A Versatile Hardware Solution for Generating Power Law Noise with Tunable Long-Term Dependency

Xihui Yuan, Yongjie Luo, Zheng Chai, Jiajia Jian, Xue Zhou, Xin Yue, Jian Fu Zhang, Weidong Zhang and Tai Min

***Abstract*—Power law noise (PLN) has promising applications in healthcare and artificial intelligence (AI). In this paper, we propose a hardware solution for generating PLN with tunable long-term dependency, by combining the magnetic tunnel junction (MTJ) based two-state Markov chain (MC) generation and modulation method. PLNs with specific parameter α can be obtained by merging multiple two-state MCs. The merged PLN, with α ranging from 0.2 to 1.8 and adjustable long-term dependency as represented by the autocorrelation function (ACF), is demonstrated by varying the voltage conditions applied to the MTJs. This method provides a versatile hardware solution for constructing stochastic signal in semiconductor IC chips.**

***Index Terms*—Power-law Noise, Markov chain, magnetic tunnel junction**

I. Introduction

RANDOM noise is a ubiquitous phenomenon in semiconductor devices. When the power spectral density (PSD), $S(f)$, of a random noise follows a power-law relationship with frequency:

$$S(f) \propto \frac{1}{f^{\alpha}} \quad (1)$$

where f represents frequency and α is an exponent ranging from 0 and 2, the noise is classified as power law noise (PLN). Specifically, PLN with $\alpha = 0$, 1 and 2 corresponds to Gaussian, flicker and Brownian noise [1] [2], respectively, as illustrated in **Fig. 1** [3]. PLN is widely applicable in various fields [4], such as healthcare electronics [5] [6] and neuromorphic computing [7] [8] [9]. Traditionally, generating PLN has relied on computation-intensive algorithms, such filtering Gaussian noise using Fourier transforms, convolution operations [10] [11] or nonlinear stochastic differential equations [12] [13]. However, with the rapid development of those applications, there is an increasing demand for hardware solutions that can generate PLN with improved efficiency.

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In the noise analysis of semiconductor devices, it is well-established that flicker noise can be modeled as a superposition of random telegraph noises (RTNs) originating from multiple defects in a device [14] [15]. Nevertheless, modulating flicker noise proves challenging due to physical limitations imposed by the inherent distribution of defects. On the other hand, it is important to note that a two-level RTN can be represented by a two-state Markov chain (MC) [16] [17]. Therefore, if the MC can be generated in a more flexible way, unconstrained by the defect distribution, it is plausible to envision the generation of PLN reasonable to speculate that generating PLN with a tunable exponent α which spans the entire range from 0 to 2.

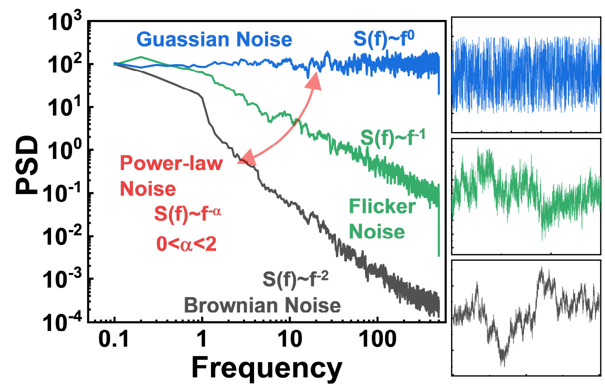

Fig.1. Power spectral density (PSD) of power law noises (PLNs) with exponents α = 0, 1, and 2, corresponding to Gaussian, flicker, and Brownian noise, respectively.

In this work, we propose a novel approach to generate PLN with a tunable exponent α ranging from 0 to 2, utilizing a small number of non-volatile memory (NVM) devices. The desired PLN is decomposed into several MCs, with each being generated and modulated by a single NVM device. This method allows for flexible control over the exponent, enabling fine-tuning across its entire range. This method provides a compact and efficient hardware solution for PLN generation, addressing the increasing demand for noise generation in next-generation information technology applications.

II. Method of Generating PLN

A. Mathematical derivation

Let us recall that the PSD of a two-state MC, consisting of state 0 and 1, is described as a Lorentzian function in the frequency domain [18].

$$S(f) = \frac{4A^2\beta}{(1+\beta)^2}\frac{\tau}{1+(f/f_c)^2} \quad (2)$$

where A is the MC's amplitude (difference between the top and base), $\beta = \tau_1/\tau_0$ is the ratio of the average dwell times (ADTs) τ_0 and τ_1 which are the average length that the MC remains at one state before switching to the other state, and $\tau = (\tau_1 \cdot \tau_0)/(\tau_1 + \tau_0)$ is the characteristic ADT. The Lorentzian curve is distinguished by a characteristic corner frequency f_c and the corresponding $S(f_c)$.

$$f_c = \frac{1}{2\pi}\left(\frac{1}{\tau_0} + \frac{1}{\tau_1}\right) \quad (3)$$

$$S(f_c) = \frac{2A^2\beta\tau}{(1+\beta)^2} \quad (4)$$

Apparently, the PSD remains constant for $f < f_c$, and becomes proportional to f^{-2} for $f > f_c$.

B. Experiment setup

In our previous work [19] [20], a technique to generate an two-state MCs with arbitrary ADTs have been developed by using a three-pulse waveform on a single magnetic tunnel junction (MTJ) (**Fig.2a**), exploiting the stochastic switching nature of MTJs [21] [22]. In the technique, the ADTs of the MC can be modulated by the conditions of the first two pulses, as explained in detail in [19] [20]; while the amplitude of the MC can be determined by the read voltage (V_{read}), as shown in **Fig. 2b** and **Fig. 2c**.

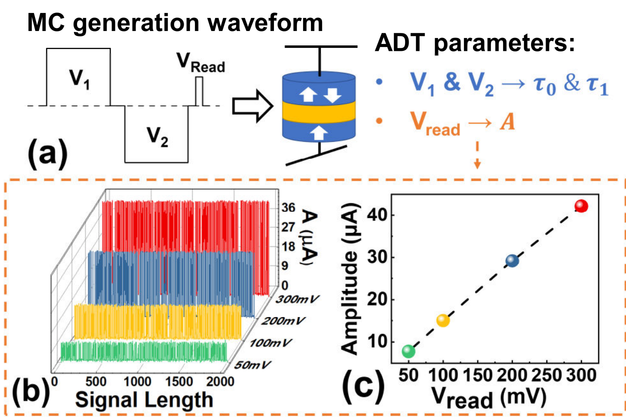

Fig.2 (a) The three-pulse waveform used to generate a two-state Markov chain (MC) with arbitrary average dwell times (ADTs) on a single magnetic tunnel junction (MTJ). (b) MCs generated under different read voltage. (c) Dependence of MC amplitude on V_{read}.

At this stage, the two-state MCs need to be combined. Since the base of an MC, corresponding to its lower state, does not affect the dynamic aspects of the PSD as it appears as a static DC component in the frequency domain, the base can be modulated flexibly without impacting the spectral characteristics. Here, the average of the high and low states for each MC is calculated and maintained at a consistent level, ensuring uniformity across the combined signals.

C. Operation procedure

Following the PLN construction methodology and MTJ based two-state MC generation technique outlined above, we propose a straightforward approach for constructing PLNs using a small number of MTJs.

As shown in **Fig. 3a**, to generate a targeted PLN $S(f) = hf_c^{-\alpha}$, a set of f_c logarithmically spaced between the targeted frequency range (f_{min}, f_{max}) should be determined, ensures a uniform contribution of each MC to the PLN. As a consequence, the $S(f_c)$ of each MC, is determined by:

$$S(f_c) = hf_c^{-\alpha}. \quad (5)$$

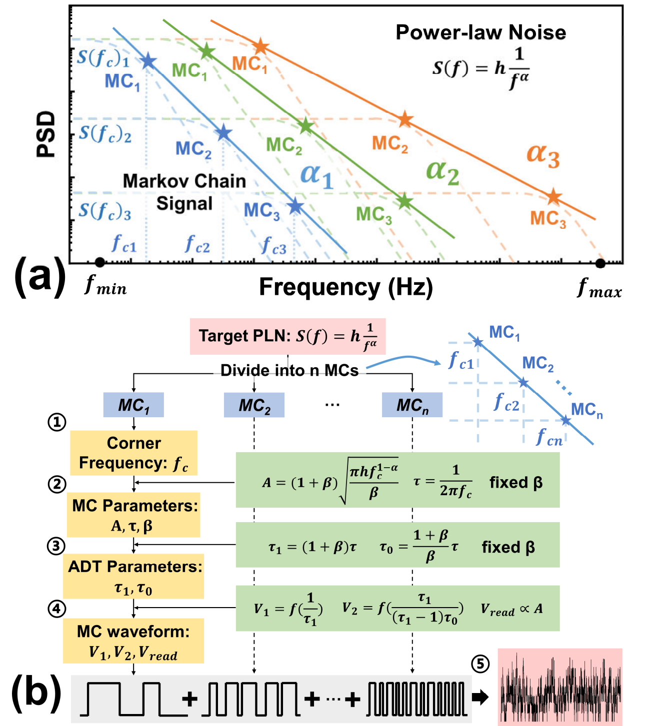

Fig.3 (a) The PSD of the PLN is approximated by the envelope of Lorentzian curves corresponding to the two-state MCs. (b) Flow chart of the proposed PLN construction procedure.

Based on the flow chart in **Fig. 3b**, we propose a step-by-step procedure to realize the generation of PLN with arbitrary α, outlined as follows:

(1) With the logarithmic f_c in the frequency range and the given α, calculate the corresponding $S(f_c)$.

(2) For each MC, with a fixed β, calculate the parameters (τ and A) according to the f_c and $S(f_c)$.

(3) Calculate the ADTs (τ_0, τ_1) of each MC.

(4) Form three-pulse waveform including V_1, V_2, and V_{read} on an MTJ device to generate MCs.

(5) Merge the MCs to form a time-domain PLN, and test if the α of this PLN equals to the target value.

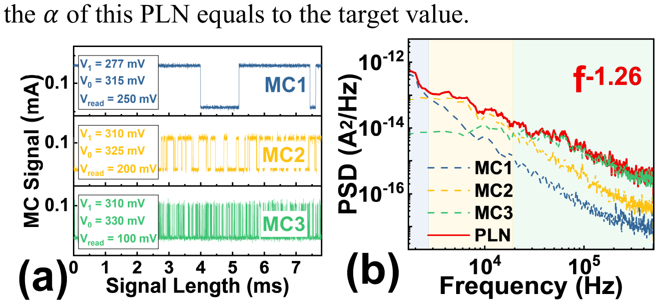

Fig.4. Demonstration of a PLN constructed with practical MTJs. (a) Three two-state MCs used to construct the target PLN. Each MC has a set of ADTs and amplitude and is generated by an MTJ. (b) PSD of the constructed PLN, with α= 1.26, agreeing with the target value.

III. Verification of the Method

Fig. 4 illustrates the construction of PLN with α = 1.26, utilizing three MTJs measured by a Keithley 4200 semiconductor analyzer. **Fig. 4a** shows the three two-state MCs with distinct ADTs generated from three MTJs with three various sets of voltage pulse (V_1, V_2, V_{read}). In the waveform applied on the MTJs, the pulse width is fixed at 500ns. As depicted in **Fig. 4b**, the PSD of the constructed PLN at a given frequency is predominantly contributed by the MC whose corner frequency is closest to that frequency.

Fig. 5a and **Fig. 5b** demonstrate the parameters τ_1, τ_0, and A, of MCs used to construct PLNs with exponent α = 0.2, 0.5, 1.0, 1.5, and 1.8, via MATLAB simulation, with each PLN constructed by 10 MCs. Note that τ_1 and τ_0 exhibit similar distributions due to the fixed parameter $\beta = 3$. The PLNs in the frequency domain with exponent α = 0.2, 0.5, 1.0, 1.5, and 1.8 are presented in **Fig. 5c**, while the PLNs in the time domain with exponent α = 0.5, 1.0, and 1.5 are shown in **Fig. 5d**.

The autocorrelation function (ACF) in **Fig. 5e** confirms that long-term dependency in PLN increases with higher α, further supported by the 95% confidence boundary in **Fig. 5f**, consistent with prior studies [23]. The ability to conveniently modulate the long-term dependency of PLN makes this method promising for a wide range of applications, such as enhancing the adaptability of healthcare electronics like brain-computer interface (BCI) [24], or improving neuromorphic computing by preventing overfitting in hardware neural networks [8].

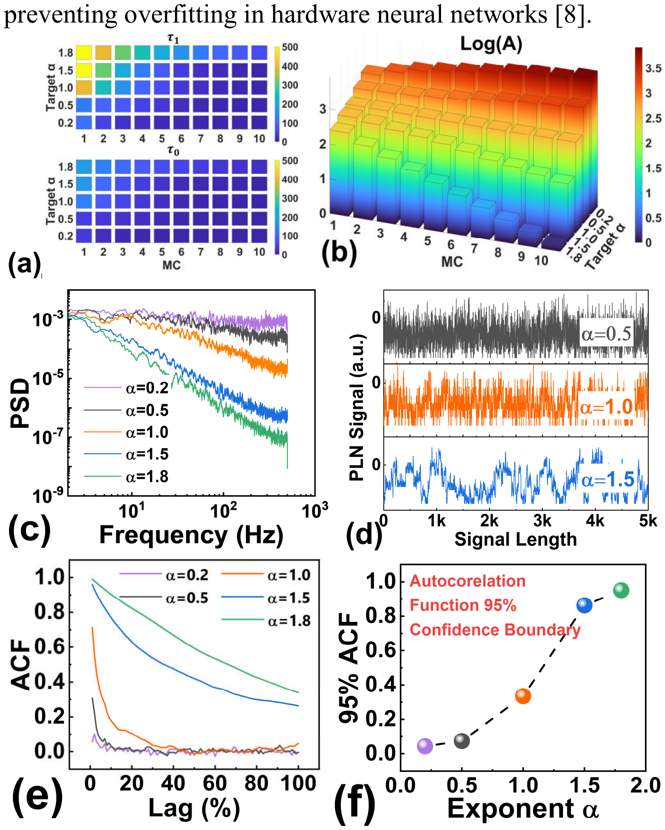

Fig.5. By using 10 sets of parameters (a) τ_1, τ_0, and (b) A of two-state MCs, PLNs with α ranging from 0.2 to 1.8 are constructed, as shown in (c) the frequency domain and (d) the time domain. (e) Autocorrelation function (ACF) and (f) corresponding 95% confidence boundary of the 5 PLNs with α = 0.2, 0.5, 1.0, 1.5, and 1.8.

IV. Conclusions

In this work, we present a versatile hardware solution for generating PLN with tunable long-term dependency by integrating multiple two-state MCs with specifically designed ADT parameters and amplitude. This approach, requiring only a few devices, has been successfully demonstrated using MTJs. This method provides a practical and scalable hardware solution for generating stochastic signals within semiconductor chips, paving the way for applications in fields such as healthcare electronics and neuromorphic computing.

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