements within those groups or intervals. This feature is particularly beneficial in engineering design, where information is often limited, conflicting, or derived from expert judgment or experimental data. By allowing for the combination of random and epistemic uncertainties, evidence theory offers potential benefits for design under uncertainty. However, its application in engineering design remains relatively limited, and its integration into design optimization frameworks is even less common. Most evidence-based methods have been employed to propagate epistemic uncertainty [141], especially in the context of large-scale engineering structures. One of the main challenges in the application of proof theory is the high computational cost [90]. To address this issue, the multipoint approximation method has been proposed as an effective solution to mitigate computational difficulties [141]. Detailed flowchart of this approach is presented in figure 5.

Design optimization methods are computationally efficient and capable of handling a mixture of aleatory and epistemic uncertainties in recent days, and the problem can be formulated as follows:

$$\begin{aligned} &\min_{\mathbf{d},\widetilde{\mathbf{d}}(\mathbf{X})} : f(\mathbf{d},\mathbf{X},\mathbf{P}) \\ &st.\ Pl\{G_i(\mathbf{d},\mathbf{X},\mathbf{P})>0\} \leqslant p_{f_i}, \quad i=1,2,\ldots,np \\ &\mathbf{d}^L \leqslant \mathbf{d} \leqslant \mathbf{d}^U \\ &\widetilde{\mathbf{d}}(\mathbf{X})^L \leqslant \widetilde{\mathbf{d}}(\mathbf{X}) \leqslant \widetilde{\mathbf{d}}(\mathbf{X})^U \\ &\mathbf{d}\in\mathbb{R}^n, \mathbf{X}\in\mathbb{R}^{nr}, \mathbf{P}\in\mathbb{R}^q \end{aligned} \quad . \tag{32}$$

The function $f(\mathbf{d},\mathbf{X},\mathbf{P})$ represents the objective function, which may correspond to the cost, weight, performance index, or any other design metric to be optimized. The vector of $\mathbf{d}$, $\widetilde{\mathbf{d}}(\mathbf{X})$, and $\mathbf{P}$ denote the vector of deterministic design variables, the uncertain design variables based on evidence theory, and the uncertain parameter, respectively. The $\mathbf{d}^L$ and $\mathbf{d}^U$ define the lower and upper bounds of $\mathbf{d}$, while $\widetilde{\mathbf{d}}(\mathbf{X})^L$ and $\widetilde{\mathbf{d}}(\mathbf{X})^U$ represent the lower and upper bounds of $\widetilde{\mathbf{d}}(\mathbf{X})$ respectively. The plausibility function *Pl* is used to account for epistemic uncertainty in the system. The function $G_i(\mathbf{d},\mathbf{X},\mathbf{P})$ describes the failure condition, which is constrained by the failure probability threshold p_{f_i}. The parameter *np* denotes the number of constrains imposed on the system. $\mathbb{R}^{\mathrm{n}}$ is the real number space for determine design variables. $\mathbb{R}^{\mathrm{nr}}$ is the real number space for uncertain variables represented by evidence theory, $\mathbb{R}^q$ is the real number space for uncertain parameters.

Notably, Pl function expresses the maximum possibility that an event could occur. In contrast, the Bel function provides a lower bound of an event occur, which might underestimate the actual failure probability. Through Pl function, the worst-case scenario can be considered, which is required in reliability engineering. It ensures that even in cases where failure probability is not fully known, the optimization remains robust. Therefore, Using Pl instead of Bel ensures a more conservative and flexible optimization process.

A geometric interpretation of EBDO problems has been provided, along with an efficient computational solution that demonstrates the proposed EBDO method through two design examples [85]. The algorithm, derivative-free optimizer, is applied to locate the neighborhood of the optimal solution. It aims to identify the optimum based on evidence. The process begins at an initial point near the optimal solution from Reliability-Based Design Optimization (RBDO) and employs a hyper-ellipse movement strategy within the original design space, similar to the approach used in RBDO. To improve computational efficiency, local surrogate models are constructed using only the active constraints, which effectively reduces computational costs [85]. To address the computational challenges of nested optimization inherent in EBDO, a decoupling approach has been proposed. This method transforms the original nested optimization problem into a sequential iterative

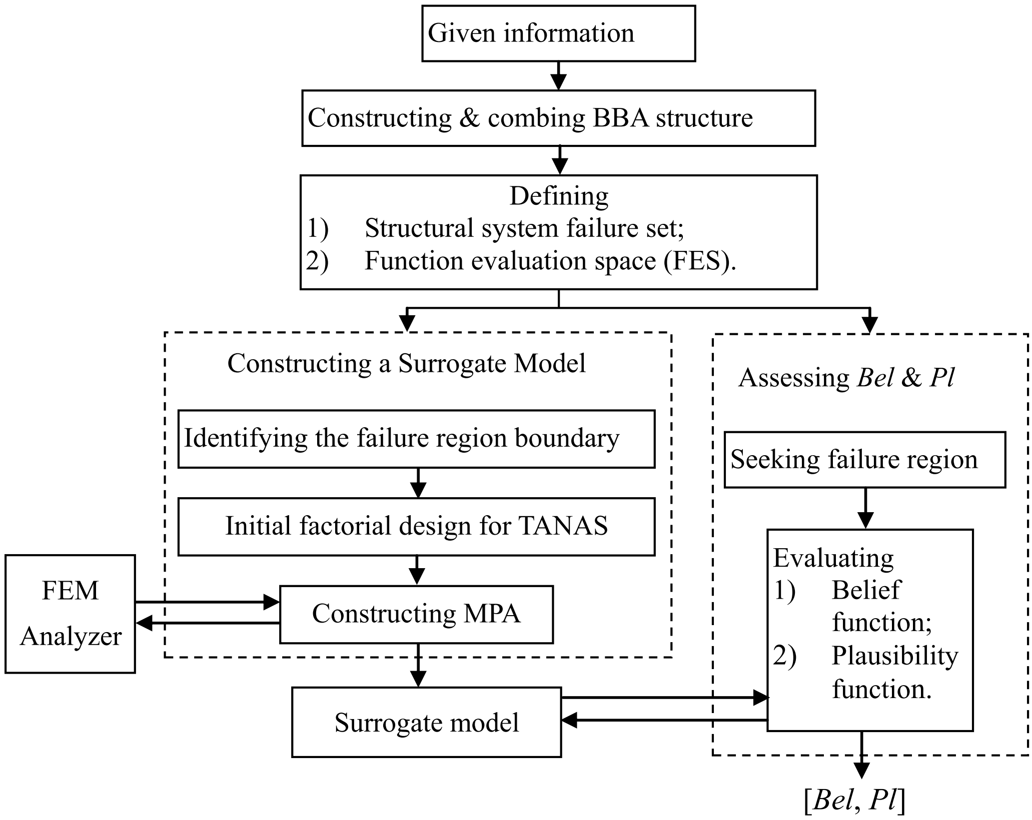

Figure 5. Uncertainty qualification approximation algorithm using evidence theory [90].

process, consisting of alternating steps for design optimization and reliability analysis [142]. In addition, an improved two-step framework had been proposed to solve EBDO problems under epistemic uncertainty [143]. A cross-scale topology optimization model was presented to address uncertainty in design [144].

4. Developing trends of possibility and evidence-based methods

Possibility theory quantifies uncertainty using linguistic terms or intervals rather than precise probability values. Unlike probability distributions, it employs a possibility distribution to describe the plausibility of different outcomes. Constraints are formulated through necessity and plausibility functions, optimizing decisions based on the most pessimistic or optimistic scenarios. Instead of probability density function, interval-based representations are used, making it well-suited for issues with limited statistical data. In summary, PBDO is widely applied in: (i) scenarios with known upper and lower uncertainty bounds; (ii) decision-making under vague and qualitative uncertainty (e.g. expert-based engineering assessments); and (iii) structural reliability optimization where failure probability is not well-defined.

DST extends probability theory by allowing belief intervals instead of precise probability values. It is suitable for both aleatory and epistemic uncertainties quantification. Bel and Pl functions are used to transform the uncertainty information into uncertainty interval bounds on event likelihoods. However, contrary to the possibility theory, DST integrates multiple sources of evidence for multi-source decision-making problems using Dempster's rule of combination. Due to its reliance on interval propagation and set-based computations, EBDO is computationally more expensive than PBDO, particularly in high-dimensional problems. In general, EBOD is commonly applied in: (i) reliability-based optimization where probability distributions are partially known; (ii) multi-source uncertainty aggregation where decisions depend on conflicting or incomplete data, and (iii) safety-critical engineering applications, such as aerospace or nuclear engineering, where reliability bounds are crucial. The main connections and differences between PBDO and EBDO are provided in table 3.

Significant advancements have been made for the development of possibility theory and evidence theory over the past two decades, the field remains highly active, with numerous opportunities for future research. The following key directions highlight areas for further exploration [145].

(1) Integrating and Perfecting Existing Integration Methods

(i) Integrating possibilistic and probabilistic approaches, such as combining the Dempster–Shafer (DS) method with other established uncertainty modeling techniques to enhance robustness and reliability [146].

Table 3. Comparison of PBDO and EBDO.

Feature	PBDO	EBDO
Uncertainty Quantification	Possibility distributions	Belief and plausibility functions
Applicable	Vague, qualitative, and imprecise uncertainties	Partial probability knowledge with conflicting evidence
Computational Cost	Lower (max-min operations on possibility distributions)	Higher (interval propagation and set-based calculations)
Mathematical Framework	Maxitive measures	Additive and non-additive measures
Flexibility	Work with incomplete data and expert knowledge	Combines multiple sources of uncertain data
Optimization Method	Necessity and plausibility measures	Belief and plausibility measures
Application scenarios	Structural optimization, qualitative decision-making	Safety-critical system design, multi-source uncertainty fusion,

(ii) Proposing hybrid algorithms, including genetic algorithms with dynamic parameter optimization, offer a promising way to reduce design iterations and narrow search intervals.

(iii) Improving computational accuracy, stability, and numerical efficiency remains a priority. Future research could focus on developing adaptive algorithms capable of dynamically fine-tuning parameters based on real-time performance feedback.

(2) Establishing Common Analytical Frameworks

(i) Establishing a common analytical framework that addresses conflicts arising from different types of uncertainties is essential for ensuring consistency in modeling.

(ii) Developing global uncertainty propagation techniques to account for the cumulative effects of uncertainties across various system components.

(iii) Constructing error-compensation feedback loops or implementing adaptive correction mechanisms in software tools will enable continuous improvement and refinement of predictive models.

(3) Soft Computing Strategies

(i) The soft computing strategies are applied to construct cooperative framework is gaining traction such as basic integrations of fuzzy logic, probabilistic reasoning, and neural networks.

(ii) Advanced combinations involving genetic algorithms, evidential reasoning, machine learning, and chaos theory offer enhanced modeling capabilities and innovative approaches to solving complex problems.

(4) Design Optimization Under Uncertainty

The development of optimization techniques is crucial for addressing complex systems that include multiple failure mechanisms and highly nonlinear state functions. These approaches will enable more accurate and reliable assessments of system performance across varying operating conditions.

(5) Accurate and Efficient Reliability Analysis

It is crucial to improve reliability analysis methods to evaluate systems with low failure probabilities under uncertainty. Future efforts should focus on refining these methods to improve both predictive accuracy and practical applicability in real-world engineering scenarios.

5. Conclusions

This paper provides a comprehensive and detailed overview of possibility and evidence theories, emphasizing their fundamental principles and applications in reliability and uncertainty analysis within engineering design. Particularly in scenarios where input data is limited or incomplete, these theories offer valuable frameworks for addressing specific uncertainties. Both possibility theory and evidence theory are instrumental in enhancing reliability analysis and design optimization, especially in the context of epistemic uncertainty under data scarcity. The distinctive representations and theoretical underpinnings of uncertainties enable engineers to navigate the complexities of uncertainty more effectively. The review also highlights the need for further exploration in the domain. There remains a gap in the development of a more unified framework that incorporates advanced performance characteristics and modified design criteria tailored to the specific challenges posed by uncertainty. This presents an opportunity for researchers to innovate and refine existing methodologies. By adopting a holistic perspective, we aim to foster a deeper understanding of current approaches while illuminating pathways for future research. The review encourage collaboration among reliability engineers, statisticians, and other stakeholders to advance the integration of these theories into practical applications. This collaboration will be crucial in addressing the multifaceted nature of uncertainty in engineering design, ultimately leading to more robust and resilient systems.

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