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Visuospatial working memory in intuitive geometry, and in academic achievement in geometry

David Giofrè*, Irene C. Mammarella**, Lucia Ronconi* and Cesare Cornoldi*

* *Department of General Psychology, University of Padua, Italy*

** *Department of Developmental and Social Psychology, University of Padua, Italy*

Correspondence concerning this article should be addressed to:

David Giofrè

Department of General Psychology

University of Padua

via Venezia 8 – 35131 – Padova - Italy

E-mail: david.giofre@studenti.unipd.it

Abstract

A study was conducted on the involvement of visuospatial working memory (VSWM) in intuitive geometry and in school performance in geometry at secondary school. A total of 166 pupils were administered: (1) six VSWM tasks, comprising simple storage and complex span tasks; (2) the intuitive geometry task devised by Dehaene, Izard, Pica, and Spelke (2006), which distinguishes between core, presumably innate, and culturally-mediated principles of geometry; and (3) a task measuring academic achievement in geometry. Path analysis models showed that some VSWM components support culturally-mediated principles of geometry, whereas no VSWM component is related to the core principles of geometry. A complex VSWM task requiring the manipulation of visual information as well as core and culturally-mediated principles of geometry directly predicted academic achievement in geometry. Our results are discussed in terms of the role of VSWM in learning geometry.

Keywords: Intuitive geometry, VSWM, Academic achievement.

Introduction

Although geometry is one of the main areas of mathematical learning, along with calculation and arithmetical problem-solving, the cognitive processes underlying geometry-related academic achievement have not been studied in detail. The psychological aspects of geometry have received attention from both developmental psychologists (e.g., Piaget, 1960; Piaget & Inhelder, 1967) and educational psychologists (e.g., Clements, 2003, 2004; Clements & Battista, 1992; Crowley, 1987; Owens & Outrhed, 2006; Van Hiele, 1986). As regards to the underlying cognitive mechanisms, the involvement of spatial abilities and imagery in geometry has also been analyzed (Bishop, 1980; Brown, & Presmeg, 1993; Piaget & Inhelder, 1967) but, to the best of our knowledge, no research has attempted to investigate the role of visuospatial working memory (VSWM) in geometry. The present study tried to fill this gap by examining the involvement of different components of VSWM in the learning of various aspects of geometry.

Geometry at school and the intuitive (core and culturally-mediated) principles of geometry

The intuitive knowledge of geometry has been examined in a number of studies. For example, Rosch (1975) showed that, when people in a Stone-Age culture with no explicit education in geometry were asked to choose the “best examples” of a set of shapes (i.e., a group of quadrilaterals and near-quadrilaterals), they usually selected a square and a circle, even when the set contained variants closely resembling them (for instance, the set containing squares also included square-like shapes that were open, or had curved sides, or contained non-right angles), suggesting that people have a preference for closed symmetrical shapes (Bornstein, Ferdinandsen, & Gross, 1981).

In the same vein, Dehaene, Izard, Pica, and Spelke (2006) devised a test to analyze the intuitive comprehension of certain basic concepts of geometry. Their test was based on a series of arrays of six images, each representing an intuitive concept of geometry: five images fitted the target concept (i.e. they were correct), while one contradicted it. Participants included native

Amazon Indians and North Americans who were asked, each in their own language, to point to the "ugly" image. The results revealed that:

a. core intuitions of geometry can be identified, since the native Amazon Indian group succeeded remarkably well with concepts of topology (e.g., connectedness), Euclidean geometry (e.g., lines, points, parallelism, and right angles) and geometrical figures (e.g., squares, triangles, and circles). Dehaene et al. (2006) consequently considered these concepts as the *core principles* (CP) of geometry;

b. adults who had received no schooling in geometry and young children (from both geographical groups) revealed a similar competence in these CP of geometry, i.e. the Amazonian children's performance did not differ from that of the American children. The American adults performed significantly better in all the tests, however, going to show that cultural differences emerge when it comes to non-core principles of geometry. To be more precise, the group of native Amazon Indian adults performed poorly (on a level comparable with the North American and Amazonian children) in items assessing geometrical transformations, when participants had to use concepts such as translations, symmetries, and rotations. The authors concluded that all of these items entail a mental transformation from one shape into another and might thus require *culturally-mediated principles* (CMP) of geometry.

Spelke, Lee, and Izard (2010) claimed that knowledge of geometry is founded on at least two distinct, core cognitive systems; the first is used to represent the shapes of large-scale navigable surface layouts and the second represents small-scale movable forms and objects. Empirical evidence of this latter system emerged from developmental studies showing that infants are sensitive to variations in angle (Schwartz & Day, 1979; Slater, Mattock, Brown, & Bremner, 1991) and length (Newcombe, Huttenlocher, & Learmonth, 1999). The system for representing small-scale movable forms and objects would therefore capture abstract geometrical information representing the shapes of objects that vary in length and angle, but not direction. The system fails to distinguish a form from its mirror image, for instance, and it reveals qualitative continuities during the course of human development (Izard & Spelke, 2009), as well as across cultures (Dehaene et al., 2006).

In sum, these studies have shown that some aspects of geometry are ‘intuitive’: (1) primitive (Rosch, 1975), (2) very early developed (Spelke, Lee, & Izard, 2010) and (3) not dependent by culture and formal instruction (Dehaene et al, 2006). Moreover, Dehaene and colleagues (2006) have shown that it is possible to assess experimentally intuitive geometry. Although, they did not explore the relationship between intuitive aspects and other aspects which are independent from culture or schooling (i.e., working memory or intelligence), or aspects dependent on formal instruction (i.e., achievement in geometry).

Cognitive processes involved in geometry

Competence in geometry could be considered not only vis-à-vis intuitive geometry, but also in terms of academic achievement in geometry (i.e., a student's ability to respond to the typical geometry questions on the mathematical curriculum). Academic achievement in geometry, especially at secondary school level, is considered one of the most important areas of mathematical learning, and it is linked to a student's future academic and professional success (Verstijnen, van Leeuwen, Goldschimdt, Haeml, & Hennessey, 1998). Pupils attending secondary schools must possess concepts, definitions, theorems, etc., and apply their knowledge to solving problems that are typically presented in language form. It therefore seems important to examine whether differences in intuitive geometry and other underlying cognitive mechanisms may have a crucial role in predicting school achievement in geometry.

The working memory (WM) system, in which specific storage components (i.e., the ‘slave’ systems) sub-serve a central component responsible for controlling information processing (Baddeley, 1986), could be involved both in the acquired part of intuitive geometry and in the geometry learnt at school. A large body of research has shown that WM predicts success in school-related tasks, such as reading comprehension (Daneman & Carpenter, 1980), mathematical achievement (Bull, Espy, & Wiebe, 2008; Fürst & Hitch, 2000; Geary, Klosterman, & Adrales, 1990; Hitch, 1978; Passolunghi, Mammarella, & Altoè, 2008) and arithmetical problem-solving (Passolunghi, Cornoldi, & Di Liberto, 1999; Passolunghi & Siegel, 2001, 2004). More specifically, the WM component involved in retaining and processing visuospatial information (VSWM) appears to be involved in children's ability to count (Kyttälä, Aunio, Lehto, van Luit, & Hautamaki, 2003), performance in multi-digit operations (Heathcote, 1994) and nonverbal problem-solving (Rasmussen & Bisanz, 2005), and mathematical achievement (Bull et al., 2008; Jarvis & Gathercole, 2003; Maybery & Do, 2003).

Although the relationship between VSWM and geometry has not been studied before, as far as we know, it has already been demonstrated that VSWM predicts a person's success in geometry-related activities. To give an example, the capacity to hold and manipulate visuospatial information has been shown to specifically predict success in architecture and engineering (Verstijnen, et al., 1998). This makes VSWM the prime candidate for seeking cognitive mechanisms supporting both intuitive geometry and school achievement in geometry, though the latter will be associated with many other variables influencing mathematical achievement at school (e.g., language, calculation, problem-solving, motivation, metacognition, and so on; Aydin & Ubuz, 2010). In addition, considering the sub-components of VSWM will make possible to understand which components of VSWM are related to intuitive geometry and achievement in geometry.

The organization of VSWM

It has been demonstrated that the VSWM system is not unitary. Many studies (see Logie, 1995) have supported a distinction between the visual and spatial subcomponents of VSWM, the former referring to the recall of shapes and/or textures while the latter referring to the recall of spatial locations and sequences. An alternative approach-that is less widely acknowledged, but has recently received support (Cornoldi & Vecchi, 2003; Mammarella, Borella, Pastore, & Pazzaglia, 2012; Mammarella, Pazzaglia, & Cornoldi, 2008; Mammarella et al., 2006; Pazzaglia & Cornoldi, 1999) - distinguishes between visual WM tasks that involve memorizing shapes, textures and colors, spatial-sequential tasks requiring the recall of a sequence of spatial locations, and spatial-simultaneous tasks demanding the recall of an array of simultaneously-presented locations. It has also been suggested that a distinction should be drawn between many different types of WM process based not only on the format/content of the information, but also on the degree of controlled attention involved. This latter distinction has been described in many ways, e.g. by differentiating between simple storage and complex span tasks (Unsworth & Engle, 2005), or between passive processes (as in simple storage tasks) and active processes (as in complex span tasks) (Cornoldi & Vecchi, 2003), where the former involves retaining information that has not been modified after encoding, while the latter requires some transformation and manipulation of the information and presumably correlate more closely with an individual's degree of success in geometrical tasks requiring the manipulation of visual information.

Study design

The present study was designed primarily to seek any relationships between VSWM, intuitive geometry, and academic achievement in geometry among secondary school students. Second, we aimed to investigate whether different components of VSWM relate differently to CP and CMP of geometry, as defined by Dehaene et al. (2006). To do so, we administered both the intuitive geometry task (Dehaene et al., 2006) and the MT advanced battery, a standardized test assessing achievement in geometry (Cornoldi, Friso, & Pra Baldi, 2010) devised for secondary school students, which includes items of the type contained in the PISA tests (OECD, 2007). We chose to test secondary school students because the PISA tests are only administered to this age group, and because these students will have presumably nearly completed their learning of the cultural and educational aspects of geometry, since any further education may well contain no geometry (in Italy at least, where this study was carried out).

To assess VSWM, we used three simple storage tasks (one visual, one spatial-sequential, and one spatial-simultaneous) and three complex span tasks. The distinction between simple storage and complex VSWM tasks was particularly crucial for the purposes of this study because performance in geometry is related not simply to maintenance, but also to the manipulation of information; complex span tasks could therefore provide important information, while the contribution of simple storage tasks could prove less relevant.

Our study thus examined the involvement of VSWM in intuitive geometry and sought to ascertain whether both VSWM and intuitive geometry affect academic achievement in geometry. Judging from previous evidence, intuitive geometry concepts can be divided into CP and CMP (Dehaene et al., 2006; Spelke, et al., 2010). We examined whether students' achievement in geometry was supported by both CP and CMP of geometry, as well as by VSWM. We also investigated whether the CMP of geometry (the learning of which is mediated by experience) require the support of VSWM.

The pattern of relationships was examined using path analysis models in successive steps to compare the adequacy of different models in describing the relationships between variables.

Method

Participants

The study involved 166 students (125 boys and 41 girls) in their last 2 years at secondary school (mean age=17.84; SD=.74) in northern Italy. The mean age of participants in 12th grade was 17.35 (SD=.73) and for those in 13th grade it was 18.03 (SD=.65). Participants were attending schools where geometry had an important role, i.e. secondary schools that focused on science or specialized in land surveying, or technical and industrial colleges.

Materials and procedures

Participants were tested in two phases, i.e. a group session in the classroom lasting approximately 20 min, and an individual session approximately one hour long in a quiet room away from the classroom.

During the first phase, we administered a school achievement test (the geometry items in the MT advanced battery) to the whole class (Cornoldi et al., 2010). In the second phase, we administered the following tasks on an individual basis in this order: the intuitive geometry task (Dehaene et al., 2006) and six VSWM tasks in the following fixed order: (1) simultaneous dot matrix task; (2) dot matrix task; (3) nonsense shapes task; (4) visual pattern test, active version; (5) sequential dot matrix task; and (6) jigsaw puzzle task.

Measures of geometry

Test on achievement in geometry. The MT advanced geometry task is a paper-and-pencil test that includes the six multiple-choice questions from the MT advanced battery (Cornoldi et al., 2010) concerning school-based geometry education. This battery was developed on the basis of the PISA tasks (OECD, 2007) and was designed for use in comparing individual performance with typical school standards in Italy. Participants were asked to solve a series of geometrical problems (see an example in Figure 1) and the mean percentage of the correct answers was considered. All the students in the class took about 20 min to complete the test.

Figure 1 about here

Intuitive geometry task. The intuitive geometry task (Dehaene et al., 2006) was programmed using E-Prime 1.1 software, and the items were randomly presented on a computer. Participants were presented with forty-three items split into seven concepts: topology, Euclidean geometry, geometrical figures, symmetrical figures, chiral figures, metric properties, and geometrical transformation. At the beginning of the procedure, a masking screen appeared for 2000 ms before the randomly presented stimuli appeared. Each stimulus remained on the screen until the participant had given a response. The items consisted of an array of six simultaneously-presented images, five of which instantiated a given concept, while one image violated it. For each item, participants were asked to identify the odd one out (which appeared in a random position among the other five images).

Three different scores were calculated: one was the total mean percentage of correct responses; the second (as in Dehaene et al, 2006) was a score representing the CP of geometry (i.e. the mean percentages of correct answers for images relating to topology, Euclidean geometry, and geometrical figures, for a total of 21 items); and the third was a score representing the CMP of geometry (i.e., the mean percentages of correct answers for images relating to symmetrical figures, chiral figures, metric properties, and geometrical transformation, for a total of 22 items). Figure 2 shows some examples of the concepts presented.

Figure 2 about here

VSWM measures

Participants were presented with six tests (4 computerized, 2 paper-and-pencil); five of them are part of an Italian standardized VSWM test battery (Mammarella, Toso, Pazzaglia, & Cornoldi, 2008), while the dot matrix test was derived from Miyake, Friedman, Rettinger, Shah, and Hegarty (2001). Three tests were passive, simple storage tasks, and three were active, complex span tasks. The simple storage tasks were classifiable as visual (the nonsense shapes task), spatial-sequential (the sequential dot matrix task), or spatial-simultaneous (the simultaneous dot matrix task) (Pazzaglia & Cornoldi, 1999; Mammarella et al., 2008). The complex span tasks were the jigsaw puzzle task (adapted from Vecchi & Richardson, 2000), the dot matrix task (drawn from Miyake et al., 2001), and active version of the visual pattern test (VPTA, derived from Della Sala, Gray, Baddeley, & Wilson, 1997). Examples of these materials are shown in Figure 3.

The six tests were administered adopting a self-terminating procedure (starting with the easiest, the tests became increasingly complex and participants continued as long as they were able to solve at least two of three problems for a given level). For scoring purposes, items on the second level of difficulty scored 2, on the third level they scored 3, and so on. The final scores corresponded to the sum of the last three correct responses. For instance, a participant who solved two problems on the fourth level and one on the fifth scored 4+4+5 = 13 (see Mammarella, et al., 2008; Mammarella, Lucangeli, & Cornoldi, 2010). Before administering each task, participants were given two practice trials with feedback. The tests were administered during a single individual session in a quiet room at the students' school.

For the simple storage tasks, participants had to decide whether a set of figures/locations was the same as, or different from a previously-presented set: after a first stimulus had been shown, either the same stimulus or one in which just one element had changed appeared, followed by a screen containing two letters, U (*uguale*=same) and D (*diverso*=different), and participants responded by pressing one of the two keys on the keyboard. The complex span tasks involved not only recognizing but also processing the information presented.

Figure 3 about here

Results

Descriptive statistics for each test are presented in Table 1. The scores are expressed as the percentages of correct responses for geometrical measures, while for VSWM they are given by the sum of the three highest levels of difficulty reached by the subject. Table 1 also shows the test reliabilities.

Table 1 about here

Model estimation

Path analysis models were computed with the LISREL 8.8 statistical package (Jöreskog & Sörbom, 1996). We used the fit indices recommended by Jöreskog and Sörbom (1993), such as the root-mean-square error of approximation (*RMSEA*), the non-normed fit index (*NNFI*), and the comparative fit index (*CFI*). Like Schreiber, Stage, King, Nora, and Barlow (2006) (see also Schermelleh-Engel, Moosbrugger, & Müller, 2003), we considered substantively interpretive models with a non-significant chi-square, an *RMSEA* below .05, an *NNFI* above .97, and a *CFI* above .97 as a good fit.

Preliminary analysis

Possible differences related to gender and school year were measured: for the former, only the effect of the dot matrix task ($F[1,164]=8.93$, $p=.003$, $\eta_p^2=.05$) was significant (males did better than females); for the latter, only the effects of the MT advanced geometry task ($F[1,164]=11.46$, $p=.001$, $\eta_p^2=.65$), and of the nonsense shapes ($F[1,164]=5.81$, $p=.017$, $\eta_p^2=.03$) were significant (13^{th} graders performed better than 12^{th} graders in both cases).

Path analysis

Normality was taken into consideration. Mardia's measure of relative multivariate kurtosis (MK) was obtained using PRELIS (Jöreskog & Sörbom, 1993). The MK was 1.09, which implies a non-significant departure from normality ($-1.96 < z < 1.96$; Mardia, 1970).

For the purposes of our analysis, we considered the VSWM tasks as independent variables and the geometry achievement test (the MT advanced geometry task) as a dependent variable. We sought the best model first (models 1 to 4), considering only the total score for the intuitive geometry task as the mediator variable, then (models 5 and 6) we distinguished between the CP and CMP of geometry (as in Dehaene et al., 2006).

Correlations between measures are given in Table 2.

Table 2 about here

We began our analysis by assessing the full model involving all the variables. Then we gradually deleted some of the variables, taking their weight and our hypotheses into account. The initial model thus involved the nonsense shapes, sequential dot matrix, simultaneous dot matrix, jigsaw puzzle and dot matrix tasks, and the VPTA tests as independent variables. The total score for the intuitive geometry task served as the mediator and the MT geometry achievement task as the dependent variable.

Path model 1 was saturated. The fit was completely adequate (Table 3, 4; Figure 4).

In Path model 2, we deleted the direct effects of nonsense shapes, sequential dot matrix, simultaneous dot matrix, dot matrix tasks and VPTA on MT geometry achievement, since the relationships between these variables and MT geometry achievement were not significant. The fit indices of the model were perfect (Table 3), but the relationships between the nonsense shapes, simultaneous dot matrix and VPTA variables, and the intuitive geometry task were not significant (Table 4).

In Path model 3, the nonsense shapes, simultaneous dot matrix and VPTA were deleted. The fit indices of the model were perfect (Table 3).

In Path model 4a, the dot matrix task and the non-significant correlation between the sequential dot matrix and jigsaw puzzle were deleted (Figure 4, Table 4). In this model, the sequential dot matrix and jigsaw puzzle, in conjunction with the mediation of the intuitive geometry task, predicted the MT geometry achievement; the intuitive geometry task and the jigsaw puzzle directly predicted MT geometry achievement. The resulting fit indices were excellent (Table 3). This model explained 14% of the MT geometry achievement variance. In Path model 4b, we attempted to delete the direct effect of the jigsaw puzzle on the MT geometry achievement task, but the fit indices became worse (Table 3). Since the model 4b was nested in the model 4a, we calculated the chi-square difference between the two models, $\chi^2_D(1)=4.11$, $p=.043$ (right tail), finding the fit of the model 4a statistically better than that the model 4b. We therefore opted for the Path model 4a.

In Path model 5a, CP and CMP of geometry were introduced as separate mediator variables (instead of single mediator variables of intuitive geometry). Based on the fit indices, this model was unacceptable (Table 3). In Path model 5b, we introduced a direct path from CP to CMP of geometry and the fit indices improved significantly (Table 3), but the path from VSWM to CP, and the direct effect of CP on the MT advanced geometry task were poor.

In Path model 6a, we considered CP as an independent variable (Figure 5). In this model, the CP, the sequential dot matrix, and the jigsaw puzzle, with the mediation of CMP of geometry, were able to predict MT geometry achievement; CP and CMP of geometry, and the jigsaw puzzle task also directly predicted MT geometry achievement (Table 4). The fit indices were very good (Table 3). This model explained 14% of the variance for the MT geometry achievement task. In Path model 6b, we attempted to delete the direct effect of the jigsaw puzzle task on the MT-advanced geometry task, and the fit indices were good (see Table 4). We also calculated the chi-square difference between the two models; the chi-square was significant (χ^2_D[1]=4.05, p=.044 [right tail]), showing that the fit for the model 6a was statistically better than for the model 6b. In Path model 6c, we tested a model including CP with a path on the sequential dot matrix and the jigsaw puzzle task with a path on the CMP of gometry with a path on the MT advanced geometry task, but the model did not converge. We consequently selected the Path model 6a (Figure 6).[1]

Figures 4, 5 and 6 about here

Tables 3 and 4 about here

Discussion

In this study, we investigated the relationships between VSWM, intuitive geometry and academic achievement in geometry in secondary school students.

In particular, we expected to find a relationship between VSWM and intuitive geometry, and we hypothesized that both intuitive geometry and VSWM could predict academic achievement in geometry. To investigate these issues, the total score obtained in the intuitive geometry task devised by Dehaene et al. (2006) was used as a mediator variable. The final path model showed that only two of the six VSWM tasks considered were significantly related to the intuitive geometry task, namely a complex span task (jigsaw puzzle) and a simple storage task assessing spatial-sequential memory (sequential dot matrix). Only the jigsaw puzzle task related directly to academic achievement in geometry (i.e., the score in the MT advanced geometry task), whereas the sequential dot matrix task indirectly predicted academic achievement in geometry.

Our second hypothesis, based on the distinction made by Dehaene et al (2006) between the CP and CMP of geometry, was that VSWM could be more implicated in the acquired principles than in the CP of geometry, while both these aspects of intuitive geometry would be related to academic achievement in geometry. In the final path model, only the jigsaw puzzle task directly predicted academic achievement in geometry. More specifically, the VSWM tasks only related to CMP of geometry, while none of them related to the CP of geometry. Both the core principles and the culturally-mediated principles of geometry were related to academic achievement in geometry, but the latter CMP attributes had a stronger (β=.24) relationship with academic achievement than the CP of geometry (β=.15). Although the total variance in academic achievement in geometry explained by the model was not particularly high (producing a result consistent with the observation that many other variables can influence achievement in geometry; Aydin & Ubuz, 2010), the final model showed a very good fit and provided a picture of the relationship between VSWM, intuitive geometrical concepts, and academic achievement in geometry that is plausible and consistent with our predictions. Our results confirm the existence of a relationship between VSWM and geometry, but introduce the novel finding that this relationship is not involved in all the tasks. Some VSWM tests did not correlate significantly with performance in geometry, showing for example that the ability to retain a shape or a pattern of locations is not crucial to success in geometrical tasks. The most powerful VSWM test for predicting performance in intuitive geometry tasks and academic achievement in geometry was the jigsaw puzzle, which requires that participants not only memorize but also manipulate visual information (Cornoldi & Vecchi, 2003). Its relationship with the CMP of geometry can be explained by the finding that the items used in the study by Dehaene et al (2006) in which the native Amazon Indian adults failed involved geometrical transformations, with participants having to rotate, translate, or mentally manipulate one shape to convert it into another. In a more recent study comparing adults with children 4-10 years old, Izard and Spelke (2009) demonstrated that it is only after adolescence that young people are able to detect directional relationships, a skill requiring discrimination of mirror and rotated images.

The jigsaw puzzle task not only supported CMP of geometry, but was also directly related to academic achievement in geometry. It is worth noting that the task we used to test academic achievement in geometry included items in which participants had to remember theorems or geometrical rules, as well as visualizing and manipulating visuospatial information to solve the geometrical problems. In contrast with the other two complex span tasks, which involved manipulating spatial locations, the jigsaw puzzle task seems the most suitable for representing operations that are also required in the task for testing academic achievement in geometry.

The second VSWM task entered in the final path model was the sequential dot matrix task, which is believed to assess passive spatial-sequential processes (Cornoldi & Vecchi, 2003). It involves recognizing increasing numbers of locations presented one after the other. This task did not directly predict academic achievement in geometry, but it did appear to be related to the CMP of geometry. The specific contribution of the test to the CMP of geometry could be due to the geometrical requirement involved in memorizing the exact sequence of successive visuospatial operations.

It is worth noting that none of the VSWM tasks was related to the CP of geometry. This may be because the CP of geometry need no support from VSWM. The CP of geometry could develop without any need for either experience or other underlying cognitive structures, as in the case of other aspects of mathematics (Spelke, 2004; Spelke & Kinzler, 2007).

A number of crucial issues would need to be considered in future research. For a start, only VSWM tasks were administered to the participants in our study, based on the assumption that VSWM processes might be stronger predictors of achievement in geometry than verbal WM processes. Further research should consider the role of verbal WM, however, given that formal education in geometry involves using verbal rules, formulas, theorems, and so on), as well as numerous other factors that presumably affect the acquisition of geometrical knowledge (Aydin & Ubuz, 2010), as indirectly demonstrated by the limited percentage of variance explained by our path models. In addition to VSWM, further studies should analyze the role of visuospatial abilities, such as spatial visualization and mental rotation skills in academic achievement in geometry. Finally, reasoning and fluid intelligence may also have a central role in accounting for a part of the variance affecting the acquisition of geometry. Second, our findings might be explained by our sample selection procedures and consequent choice of task for assessing academic achievement in geometry. As previously mentioned, we chose secondary school students because they have received the highest level of compulsory schooling in geometry, and we were thus able to study the role of both CP and CMP of geometry. It would be reasonable to expect different results when testing young children, for instance, when their cultural background and schooling would have a lower weight. Our students were also attending schools where geometry had an important role, so our findings cannot be generally applied to pupils at different types of school. Because the types of secondary school that we considered are attended mainly by boys, our sample also contained more males than females, though the only significant effect of gender was found in the dot matrix task, which was not included in our final path models. This aspect may nonetheless be a limitation of our study.

Finally, our findings also have educational and clinical implications. First of all, they can provide teachers and educators with information on which cognitive processes support students learning geometry. To give an example, knowing that complex VSWM tasks can directly predict academic achievement in geometry could help teachers to suggest activities that do not overshadow their students' VSWM capacity. Secondly, shedding light on the mechanisms influencing academic achievement could help us to understand why some students fail in geometry and how we can help them to cope with their difficulties. Thirdly, assessing visuospatial abilities in general, and VSWM in particular, could make it easier to identify children who might meet with difficulties in learning geometry later on. Consistently with these observations, research is underway to examine the cognitive deficits underlying difficulties in learning geometry. In particular, Mammarella, Giofrè, Ferrara and Cornoldi (2012) found that young children with poor visuospatial skills failed in both intuitive geometry and VSWM tasks; and Hannafin, Truxau, Vermillion and Liu (2008) found that students with weak spatial abilities performed worse than students with strong spatial abilities in terms of their academic achievement in geometry.

In conclusion, our study shows that the academic achievement in geometry of secondary school students can be predicted: (1) indirectly by VSWM tasks which support CMP of geometry; (2) directly by a complex VSWM task (the jigsaw puzzle task); and (3) by CP and CMP of geometry, the latter showing a stronger relationship with academic achievement than the former.

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Footnotes

1. To control for grade- and sex-related effects, we cleared variables of grade- and gender-related variance, by extracting statistical regression residuals in each variable and by removing the variance shared with grade and gender. These residuals were used in path models 4a and 6a. In particular, path model 4a (χ^2_M[2]=0.09, p=.96; *RMSEA*=0, *95% CI* (0,0); *NNFI*=1,14; *CFI*=1.00), and path model 6a (χ^2_M[4]=0.21, p=.99; *RMSEA*=0, *95% CI* (0,0); *NNFI*=1,15; *CFI*=1.00) provided a very good fit of the data also controlling for grade and gender.

Figure captions

Figure 1: An example of the MT advanced geometry task (Cornoldi et al., 2010).

Figure 2: Examples of each geometrical concept in the intuitive geometry task (Dehaene et al., 2006). The odd one out is shown here in the upper panel of each image for easy reference, but the real test procedure involved identifying the odd one out when it was presented in a random position among the other five images.

Figure 3: Examples of the materials used to assess visuospatial working memory.

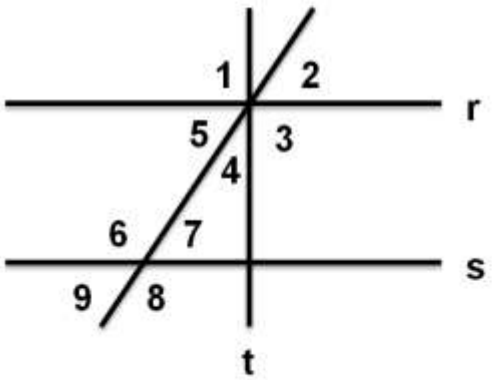
Figure 4: Conceptual diagram of path model 1.

Figure 5: Standardized solution of path model 4a.

Figure 6: Standardized solution of path model 6a.

Figure 1

'r' and 's' are parallel lines cut by a perpendicular line 't'



Which of the following is false:

(a) The sum of ∠ 3 and ∠ 4 is congruent to ∠ 8;
(b) The sum of ∠ 7 and ∠ 4 is a right angle;
(c) The sum of ∠ 6 and ∠ 5 is a straight angle;
(d) The sum of ∠ 3 and ∠ 4 is congruent to the sum of ∠ 3 and ∠ 5.

Figure 2

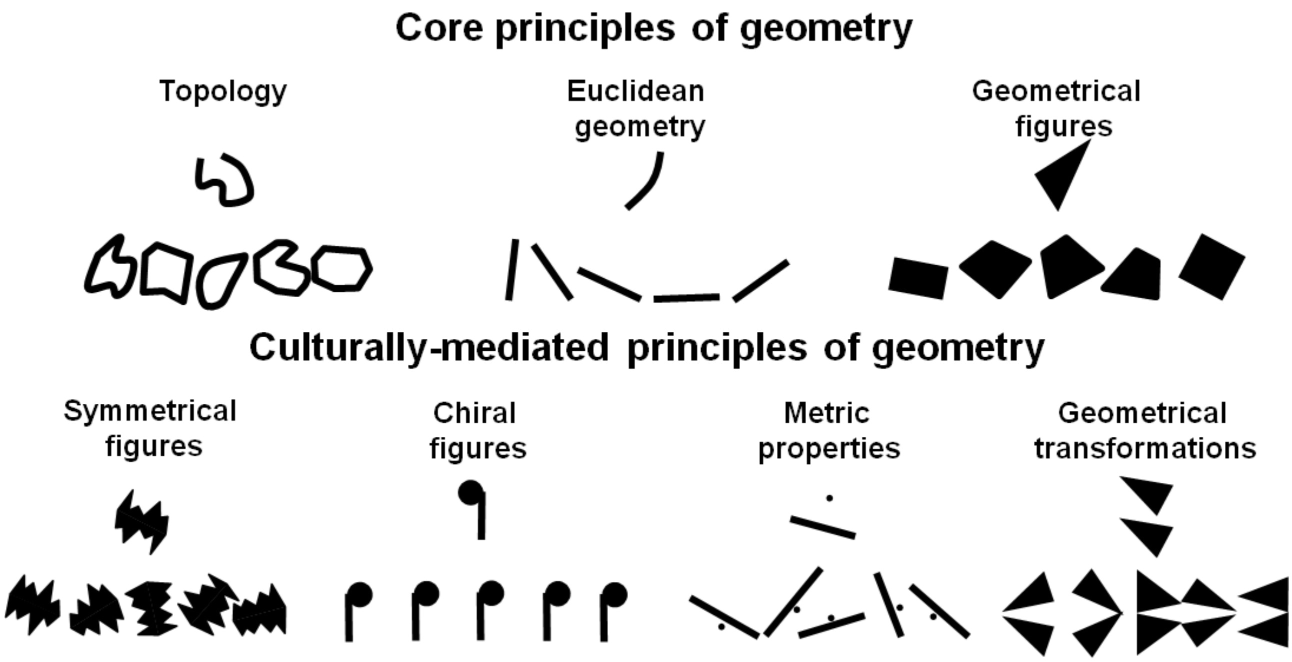

Core principles of geometry
Topology
Euclidean geometry
Geometrical figures
Culturally-mediated principles of geometry
Symmetrical figures
Chiral figures
Metric properties
Geometrical transformations

Figure 3

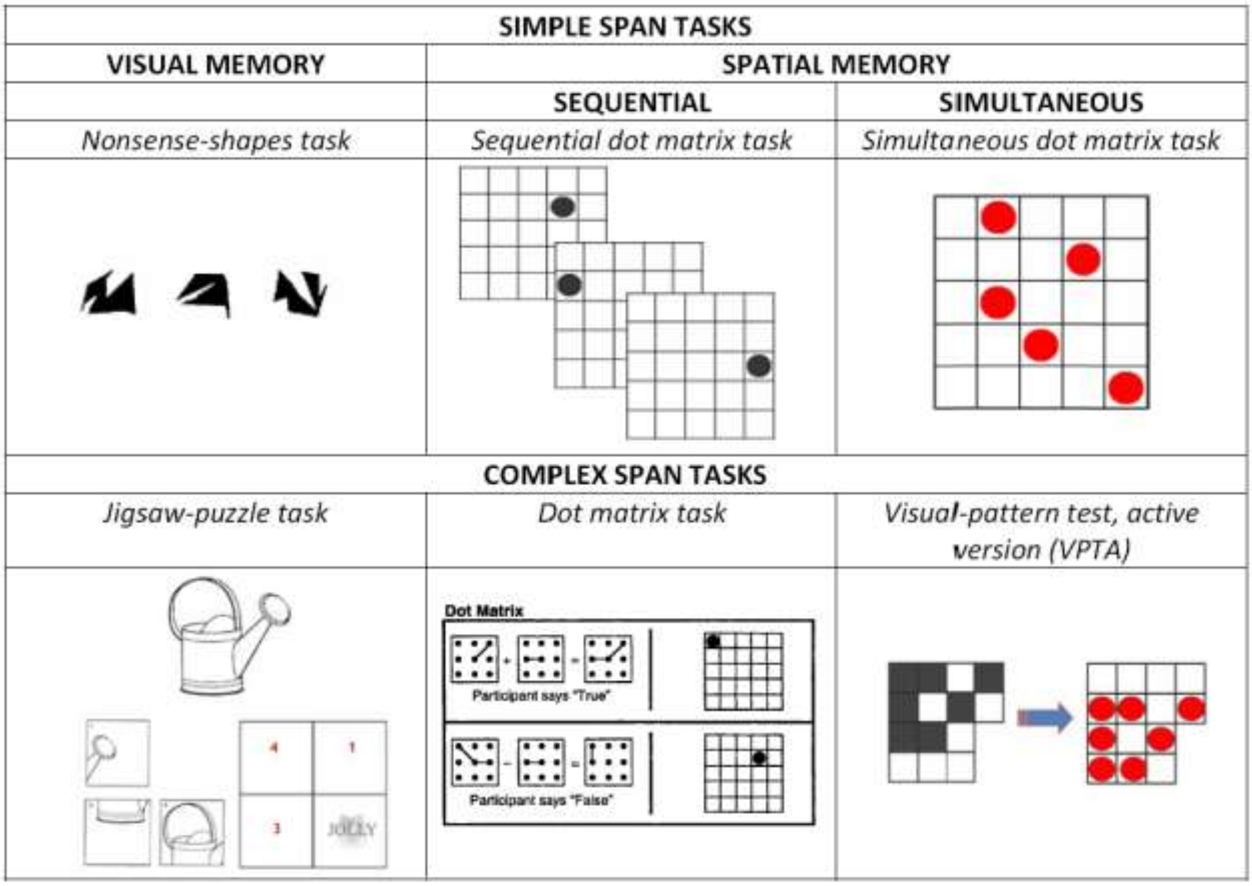

SIMPLE SPAN TASKS
VISUAL MEMORY
SPATIAL MEMORY
SEQUENTIAL
SIMULTANEOUS
Nonsense-shapes task
Sequential dot matrix task
Simultaneous dot matrix task
COMPLEX SPAN TASKS
Jigsaw-puzzle task
Dot matrix task
Visual-pattern test, active version (VPTA)
Dot Matrix
Participant says "True"
Participant says "False"

Figure 4

First step in path model 1 intuitive geometry task

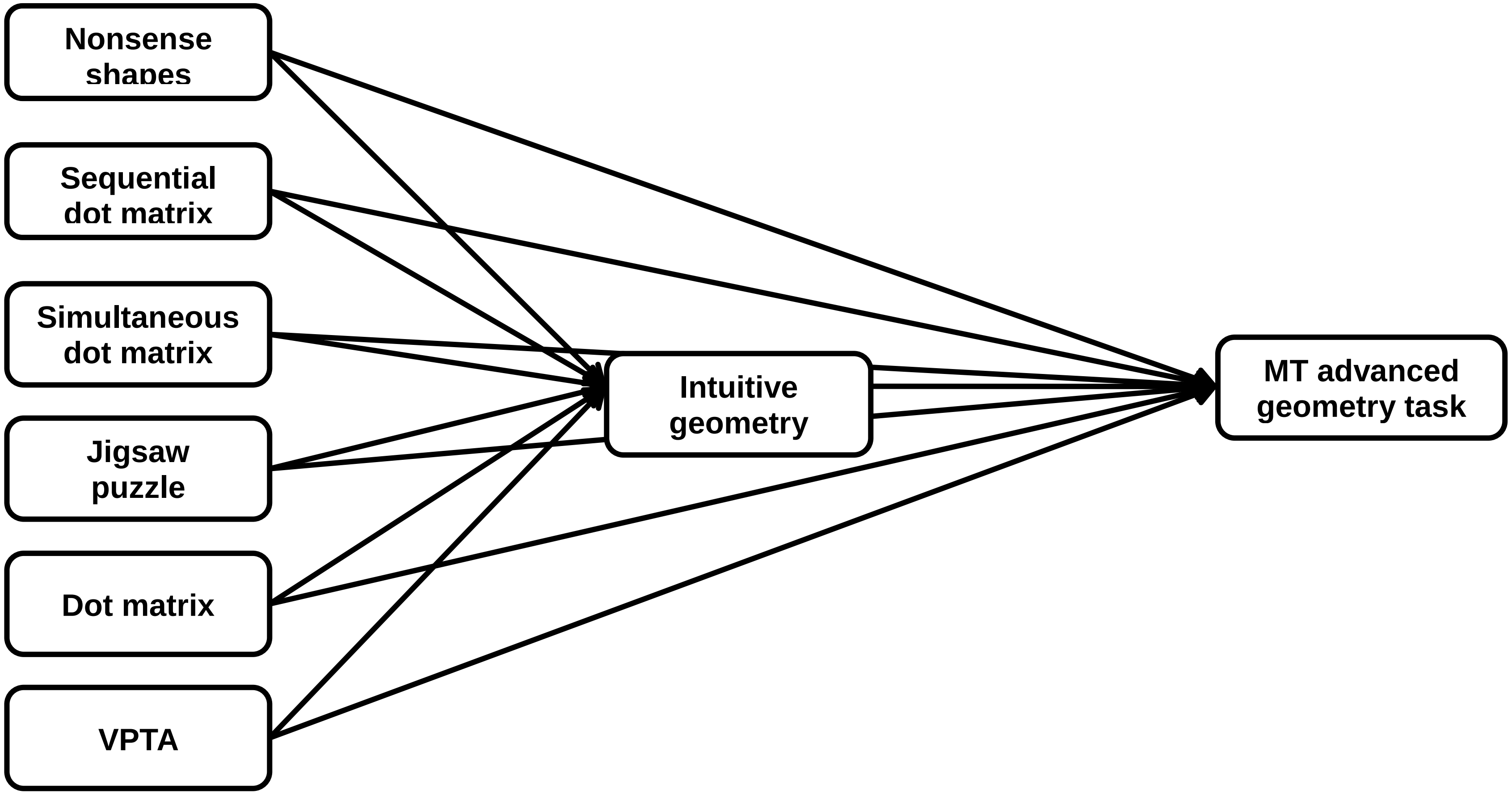


Figure 5

First step in path model 4a

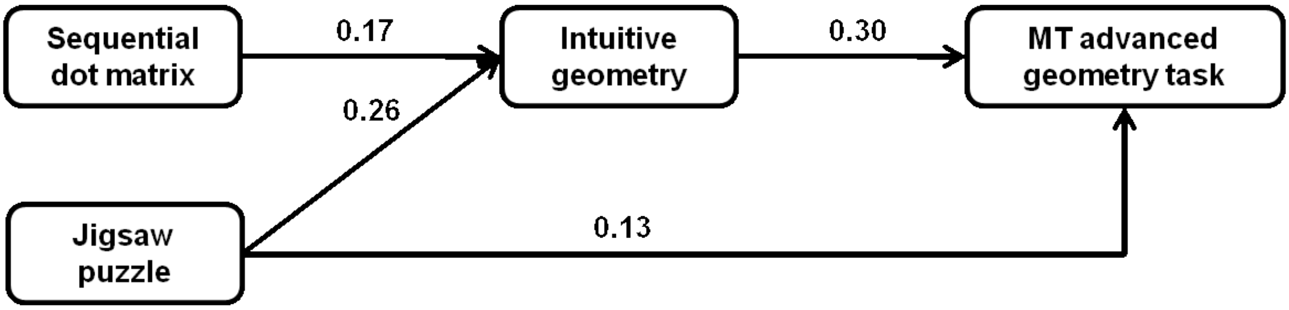


Figure 6

Second step in path model 6a

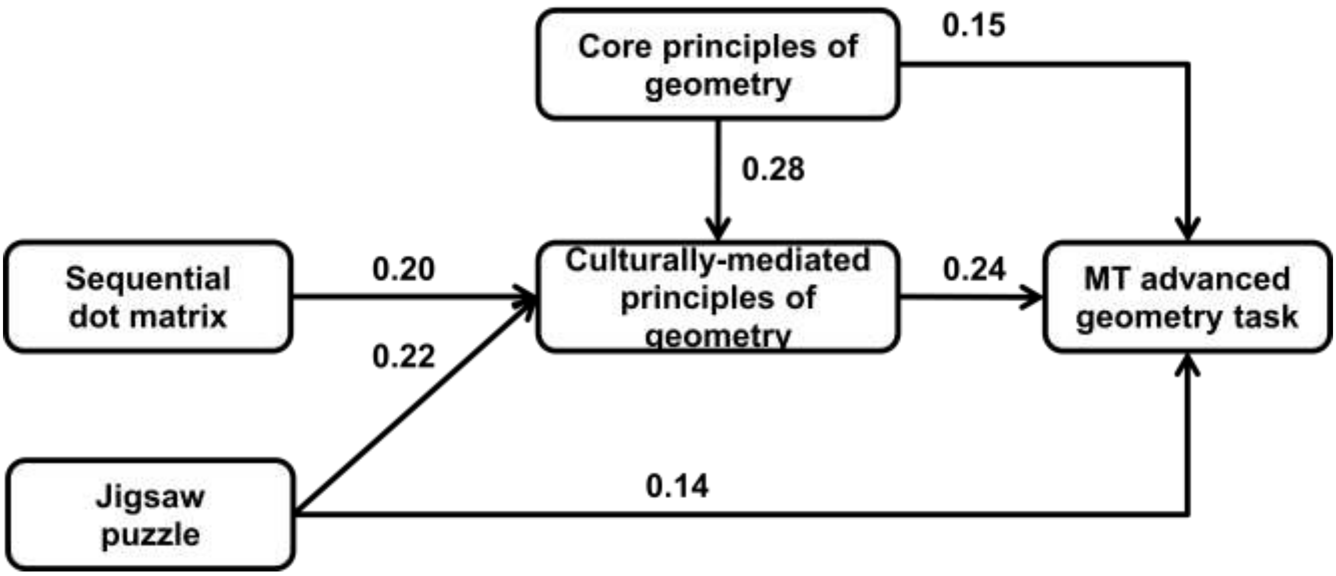


Table 1.

Descriptive statistics and reliability

	Tasks	Reliability	M	SD	Skewness	Kurtosis
Geometry	MT advanced geometry task[a]	.66	66.77	21.98	-.41	-.53
	Intuitive geometry task	.65	86.41	7.32	-.71	.46
Simple storage tasks	Nonsense shapes	.89	13.55	6.00	-.08	-.74
	Sequential dot matrix	.91	18.75	4.51	-1.12	1.77
	Simultaneous dot matrix	.90	21.20	4.80	-1.74	2.02
Complex span tasks	Jigsaw puzzle	.84	26.65	4.20	-.90	-.88
	Dot matrix task	.79	10.49	1.79	-2.02	4.49
	VPTA	.89	24.66	4.64	-.68	-33

[a] Dependent variable in percentage

Table 2.

Correlation matrix for MT advanced geometry task; intuitive geometry, core and culturally-mediated principles of geometry; and VSWM tasks

Variables	1	2	3	4	5	6	7	8	9	10
Achievement in geometry										
1. MT advanced geometry task	1									
Intuitive geometry										
2. Intuitive geometry	.35**	1								
3. Core principles of geometry	.24**	.54**	1							
4. Culturally-mediated principles of geometry	.32**	.96**	.30**	1						
Simple storage tasks										
5. Nonsense shapes	.08	.13	.01	.15	1					
6. Sequential dot matrix	.07	.19*	.00	.22**	.06	1				
7. Simultaneous dot matrix	.08	.13	.10	.12	.17*	.11	1			
Complex span tasks										
8. Jigsaw puzzle	.22**	.26**	.10	.27**	.10	.08	.10	1		
9. Dot matrix	.09	.17*	.10	.16*	.12	.17*	.04	.09	1	
10. VPTA	.14	.11	.11	.09	.03	.08	.14	.33**	0.16*	1

Note:

* *p< .05.*

** *p< .01.*

Table 3. Values of selected fit statistics for path models

Model	χ^2_M	df_M	p	RMSEA	RMSEA CI 90% LL	RMSEA CI 90% UL	NNFI	CFI
1	0	0	1	0	0	0	1	1
2	1.19	5	.95	0	0	.01	1.26	1
3	0.12	2	.94	0	0	.03	1.19	1
4a	0.005	2	1	0	0	0	1	1
4b	4.12	3	.25	.04	0	.14	.95	.97
5a	14.76	3	.002	.15	.08	.23	.37	.81
5b	0.001	2	1	0	0	0	1	1
6a	0.002	4	1	0	0	0	1	1
6b	4.05	5	.54	0	0	.09	1.03	1

Table 4: Direct and indirect effects predicting academic achievement in geometry, and total standardized regression weight (R²) in path models from 1 to 4.

Dependent variable	Independent variable	Direct effect		Indirect effect		Total
	Path model 1	*β*	*Z*	*β*	*Z*	*R²*
	Intuitive geometry	.31	3.93**			
	Nonsense shapes	.02	.33	.02	.96	
MT advanced geometry task	Sequential dot matrix	-.01	-.17	.04	1.70*	
	Simultaneous dot matrix	.02	.23	.02	1.02	.15
	Jigsaw puzzle	.11	1.36	.07	2.33**	
	Dot matrix	.01	.18	.03	1.39	
	VPTA	.07	.89	.00	-0.10	
	Path model 2	*β*	*Z*	*β*	*Z*	*R²*
	Intuitive geometry	.31	4.10**			
	Nonsense shapes			.02	.96	
MT advanced geometry task	Sequential dot matrix			.04	1.71*	
	Simultaneous dot matrix			.03	1.03	.14
	Jigsaw puzzle	.13	1.76*	.07	2.37**	
	Dot matrix			.04	1.39	
	VPTA			.00	-0.10	
	Path model 3	*β*	*Z*	*B*	*Z*	*R²*
	Intuitive geometry	.31	4.16**			
MT advanced geometry task	Sequential dot matrix			.05	1.83*	.14
	Jigsaw puzzle	.13	1.78*	.08	2.54**	
	Dot matrix			.04	1.50	
	Path model 4a	*B*	*Z*	*B*	*Z*	*R²*
MT advanced geometry task	Intuitive geometry	.31	4.17**			
	Sequential dot matrix			.05	2.03**	.14
	Jigsaw puzzle	.13	1.79*	.08	2.61**	
	Path model 5a	*B*	*Z*	*B*	*Z*	*R²*
MT advanced geometry task	Core principles of geometry	.15	2.06**			
	Culturally-mediated principles of geometry	.24	3.20**			.13
	Sequential dot matrix			.05	1.79*	
	Jigsaw puzzle	.14	1.83*	.08	2.57**	
	Path model 6a	*B*	*Z*	*β*	*Z*	*R²*
MT advanced geometry task	Core principles of geometry	.15	1.97**	.07	2.41**	
	Culturally-mediated principles of geometry	.24	3.06**			.14
	Sequential dot matrix			.05	2.09**	
	Jigsaw puzzle	.14	1.83*	.05	2.18**	
	Path model 6b	*B*	*Z*	*β*	*Z*	*R²*
MT advanced geometry task	Core principles of geometry	.15	1.90*	.07	2.57**	.12
	Culturally-mediated principles of geometry	.27	3.43**			
	Sequential dot matrix			.05	2.20**	
	Jigsaw puzzle			.06	2.30**	

Note: *p < .05 **, p < .01 (one tailed)