Predetermined Time Constant Approximation Method for Optimising Search Space Boundary by Standard Genetic Algorithm

Kumaran Rajarathianm, J. Barry Gomm, Karl O. Jones, DingLi Yu

Abstract: In this paper, a new predetermined time constant approximation (Ts_p) method for optimising the search space boundaries to improve SGAs convergence is proposed. This method is demonstrated on parameter identification of higher order models. Using the dynamic response period and desired settling time of the transfer function coefficients offered a better suggestion for initial Ts_p values. Furthermore, an extension on boundaries derived from the initial Ts_p values and the consecutive execution, brought the elite groups within feasible boundary regions for better exploration. This enhanced the process of locating of the optimal values of coefficients for the transfer function. The Ts_p method is investigated on two processes; excess oxygen and a third order continuous model with and without random disturbance. The simulation results assured the Ts_p method's effectiveness and flexibility in assisting SGAs to locate optimal transfer function coefficients.

Key words: *Predetermined Time Constant Approximation; Genetic Algorithms; Search Space Boundary Constraints; Premature Convergence.*

INTRODUCTION

Search space boundary constraint is one of the common phenomena that lead to premature convergence in standard genetic algorithms (SGAs). An optimisation process has prematurely converged to a local optimum if it is no longer able to explore other parts of the search space region than the area currently being explored and there exists another region that may contain a superior solution [1]. This is especially true when the optimum values are located near to the boundary region or outside the boundary region. Therefore, an optimum search space region is required for better exploration and to avoid premature convergence. Parameter identification of continuous higher order models where the model parameters distinguish the dynamic characteristics of system are of particular concern. Without any prior knowledge of the transfer function coefficients, it is highly infeasible to predict the search space upper (SB_{Upper}) and lower boundaries (SB_{Lower}).

Significant work has been undertaken to improve the defined search space to an optimal solution. Based on the complex Box technique, a boundary search method for optimisation problems in the case of the optimal solution at the boundary was proposed [2]. A technique for resolving the structural optimisation difficulties in quantising the subjective uncertainties of active constraints are proposed by fuzzy logic formulation [3]. A new approach called the self-adaptive boundary search strategy for penalty factor selection within SGA was proposed [4]. This approach guides the SGA to preserve around constraint boundaries and improves the efficiency of attaining the optimal or near optimal solution.

Another method to improve the prematurity and to sustain the diversity population was proposed by Niche Genetic Algorithm (NGM) associated with isolation mechanism [5]. A comparison study was done on NGM and Annealing Genetic Algorithm (AGA) where the AGA has better premature convergence [6], however it is time consuming. Another method, named Accelerating Genetic Algorithm (AGM) was proposed to resizing the feasible region into the elite individual's adjacent region for better local searching and convergence [7]. Search space boundary reduction for the candidate diameter for each link by pipe index vector and critical path method along with modified genetic operator's derivatives was proposed [8][9]. Further, an improved AGM based on the saddle distribution by which adding random individuals into initial population to increase the searching ability of optimal solution was proposed [10]. Literature survey discloses that most techniques are considered based on limited or confined search space boundaries and involves complex mathematics. Also, the discussed research information has an initial approach about the search parameter and inevitably is time consuming for convergence.

This paper introduces and investigates the predetermined time constant approximation (Ts_p) method to improve the SGAs exploration and exploitation towards global optima. This method employs a novel search space boundary extension technique by Ts_p which guides the search to concentrate on optimal value within the boundaries of the feasible region of the solution space. The structure of this paper is as follows; first, 3 SGAs convergence states for an optimal value by search space boundary constraints are discussed. Second, the approximation process of predetermined time constant methods is discussed. Further, search space boundary extensions for better exploration and for optimal exploitation are discussed here. Third, the effectiveness of predetermined time constant approximation method is assessed with two processes.

POLYNOMIAL COEFFICIENTS

Consider a system can be modelled by the general order differential equation,

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + y = K_p f(t-\theta) \quad (1)$$

where $f(t - \theta)$ is the input signal or forcing function with time delay, $y(t)$ is the output signal and K_p is process gain. Assuming zero initial condition, $y(0)=0$, $y'(0)=0$, and taking the laplace transform of equ. 1 gives the general order transfer function is of the form,

$$G(s) = \frac{Y(s)}{F(s)} = \frac{K_p}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1} e^{-\theta s} \quad (2)$$

where $a_n \ldots a_1$ are coefficients of the denominator polynomial. The denominator polynomial coefficients provide a foundation for determining a system's dynamic response characteristics. In particular the system's poles directly define the components in the homogeneous response. Thus, optimal poles identification is primarily considered here.

CONVERGENCE CONSTRAINTS OF SGAs BY SEARCH SPACE BOUNDARIES

In most situations, selecting the search space boundaries is delicate if there is no prior knowledge of optimum value location. Thus, randomly selected search space boundary is a significant factor which leads the SGAs are often converged and trapped in local optima, resulting suboptimal solutions. Particularly, it locates near the boundary or outside of boundary.

As shown in figure 1, the SGAs convergences by search space boundary constraints can be classified by three states;

- State 1 – If the optimal value (X_i) located within uniformly distributed elite group around boundary region [$X_i - \Delta_{GO}$, $X_i + \Delta_{GO}$], the genetic operators have higher probability of converging to global optimum. Thus, the randomly generated initial population within well distributed elite group search boundary has higher probability exploring and exploiting a better parent chromosome. Further, the selected parent chromosome will be evaluated by genetic precision process (selection, crossover and mutation) to produce fitter offspring without any convergence constraint.
- State 2 – If the X_i located near ([SB_{Lower}, $X_i - \Delta_{GO}$], [$X_i + \Delta_{GO}$, SB_{Upper}]), the SGAs possibly will converge to local minima. The elite group which is distributed near the boundary may have located a part of elite group at outer boundary. If the elite group at outer part may have the genetic information of optimal value, the genetic operators

will suffer to exploit the optimal value and the exploration process will retard. As a result, the search space boundary constrains will lead the SGAs to converge to local minima.

- State 3 – If the X_i located outside the boundary region [$SB_{Lower} > X_i > SB_{Upper}$], the SGAs will fail to explore and exploit the optimal value. The simulation may retarded and stopped.

where SB_{Lower} is lower search boundary, SB_{Upper} is upper search boundary and Δ_{GO} is the genetic operator for convergence precision.

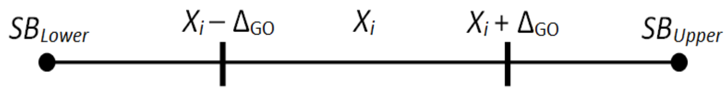

Figure 1: Schematic diagram of feasible serach space boundary region

By approximating the distribution of the elite group in a boundary region at the initial stage, gives the genetic operators opportunity to locate the optimal value rapidly without any constraint. To improve searching space boundaries for optimal model identification, a straightforward trial and error technique without a mathematical constraint is introduced here, named predetermined time constant approximation (T_{Sp}). The approximation process can be simplified as follows;

- Selecting σT_s, where σ is the settling band in %. (σ = 3, 4 and 5). The selection of desired σ is according to the raggedness of dynamic response.
- Estimating process's dynamic response period $(DR_P)_{(\tau 2-\tau)}$. At $C(t) = 0_{(T=\tau)}$ to $C(t) = 1 \pm \sigma\ (\%)_{(T=\tau 2)}$, where $C(t)$ is desired settling point.
- Approximating an initial $\tau_1 = DR_{P(\tau 2-\tau)} / \sigma$.
- Calculating initial T_{Sp} by identified τ_1 according to the respective transfer function coefficients $(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1)$.

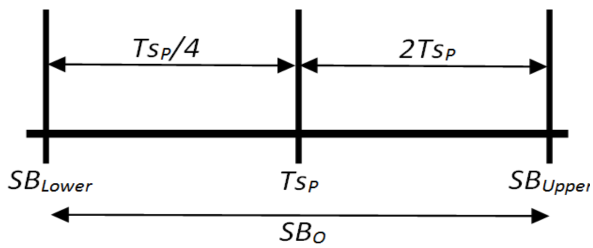

Figure 2: Optimising the search space boundary by Ts_p

where SB_O is optimum search space boundary, SB_{Lower} is lower search boundary and SB_{Upper} is upper search boundary. An optimum search space boundary can be approximated by T_{Sp}, as illustrated in figure 2 and can be expressed as;

$$SB_O = \{SB_O; SB_{Lower} \le T_{S_P} \le SB_{Upper}\} \quad (3)$$

For an SB_O, the SB_{Upper} and SB_{Lower} are extended by 100% and 75% from T_{Sp}, respectively. Especially, 100% of extension for SB_{Upper} is required as the optimal solution mostly located near to the upper boundary region. Such a search space extension is required for SGAs to explore the elite groups which are uniformly distributed within boundaries and to exploit the X_i.

SIMULATION STUDIES

To illustrate the non-complexity and effectiveness, the proposed Ts_p method is applied on two industrial processes; excess oxygen (EO_2) and 3rd order transfer function.

Process 1 – Excess Oxygen (EO_2)

A raw numerical data of EO_2 is collected from a real industrial furnace by empirical technique for 1000 seconds with 5 seconds interval. As illustrated in fig. 3, the process response of EO_2 is exhibiting the first-order plus dead-time (FOPDT) dynamic system. The data was gathered by the step input of increasing air ratio from 9.5 to 10.5 in volumetric.

As discussed earlier, the time constant (τ_S) of transfer function are primarily considered here for an optimal model identification by T_{Sp} method. Whereas, the process gain (K_p) and transport delay (θ) can be approximated by close observation on transient response. As illustrated on the transient response of EO_2, the $K_p \approx 1.54$ and $\theta \approx 160$s. As a result, an extension on the search space boundaries are approximated for $K_p \in [1:2]$ and $\theta \in [50:200]$.

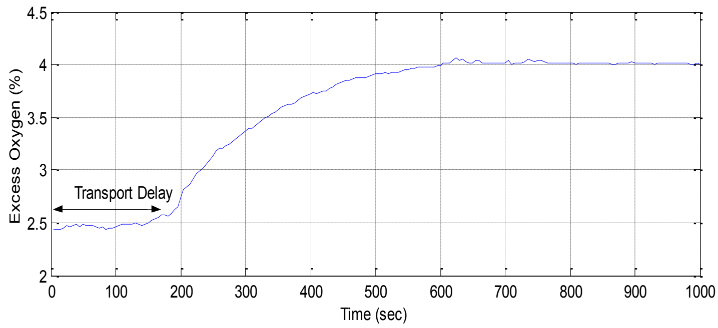

Figure 3: Step response of EO_2

According to the EO_2 response, the $DR_{P(\tau 2-\tau)}$ = 700s – 100s = 600s. Selecting σT_s = 5T_s, as the desired T_s is 1% settling band, gives the initial τ_1 as 120s. For EO_2, the selection of an optimal model is a 3rd order transfer function. Therefore, the T_{Sp} for the 3rd order polynomial coefficients can be approximated,

$$\tau_1 s = 120; - \to (T_1 s)^3 + 3(T_1 s)^2 + 3T_1 s + 1 \\ = 1.728e^6 s^3 + 4.32e^4 s^2 + 3.6e^2 s + 1 \quad (4)$$

Process 2 – 3rd Order Transfer Function

For simulation study, the transfer function of a 3rd order process is selected with process gain (K_p = 10),

$$G(s) = \frac{10}{15s^3 + 78s^2 + 6s + 1} \quad (5)$$

The particular motive of selecting this 3rd order transfer function is that it has a real pole at -5.1245 and a pair of complex poles at -0.0378 ± 0.1076i which are exhibiting a significant oscillatory response. Also, to assess the T_{Sp} method's flexibilities and effectiveness, the 3rd order transfer function coefficients are moderately small parameters. So, an appropriate search space boundary extension is required.

According to the 3rd order process step response (Fig. 3), the $DR_{P(\tau 2-\tau)}$ = 123s – 0s = 123s. Selecting σT_s = 5T_s, as the desired T_s is 1% settling band, gives the initial τ_1 is 24.6s. Therefore, the T_{Sp} for the 3rd order polynomial coefficients can be approximated by,

$$\tau_1 s = 24.6; - \to (T_1 s)^3 + 3(T_1 s)^2 + 3T_1 s + 1 \\ = 14887s^3 + 1815.5s^2 + 73.8s + 1 \quad (6)$$

DISCUSSION

Generally, the all process of search space boundary adjustment and an optimal X_i identification can be simplified as follows;

1. Initial attempt – Identified T_{Sp} according to the respective transfer function coefficients are applied with 100% extension on SB_{Upper}. The SB_{Lower} is extended to approximately 95% (10) instead 75% for better exploration at beginning stage. Execute the SGAs.
2. Second attempt – Genetically identified T_{Sp} of respective transfer function coefficients by initial attempts are extended accordingly (SB_{Upper} to 100% and SB_{Lower} to 75%) to optimise SB_O. Execute the SGAs.
3. Subsequent attempt – Continuing the SGAs execution with unchanged boundary search approximation by second attempt, until optimal X_i and minimum sum of square error (*SSE*) attained.
4. *Subsequent attempt – If the extended boundary in second attempt is not a SB_O, consecutive boundary adjustment is essential until SB_O achieved. Then, continuing the SGAs execution until optimal X_i and *SSE* attained.

Simulation Results of EO_2

As illustrated in table 1, the SGAs is explored well the entire search space boundaries and exploited the elite group within boundary region $[X_i - \Delta_{GO}, X_i + \Delta_{GO}]$ for T_{Sp} values of S^2 and S^1 at initial attempt. This can be seen the consistency of the T_{Sp} values of S^2 and S^1 in further execution with readjusted boundaries at 2nd attempt. This has enhanced the exploitation an optimal X_i at each subsequent attempted by SGAs.

Table 1: Simulation Results of EO_2 Executions

Execution Cycle	***S^3***		***S^2***		***S^1***		***Ts_p (S^3)***	***Ts_p (S^2)***	***Ts_p (S^1)***	***SSE***	***Gen***
	SB_U	***SB_L***	***SB_U***	***SB_L***	***SB_U***	***SB_L***					
1	3.5e6	10	8.6e4	10	7.2e2	10	8088.2	10085	178.73	0.86796	70
2	1.6e4	2e3	2e4	2e3	3.5e2	40	4039.7	14074	180.02	0.49128	20
3	1.6e4	2e3	2e4	2e3	3.5e2	40	2699.7	13304	180.38	0.51873	40
4	1.6e4	2e3	2e4	2e3	3.5e2	40	4875.7	14995	183.64	0.49413	40
5	1.6e4	2e3	2e4	2e3	3.5e2	40	8187.7	14524	181.41	0.48654	20
6	1.6e4	2e3	2e4	2e3	3.5e2	40	8079.1	16513	184.16	0.53421	35
7	1.6e4	2e3	2e4	2e3	3.5e2	40	4330.5	14555	177.2	0.5109	90
8	1.6e4	2e3	2e4	2e3	3.5e2	40	4137.2	15028	181.88	0.48758	22
9	1.6e4	2e3	2e4	2e3	3.5e2	40	9903.9	16043	182.3	0.51771	80

Based on initial attempt, the elite groups of T_{Sp} value of S^3 are uniformly distributed around $X_i - \Delta_{GO}$ region. As illustrated table 1, the T_{Sp} value of S^3 is still continuously evolving within boundary SB_O region at each execution. Therefore, further readjustment on SB_O boundaries is not required as the elite groups are still within the boundary range (state 1) as discussed section 3. For 3rd order model of EO_2, the T_{Sp} values by 5th iteration are selected as the *SSE* and *Gen* (generation) is minimum and optimal.

However, the inconsistency of S^3 shows that there are two global optimal X_i (X_i = 8187.7; 4137.2), which are frequently appears within the SB_O region at 1st, 2nd, 4th, 5th, 6th, 7th and 8th iterations. This has been verified by simulation results in Fig. 4 and 5 of both global optimal X_i values of S^3 and minimum *SSE*.

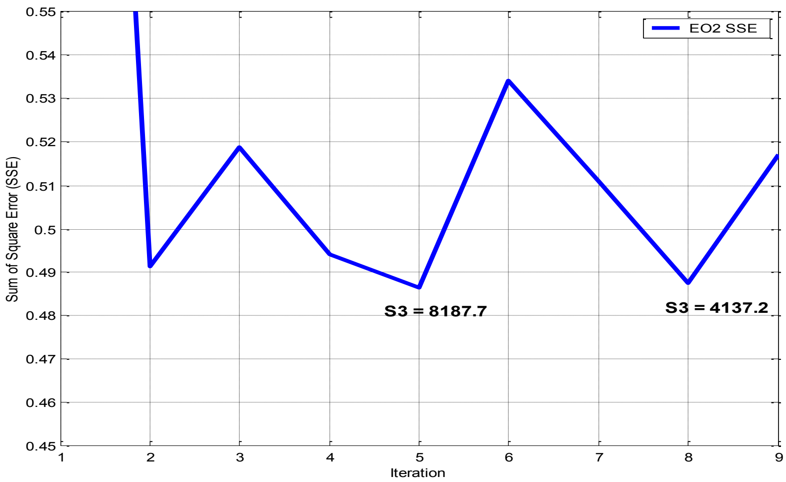

Figure 4: Two optimal values of S^3 for EO_2

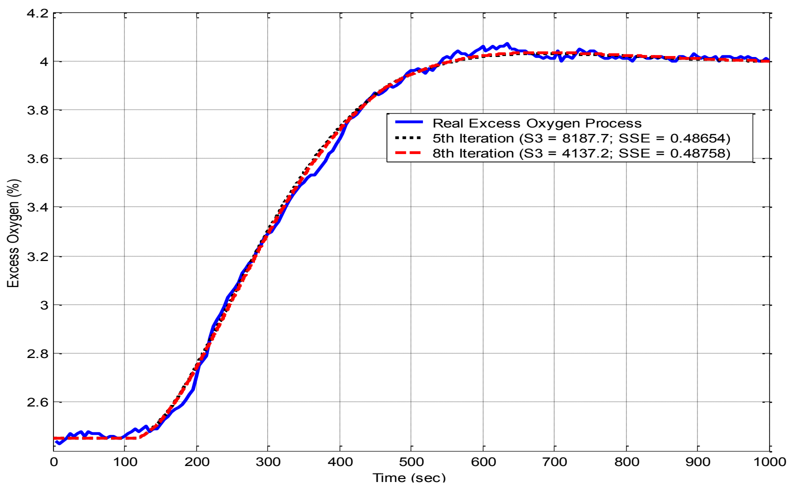

Figure 5: Transient responses of 2 global optimal values with real process of EO_2

$$G(s)_{EO_2} = \frac{1.555}{8187.7s^3 + 14524s^2 + 181.41s + 1} e^{-109.36s} \quad (7)$$

Simulation Results of 3rd Order Transfer Function

According to the table 2, the distribution of elite groups within boundary region [$X_i - \Delta_{GO}$, $X_i + \Delta_{GO}$], the exploitation of optimal X_i and the consistency of the T_{Sp} values of S^2 and S^1 in further execution by SGAs are exhibiting similar process characteristics as EO_2.

On other hand, the simulation result reveals that the elite group of T_{Sp} values of S^3 are distributed near to SB_{Lower} region. This is clearly noticeable at 1st, 2nd and 3rd iterations results that the T_{Sp} value of S^3 is remain exploiting at SB_{Lower}. This caused the SGAs suffered to exploit an optimal X_i and converged to local minima as a part of elite group is located at outside of SB_{Lower} (state 2). As a result, 3 adjustments on boundaries, especially on SB_{Lower} are required to optimise the SB_O and to bring the elite groups within feasible boundary region. As expected, the boundaries are optimised and explored the elite groups well at 4th iteration. Further SGAs execution enhanced an optimal X_i exploitation.

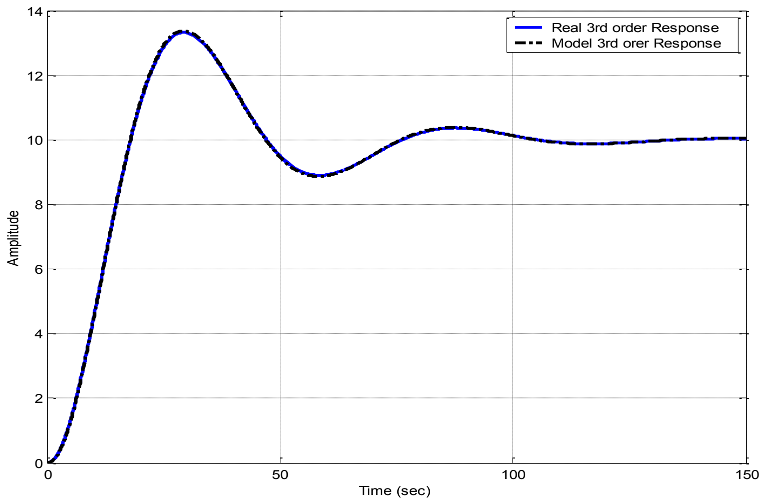

Figure 6: Transient response of 3rd order transfer function real and model process

Table 2: Simulation Results of 3rd Order Transfer Function Iterations

Iteration	S^3		S^2		S^1		Ts_p (S^3)	Ts_p (S^2)	Ts_p (S^1)	SSE	Gen
	SB_U	SB_L	SB_U	SB_L	SB_U	SB_L					
1	29774	10	3630	10	98	0	141.3	76.75	7.439	60.092	70
2	280	35	150	20	15	2	42.55	77.73	6.281	8.4924	50
3	85	12	150	20	15	2	23.25	77.67	6.182	7.7894	30
4	50	5	150	20	15	2	22.98	77.69	6.179	7.7899	20
5	50	5	150	20	15	2	21.23	77.67	6.157	7.8149	20
6	50	5	150	20	15	2	22.18	77.67	6.189	7.7915	30
7	50	5	150	20	15	2	21.98	77.68	6.197	7.6025	25
8	50	5	150	20	15	2	21.41	77.69	6.171	7.6171	35
9	50	5	150	20	15	2	23.53	77.67	6.186	7.7898	25
10	50	5	150	20	15	2	22.62	77.68	6.175	7.7914	15
11	50	5	150	20	15	2	23.49	77.69	6.183	7.7895	20

Table 3: Simulation Results of 3rd Order Transfer Function with 5% Disturbance Iterations

Iteration	S^3		S^2		S^1		Ts_p (S^3)	Ts_p (S^2)	Ts_p (S^1)	SSE	Gen
	SB_U	SB_L	SB_U	SB_L	SB_U	SB_L					
1	29774	10	3630	10	98	0	380.4	82.03	11.27	150.832	90
2	760	95	165	20	22	3	95.15	77.78	6.296	60.1486	78
3	190	24	155	20	13	2	25.29	77.57	6.211	33.4558	43
4	50	6	155	20	13	2	24.02	77.57	6.196	33.4456	37
5	50	6	155	20	13	2	24.67	77.58	6.049	33.4481	32
6	50	6	155	20	13	2	24.05	76.33	6.398	33.4452	28
7	50	6	155	20	13	2	26.14	77.91	6.215	33.4627	22
8	50	6	155	20	13	2	24.25	77.51	6.198	33.4459	30
9	50	6	155	20	13	2	22.99	77.58	6.186	33.4503	21
10	50	6	155	20	13	2	22.89	77.58	6.183	33.4511	42
11	50	6	155	20	13	2	22.76	77.84	6.114	33.4596	34

Further, the flexibilities and effectiveness of T_{Sp} methods is assessed on 3rd order transfer function model with 5% disturbance. Initially identified transfer function coefficients without the disturbance are applied on 3rd order model with disturbance. The simulation result in figure 7 reveals that the exploration of elite groups and exploitation of an optimal X_i for 3rd order model with disturbance is immensely similar process with 3rd order model without disturbance.

Thus, the effectiveness of T_{Sp} methods is well demonstrated in optimizing the SB_O and exploiting the X_i with or without disturbance. By comparing the identified T_{Sp} coefficients with 3rd order transfer function model's coefficients, the S^2 and S^1 values have 99% similarity. But, the S^3 value only has 54% of similarity. Nevertheless, the identified model responses, with and without noise, closely match the response of the actual system as illustrated in figure 6 and 7. Based on minimum *SSE*, the selected 3rd order model transfer function without disturbance is;

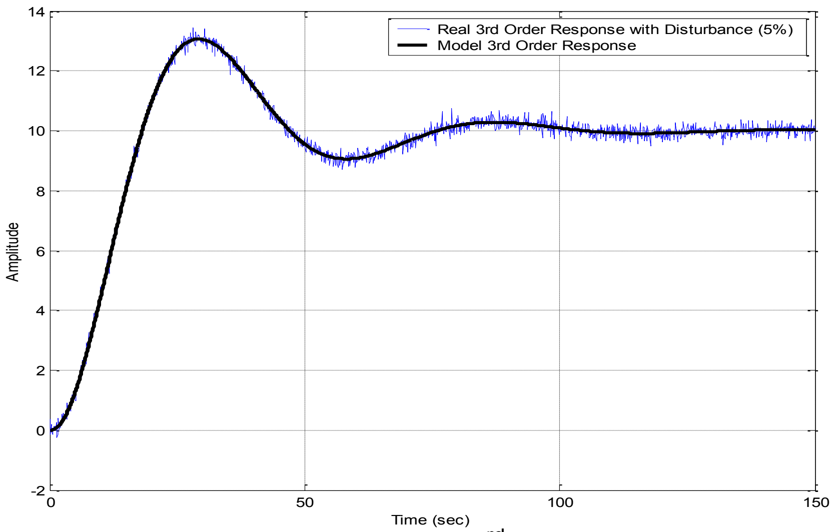

Figure 8: Transient responses of 3rd order transfer function real and model with 5% disturbance

$$G(s) = \frac{9.997}{21.98s^3 + 77.68s^2 + 6.197s + 1} \tag{8}$$

and with 5% disturbance is;

$$G(s) = \frac{9.976}{24.05s^3 + 76.33s^2 + 6.398s + 1} \tag{9}$$

CONCLUSIONS AND FUTURE WORK

The proposed predetermined time constant (T_{Sp}) method enhanced the optimization of search space boundaries for global optima convergence. The response's dynamic period and settling time provide better presumption of an initial T_{Sp} for search space optimisation. The extended SB_{Upper} and SB_{Lower} for an optimal search boundary (SB_O) derived from initial T_{Sp} brought the elite group within a feasible bounded search region. Further, SGAs execution improved the exploration of elite groups to locate exploit the optimal values for the identified model parameters. As expected, the polynomial coefficients (for S^1, S^2 and S^3) of both (EO_2 and 3rd order TF) processes are optimised well by SGAs with optimised boundaries. Future work will be carried out on designing the Matlab coding for automatic self adjusting boundary and identification of parameters of more complex models with poles and zeros.

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