The Assessment and Control of Risk of Collision

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July-1984

To the next generation-

Ahmed and Hatem Abou El-Atta

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Waddah Abou El-Atta
1984

UNIT FOUR

EQUI-POTENTIAL AREA OF RISK

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UNIT SIX

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ABSTRACT

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This thesis, as its title implies, is a unique step in the management of risk of collision, which may arise in a two-ship encounter. This work advances the development established in the author's M . Phil thesis. It is suggested that the contemporary technique of collision avoidance based solely on CPA criteria is inadequate for risk analysis. The proposed strategy for handling risk of collision revolves around the introduced hypothesis for dealing with risks having various probabilities of occurrence resulting in various degrees of severity. The risk values are obtained by computing the geometrical probability of collision based on the following definition :-

> " The risk of collision can be measured by the ratio of the ways available for a collision to occur to all the possible ways that could be considered by the observing vessel."

Based on this hypothesis several approaches to the presentation of the risk of collision are described separately in the thesis's units together with their application, merits and demerits.

It is found that the introduction of the assessment and control of the risk of collision by means of the proposed risk criteria has converted the vague awareness given by the traditional methods, to a definite risk criteria which could provide alternative ways of assessing any situation. The restructuring of the information clearly provided to the mariner gives him a much greater insight into the level of the risk which he is accepting in any situation.

A strong risk controllability can be achieved if it is characterised by a relatively high degree of constraints in the form of regulation. These regulation should acquire features that will permit them to discriminate, act upon, and respond to aspects of the situation variety. Due to the fact that certain statements within the International Rules are not clearly defined and are thus open to individual interpretation, some mathematical definitions of risk of collision and close range situations are established. The analysis and testing of specific examples has proved that these methods work and are able to provide the mariners with a common language in resolving collision avoidance problem.

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Background

The usual question asked regarding the impact of the Automatic Radar Plotting Aid (ARPA) upon the collosion avoidance problem is:

> " Will the Application of ARPA to marine use provide a lasting answer to the collision avoidance problem ?"

Although existing APRA'S are an improvement on the reliance on radar for collision avoidance, they also have limitations. All existing systems incorporate a static vector analysis methodology for assessing risk. The concept is based on the calculation of risk via measurement of the closest point of approach (CPA), and time to CPA (TCPA). These methods are useful guides in some circumstances, but they may lead to large, persistent biases with serious implications for decision making. There is still a need for a comprehensive risk model in order to clarify the objectivities of this study. It is valuable to characterize the (C/A) problem and to give a brief review of the current method used to assess the risk of collision.

One way of characterizing the collision avoidance problem is illustrated in fig. (i-a). In this figure a real situation and a conceptual situation have been identified.The external world of the problem is the reality, wherein another ship proceeds towards the observing vessel and enters the range of observation. The navigator of the observing vessel takes note of the compound event of the two ship encounter involved in a possible collision.

The conceptual world is the world of the intellect. This is the world which is within the navigator's mind and which he uses when trying to understand what goes on in the other real world. This conceptual world has been divided into three stages, observations (tracking), processing (data transformation) and risk predictions. The first stage deals with observations, in which radar equipment, gyro compass and speed log are used. This stage of the problem handling is devoted to the gathering of raw data, which is a chronological set of target's relative positions;

$(\sum_1^n \; B_i, R_i, T_i)$

Where

R = Range of other ship.

B = Bearing of other ship.

T = Time of observation.

i = 1 , 2 , 3 ....n

The information retrieval process can be done most sensibly, and most efficiently if the navigator has some idea of what he is looking for, that is if he has some mathematical model that can be used as a guide to tell him what to expect from the existing situation.

The second major stage in the development of the conceptual part of the problem is the development of the collision risk model to analyize situations. The risk model is of central importance for advancing the observations as well as making predictions. Regarding the existing collision-avoidance systems, the risk model is based on prediction of CPA and TCPA. Given a chronological set of other ship relative positions together with course and speed of the observing vessel (Co,Vo) the computer then uses these data to provide a graphic display of 'risk' information.

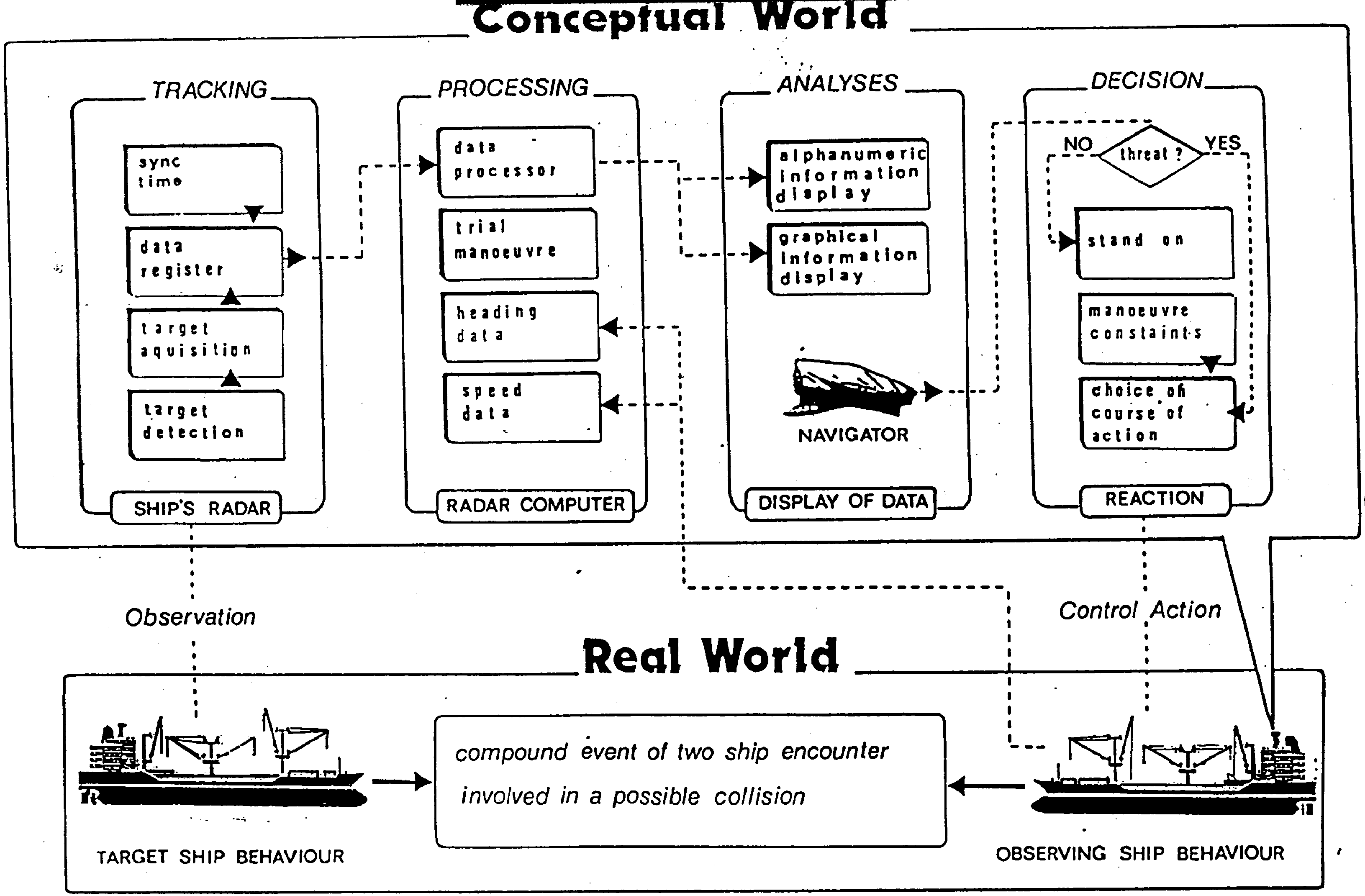


Figure (i-a) Collision avoidance system analyses.

To provide these functions requires the processor to compute periodical (CPA) and (TCPA) values for the other ship, and warn the navigator if these values fall below a selected safety criteria. At the same time, the position of the tracked targets, their true or relative speed vectors are presented on the screen attached to each target position. The end of the vector indicates where the target will be in a few minutes as set by a vector length command. The vector length can be varied to extend its displacement to pass the display origin, and thereby indicate CPA and TCPA, as shown in (FIG. ii-a).

The final stage is that of prediction, and as it has been already seen, prediction is intricately tied into the observation and modelling processes. Risk models can also be used to provide information about situations that have not been observed in more concrete terms. The model allows the observer to analyze the effect of proposed course and/or speed change on the situation before it is executed. The navigator proposes a certain evasive manoeuvre (course and/or speed change) and the system indicates, either by means of relative vectors or in a speeded-up motion on a radar screen, if the consequences of the manoeuvre are safe, that is to say whether it keeps the target at or beyond a certain minimum distance. If it does not, he has to search for a better solution by trial and error. This can be done with great confidence if the navigator is careful to remember the assumptions inherent in the model. Thus in the assessment process, a risk model plays a crucial role, not only in the search for an optimum solution, but also in the kind of modified predictions which allow the navigator to assess the consequences of his action in advance.

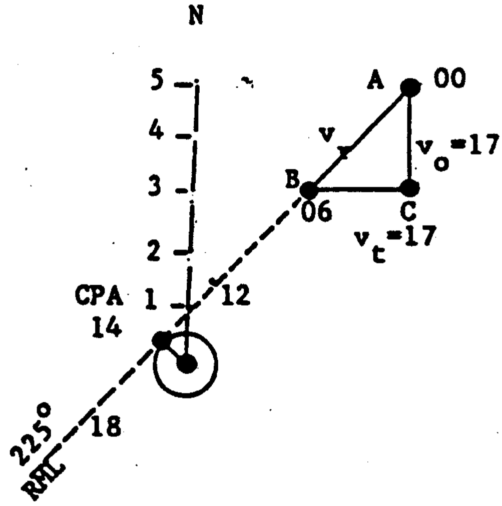
In the absence of comprehensive risk assessment, the navigator may increase or decrease the perceived risk of collision.
A consideration of each navigator's capabilities and limitations thus becomes critical to a successful collision - avoidance system operation.

DETERMINATION OF (CPA),(TCPA) AND TARGET TRUE MOTION VECTOR BY MEANS OF PLOTTING ON RELATIVE MOTION.

Target ship's position at zero time is marked at (A), and time starts counting in minutes.

A relative marker remains at (A), while target moves to (B) in (6) minutes, and the true motion vector of own ship in (6) minutes interval is plotted at (A) from (C), forming the velocity triangle.

AB =Relative motion vector 24 Kt
CA =True motion vector of own ship 17 Kt.
CB =True motionvector of target ship 17 Kt.
CPA =Closest point of approach read-out of 0.5 n.mile.
TCPA=Time to CPA is measured to be 14 minutes.
RCo =Relative course.



DETERMINATION OF EVASIVE COURSE OR SPEED TO CLEAR THE TARGET AT SAFE CPA.

For course change, rotate own ship vector around point (C) till the resulting relative motion line becomes tangent to the safe CPA circle.in this case vector CA' (40°/17 Kt). New relative motion vector A'B= 30 kt.
New CPA chosen = 1.5 n.m
New TCPA indicated by 6 minutes time segments is = 12 minutes.
A speed reduced to 12 Kt,(position A" on the original vector of own ship), gives the same safe CPA as that of the 40° alteration to starboard

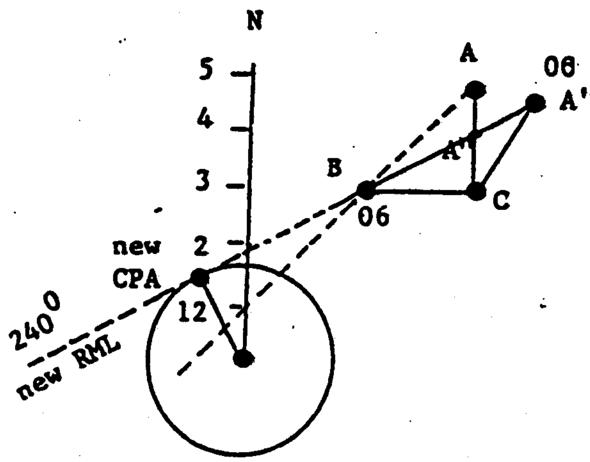


Figure (ii-a) Conventional method of assessment and control risk of collision.

In this field there are obviously many subleties and philosophical points that can be the subject of endless debate. It is not the intention of this background to compress into a few paragraphs the entire subject matter of the collision avoidance problem. However, this background is important because it sets the stage for what this thesis specifies. The main concern in this work is with the proper risk modelling process, and in particular with the introduction of a new set of criteria for risk assessment and control and how they can be used as predictors in the collision avoidance context.

At the present time there is a considerable interest in the mariner's role as a decision maker. That the navigator must be an analyst, evaluator and a user of navigation aids in any given situation is self-evident. Prior to acting in any situation the navigator must engage in an intellectual process for which risk assessment is an important input. The contemporary collision avoidance problem is tackled by reducing it to the simpler one of increasing CPA. This thesis suggests that such simplicity is inadequate for risk analysis, the danger being that in relying solely on these elementary criteria the navigator may not realise how little he knows, and how much additional information is needed to successfully resolve the encounter.

Hypothesis

Decision-making under certainty, that is, when all the outcomes are known, is difficult when each alternative choice is characterised by several attributes which are not directly comparable. Considering the development within the closing phase of a two-ship encounter, there is a necessity for risk criteria which related threats to each other by degree of importance or "priority". The integer scale can serve as the simplest example, for with any two distinct integers, it is feasible to determine which of them is greater than the other. This is the case where the value dynamics of developing risk within an encounter are arranged in order of magnitude.

Probability is an important variable in the discussion of risk. The measurement of probability has many forms which range from the classical notion to the subjective or judgemental view. Regarding the case of a binary encounter, the estimates of the risk of collision might be formal and objective. Hence, the classical notion of the collision probability which was previously established in the author's M.Phil thesis is probably most appropriate for evaluating the risk of collision. To satisfy this need and explain this point of view, the following general relationship is stated to define the risk of collision as:

> *"The risk of collision can be measured by the ratio of the ways available for a collision to occur to all the possible ways that could be considered by the observing vessel."*

Then the hypothesis of the risk of collision is

$$P = M/N \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where

- P = The risk of collision in a given time and space
- M = The ways available for a collision to occur.
- N = All the possible ways that could be considered by the observing ship.

The definition of (P) is inappropriate if (M) is greater than the value of (N); this condition must always be fulfilled in practice. Thus risk function is always a number between (0) and (1) or $0 < P < 1$; for the purpose of this thesis it would be better if it is expressed as percentage, $0 < P < 100\%$.

Based on relation (1) a maximum risk matrix can be constructed. It is simply a measurement of the maximum risk as a cross tabulation of the alternatives (C) and (V), i.e. all the courses and speeds possible to the observing ship.

$$
\begin{array}{ccc}
 & & \begin{array}{cccccc} C_1 & C_2 & & C_j & & C_n \end{array} \\
 & & \text{Course alternatives} \\
\begin{array}{c} V_1 \\ V_2 \\ V_i \\ V_m \end{array} & \text{Speed alternatives} &
\boxed{\begin{array}{cccccccc}
P_1 & P_2 & \cdot\cdot & P_{1j} & \cdot\cdot & P_{1n} \\
P_{21} & P_{22} & \cdot\cdot & P_{2j} & \cdot\cdot & P_{2n} \\
P_{i1} & P_{i2} & \cdot\cdot & P_{2j} & \cdot\cdot & P_{in} \\
P_{m1} & P_{m2} & \cdot\cdot & P_{mj} & \cdot\cdot & P_{mn}
\end{array}}
\end{array}
$$

Risk matrix

The cells in the maximum risk matrix represent the maximum value of the collision probability outcome that results from the choice of a particular course and speed as defined by risk transition distribution derived in the M. Phil thesis.

Having defined the maximum risk matrix, which represents all the possible outcomes of maximum risk for each alternative, then this matrix will be the criterion for selection. Hence the object of risk control is to minimize the maximum risk, and the hypothesis of risk control will be :

$$\text{Minimize} \quad P_{ij} = (M_{ij} / N)$$

UNIT_1

Foundations

1.1 INTRODUCTION

This unit is devoted to detailing tools of mathematical expressions that will be useful in developing and checking mathematical abstractions of the physical or otherwise "real" world of the Collision Avoidance (CA) problem. Questions concerning the nature of the basic concept of CA problem or their relation to the real world do not belong to the mathematical expressions concerned. The following foundations only lay down certain relationships between the fundamental concepts of the problem. The material will be used as the basis of risk modeling and analysis. A number of derived equations describing the two-ship encounter and the related parameters are summarized in the following table and explained in detail later:-

1 - Navigation Coordinate System (NCS).
2 - Equations of Relative Motion.
3 - Equations of True Motion.
4 - Equations of Closest Point of Approach (CPA).
5 - Equations of a Buffer Circle.
6 - Equations of Range and Bearing Rates
7 - Equations of Changing Position on a Moving Ship.
8 - Equations of Exact Collision.
9 - Equations of Collision Probability.
10- Equations of the turning circle.

1.2 NAVIGATIONAL COORDINATE SYSTEM (NCS)

The main idea of the (NCS) is that geometric investigations of collision avoidance problem can be carried out by means of algebraic calculations. This method provides a direct application of navigational observations. The connection between the plane motion of ships as sets of points and (NCS) may be established in accordance with the following:

A line in the plane, extending indefinitely upward and downward is chosen, and is defined to be the North-South direction; called the X-axis. A line at right angle to this axis extending indefinitely to the left and to the right is termed the Y-axis. A point of origin (O) on the intersection of these axes represents the position of the observing ship. The two axes divide the plane into four quadrants, called the first quadrant, second etc., following the direction of azimuth measurement, as in fig.(1.1).The

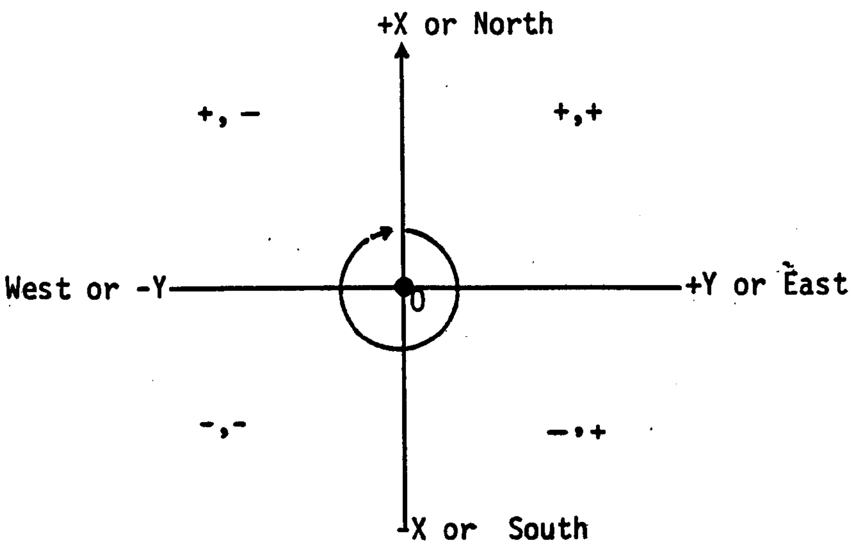
position of other ship (A) may be defined in the plane either by rectangular coordinates (X, Y) or by polar coordinates are given as follows:-

$$X_a = R. \cos (B)$$

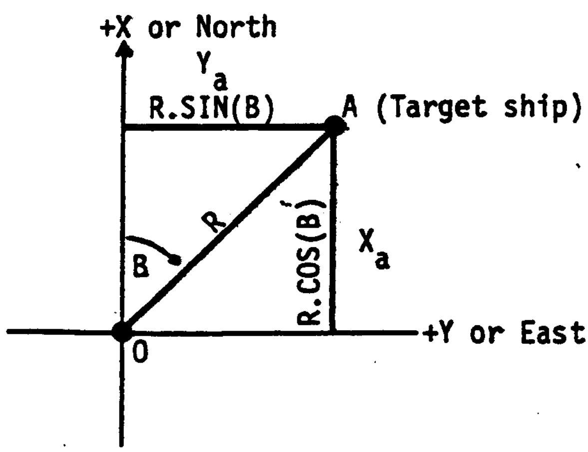
$$Y_a = R. \sin (B)$$

The angle (B) is called the true bearing or azimuth; it can take values from 0^{o} up to 360^{o} through which the axis N-S must be rotated in a clockwise direction so as to pass through (A). The length OA = R is called the range, as in Fig. 1.2. It can be seen from the unit circle, in particular, that all possible combination of signs of X and Y can be obtained in the various quadrants by making B take all values from zero to 360^{o} (Fig. 1.2).

Frequently it is desirable to change from one coordinate system to another, as the parallel displacement of the NCS and a rotation through an angle equal to the ship's true course. Hence a true position can be related to ship's horizontal axes and conversely.



Figure(1-1)
Navigational Coordinate System.



Figure(1-2)
Relation between cartesian and
polar Nav.coordinate system.

Two navigational coordinate systems NCS_0 with coordinates X and Y and NCS with coordinates ξ and η are related in such a way that the corresponding axes are parallel to one another and the origin A of NCS has coordinates (a,b) in NCS_0, (see Figure (1.3)). The same point P then has coordinates (X , Y), in NCS_0 and (ξ , η) in NCS ,where:-

$$
\begin{aligned}
X &= a + \xi \\
Y &= b + \eta
\end{aligned}
\qquad 1.2
$$

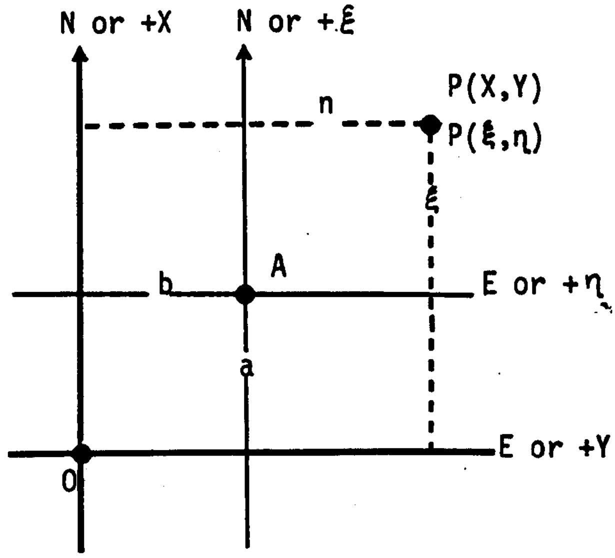
$$
\begin{aligned}
\xi &= X - a \\
\eta &= Y - b
\end{aligned}
\qquad 1.3
$$

Suppose that the NCS is rotated (keeping the origin fixed) in the navigational positive sense through an angle (B) into a (ξ , η) in the new system, (See figure (1.4)). Hence the NCS satisfies the following transformation equations:

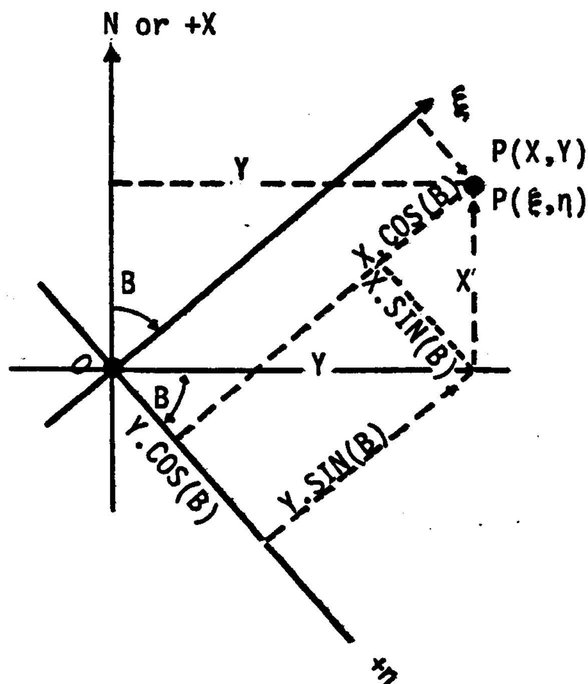
$$
\begin{aligned}
X &= \xi . \cos B - \eta . \sin B \\
Y &= \xi . \sin B + \eta . \cos B
\end{aligned}
\qquad 1.4
$$

$$
\begin{aligned}
\xi &= X . \cos B + Y . \sin B \\
\eta &= - X . \sin B + Y . \cos B
\end{aligned}
\qquad 1.5
$$

The formula of X and Y are obtained by rotating the (ξ , η) system through an angle (-B).



Figure(1-3)
Parallel transformation of Navigational
Coordinate System.



Figure(1-4)
Rotation of the Navigational Coordinate System.

.3 EQUATIONS OF RELATIVE MOTION

In nearly every problem that involves moving ships, it is essential to find the course and speed of one ship relative to another. To an observer on board ship 0 the apparent track of (A) is the segment connecting the two positions of (A). This apparent movement as seen in Fig. (1.5), is called the relative movement. Provided that (X_1, Y_1) and (X_2, Y_2) are the coordinates of the two positions (A_1) and (A_2), then they have changed by the increments (DX and (DY), then:

$$DX = X_2 - X_1$$

$$DY = Y_2 - Y_1 \qquad 1.6$$

Using the pythagorean theorem the distance travelled (S_r) can be found as seen in Fig. 1.5:-

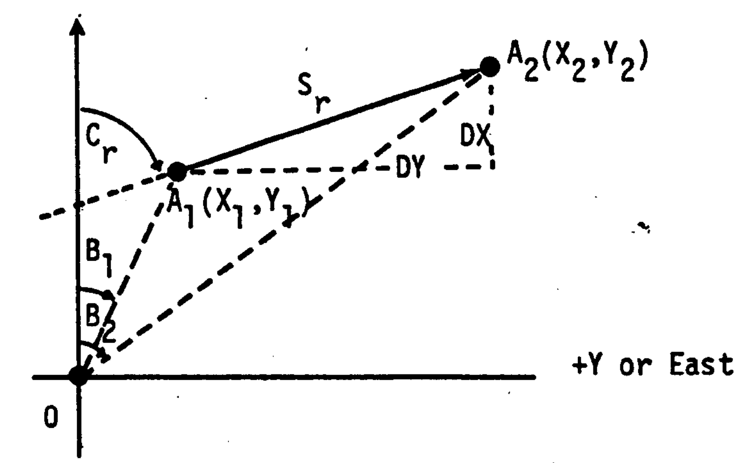
$$S_r = (DX^2 + DY^2)^{\frac{1}{2}} \qquad 1.7$$

If Dt is the interval of time, the position has undergone a displacement (S_r), then the relative speed is given by:-

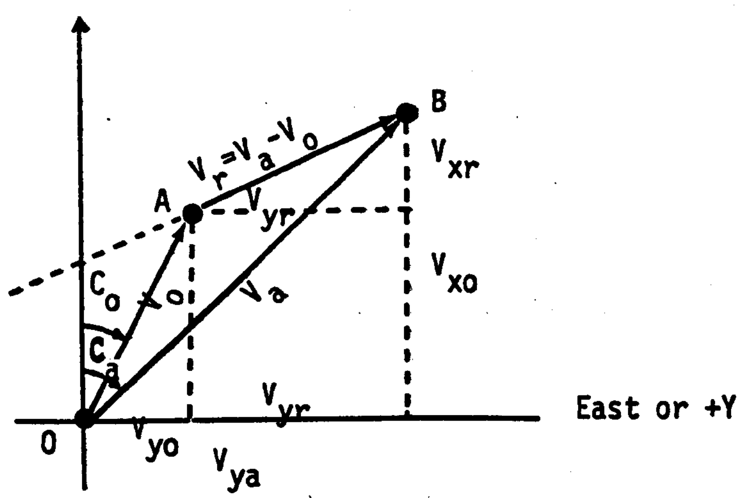
$$V_r = S_r \,/\, Dt \qquad 1.8$$

The direction of the relative motion line is determined by the direction from initial position (A_1) to terminal position (A_2). The angle made by this line and the north direction measured in clockwise direction is defined as the relative course (C_r). This angle may have any value from 0° to 360°. It is related to the slope of the line as follows:-

C_r	$=$ ATAN (DY/DX)	if for $+$ DX, $+$ DY
	$=$ 360 + ATAN (DY/DX)	$+$ DX, $-$ DY
	$=$ 180 + ATAN (DY/DX)	$-$ DX, $\pm$ DY
	$= 90^\circ$	0, $+$ DY
	$= 180^\circ$	$-$ DX, 0



Figure(1-5)
Graphical representation of relative motion.



Figure(1-6)
Graphical representation of the elements of relative motion.

$$C_r = 270^o \qquad \text{for } 0, \ -DY \qquad 1.9$$

Apart from the above relations, the relative velocity (C_r, V_r) can be found from the knowledge of the true motion of both observing and other vessel, (Co , Vo) and (Ca , Va) respectively. It is obvious as seen in Fig. 1.6 that the relative velocity is the vector subtract of the two true motions, $(V_r = Vo - Va)$. An analytic solution can be done by resolving of the velocities (Va) and (Vo). Hence if the relative velocity of other ship (A) with respect to observing ship (O) is considered :-

$$VX_r = VX_a - VX_o$$

$$VY_r = VY_a - VY_o$$

$$VX_r = V_a.COS\ (C_a) - V_o.\ COS\ (C_o)$$

$$VY_r = V_a.SIN\ (C_a) - V_o.\ SIN\ (C_o)$$

$$V_r = (VX_r + VY_r)^{\frac{1}{2}}$$

$$C_r = ATAN\ (VY_r / VX_r) \qquad 1.10$$

Similar constraint to that mentioned in equation 1.9 applies to the calculation of the relative course (C_r).

1.4 EQUATIONS OF TRUE MOTION

The elements of motion of another ship can be found from its relative motion together with observing ship velocity. Following a similar procedure to that of equation 1.6 the following equations of other ship's true motion can be derived, as seen in Fig. 1.6:-

$$VX_a = VX_r + VX_o$$

$$VY_o = VY_r + VY_o$$

$$VX_a = V_r . COS(C_r) + V_o.COS(C_o)$$

$$VY_a = V_r . SIN(C_r) + V_o.SIN(C_o)$$

$$Va = (VX_a^{\,2} + VY_a^{\,2})^{\frac{1}{2}}, \quad Ca = ATAN(VY_a / VX_a) \qquad 1.11$$

Similar constraints to those mentioned in equation 1.9 apply to the calculation of the other ship's true course.

Having decided on an arbitrary relative motion, and provided a pre-knowledge of other ship's true velocity, the observing vessel's true velocity which satisfies this condition can be found, as above:-

$$VX_o = VX_a - VX_r$$

$$VY_o = VY_a - VY_r$$

$$VX_o = V_a.COS(C_a) - V_r.COS(C_r)$$

$$VY_o = V_a.SIN(C_a) - V_r.SIN(C_r)$$

$$C_o = ATAN(VY_o / VX_o) \qquad 1.12$$

Similar constraints to that of equation 1.9 apply to the calculation of the observing ship's true course.

1.5 EQUATIONS OF CLOSEST POINT OF APPROACH AND A BUFFER CIRCLE

Regarding the closing phase of a rapidly changing situation, a periodic determination of the (CPA) from a set of raw data is essential. The following expressions which define the (CPA) in terms of bearing and range observations are given for this purpose. Now if (A_1) and (A_2) are two successive positions of other ship defined in the (NCS) of the observing vessel from their bearings and ranges (B_1, R_1, B_2, R_2) as shown in Fig. (1.5), and the two positions are joined by a line which is extended in the direction of the relative movement, the point (M) at which the line passes closest to the center point (O) will be the (CPA) (See Fig. 1.7).

If the perpendicular (d) is laid from point (A2) to the first line of bearing and intersecting at point (C); then two triangles are formed (A_1. M. O) and A_1. C. A_2). From the similarity of the two triangles the following equation is formed:

$$R_m/d = R_1/S_r$$

$$R_m = d.R_1/S_r$$

$$= R_1 . R_2 . \text{SIN}(B_2 - B_1)/(R_1^2 + R_2^2 - 2.R_1.R_2.\text{COS}(B_2 - B_1))^{\frac{1}{2}} \qquad 1.13$$

Provided that:

$$R_2 \neq R_1 \quad \text{and} \quad B_2 \neq B_1$$

Having defined the relative motion by any method as described in (Section 1.2) which passes through a given position, (X_i, Y_i), then the following relation can be derived, (see Fig.(1.5)):-

$$\text{Tan}(C_r) = (Y_i - Y)/(X_i - X)$$

$$Y = \text{Tg}(C_r).X + Y_i - \text{Tg}(C_r).X_i$$

$$Y = m.X + Y_i - m.X_i \qquad 1.14$$

Where (X_i , Y_i) is the known initial position in the (NCS) of ship (O).

(m) is the inclination of the relative track of the true north direction, it equals tan (relative course (C_r).

(X . Y) are the coordinates of a position on the relative motion line.

Now suppose that a circle of radius (Ra). representing a buffer circle, is centred at the origin of the (NCS), then its equation is given by:

$$X^2 + Y^2 - R_a = 0$$

Then the coordinates (X , Y) of a point of intersection (A_3) must satisfy the equation of the circle and the equation of the R.M. line, and a system of equations is formed. By substituting for (Y), squaring and collecting terms together the following quadratic equation is given:-

$$(m^2+1).X^2 + 2.m. (Y_i - m.X_i).X + (Y_i - m.X_i)^2 - Ra^2 = 0$$

Let

$$a = (m^2 + 1) \qquad\qquad b = 2.m\ (Y_i - m.X_i)$$

$$c = (Y_i - m.X_i)^2 - Ra^2$$

Then $X = (-b \mp (b^2 - 4.a.c)^{\frac{1}{2}} /2.a$ 1.15

As the value of (X) is determined from equation (1.15), then equation (1.14) can be used to calculate value of (Y).

According to the sign of the discriminent $(b^2 - 4.a.c.)$, it has two real roots, one real root, or two conjugate complex roots. Geometrically this means that the R.M. line has two, one, or no points in common with the assigned buffer circle. Henceforth, the discriminant can be used to predict a safe pass or unsafe pass with regard to a given buffer circle.

Apart from the equation (1.13), which is based on raw data, sometimes it may be necessary to determine the (CPA) in advance as a consequence of a given relative motion (Cp , Vp), together with a known position (X_i , Y_i). This can be formed by considering the equality condition of the discriminent "$b^2 = 4.a.c$", then the tangent circle to the given R.M. line can be defined, and hence the (CPA).

Let $X_m = X$, $b^2 = 4.a.c.$, and substituting in Equ.1.15:-

Then $X_m = -b/2.a$

$$X_m = 2.M\ (M.X_i - Y_i) /\ 2.(M^2 + 1)$$

$$= M.\ (M.X_i - Y_i)\ /\ (M^2 + 1) \qquad 1.16$$

$$Y_m = M.X_m + Y_i - M.X_i$$

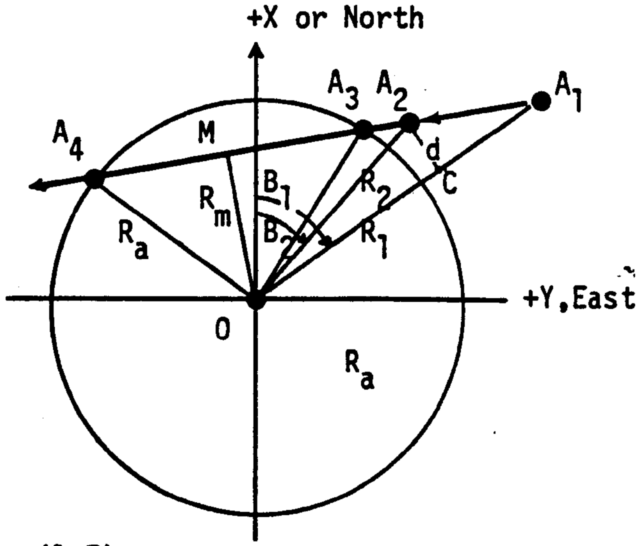
The distance to the (CPA) can be given by the relation

$$R_m = (X_m^2 + Y_m^2)^{\frac{1}{2}} \qquad 1.17$$

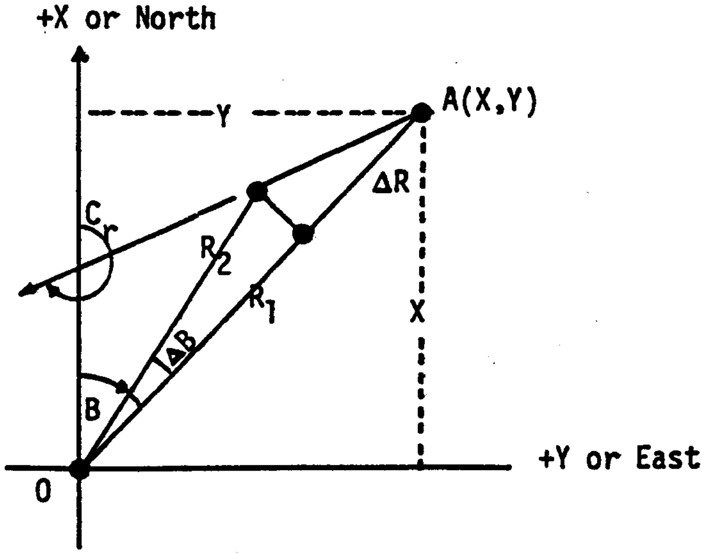
The bearing of the (CPA) can be given by the relation

$$B_m = \text{ATAN}\ (Y_m\ /\ X_m)$$

Having a similar constraint as in relation (1.9).



Figure(1-7)
Relation between relative motion and buffer circle.



Figure(1-8)
Changing of range and bearing.

1.6 EQUATIONS OF RANGE AND BEARING RATES

Range rate is an important factor in characterising the closing phase of a two ship encounter. Range rate together with bearing rate can be used as an index for a rapidly deteriorating situation. Henceforth the following derivation of range and bearing rate equations are given:-

Let other ship's position (A) being described relative to the observing ship (O) by the (NCS) as illustrated in Fig. (1.8), and being also defined by the equation of the position vector as follows:-

$$R^2 = X^2 + Y^2 \qquad 1.18$$

On differentiating with respect to time (T), then:-

$$dR/dT = (X.dX/dT + Y.dY/dT) \;/\; (X^2 + Y^2)^{\frac{1}{2}}$$

But $\quad dX/dT = V_r.COS\ (C_r)$, and $dY/dT = V_r.SIN\ (C_r)$

then $\quad R^{\bullet} = V_r\ (X.COS\ (C_r) + Y.\ SIN\ (C_r))/(X^2 + Y^2)^{\frac{1}{2}} \qquad 1.19$

Let $\quad F = X.\ COS\ (C_r) + Y.\ SIN\ (C_r) \qquad 1.20$

The expression "F" may be used to determine if the range is constant, decreasing, or increasing depending on its sign; zero, negative, or positive respectively.

Considering Fig. (1.8), the bearing of ship (A) can be given in terms of its coordinates (X , Y) in the following form:-

$$B = ATAN\ (Y / X) \qquad 1.21$$

Similar constraints to those mentioned in equation (1.9) apply to the calculation of the true bearing (B).

On differentiating equation (1.21) with respect to time (T), then

$$dB/dT = \cos^2(B). \quad (X.dY/dT - Y.dX/dT) / X^2$$

$$= (X. V_r. \sin(C_r) - Y. V_r. \cos(C_r)) / R^2$$

$$= (V_r/R). [(\cos.(B). \sin(C_r)) - (\sin(B).\cos(C_r))]$$

$$= (V_r/R). (\sin(C_r - B))$$

Or

$$B^{\bullet} = 0.955 \quad (V_r/R). \quad \sin(Q_r) \qquad 1.22$$

Where

$B^{\bullet}$ = rate of change of bearing in degrees per min.

Q_r = the relative aspect which equals $(C_r - B)$

V_r = the relative speed in knots.

R = other ship's range in miles.

1.7 EQUATIONS OF CHANGING POSITION ON A MOVING SHIP

Refer to Fig. (1.9), showing a geometrical figure of a possible two ship encounter. It is assumed that ship (O) has a constant speed (Vo) and a ship (A) has a forward constant velocity (Ca,Va). Ship (O) is required to set her course (C_o) to take a new position on the other ship (A).

The ships' paths and their relative positions are referred to (NCS), the navigational right - hand orthogonal system (O, X, Y). Now let (C) be an arbitrary point of a fixed position relative to Ship (A). The arguments of which are given in terms of aspect and range (Q,D). Hence its coordinates in accordance with the (NCS) centred at (O) are in the form:-

$$Xc = Xa + D.COS\ (Ca + Q)$$

$$Yc = Ya + D.Sin\ (Ca + Q) \qquad 1.23$$

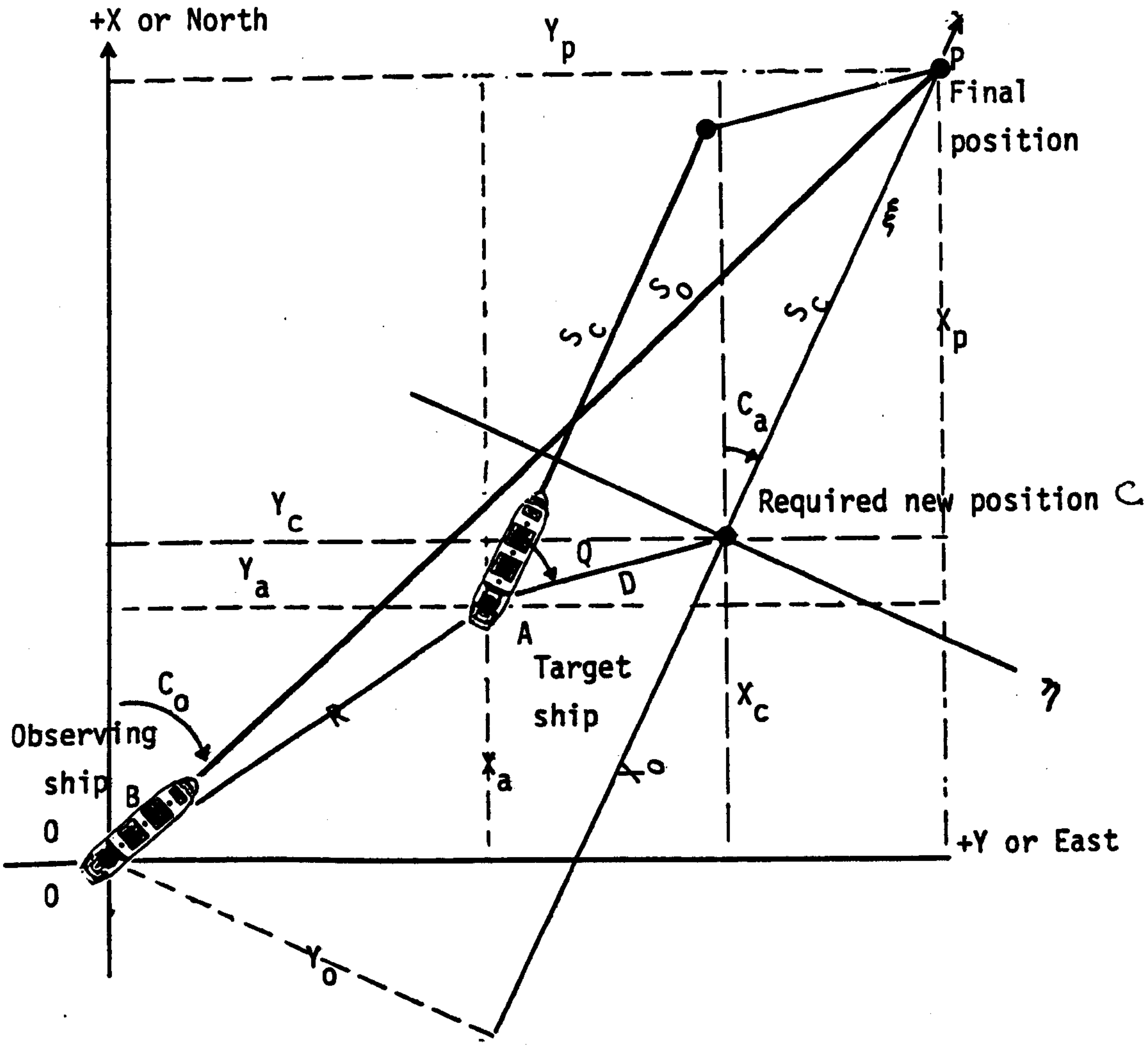
Consider that the coordinate is translated to pointed (C) and rotated in the navigational positive sense of direction through an angle (Ca) into a (ξ, η) system. The coordinates (ξ, η) of observing ship (O) satisfy the inverse transformation equations:-

$$\xi = Yc.\ SIN\ (Ca) - Xc.\ COS\ (Ca)$$

$$\eta = Yc.\ COS\ (Ca) + Xc.\ SIN\ (Ca) \qquad 1.24$$

With the course of ship (O) being set to intercept the moving (C) at a future position point (P), which will be on the positive axis of the (ξ, η) - system, then the following ratio holds constant:-

$$So/Sc = Vo/Va = E$$



Figure(1-9)
Changing position of observing ship relative to a moving target.

Where So = is the distance run of ship (O) to the future point (P)

Sc = is the distance run of point (C) to the future point (P)

Vo = is the speed of observing ship (O)

Va = is the speed of ship (A)

E = is the speed ratio between ship (O) and Ship (A).

Squaring $S_c^2 . E^2 = S_o^2$

$$S_c^2 . E^2 = \eta^2 + (-\xi + Sc)^2$$

$$= \eta^2 - 2.\xi .Sc + Sc^2 + \xi^2$$

then $(1 - E^2). Sc^2 - 2. \xi \ Sc + \eta^2 + \xi^2 = 0$ 1.25

Let $a = 1 - E^2$

$b = -2.\xi$

$c = \eta^2 + \xi^2$

then $Sc = (-b \mp (b^2 - 4.a.c.)^{\frac{1}{2}})/2.a$ 1.26

Having defined displacement (Sc), then the future point (P) can be found in the (NCS) of ship (O) by the following relations:-

$$Xp = Xc + Sc.COS \ (Ca)$$

$$Yp = Yc + Sc.SIN \ (Ca) \qquad 1.27$$

The system of equations from (1.23) to (1.27) can be used to determine the coordinates of the future point of possible collision. If, however, point (C) is initially taken at the point (A), so that distance (D) is nil, then (Xc = Xa), and (Yc = Ya), and equations (1.24) become:-

$$\xi = -Ya. SIN \ (Ca) - Xa.COS \ (Ca)$$

$$\eta = -Ya. COS \ (Ca) + Xa.COS \ (Ca) \qquad 1.28$$

1.8 EQUATION OF COLLISION

Consider the dynamic situation of a two-ship encounter involved in exact collision. Such a situation appears in Fig. (1.10) which illustrates the geometry of a collision situation between two ships on converging courses. At an instante (T_1) the two ships (O) and (A) at a distance (R_1) and are moving according to the speed vectors (Vo) and (Va). For the sake of simplification the geometric figure is regarded as a set of points and the two true velocities are assumed to be uniform. The relative bearing of ship (A) in relation to ship (O) is the angle (Q) or the aspect.

If both ships maintain their velocity they will collide at point (Pc). The intersection angle at this point is the relative heading (H), and the following relation holds constant:

$$Vo/Va = So/Sa = E$$

Where E = the speed ratio

So = distance run of ship (O) to the collision point.

Sa = distance run of ship (A) to the collision point.

From the two triangles (O.P.F) and (A.P.F.)

$$Sa = b.CSC\ (Q) \quad \text{and} \quad So = b.CSC\ (Q + H)$$

then

$$\begin{aligned}(1/E) &= b.\ SIN\ (Q + H)/b.\ SIN\ (Q) \\ &= (SIN\ (Q).\ COS\ (H) + COS\ (Q)\ SIN\ (H)/SIN\ (Q) \\ &= COS\ (H) + COT\ (Q).\ SIN\ (H)\end{aligned}$$

and hence

$$TAN\ (Q) = E.SIN\ (H)\ /\ (I-E.\ COS\ (H)\ ) \qquad 1.29$$

Equation (1.29), gives a simple definition of a collision situation in a two-ship encounter in terms of two independent variables (E) and (H).

The following are two alternative expressions defining the same function:-

$$COS (Q) = (1-E. COS (H) / (1-2.e.COS (H) + E^2)^{\frac{1}{2}} \qquad 1.30$$

$$SIN (Q) = (E. SIN (H) / (1 - 2.E. COS (H) + E^2)^{\frac{1}{2}} \qquad 1.31$$

The simple expression (1.29) was investigated in the M.Phil thesis. However, as it constitutes a basis for the collision probability, it is worthwhile to give a brief review of the collision function through a graphical presentation.

The equation (1.29) may be interpreted as defining a surface in the EHQ-Space. In particular the equation might be imagined to present the elevation points on a three dimentional space, above the plane Q=0, as in figure (1.11). If the surface is cut with a plane Q=Qo, a constant, a contour line is generated on the surface, and if this contour line is projected straight down onto EH-plane, an isoaspect curve is obtained in the domain of Q. The contour curve consists of the points in the domain where Q has the value Qo. An equation for the contour curve is obtained by setting f(E,H) = Qo. By giving other values to Q, other contours are obtained. The generated figure is called the EH - diagram of the collision function, see fig (1.12).

The investigation of the collision equation and the related behaviour can be achieved through the study of the derivatives and the related incremental relations. It is not difficult to derive the following relations by differentiation of the collision equation:-

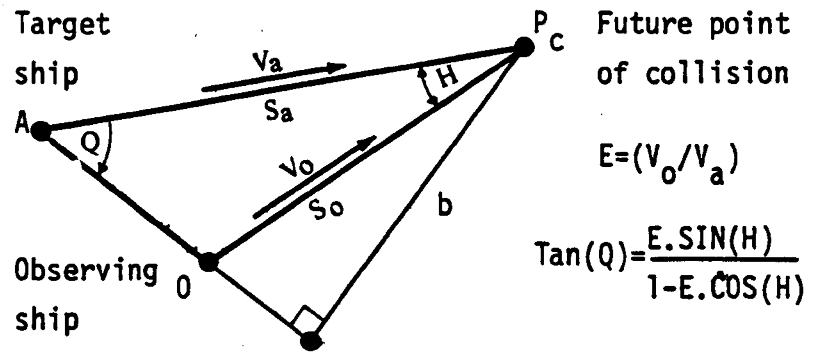
$$\Delta Q/\Delta H = E. (COS (H) - E)/(1-2.E.COS (H) + E^2) \qquad 1.32$$

$$\Delta Q/ \Delta E = SIN (H) / (1-2.E.COS(H) + E^2) \qquad 1.33$$

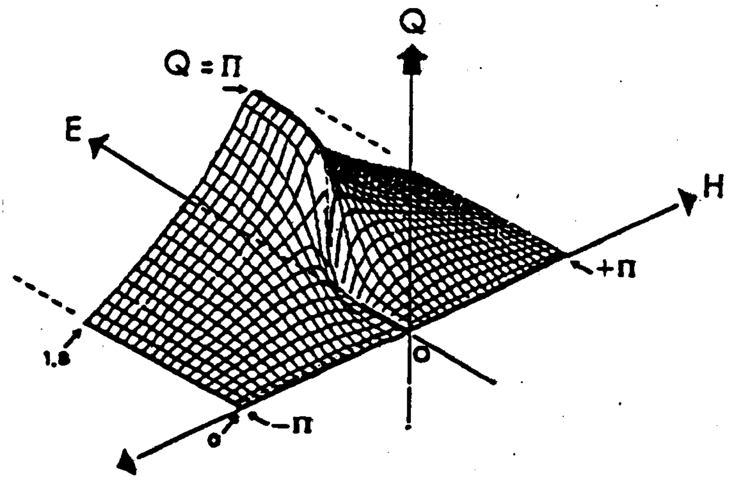
$$\Delta B \quad = - (\Delta H + \Delta Q) \qquad 1.34$$

$$\Delta H \quad = \Delta B. (1-2.E. CO (H) + E^2)/ (1-E.COS (H) \qquad 1.35$$

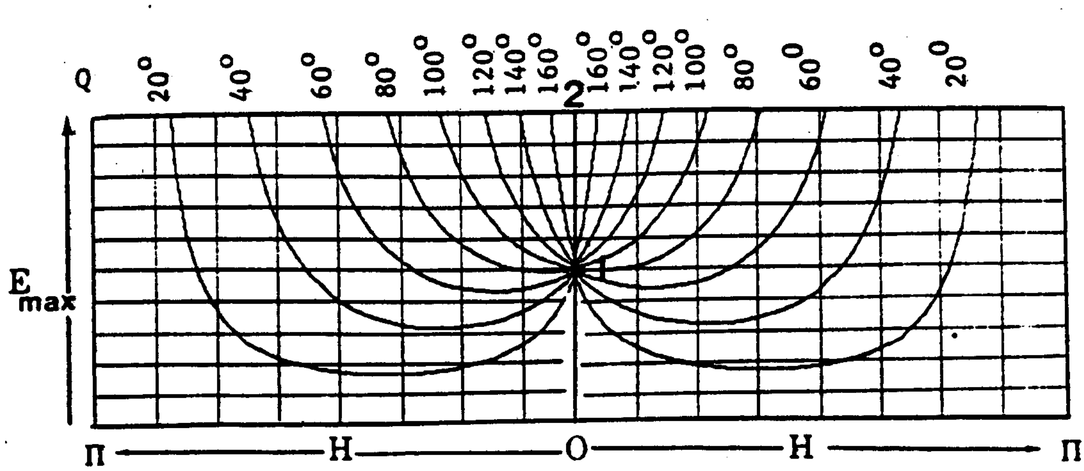
(See Appendix A2)



Figure(1-10)
Geometric definition of an exact collision situation.



Figure(1-11)
Collision Surface.



Figure(1-12)
E.H-Diagram:Projection of the Collision Surface
on the E.H-plane.

1.9 EQUATIONS OF COLLISION PROBABILITY

If the encounter dimension should occupy a sea area of a finite radius (Ra) not a point, it would be geometrically defined in the (E-H) diagram by a set of aspects confined between two limiting aspects (Q_1) and (Q_2), Fig. (1.13). The intersection of the speed ratio line $(E = E_{max})$ with the two curves of the limiting aspects will describe an area. This area (M) defines a subset of particular combination of heading and speed ratios contained in a discrete set which consists of all possible combinations of the heading and speed ratio. Such a concept can be seen in Fig.(1.14) and can be interpreted as:-

> *" There may exist a set (M) of different ways for a collision to occur out of a total number of (N) possible courses and speeds that can be considered by the observing vessel. "*

The set of all possible ways open to the observing vessel is defined by the area of the given (E-H) diagram, which is equal to the value of:-

$$N = 2.\ \pi\ E_{max}$$

Where $\pi = 3.1415927$

Then it is reasonable to suppose that the collision probability (Pc) is proportional to the areas (M) and (N),and is expressed in the following relationship :-

$$Pc = M / N$$

$$= M / 2.\ \pi\ E_{max} \qquad 1.36$$

Area (M) is determined in accordance with the maximum speed ratio: $E_{max} < 1$, $E_{max} = 1$ or $E_{max} > 1$. The following equations are given and can be traced (see the appendix (A.4)):-

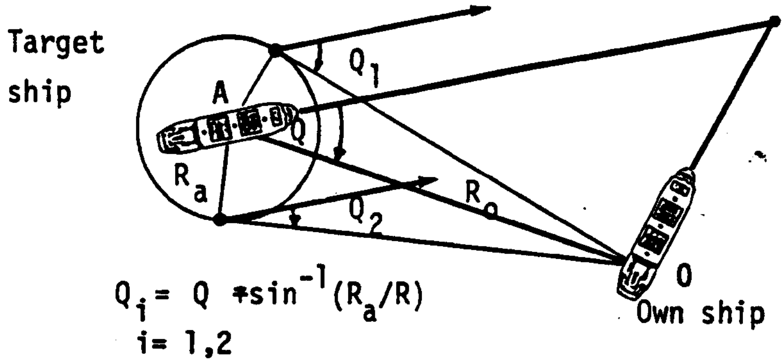


Figure 1.13 Definition of the limiting aspects (Q_1) and (Q_2) by tangents of the critical circle of radius (R_a)

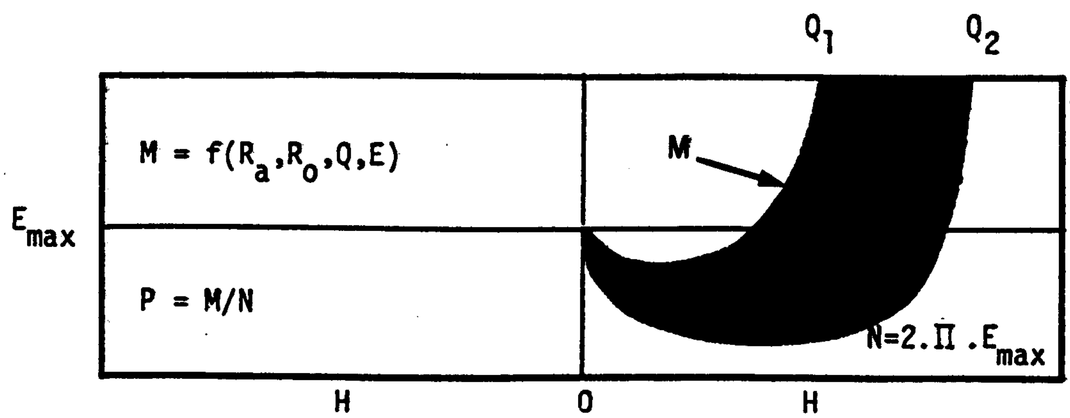


Figure 1.14 Definition of collision probability in (E/H)-plane.

In case of $(Emax \geq 1)$,

$$M = \{H_1 . Emax - SIN(Q_1). Ln(TAN\tfrac{1}{2}(H_1 + Q_1) / TAN(\tfrac{1}{2}Q))\} \pm$$

$$\{(H_2 . Emax - SIN(Q_2). Ln(TAN\tfrac{1}{2}(H_2 + Q_2)/TAN(\tfrac{1}{2}Q_2))\} \qquad 1.37$$

In case of $(Emax < 1)$,

$$M = \{H_1 . Emax - SIN(Q_1). Ln(TAN\tfrac{1}{2}(H_{12} + Q_1) / TAN\tfrac{1}{2}(H_{11} + Q_1))\} \pm$$

$$\{(H_2 . Emax - SIN(Q_2). Ln(TAN\tfrac{1}{2}(H_{22} + Q_2)/TAN\tfrac{1}{2}(H_{21} + Q_2))\} \qquad 1.37$$

Where

$Q1 = Qo + SIN^{1}(Ra / Ro)$

$Q2 = Qo - SIN^{1}(Ra / Ro)$

Ra = accepted CPA

Ro = initial range

Qo = initial aspect angle.

.10 EQUATIONS OF THE TURNING TRAJECTORY OF A SHIP

Ship manoeuvrability has been, and still is, an important practical consideration when ships are manoeuvring for collision avoidance. The parameters of a ship's turning trajectory are useful for characterizing the collision avoidance manoeuvres at close range in the open sea. A simplified description of ship's circular motion is as follow.

Suppose that a ship is advancing on a straight path, when her rudder is deflected and held at a fixed angle. The trajectory of the ship after this even may be divided into three phases as shown in figure (1.13). Based on starboard turn the first phase starts at the instant that the rudder is begun to be laid over and may be extended after the time the rudder reaches its full deflection angle. During this period the speed and the path are considered constant and straight respectively. When the rotation commences the ship enters the second phase of turning. The important event that takes place in this phase is the coexistence of deceleration and hence the speed of ship is reduced exponentially. Finally, after a new equilibrium of forces is reached, the ship settles down to the steady turn of the third phase.

Assuming a perfect circle of turn the following relations can be established provided that the ship's tactical radius (R_t) and her rate of turn ($\dot{\phi}$) are known from ship's trial maneuvres.

If (P_i) and (P_{i+1}) are two successive positions of the ship on the turning circle then the position (X_{i+1}, Y_{i+1}) can be related to the previous one in the (NCS) by the following equations which can be deduced from the figure(1-16)

$$X_{i+1} = X_i + 2.R_t. \text{SIN} (\tfrac{1}{2}\ \phi).\text{COS} (Co+\tfrac{1}{2}\ \phi)$$

$$Y_{i+1} = Y_i + 2.R_t. \text{SIN} (\tfrac{1}{2}\ \phi).\text{SIN} (Co+\tfrac{1}{2}\ \phi) \qquad 1.38$$

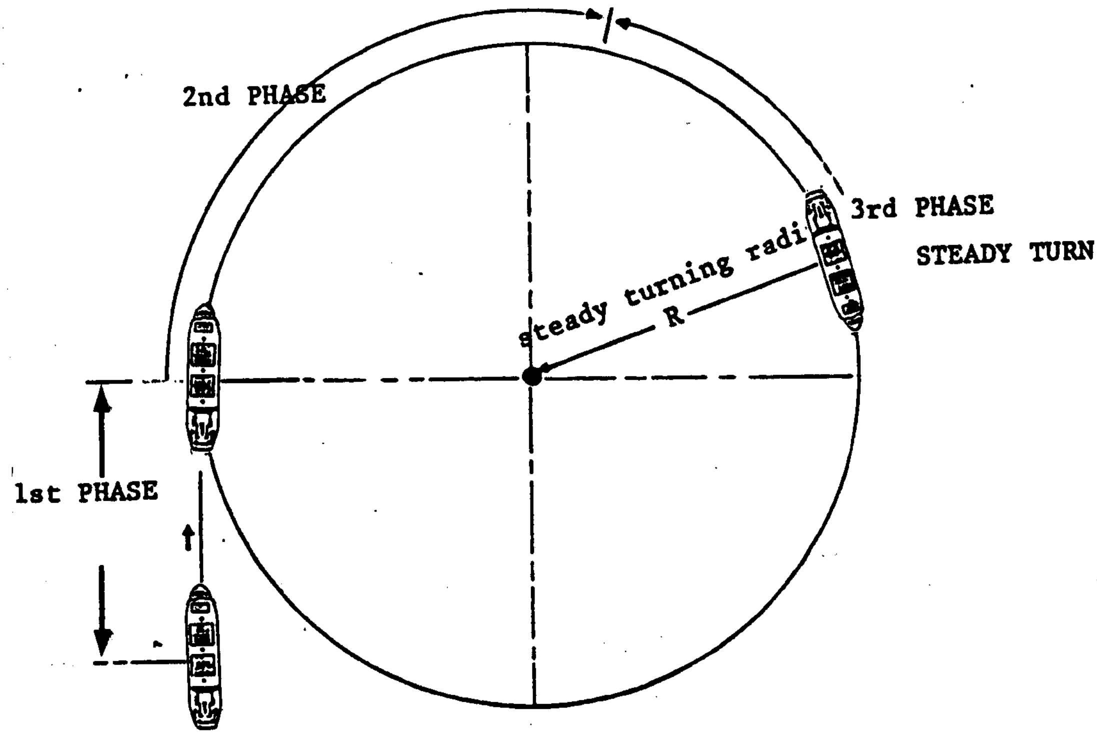
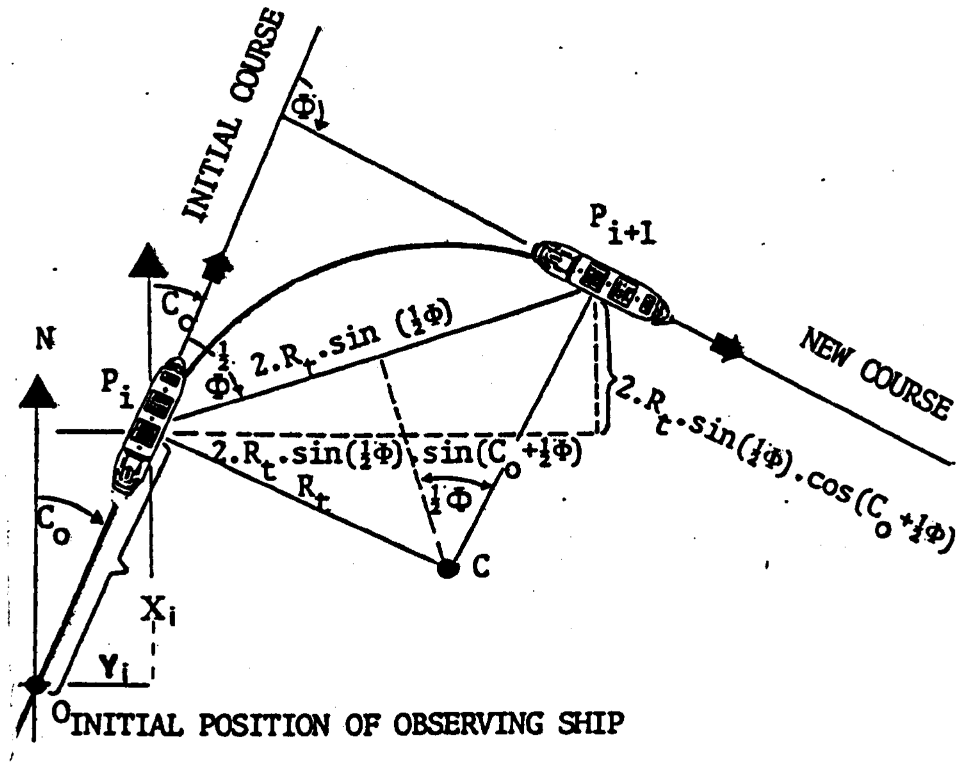


Figure (1-15). A simplified turning path of a ship.



Figure(1-16)Definition of the coordinates of a point on the turning circle of a ship relative to the initial position.

Where;

R_t = Turning radius

Co = True course of the ship at position (P_i)

ϕ = Amount of course alteration in the time interval $(T_{i+1} - T_i)$.

Value of (ϕ) can be determined from ship's tangential velocity and her tactical radius:

$$\phi = 57.3 \; (T_{i+1} - T_i) \; Vo / R_t \qquad 1.39$$

In considering the ship's tangential velocity (Vo), the empirical exponential formula advised by the (D.O.T) specification for a Marine Navigational Equipment Simulator (1980) can be used;

$$Vo = (Vmax - Vmin) . e^{-Tn/K} + Vmin \qquad 1.40$$

Where;

Vo = Speed at Tn minutes after alteration has been made.

K = A constant which simulates the chosen type of ship and is either a fixed value between (2) and (25) or is continuously variable over this range.

UNIT–2

A Risk Prediction Model for Collision Avoidance

.1 INTRODUCTION

Increasing attention has been paid to ship collision avoidance systems during the past ten years. Inspite of (ARPA) introduction into marine use, which is considered a vital contribution, the navigator is not, however, and will probably never be capable of predicting accurately the risk of collision only from closest point of approach, the basic criteria in all collision avoidance systems in use.

This unit surveys a new approach for risk prediction, assessment and control. A mathematical model of terminal risk assessment was designed to performe risk simulation on the computer. A set of equations which was derived in Unit 1, "The Foundation", is used in building these models. Risk information is presented in a special pattern which is considered to be practical. In order to clarify the effect of time on risk assessment, the transition distribution of risk values was plotted whenever necessary. For the sake of completeness a more appropriate model based on maximum risk values is also introduced in the study.

In order to obtain these models, the assumption was made that the speed of the observing ship and the velocity of other ship are taken as constant values during the encounter. The results that were obtained are considered to be realistic, although, it was recognised that more than one pattern would be necessary for full assessment of the relevant parameters.

...

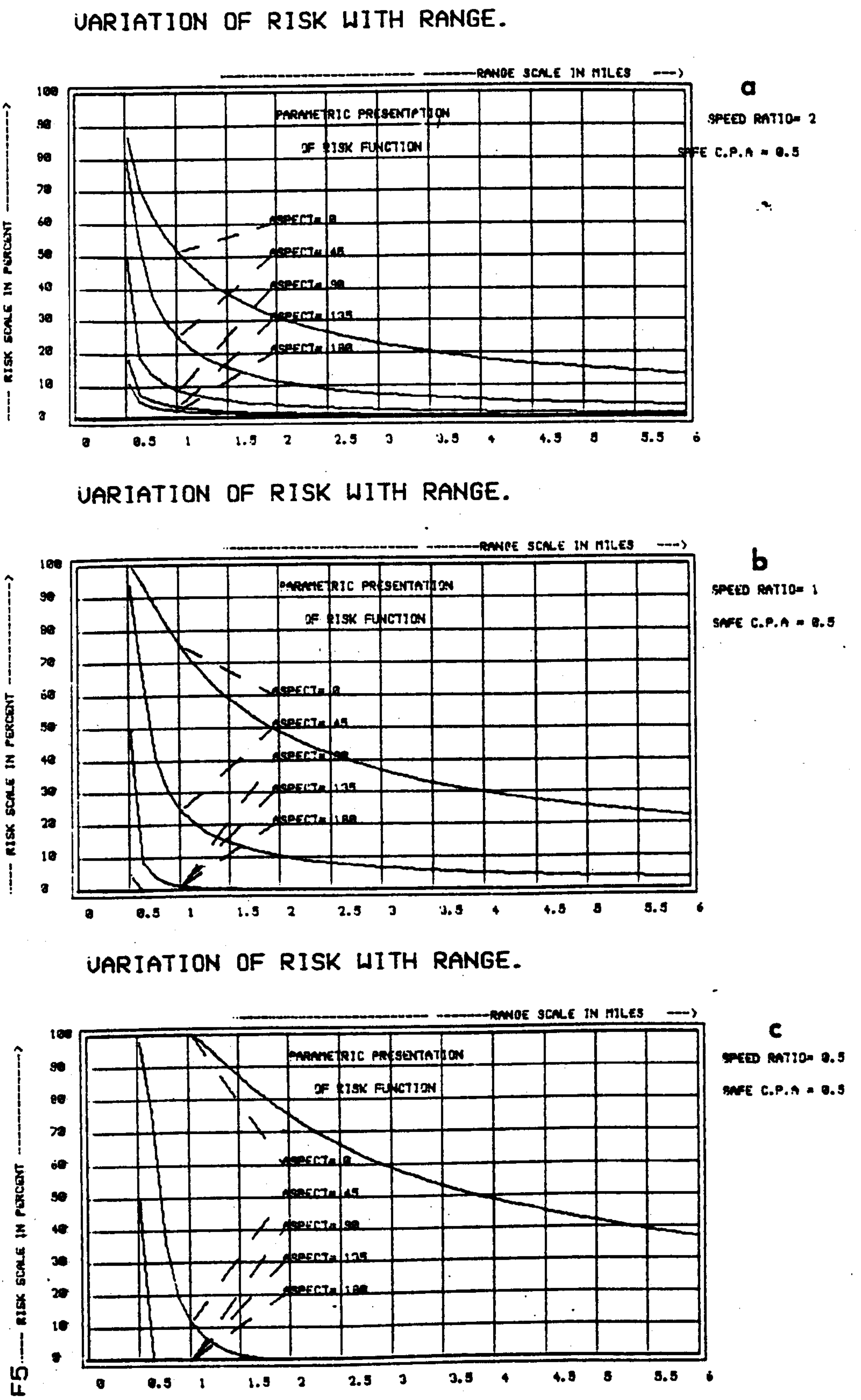
2.2 GRAPHICAL DESCRIPTION OF RISK FUNCTION

To help illustrating the basic steps in understanding the risk function the equation (1.36) is used to describe the overall features of the risk of collision which may arise in a binary encounter.

For such an encounter at a given space and time, the value of risk is given by the risk function. This risk depends on the ship's dimension (Ra), speed ratio (E), initial range (Ro) and initial aspect (Qo). The value of risk may vary significantly from stage to stage of the encounter. For an encounter of constant parameters (Ra) and (E), the influence of the variables (R) and (Q) can be solely investigated by means of graphical representation of the risk function.

Figures (2.1 a, b, c) show the risk graphs based on fixed values of (Ra = 0.5 miles) and (E = 2, 1, 0.5). The horizontal scale of each graph is in nautical miles and covers an interval of (6) miles. The vertical scale shows the risk percent, covering values from zero to 100%. The risk-range curves are computed for five values of aspect taken as a constant parameter of (0^o, 45^o, 90^o, 135^o and 180^o). The resulting set of curves is an indication of the way the range-risk relation varies.

Figures (2.2, a, b, c) show another three risk graphs based on the same fixed values of (Ra) and (E) as above. The range variable (R) is taken as a constant parameter of the values (R= 1, 2, 3, 4, 5, 6) n.miles. The horizontal scale of each graph is in degrees and covers the full range of aspect angle from (0^o) to (180^o). The resulting set of curves illustrate the overall influence of aspect change on the value of risk of collision.



Figure(2-1). Graphical representation of Risk Function on the (risk range)-plane. Each diagram is computed for one particular speed ratio and five different angles of aspects.

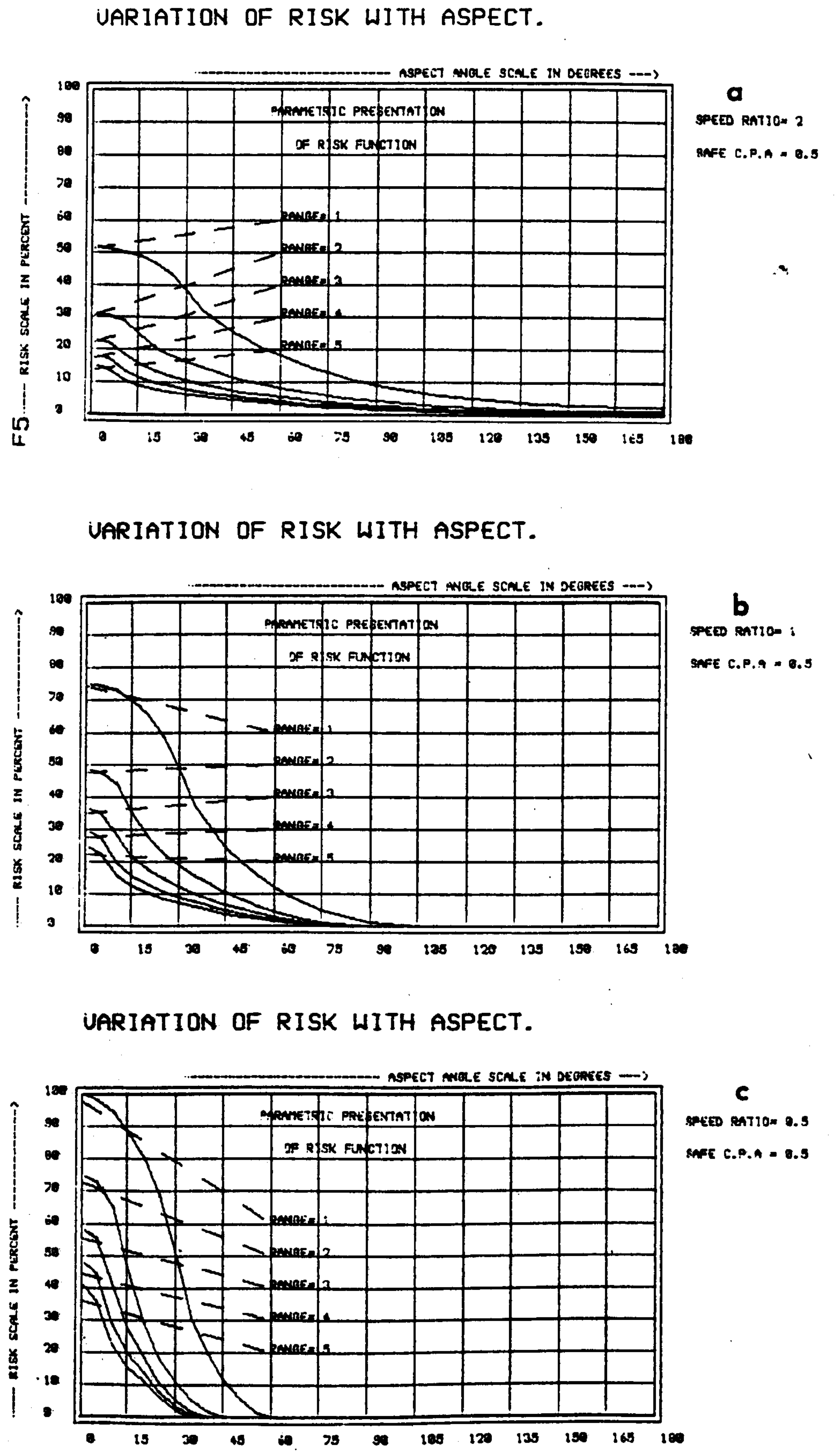


Figure (2-2). Graphical representation of Risk Function on the (risk/aspect)-plane. Each diagram is computed for one particular speed ratio and five different ranges.

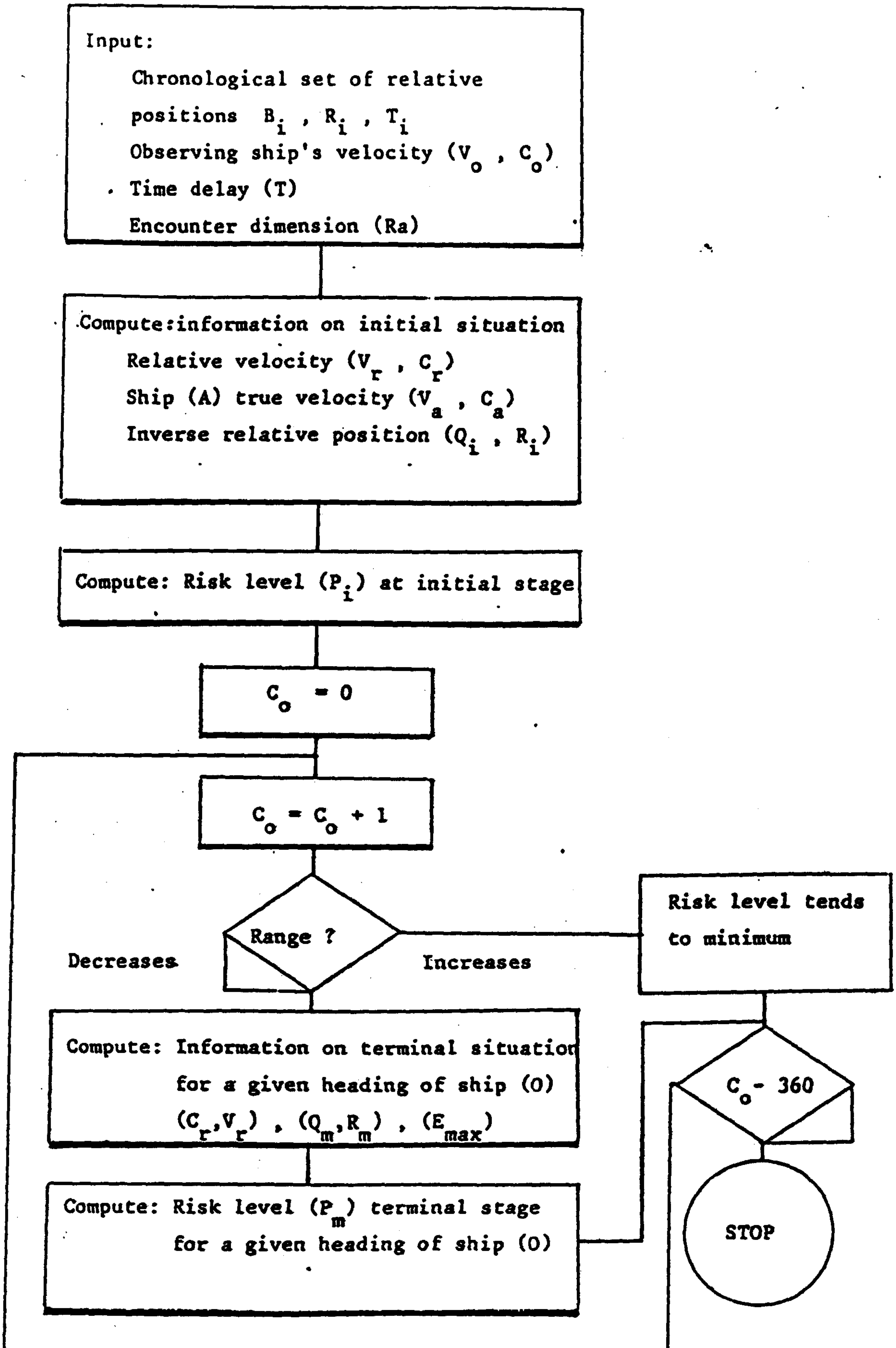
2.3 FLOW CHART AND PROGRAM FOR TERMINAL RISK

The computer program was written in BASIC as shown in appendix (B-1) listing (Risk/Course). A simplified flow chart of this program is shown in Fig. (2.3). It can be seen that it requires as inputs a chronological set of relative positions (Bi, Ri, Ti), the velocity of observing ship (Co, Vo), encounter dimension (Ra), and an arbitrary time delay (T).

Program routines require three subroutines giving in listing (Comvector) (ASPECT), and (C.RISK). The first step concerns the computation of information on the initial geometry of the situation. The inputs to subroutine (COMVECTOR) are the components of relative motion computed by equations (1.1, 1.2). The returned results of relative vector (Cp, Vp) together with the true motion vector of the observing ship (Ca, Va) are used to compute the vector (Ca, Va) of the other ship's true mption, using equations (1.7, 1.8).

To allow for the expected time of computer solution, a second step is to predict the initial risk of collision after an arbitrary interval of time. This routine is set up to first compute the relative position of ship (A) at the end of the time interval by applying equations (1.2, 1.3, 1.4) in an inverse sense. The computed argument (Bi) and other ship true course (Ca) are entered to subroutine (ASPECT) which returns the value of the aspect at the time specified. Now all the parameters necessary for computing the risk at the specified time (P) are available These data (Vo, Va, Ra, Ri, Qi) are the inputs of subroutine (C2- Risk) which returns the value of collision probability and hence the value of risk of collision.

The third step is to generate a matrix of terminal risk which covers all the courses available to the observing ship. This routine is set up to first compute the (CPA) and its associated aspect (Qm), that is for all converging courses using equations (1.12, 1.13, 1.14). Following a similar procedure to that of the second step the value of risk when the ship reaches the (CPA) is computed. To recognise when a divergence in relative motion may occur, use is made of equation (1.20). Risk of Collision is considered diminishing in case of diverging courses.



Figure(2-3). A simplified program flowchart of risk prediction model.

All of the things which have been done so far are numerical computations of the initial and terminal risk matrix for a given situation, but for better digestion of this information and quick interpretation a visual presentation is needed, which is the subject of the next paragraph.

...

2.4 RISK PRESENTATION PATTERN

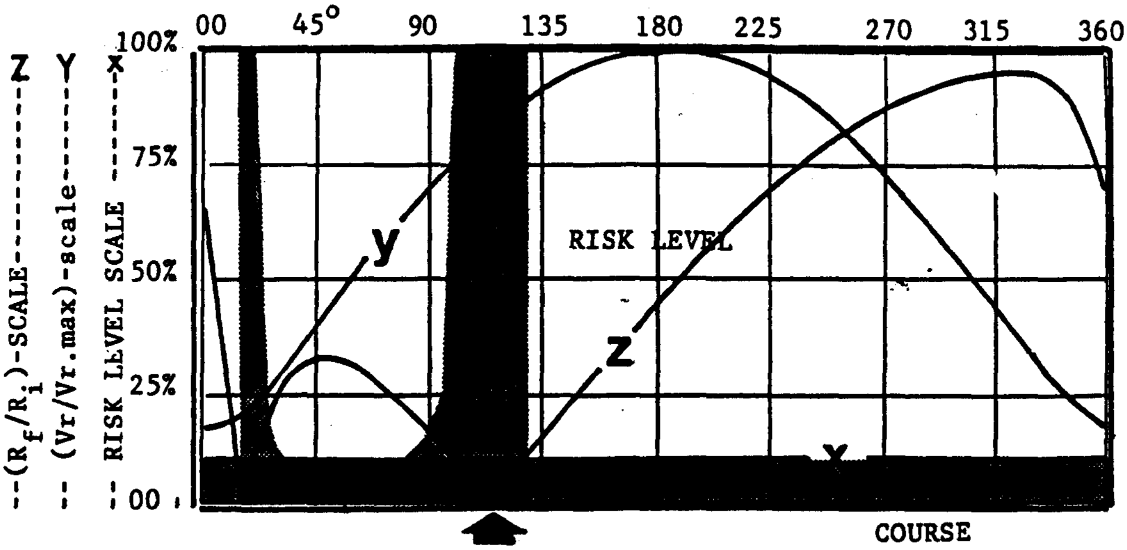
A clear picture of the shape of the graphs of the developed risk function can be obtained by introducing a cartesian coordinate system in which the true course of the observing ship (Co) is taken as abscissa and the values of respective risk function are taken as ordinates. Fig. (2.4) illustrates this presentation. A Computer program is set up to perform the plotting routine. It is included in the main computer program listing (RISK / COURSE), in appendix (B-1). The pointwise construction of the terminal risk graph for the courses at intervals of 5^{o} can be seen in the diagram. The range of risk values in a given situation thought of as being characterised by four levels:

- Zero tolerance risk in range of 00 - 25 %
- Accepted risk in range of 25 - 50 %
- Potential risk in range of 50 - 75 %
- Extreme risk in range of 75 -100 %

The levels of risk are shown in the diagram as four bands so that the user can quickly figure out the difficulty in the situation. In order to perceive the future change in the value of risk the initial risk is plotted. The initials risk curve (X) appears as a horizontal straight line as a result of course - independance of the initial risk.

Having the values of relative velocity (V_r) and distance to CPA, (Rm) already computed two other graphs (Y) and (Z) are made possible on the same diagram. Graph of (Y) represents the variation of the relative velocity normalised by its maximum value ($V_r / V_{r \cdot max}$) with course changes. This curve can be used to visualise how quickly or slowly the risk level is changing. The third graph (Z) represents the variation of distance to CPA normalised by the initial range (Ro) with course changes. The idea behind plotting of curve (Z) is to establish a comparative index between the assessment based on CPA criteria and the risk function developed.

A numerical printout of input data and results on initial situation are provided as a record. These data can also be used to view the conventional side of the problem.



VARIATION OF TERMINAL RISK WITH COURSE.

Curve (X) represents present risk

Curve (Y) represents the relation of (Vr/Vr.max) with course

Curve (Z) represents the relation of (R_f/R_i) with course

OBSERVED DATA :-	
First range	=7
Second range	=5
First bearing	=160
second bearing	=160
Time of obs.(1)	=00
Time of obs.(2)	=06
Observer course	=120
Observer speed	=10

ARBITRARY DATA :-	
Accepted CPA	= 0.5
Present range	= 5.0
present time	= 06

COMPUTED INFORMATION :-	
Initial risk	=08.1%
Target speed	=13.9
Target course	=07.5
Target aspect	=332.4
Vr.max	=23.9

Figure (2-4). A typical risk presentation pattern for the assessment and control of the risk of collision.

2.5 GENERAL CHARACTERISTICS

It is now possible to take this risk hypothesis about the encounter of two ships to appreciate the consequences. However, it would be necessary before discussing the following applications to notify the parameters and variables used to define risk of collision, and see how they contribute to the behaviour of the values of risk. These factors are; the range (R) the aspect of (Q), the speed ratio (Vo/Va), and the encounter dimension (Ra).

The following general findings are revealed by investigation of the risk function (1.29) and the relevant graphs shown in Figures (2.1) and (2.2).

(a) - With a given speed ratio and aspect, it is seen that risk level increases as range decreases.

(b) - With a given speed ratio and range it is seen that the risk level decreases as aspect decreases.

(c) - With a given aspect and range the risk increases as speed ratio decreases. This is true for small angles of aspect. However for large aspects the risk value may diminish as speed ratio decreases.

(d) - Risk value exhibits exponential behaviour over different ranges.

(e) - Initial value of risk is independent of the heading of the observing ship.

(f) - With a given speed ratio it is evident that future values of risk depend on the future position resulting from a certain course. In other words the future risk of collision depends on range and bearing rates.

(g) - In a dynamically changing situation it is not practical to consider separately the effect of range or aspect changes on the problem. Since their rates are mutually dependent, as can be seen from equation (1.22), the overall response has to be considered.

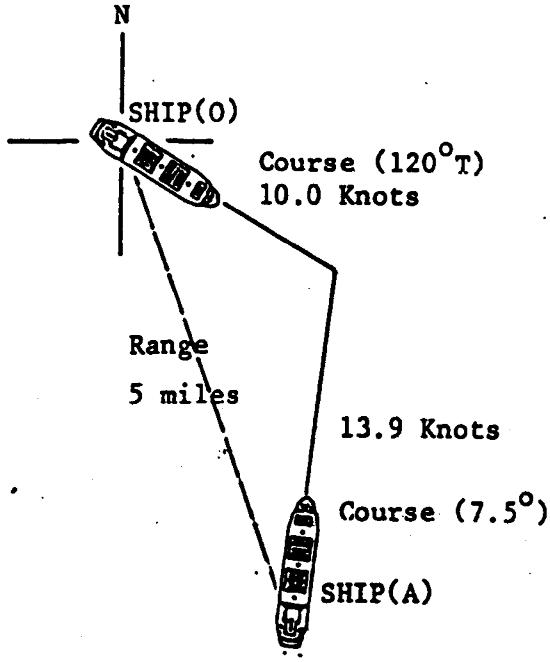
2.6 TESTING THE TERMINAL RISK MODEL

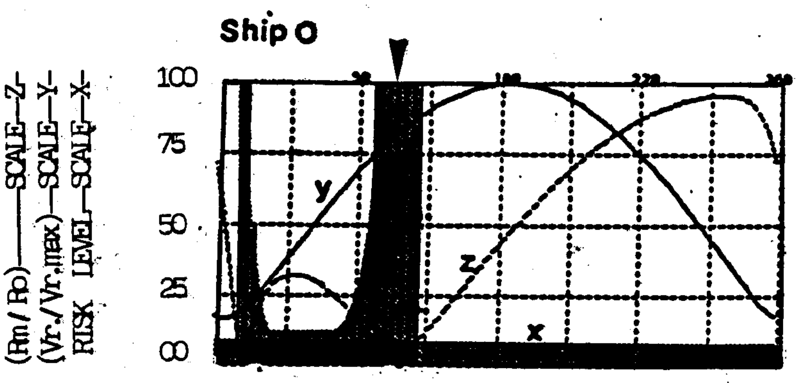
Let the pair of ships (O) and (A) be positioned as indicated by the four examples and shown in the Figures (2.5), (2.6), (2.7) and (2.8). These situations are defined geometrically by a set of observations of bearings and ranges together with computed data. This information is presented by a numerical print-out attached to each diagram. The diagram comprises four graphs: terminal risk, initial risk, normalised CPA, and normalised relative speed. Given this information, problems of collision can be identified if they rise above the acceptable level of risk.

It is of interest, however, to see by inspection of risk profiles that equal risks only exist for both ships in cases of equal speeds. There is the basic dilemma of the slower ship where double peak together with higher risk courses may arise. It is sometimes striking that the faster ship dominates the situation controlwise.

The location of 100% peak value are cases where, on following the corresponding courses, the two critical circles representing the two ships overlap. However the range of risk values about 75% is considered to be indication of extreme danger and hence the corresponding courses must be avoided. When range is available to allow manoeuvring to achieve a fixed low level of risk (50%), the manoeuvre can be selected which minimises the off-course penalty. When insufficient range is available to achieve the desired low level of risk with minimum off-course deviation, the manoeuvre is chosen which minimises risk of collision.

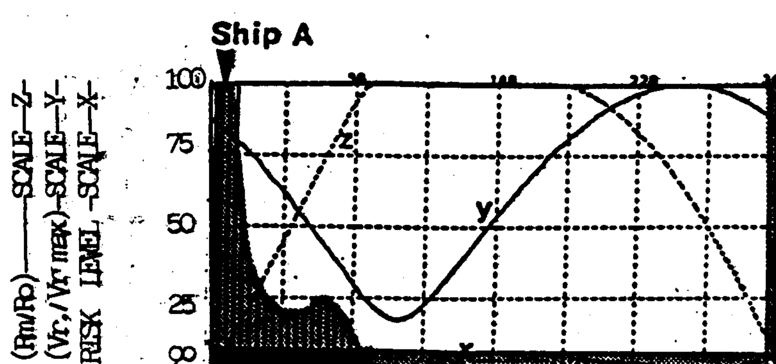
A terminal risk profile diagram gives an erroneous impression that the weighting of objectives is static. The weighting must be dynamic so that the contribution of various parameters and variables to risk prediction can be evaluated over the time span of the encounter. In order to disclose this valuable information, which is hidden in the presentation of risk rate, a criteria is needed to weight the indication of terminal risk. In other words, it is necessary to investigate the behaviour of risk transition from initial state to terminal situation. This is the subject of the following paragraph.





INPUT			
R1	:7	CPA	:0.5
R2	:5	Ro	:5
B1	:160	To	:06
B2	:160	OUTPUT	
T1	:00	Po	:06.8
T2	:06	Ca	:007.5
Co	:120	Va	:13.9
Vo	:10.0	Qa	:332.4
		Vr.max	:22.9

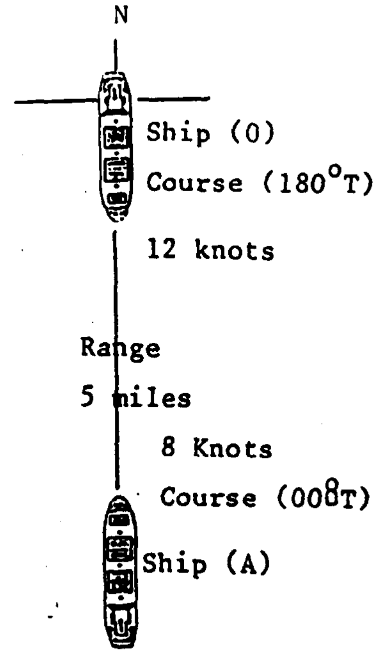
Variation of the terminal risk with course change.

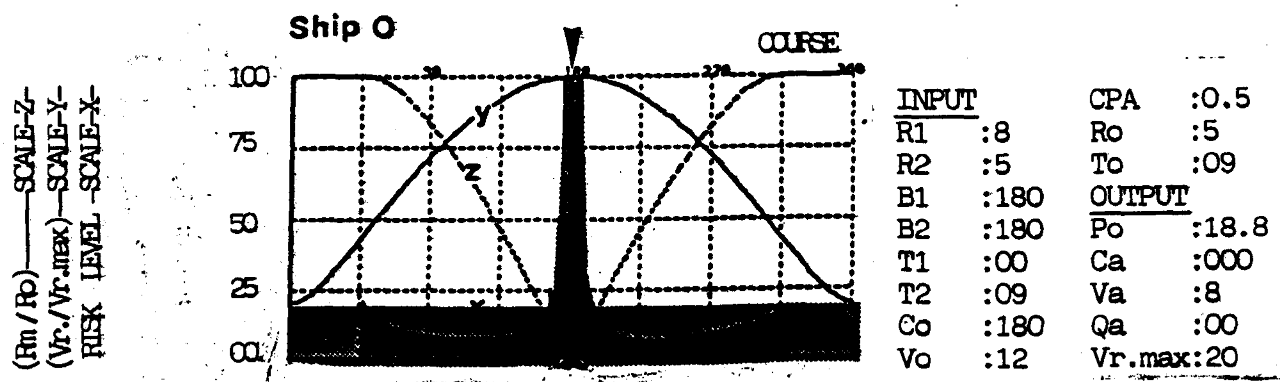


INPUT			
R1	:7	CPA	:0.5
R2	:5	Ro	:5
B1	:240	To	:06
B2	:240	OUTPUT	
T1	:00	Po	:05.2
T2	:06	Ca	:120
Co	:007.5	Va	:10
Vo	:13.9	Qa	:29.2
		Vr.max	:22.9

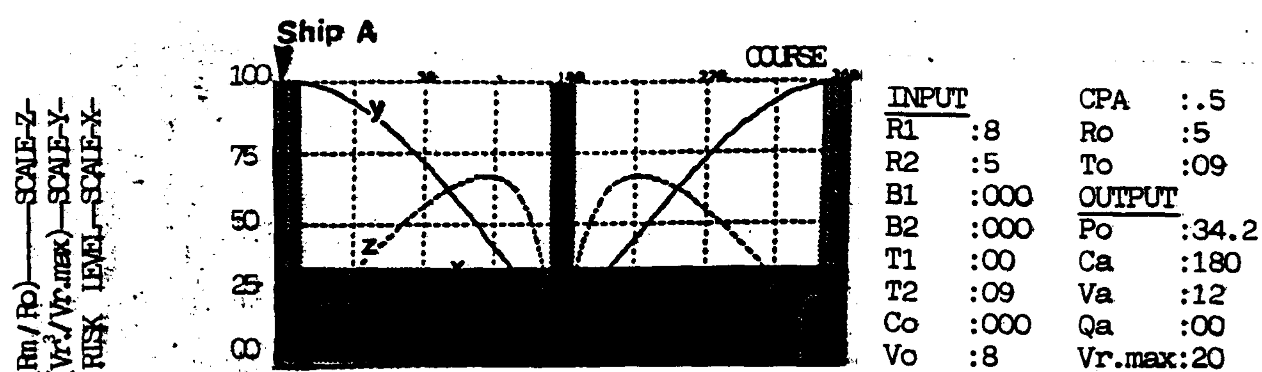
Variation of the terminal risk with course change.

Figure (2-5). Two-risk presentation diagramsof two crossing ships involved in a possible collision.



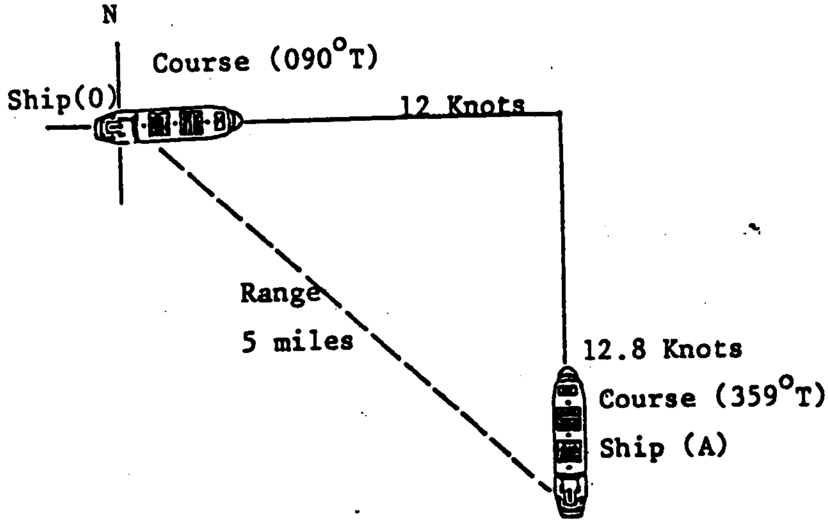


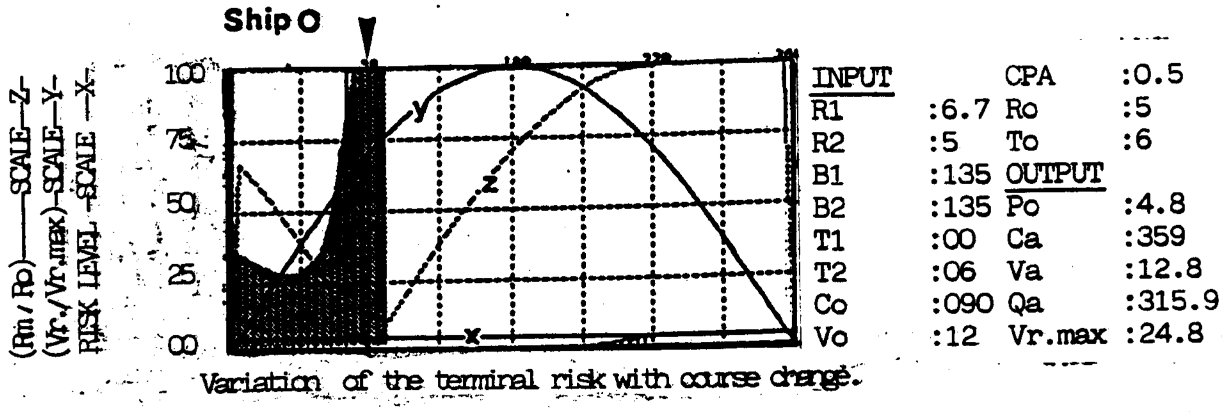
Variation of the terminal risk with course change.



Variation of the terminal risk with course change.

Figure (2-6). Two risk presentation diagrams of two ships meeting end-on involved in a possible collision.





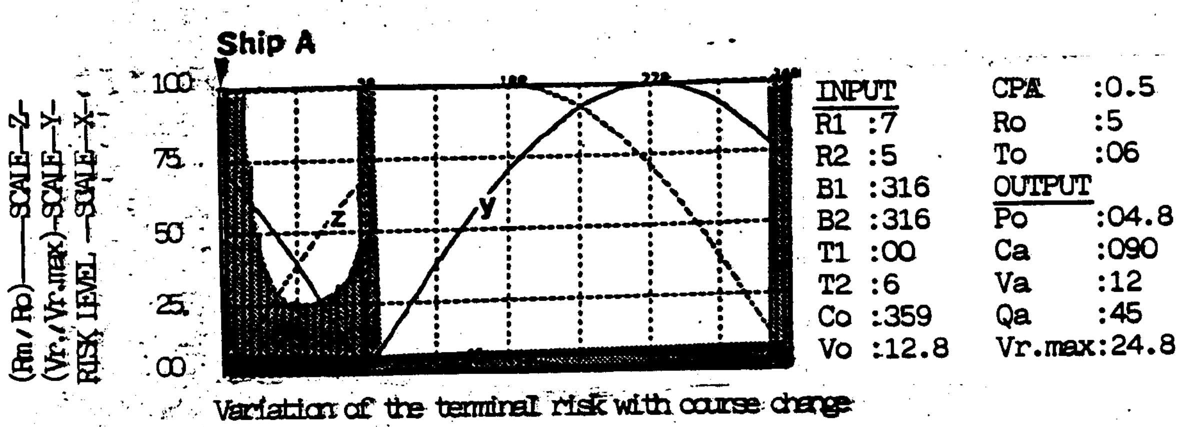


Figure (2-7). Two risk presentation diagrams of two ships crossing at right angles involved in a possible collision.

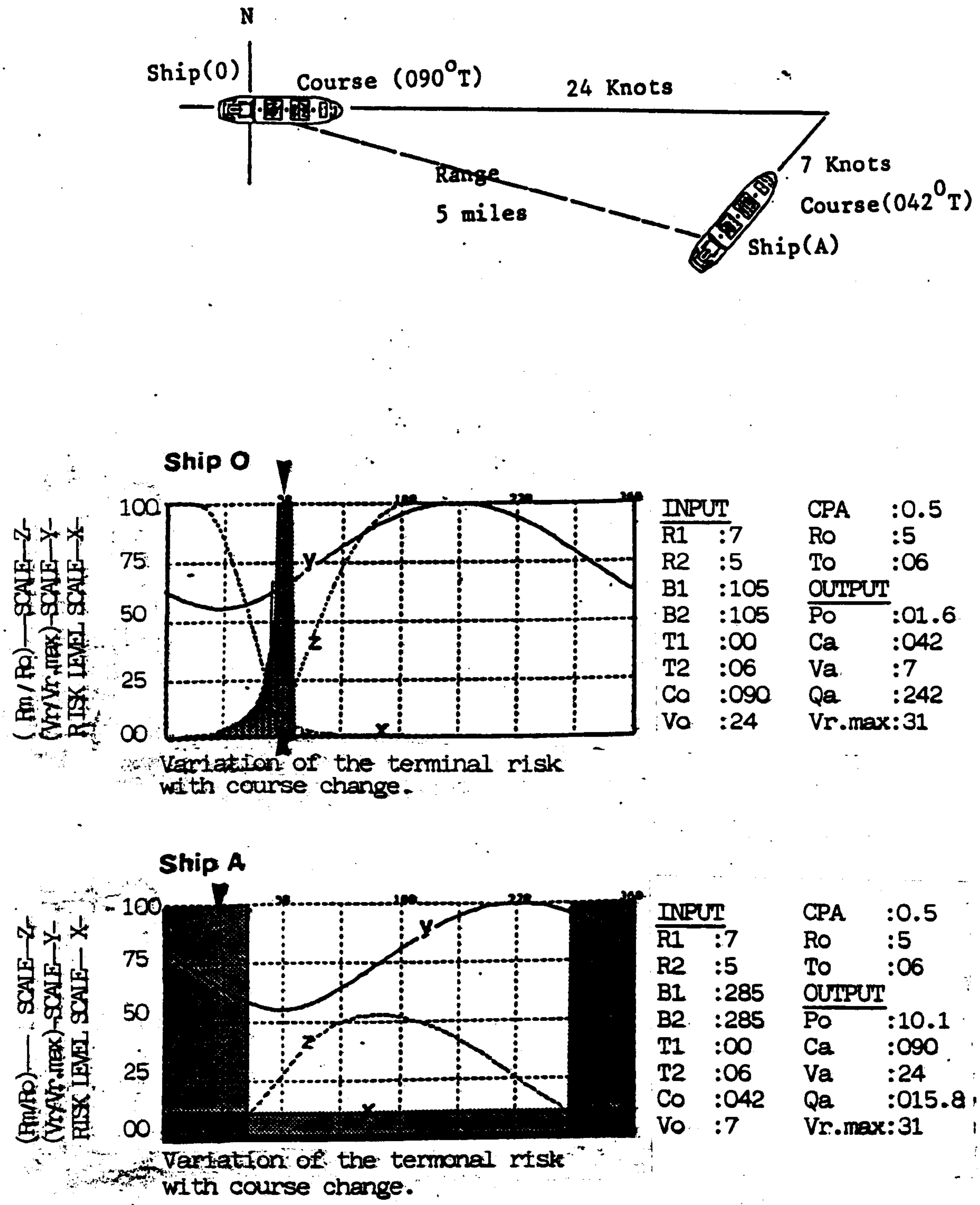


Figure (2-8). Two risk presentation diagrams of two crossing ships of different speeds and involved in a possible collision.

Regarding a dynamically changing situation, when the parameters of ship's size and speed are considered unchangable, the variation of risk value depends on the changes in range and aspect angle. Since these changes are a function of time, the risk-time relationship can be established for a specific course. A program that performs the calculation of risk as function of time is shown in listing (RISK / TIME), appendix (B-2). The associated flowchart is given in Fig. (2.9). Although the flowchart is self-explanatory, the basic idea is to find the relative position in terms of aspect and range after short intervals of time, following the same procedure described in para (2.2).

Thus by, stepping time interval in a do-loop till (TCPA), or for any interval of time, the corresponding values of risk are computed. For the sake of illustration Fig. (2.10) is an enlarged graph of the computed relations (Risk/Time), (Range/Time) and (Aspect/Time). Figure (2.11) plots typical results from a computed calculation. Four graphs (a), (b), (c) and (d) are computed for courses 120^{0},100^{0} 45^{0} and 135^{0}. The initial situation is based upon the example illustrated in figure (2.5). Since range and aspect can contribute differently to the variation of risk values, it was necessary to add separately on the same diagram the variation of range and aspect with respect to time.

The investigation of these graphs shows, as expected, greatly varying shapes depending on the interrelated rates of change of range and aspect. The behaviour of the transition distribution of risk values in a typical collision course is presented in Figure (2.11a). The risk profile shows a regular transition in risk values, where the range is steadily decreasing with constant aspect. The effect is gradual increase of risk value at large range and then more rapidly at close range, which is a typical characteristic behaviour of an exponential function. In this case of constant aspect the range is the only parameter which contributes to the increase of risk values. However, the aspect puts influences on risk values at the initial stage of the situation. As a consequence of these factors the maximum value of risk occurs at CPA.

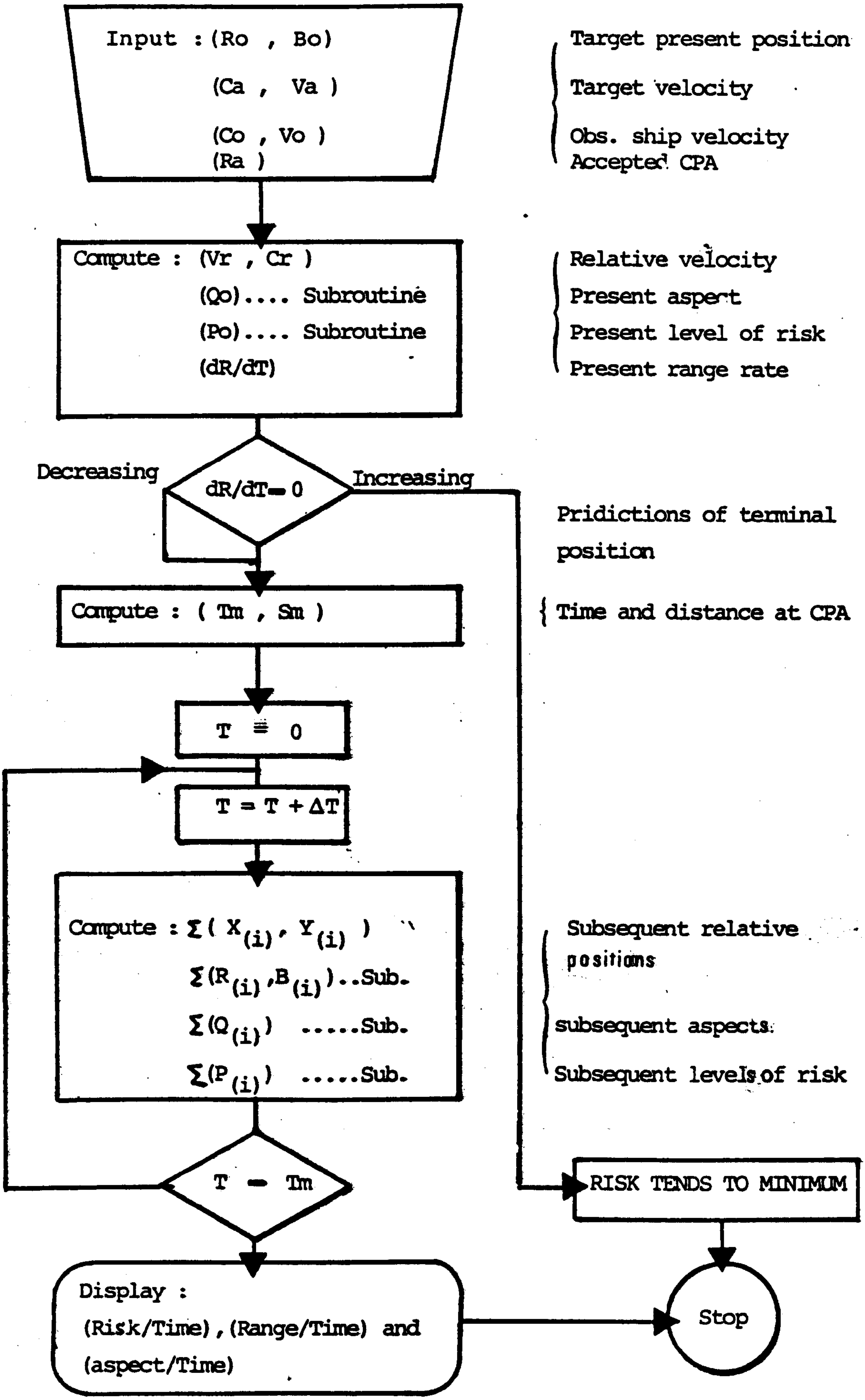
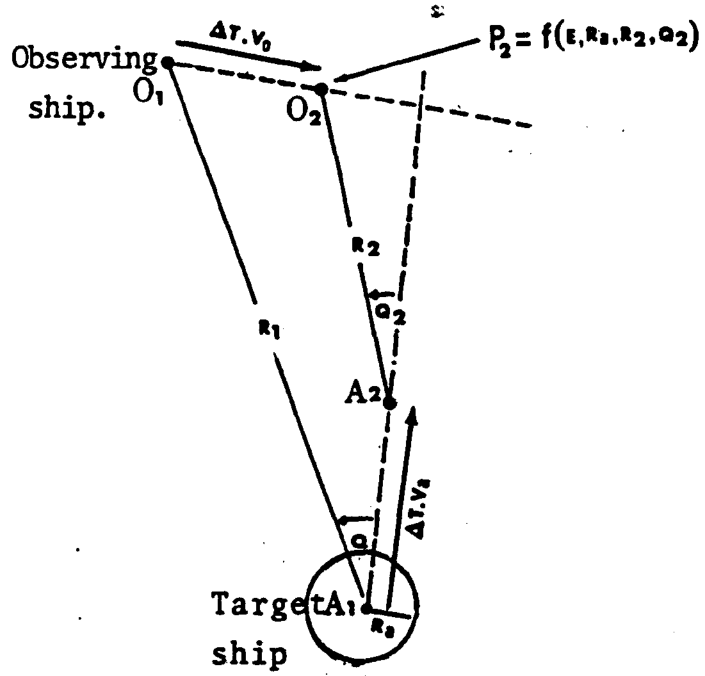
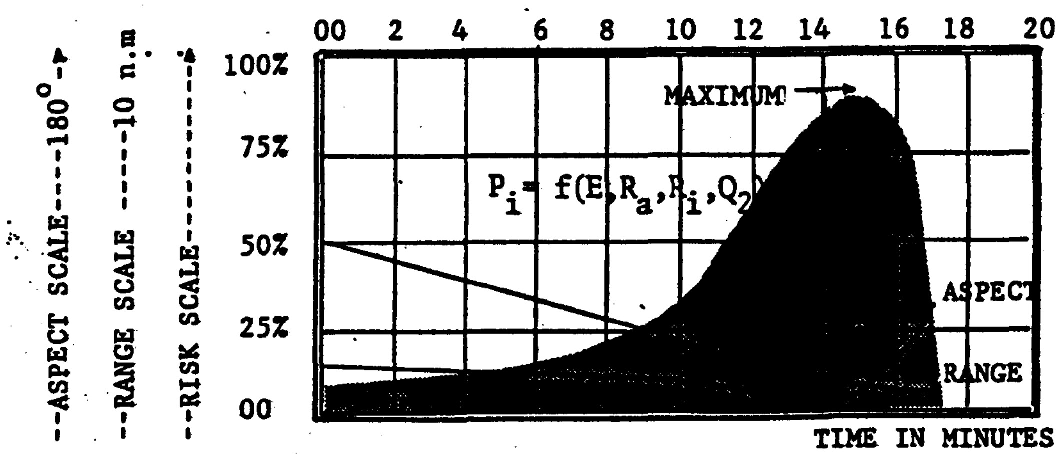


Figure (2-9). A simplified flowchart of risk transition distribution.





VARIATION OF RISK WITH TIME

INPUT DATA :-

Initial bearing	= 160°T
Initial range	= 5 n.m
Obs.ship course	= 100°T
Obs.ship speed	= 10 kts
Target course	= 007.5°T
Target speed	= 13.9 kts
Accepted CPA	= 0.5 n.m

NUMERICAL PRINTOUT:-

Relative speed	= 17.5kts
Distance to CPA	= 2.3 n.m
Initial aspect	= 332.5°
Aspect at CPA	= 54.0°
Time to CPA	=17 min
Initial risk	= 08.0%
Risk at CPA	= 18.0%
Maximum risk	= 89.0%
Time of max.risk	= 15 min
Range at max.risk	= 1.6 n.m

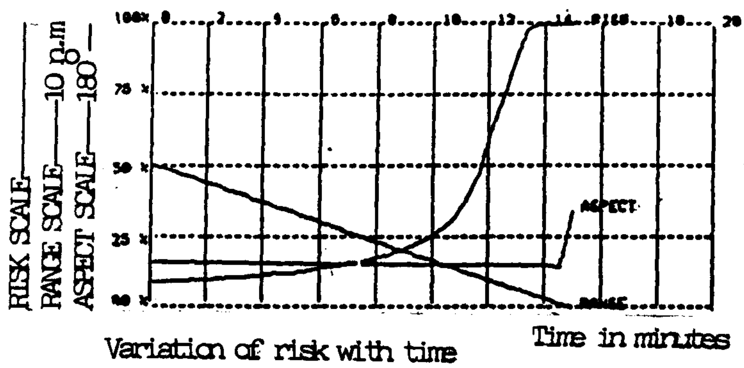
Figure (2-10). An enlarged graph of the computed relations "risk/time","range/time", and "aspect/time".

Figure 2.11(b) shows the change of risk value if 100° course is to be steered. In this case both range and aspect contribute positively to the increase of value of risk. The effect is a gradual increase of the value at large range and more rapidly at closer range due largely to the sudden decrease in aspect with faster rate. It is noticed in this case that the maximum value of risk is more than twice the value of risk which occurs at CPA, and occurs about 2.5 minutes before TCPA.

A third course of 045° is taken for investigation, the result of which is illustrated in Fig. 2.11(c). The computed risk profile shows a similar behaviour to that of course 100°, but with a lower value of maximum risk. On proceeding along this track ship (O) crosses ahead of ship (A) with a later CPA of 1.6 n.m., the value of which is three times bigger than that of the encounter dimension. However this peak value has to be considered as long as it is in the potential risk interval. This occurs at or nearly at zero aspect, the time when ship (O) crosses ahead of ship (A).

Having passed ahead then the effect of increasing aspect overcomes the effect of the slowly decreasing range. As a consequence of these effects the risk value is reduced rapidly in eight minutes from a peak of about 60% down to a value of 8% at CPA. Such a behaviour, which has been also shown in Figure 2.11 (b), is due to the greater influence of aspect change at this stage of the encounter. It is perhaps surprising that reaching CPA position is in itself an indication of diminished risk, and does not represent the extremis.

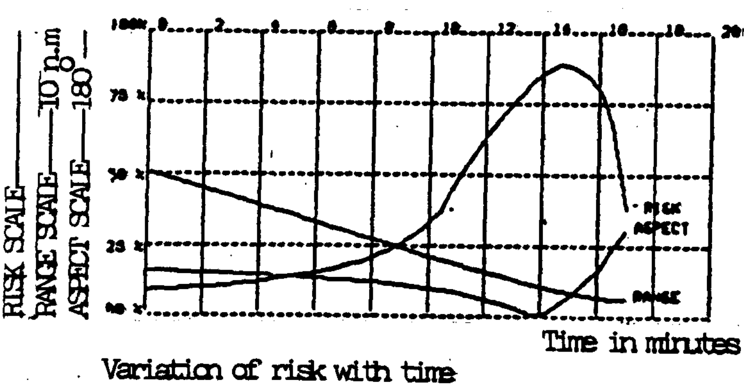
Based on a fourth course of 135°, another set of risk values is obtained. The corresponding shape of risk profile is shown in Fig. 2.11(d). A unique case of balancing effects between a deceasing range and increasing aspect is apparent in the graph. The result is a condition where a low value on the initial risk is nearly maintained constant throughout the interval of the closing range. This important fact may prove to be so, in as much as the CPAs of these particular courses are always greater than the encounter dimension. The computed data of this course shows, as expected, that the interval of the encounter is relatively smaller than that of crossing ahead courses.



a

INPUT

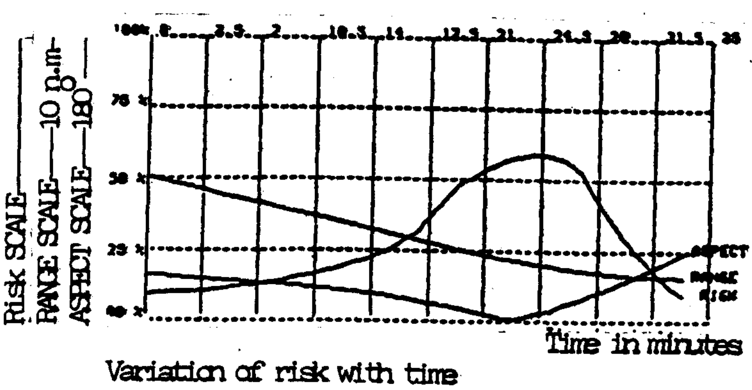
Bo	:160
Ro	:5
Co	:120
Vo	:10
Ca	:007.5
Va	:13.9
ACPA	:0.5
E	:0.7



b

INPUT

Bo	:160
Ro	:5
Co	:100
Va	:10
Ca	:007.5
Va	:13.9
ACPA	:0.5
E	:0.7



c

INPUT

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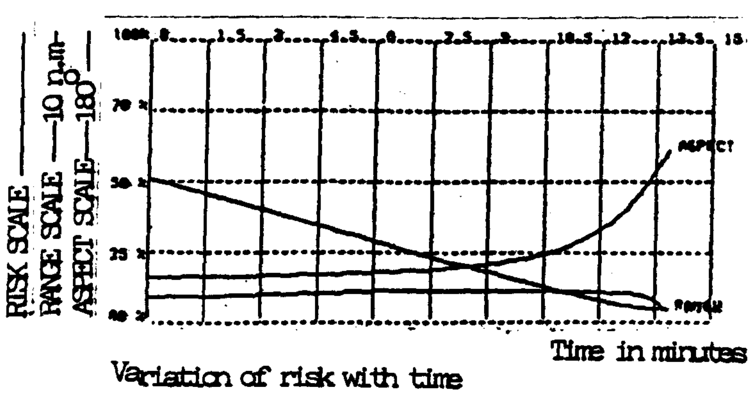
Ro	:5
Co	:045
Va	:10
Ca	:007.5
Va	:13.9
ACPA	:0.5
E	:0.7



d

INPUT

Bo	:160
Ro	:5
Co	:135
Vo	:10
Ca	:007.5
Va	:13.9
ACPA	:0.5
E	:0.7

Figure (2-11). Different features of risk transition distribution.

2.7 IMPROVED RISK MODEL

In view of a possible misinterpretation of terminal risk due to peak values not occuring at C.P.A., the improvement of the risk diagram is essential to include the effect of transition. Therefore the risk profile has to present the variation of maximum risk with respect to course changes. Determination of the value of maximum risk for each course is obtained from a set of risk transition distributions, this performed simply by numerical sorting techniques; as long as the distribution of risk value for constant course does not show a local minimum. This technique is tedious and practically restricted due to the large computational requirements. However the efficiency may be improved by using more refined numerical methods. The existence of such measures even when not practical or easy to apply provides an objective standard for which an index can be sought.

Graphs shown in Figures (2.12) and (2.13) are risk profiles of the maximum values. For the sake of comparative investigation the computations are based upon the same initial situations previously discussed and illustrated in Figures (2.5). (2.6). (2.7), and (2.8).

Fig. (2.12a) presents the maximum risk being looked at by ship (O). It is seen that risk level when steering course (120^{o}) increases up to (100%) risk. This value can be reduced to zero level by a small alteration of course to starboard on to a course of (135^{o}). This new course is also associated with higher relative velocity, the merit of which is a fast diminishing risk. Now consider the effect of trying to pass ahead and maximising the CPA by changing the heading to steer (045^{o}). This means a large alteration of course. Selecting such a course is in contrast to the one just discussed, where the future maximum risk will be reduced below the range of potential risk (75% to 50%), with a longer interval of encounter time. As a consequence of selecting this course, the risk value will increase up to potential level and the difficulty of the situation lasts longer. Regarding the risk assessment viewed by the other ship (A) in the same encounter, Figure 2.12 (b) shows that the exact collision course is (007.5^{o}). On steering course (355^{o}) by altering the course about (13^{o}) to port the risk of collision is reduced to minimum of (8%) risk.

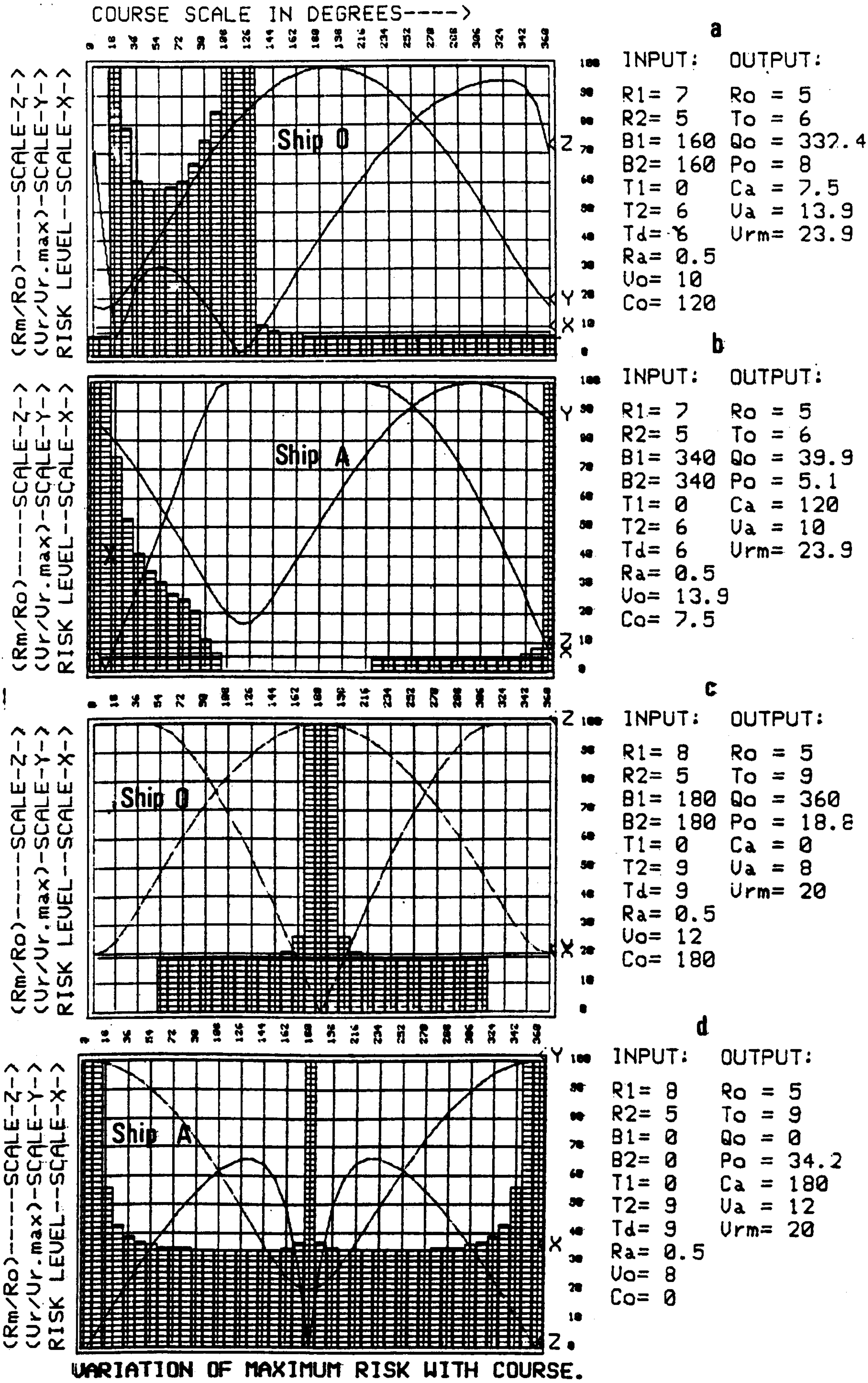


Figure (2-12). Maximum risk presentation patterns of two different binary encounters.

Meanwhile, in order to reduce the risk of collision below (25%) risk level with starboard course alteration, necessitates (60^{o}) alteration of heading. A final observation on this case is that the slower ship (O) exhibits a larger sector of risky courses than that of ship (A).

The discussion on the effect of aspect value on the risk value as given in paragraph 2.5, has proven to be significant in risk control. Furthermore the maximum risk values in case of zero aspect, show a symmetrical configuration with respect of course changes. Should an extreme risk of collision exist in such a case, there is no preference, as would be expected, to which side the risk control is best performed, see Fig. 2.12(c) and 2.12(d).

In order to show the effect of speed ratio (Vo / Va) parameter on the value of maximum risk in a binary encounter, the comparison of maximum risk values are carried out between each pair of ships by analysing their associated risk graphs. Concerning the case of equal speeds, shown in the Fig. 2.13(a) and 2.13(b), the maximum risk diagrams show similar risk profiles, which are computed at the range of (5) miles. This similarity holds good no matter what courses are steered by either ship in the encounter at the initial stage. It is only upset once the speed differs.

Regarding the case of different speeds, as shown in Figures (2-13,c) and (2-13,d). It is clearly illustrated how the speed ratio significantly affects the maximum risk profile. On comparison of risk control action, ship(O) will bring her risk down to zero level by altering her course (5^{o}) to starboard or (10^{o}) to port. Meanwhile, the slower ship (A) has to alter her course (20^{o}) to starboard or (90^{o}) to port to achieve the same control effect as ship (O).

In general, if extreme risk values are expected in any situation, the slower ship is often burdened with a wider sector of risky courses and hence has less room to perform risk control.

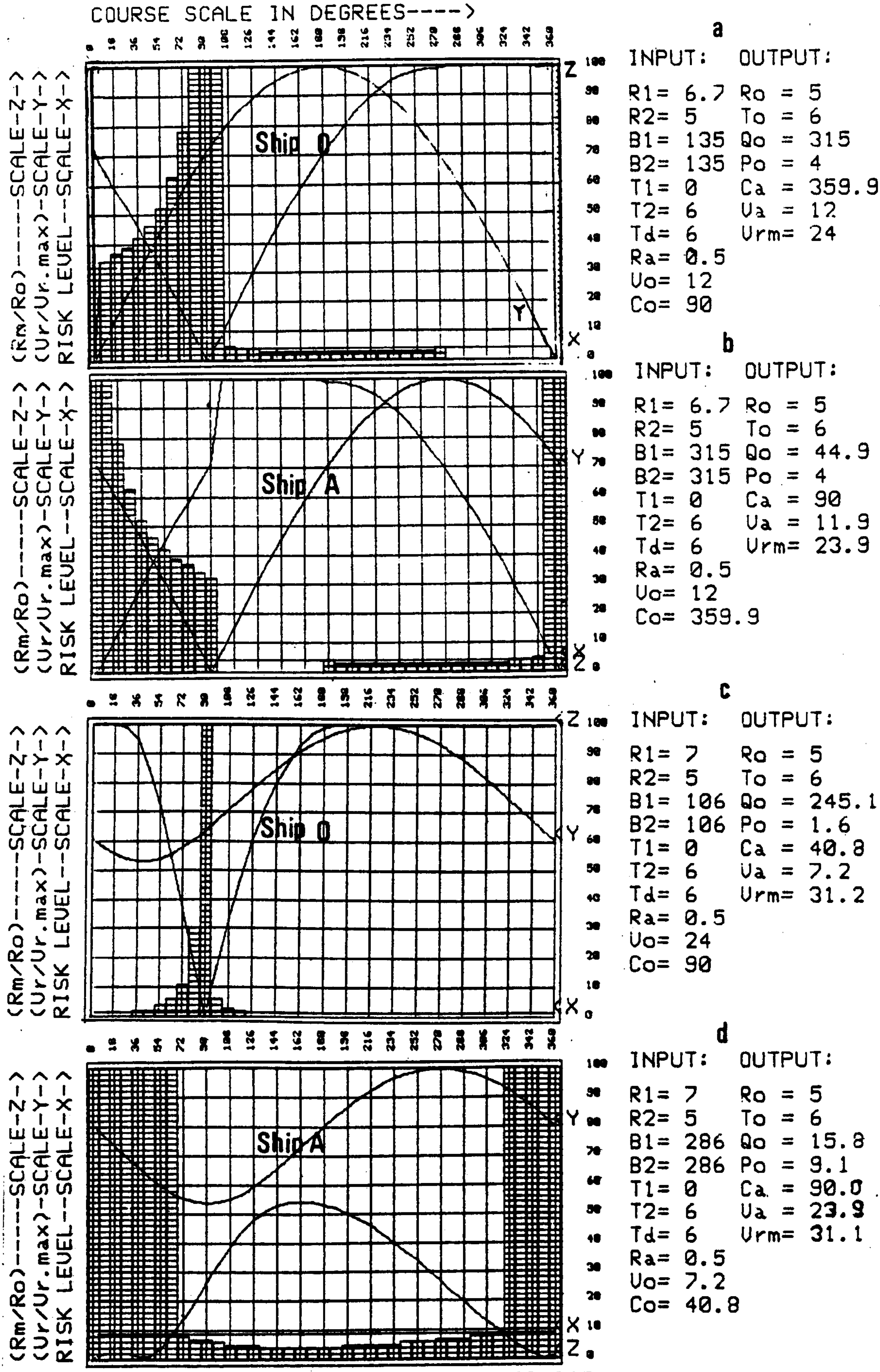


Figure (2-13). Maximum risk presentation patterns of two different binary encounters.

2.8 SUMMARY AND CONCLUSION

In this unit a risk assessment and control model has been presented taking into account all the relevant parameters and variables which affect risk of collision.Given the definition and interpretation of risk assessment and control, which is based upon the classical geometrical probability, then the general formulation was found to be straightforward within the assumption that the observing ship maintains regular speed and the other ship maintains regular velocity.

Risk assessment of an encounter between a pair of ships at sea is a very complex characteristic to define by a single rule. The value of risk is not even the same for two ships approaching each other, with the exception of the symmetrical situation when ships steam with equal speeds. It is worth noting that the slower ship is often burdened with higher risk and less room to manoeuvre.

The use of terminal risk patterns together with risk transition distribution along a selected track will enable estimates to be made of, first the number of options being considered acceptable to risk control and second, the amount and complexity of information defining the problem. On varying the amount and type of situations, insight can be gained into the overall decision making process.

Risk control is an integral part of the pattern being presented, and not an aftersight, as often happens in most C/A systems. The risk pattern focusses attention on the wider questions of where the control needs to be exercised. This model, suitably modified to account for maximum risk, has been used successfully to qualify the outcomes of alternative actions.

The theoretical approach of risk assessment is a new technique for quantitive evaluation of risk of collision and gives a fair understanding of C/A problems. This technique can also be used to form the basis for evaluating the training in C/A simulation. On setting aside the question as to which method is the correct one, the opinion that the risk assessment as a control function is a superior mean of tackling the collision avoidance problems becomes evident.

UNIT_3

Use of Speed Manoeuvre in Control of Risk of Collision

3.1 INTRODUCTION

So far a unique method of risk assessment and control to deal with the collision avoidance problem has been introduced in a form of a maximum risk matrix. The generated pattern enables the user to assess and control risk of collision by means of course variability. The computation of the maximum risk associated with each course has been found to be a useful method for generating alternative ways of looking at the resulting risk outcomes in a given situation.

In this unit the use of speed variation as a control variable in the assessment and control of risk will be investigated. It does not seem difficult to extend the previous work so that risk of collision along a given course in the same situation can be checked with speed variability. However, speed variation is not a popular manoeuvre amongst mariners due to the time penalty incurred in speed reduction and the reluctance of the inexperienced mariner to use a reduction of speed as a means of avoiding action.

It has been made clear that course controllability is highly effective in risk reduction, but this has certain limitations which have been shown in the previous work of unit (2). The inspection of the (E/H) diagram and the risk function show that the solution in the range of ($Vo/Va \leq 1$) is always constrained, especially in case of small angles of aspect.

Furthermore the introduction of the separation schemes together with geographical constraints in restricted waters impose more limitations. Hence it is essential to investigate speed controllability in the assessment and control of risk.

...

3.2 FEASIBLE REGIONS

In the selection of a method for the assessment and control of the risk of collision, the controller must concern himself with the question of the range of applicability of the method, i.e, how effective is a variation of speed in reducing the risk of collision ? Some insight into the problem can be obtained by studying the behaviour of the risk function with respect to the variation of the speed ratio in the graphical form. Figure (3-1) illustrates the behaviour of the risk function at the terminal stage of a binary encounter throughout the range values of (Emax), the available maximum speed ratio. In order to investigate the speed parameter as well, a family of curves are plotted for (7) values of aspects in the range under consideration.

In the given example (shown in figure (3-1)), the values of risk can be seen to be less than (50%) in the range of ($E = 1.7$). In case of aspect angles which are greater than (25^o) the values of risk levels are found to be less than (50%) over the full range of the speed ratios for these same aspect angles. It is also seen that the curves exhibit flatness in the range of ($E > 1$) with a sharp fall in the values of risk in the range of ($E < 1$) where they tend towards zero. This sharp decrease of risk is an indication of the effectiveness of speed reduction in this region.

On considering the smaller values of aspect, the levels of risk are higher than the case of the larger aspects and increase to the (100%) level of risk. The flat parts of the curves shown in the range of the higher speed ratios indicate the ineffectiveness of speed variation in this range. However in the case of the smaller aspects the level of risk is much reduced by increasing the speed ratio.

In the foregoing it has been shown how the aspect parameter in any situation can change the effect of the speed ratio variability on the risk of collision. The next step is to determine the range of applicability. In doing so it is necessary to know both the range of the speeds and courses at which the risk of collision cannot be avoided by speed variation alone.

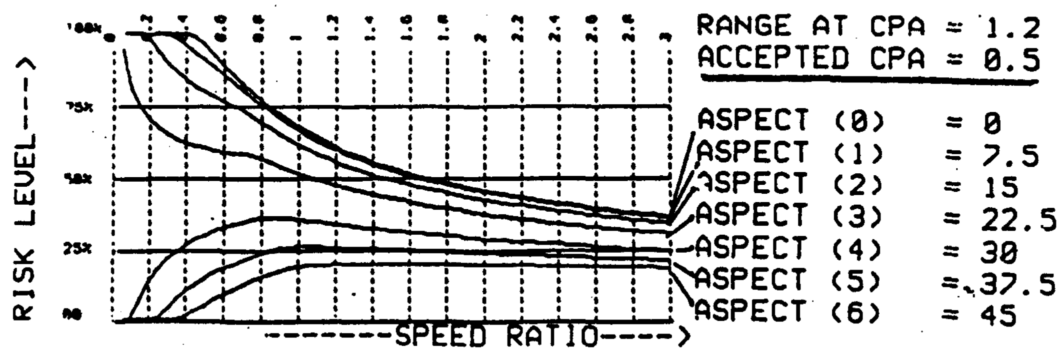


Figure (3-1). Graphical characteristics of collision function with respect to speed variation.

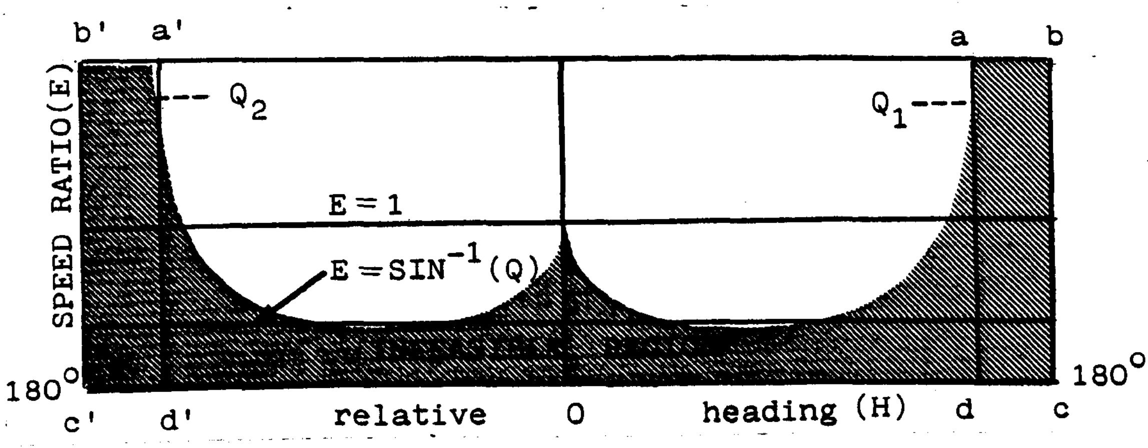


Figure (3-2). Graphical representation of the infeasible region of speed variation on the (H/E)-plane.

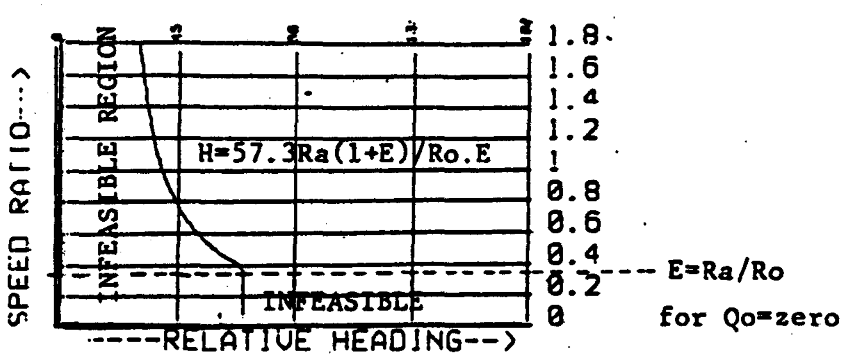


Figure (3-3). Graphical representation of the infeasible region of speed variation on the (H/E)-plane using an approximation formula.

This infeasible range is generally found in situations that have small angles of aspect. It can be best visualised by its presentation on the (E/H) diagram as can be seen in Figure (3-2). Such an infeasible domain is recognised in the interior of the shaded area defining the possible ways for collision to occur. Assuming the limiting aspects (Q_1) and (Q_2) are determined from the following known expression;

$$Q_i = Q_o \pm \sin^{-1} (Ra / Ro) \; ; i = 1,2$$

Then the local minimum of the iso-aspect curve can be determined from the following relation which is derived from the collision function.

$$E = \left| (\sin (Q_1)) \right|$$

$$= \left| (\sin (Qo + \sin^{-1}(Ra / Ro)) \right| \qquad 3.1$$

Considering the condition of (Qo = zero) then

$$E = Ra / Ro$$

or $$Vo = Va. \; Ra/Ro \qquad 3.2$$

Now if the maximum available speed of the observing ship does not exceed the value taken by the expression (3-2), then the risk of collision cannot be reduced only by speed variation regardless of ship's heading. However, if the maximum available speed exceeds the value taken by the expression (3-2), then another region to be considered exists in a narrow sector of courses bisected by the sight line. This sector of headings can be traced in the Figure (3-2) as the two rectangular areas (abcd) and (a'b'c'd'). The determination of the boundaries of these sectors is obtainable by solving the collision function for a given speed ratio and aspect. However, a good approximation is achievable by using the collision derivative (See Appendix (A-2)).

$$\Delta H = \Delta Q \ (1-2.E.\cos H + E^2)/E.(\cos H - E)$$

Setting

(H = 180) which is the case under consideration then,

$$\Delta H = - \Delta Q.(1 + E) / E$$

$$= - \sin^{-1} (Ra / Ro). (1 + E) /E \qquad 3.3$$

Provided that $Q \leqslant 20^0$ then

$$\Delta H = \pm 57.3 \ Ra \ (1 + E) / (Ro.E) \qquad 3.4$$

Where

Ra = accepted CPA

Ro = present range

E = Vo / Va = speed ratio

ΔH = the sector of courses bisected by the sight line.

This expression can be investigated by plotting speed ratio against Relative Heading, and thus obtaining a more complete picture of the infeasible range of headings throughout the range values of (E). Fig. (3-3) indicates this behaviour. Values of (R= .5) and (Ro= 1.5) are taken as arbitrary input parameters similar to those which have been used to plot the collision function of the (E/H) diagram shown in figure (3-2). Comparing the two curves indicates that expression (3-4) is a good approximation of the collision function. The applications of the constraint formulas (3-1) and (3-4) are straight forward as long as the condition of small angles of aspect are considered. These two formulas can be used to define the regions of the infeasibility and hence the area of the speed control application can be subsequently defined.

3.3. FLOW CHART AND PROGRAM FOR RISK/SPEED PRESENTATION

The computer program was written in BASIC as shown in appendix (B-3) listing (RISK / SPEED). A simplified flowchart of this program is shown in figure (3-4). The program provides a set of risk results for any course considered by the observing ship. Each value represents the predicted maximum risk level being recognised along a fixed track for each speed under consideration. The main program is sequentially executed and is controlled by a step increase with speed of ship (O) by one knot at a time, passing through the speed range (Vo) from zero to (Vmax). The program algorithm and the associated subroutines are identical to that given in unit (2). Once the total number of the subsets $(\sum p_i, v_i)$ of the given encounter have been generated, then a graphical risk presentation is possible.

Next the plot of the dimensionless values of (Vr. max/ Vr) and (Rf / Ro) are added to the diagram, which shows the relationships between the relative speed, distance to CPA and ship's speed. Both plots are intended to illustrate how these values are related to the maximum risk values. The results of the generated risk pattern will be described in the following case studies.

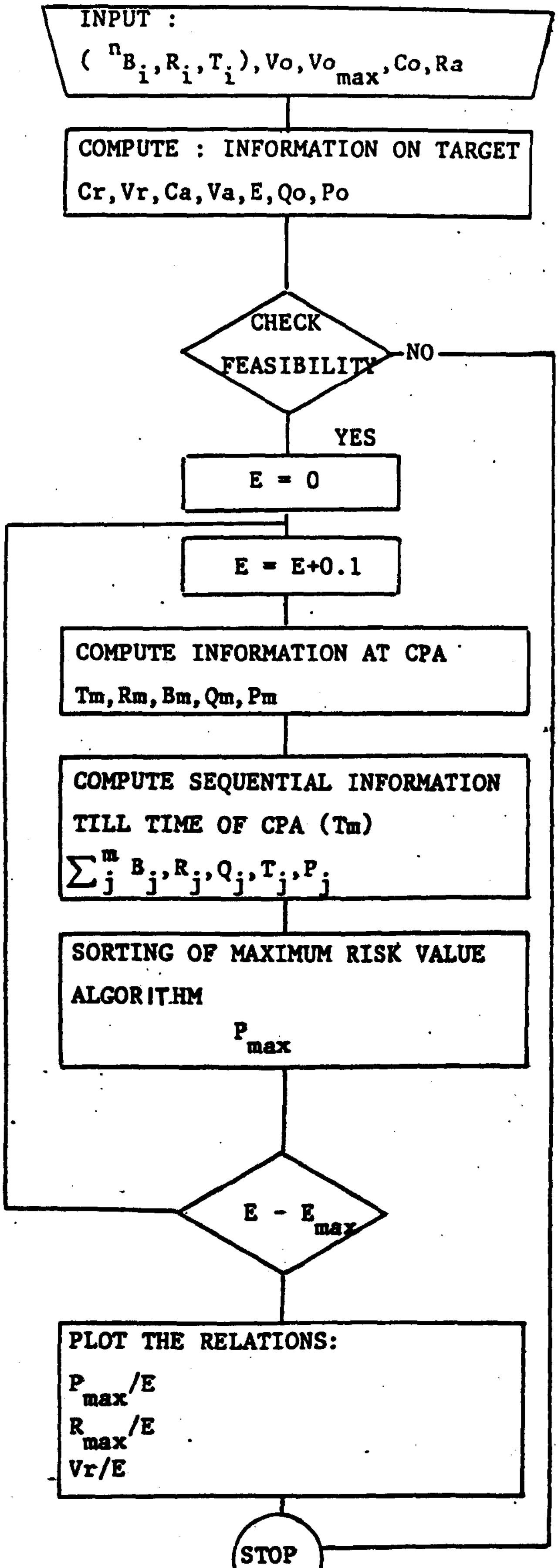


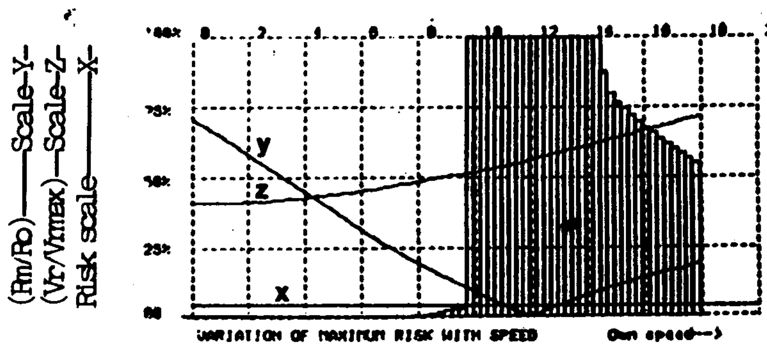
Figure (3-4). A simplified flowchart of a risk prediction model based on speed variation control.

3.4 CASE STUDY

A number of examples are now provided which describe the risk assessment as a function of ship's speed and illustrate the effectiveness of risk control by means of speed variation. Figure (3-5) demonstrates a risk pattern being recognised by an observing ship in a binary encounter. This ship is sailing on course (090^{o}) with a speed of (12) knots. The preliminary processing of the chronological observations indicates that target ship is steering due north with speed of (12) knots. The legend is a typical crossing situation. The risk profile shows how the maximum risk varies with speed variation along the (090^{o}) course of ship (O). The inspection of the risk graph indicates that the risk of collision can be eliminated by reducing ship's speed to below (10) knots. It is also seen that the risk of collision can be reduced to almost the (50%) level by means of an increase in the speed of the ship to (18) knots.

Figure (3-6) shows a similar pattern describing the risk behaviour in a crossing situation but for a slower target ship which is found steering (346^{o}) with speed of (5.3) knots. It is seen that the risk of collision can be avoided by reducing ship's speed from (18) to (12) knots. Figure (3-7) represents the risk pattern as computed for an observing ship sailing due north with a speed of (12) knots. The processed observations indicate that the target ship is sailing on course (332^{o}) with a speed of (24.5) knots. The legend is a binary encounter involving an overtaking situation where the initial risk is expected to increase from (9.1%) to (100%). It is shown from the presentation that the risk of collision can be avoided by reducing the ship's speed to below (10) knots. It is quite apparent that the reduction of risk below the (50%) level is another possibility which can be achieved by increasing the speed to (18) knots.

Figure (3-8) demonstrates an example of what might be called a complex situation consisting of three binary encounters as observed from one ship.



OBSERVED DATA:		ARBITRARY DATA:	
First Range	= 6.7	Accepted CPA	= 0.5
Second Range	= 5	Initial Range	= 5
First Bg.	= 135	Initial Time	= 6
Second Bg	= 135	COMPUTED DATA :-	
Time of Obs1	= 0	Maximum Risk	= 100%
Time of Obs2	= 6	Initial Risk	= 4
Vo.max	= 18	Target Speed	= 12
Own Course	= 90	Target Course	= 359.9
Vr.max	= 30	Target Aspect	= 315

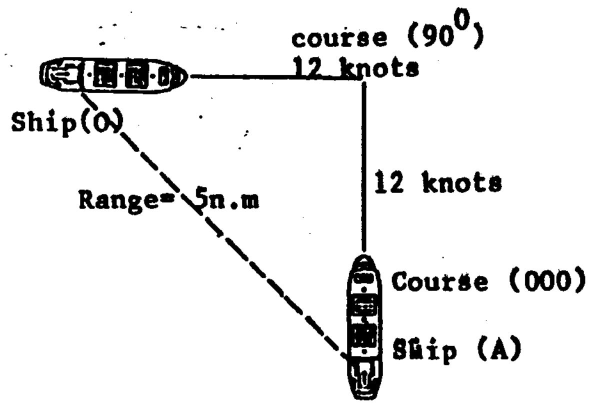
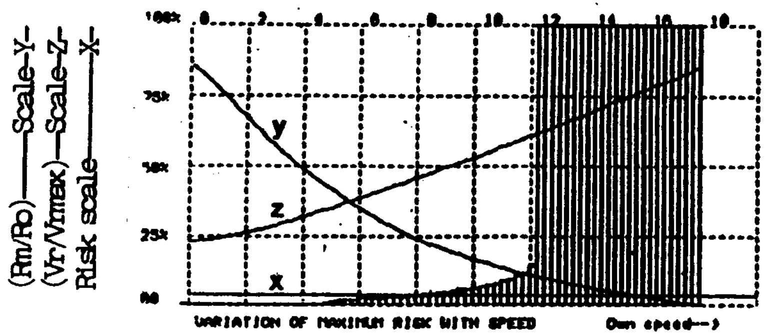


Figure (3-5). Risk presentation diagram of two ships crossing at right angle.



OBSERVED DATA:		ARBITRARY DATA:	
First Range	= 7	Accepted CPA	= 0.5
Second Range	= 5	Initial Range	= 5
First Bg.	= 105	Initial Time	= 6
Second Bg	= 105	COMPUTED DATA :-	
Time of Obs1	= 0	Maximum Risk	= 100
Time of Obs2	= 6	Initial Risk	= 3.4
Vo.max	= 18	Target Speed	= 5.3
Own Course	= 90	Target Course	= 345.7
Vr.max	= 23	Target Aspect	= 299.2

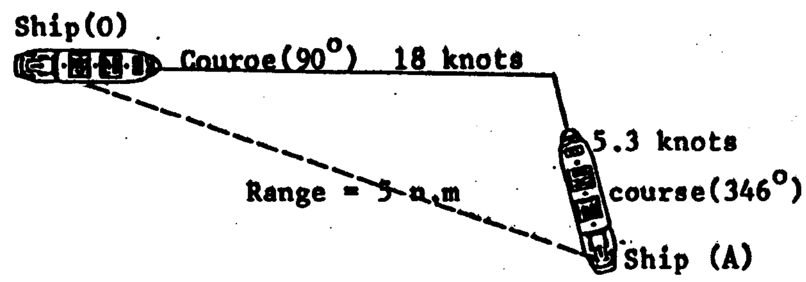
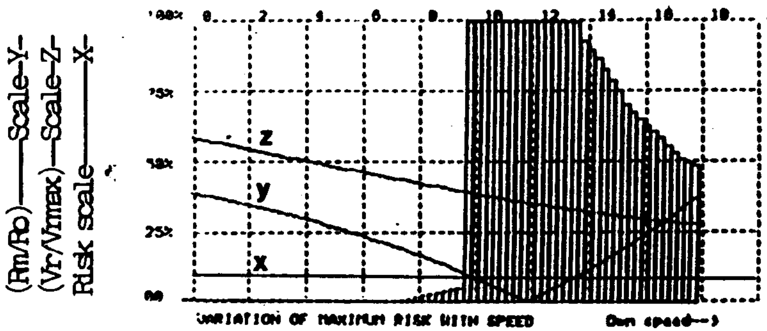


Figure (3-6). Risk presentation diagram of two ships crossing at large angle.



OBSERVED DATA:	ARBITRARY DATA:
First Range = 7	Accepted CPA = 0.5
Second Range= 5	Initial Range= 5
First Bg. = 130	Initial Time = 8
Second Bg = 130	COMPUTED DATA :-
Time of Obs1= 0	Maximum Risk =100%
Time of Obs2= 8	Initial Risk =9.1%
Vo.max = 18	Target Speed = 24.5
Own Course = 0	Target Course= 332
Vr.max = 42	Target Aspect= 337.9

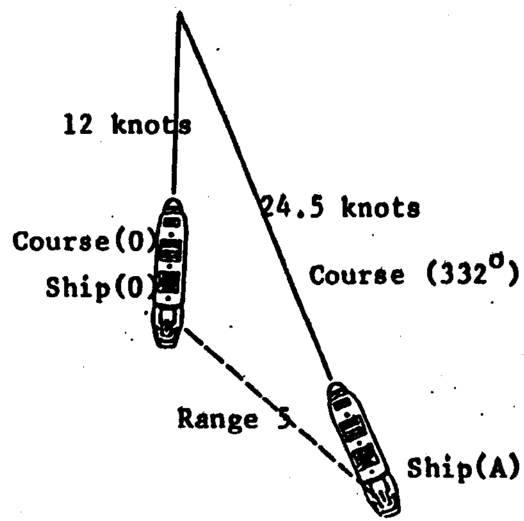
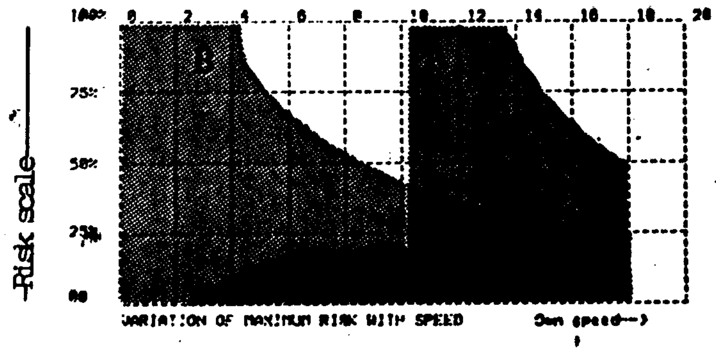
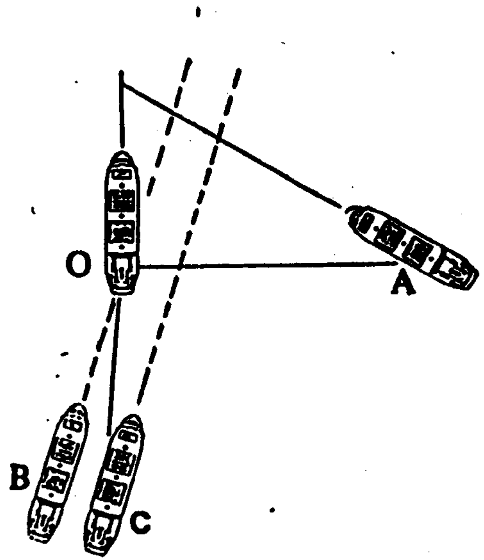


Figure (3-7). Risk representation diagram of two ships in an overtaking situation.





OBSERVED DATA: A		ARBITRARY DATA:	
First Range	= 8	Accepted CPA	= 0.5
Second Range	= 6	Initial Range	= 6
First Bg.	= 90	Initial Time	= 6
Second Bg	= 90	COMPUTED DATA :-	
Time of Obs1	= 0	Maximum Risk	=100%
Time of Obs2	= 6	Initial Risk	= 1.6%
Vo.max	= 18	Target Speed	= 23.3
Own Course	= 0	Target Course	= 300.9
Vr.max	= 41	Target Aspect	= 329

OBSERVED DATA: C		ARBITRARY DATA:	
First Range	= 6	Accepted CPA	= 0.5
Second Range	= 5	Initial Range	= 5
First Bg.	= 190	Initial Time	= 6
Second Bg	= 185	COMPUTED DATA :-	
Time of Obs1	= 0	Maximum Risk	= 20.3%
Time of Obs2	= 6	Initial Risk	= 20.3%
Vo.max	= 18	Target Speed	= 22.1
Own Course	= 0	Target Course	= 15.8
Vr.max	= 40	Target Aspect	= 349.1

OBSERVED DATA: B		ARBITRARY DATA:	
First Range	= 6	Accepted CPA	= 0.5
Second Range	= 5	Initial Range	= 5
First Bg.	= 200	Initial Time	= 7
Second Bg	= 196	COMPUTED DATA :-	
Time of Obs1	= 0	Maximum Risk	= 36.8%
Time of Obs2	= 7	Initial Risk	= 36.8 %
Vo.max	= 18	Target Speed	= 19.9
Own Course	= 0	Target Course	= 16.8
Vr.max	= 37	Target Aspect	= 359.1

Figure (3-8) Risk representation pattern of a complex situation of 3-binary encounters.

The observing ship (O) is sailing on course (000^o) with speed of (12) knots. The figure illustrates that ship (O) is crossing another ship (A) which is (6) miles away on the starboard beam and is steering a course of (301^o) with a speed of (23.3) knots. Another two ships (B) and (C) are located nearly astern at a distance of 5 miles.

It is quite clear from the risk presentation displayed to ship (O) that target ship (A) is the main threat where the maximum risk will increase from an initial value of (1 %) to the (100 %) level. Regarding the other two binary encounters, the risk levels are diminishing for the given course and speed. However the risk will increase if the speed of observing ship is reduced with respect to target ship (B), while target ship (C) is considered nonhazardous irrespective of any speed steamed by ship (O). In order to control the risk of collision in such a situation the following control actions are examined:-

(a) On reducing ship's speed from (12) knots to (10) knots the risk of collision with target (A) will be avoided while retaining some risk with target ship (B). However, it is still maintained below the (50 %) level.

(b) On increasing ship's speed from (12) knots to (18) knots the risk of collision with target ship (A) will be reduced to nearly (50%).

3.5 SUMMARY

In this unit the introduced risk speed pattern is straight forward and follows directly from the work completed in unit (2). It is considered a complementary work, as it covers the demerits of using the course alteration as the only control action. In this unit the levels of the maximum risk are computed and plotted as a function of the observing ship's speed. The merit of such a structure is that it offers the possibility of checking at a glance the effectiveness of the risk control by means of the speed variation.

The risk/speed pattern introduced has been developed to deal with more than one binary encounter. The study of the introduced risk profiles as applied to the given examples show, as expected that, the control of the risk of collision by the speed variation is effective, especially in crossing situations. It is also clear from an inspection of the cases studied that the control is much higher with speed reduction, while increasing ship's speed implies the retaining of a considerable level of risk as the observing ship is attempting to cross ahead of target ship. However, a higher level of control is always available to the faster ship in the feasible region. A better understanding of independent control variables has been added by introducing the second independant variable i.e. speed. This has been done without resorting to new techniques not already used in unit (2). It has now been seen how useful it can be to work with speed as well as course in the presentation of risk analysis. This additional and complementary work has produced a greater insight into the C/A problem.

UNIT_4

Equipotential Area of Risk

4.1. INTRODUCTION

Having examined the properties of the collosion probability function and its implication for the risk model introduced in unit (2), a further study will look at other finer implications to generate a more comprehensive risk model. Indeed it follows directlv from the definition of the risk function that a given risk can result by several different combinations of the parameters (Q), the aspect, (R), the range and (E) the speed ratio. Each of these combinations may produce risk across a wide spectrum level. Conversely, for a fixed level of risk all the constituent elements may be classified by wide bands. Providing the solution to control a risk ~~efficiently~~ and reach a stated goal, requires a search to be made of many solutions, resulting from varying the three constituent elements individually. This provides the motivation for a search through many alternative designs, even after a possible solution is found, such as that being introduced in unit(2)

This unit presents both a developed conceptual model and an analysis method for assessment and control of collision risk as applied to ships. It is a statement of what methods can reasonably be applied in this problem. Although such a work is mathematically tedious, much effort has been applied in reducing it to a relatively simple and practically applicable procedure. In addition, considerable work has been presented to provide guidance for interpretation of the results.

4.2 EQUI-RISK CONTOURS

On way of giving a geometrical interpretation of risk of collision is by means of plane figures with equi-risk contours. The risk levels are defined for each point relative to the target ship in a two - dimensional plane with reference to the (E/H) diagram and by previous work. It was shown that the probability of risk at a given point is own course independent and solely controlled by R, Q and E. If all the points $\sum(Q_i , R_i)$ at which the risk has the same value (P) are joined, the equi-risk contour U (Q, R) = P is obtained. By varying (P) the given figure represents a 1-parameter family of curves. (See figures (4-2) and (4-3) for illustration).

It has proved possible to determine directly the values of risk being looked at by an observing ship for any position relative to target ship with a given speed ratio (E). The calculations are based on the collision probability function. The problem of finding the relative position for a given risk level with a given speed ratio is obviously the inverse. Unfortunately it has been found that the inverse application of the function is not feasible, and hence an interpolation technique had to be applied. There are many iterative methods which can be used to facilitate the computation of the required function. However the polynomial LAGRANIAN method is used because it is quite easy to evaluate, particularly by the process of nested multiplications, and does not necessitate the use of equal intervals for interpolating arguments.

When considering values of speed ratio greater than one, the risk function is continuous. It neither exhibits a local maxima nor shows a local minima. Regarding this case, LAGRANIAN interpolation method can be applied on any interval of risk levels. However, when values of speed ratio are equal to or less than one, the risk function exhibits some peculiarities. Based on head-on or nearly head-on situations, the risk curves show a flat portion in the region of ($Qs = \cos^{-1}(E)$) (See figure (2-1)). In this case the collision is unavoidable and (100 %) values of risk are given in the interval of range defined by the expression:

$$R = Ra/\sin\,(Qs + \sin^{-1}E\,)$$

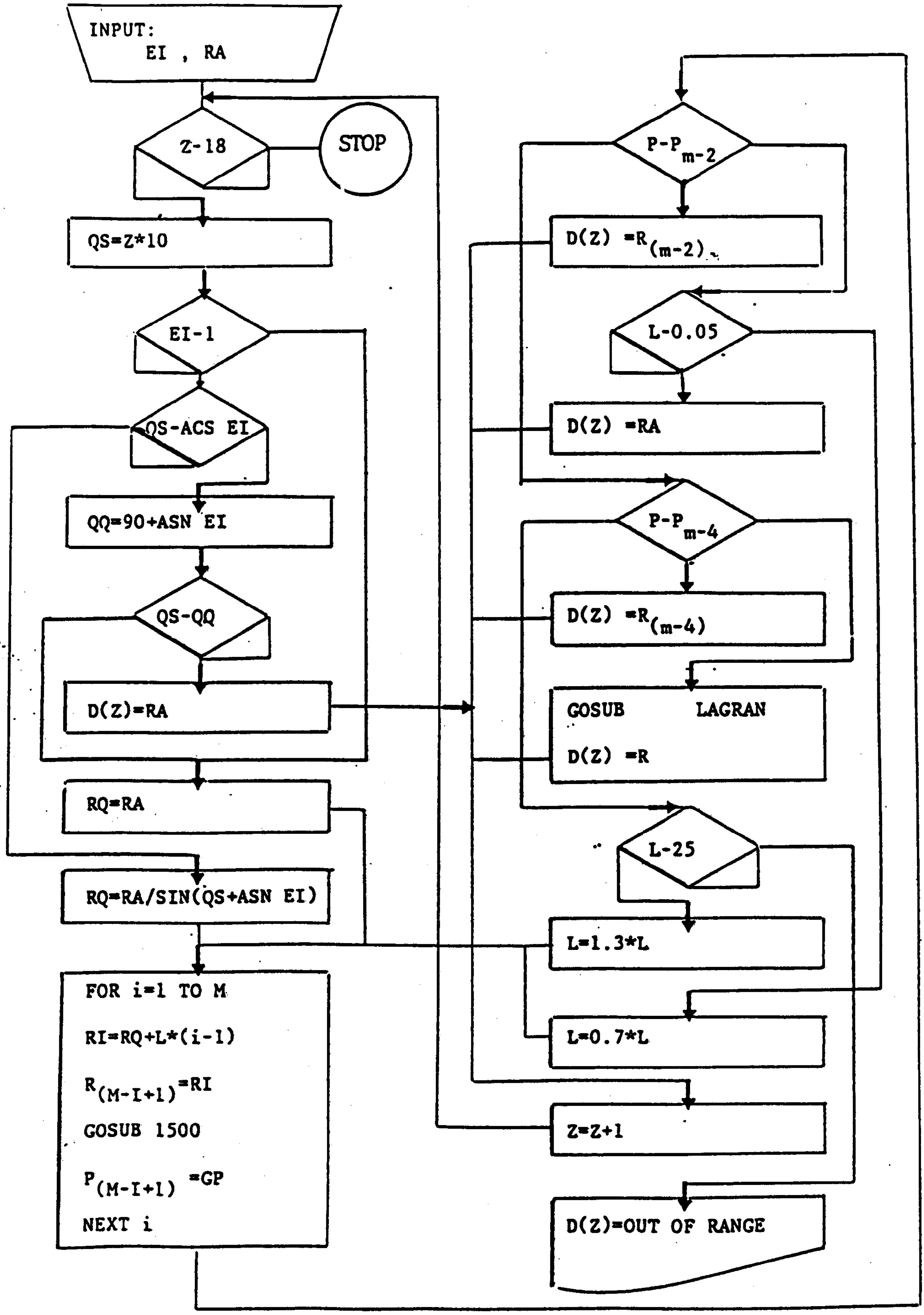


Figure (4-1) A simplified flowchart program to compute the equirisk contours which surround target ship.

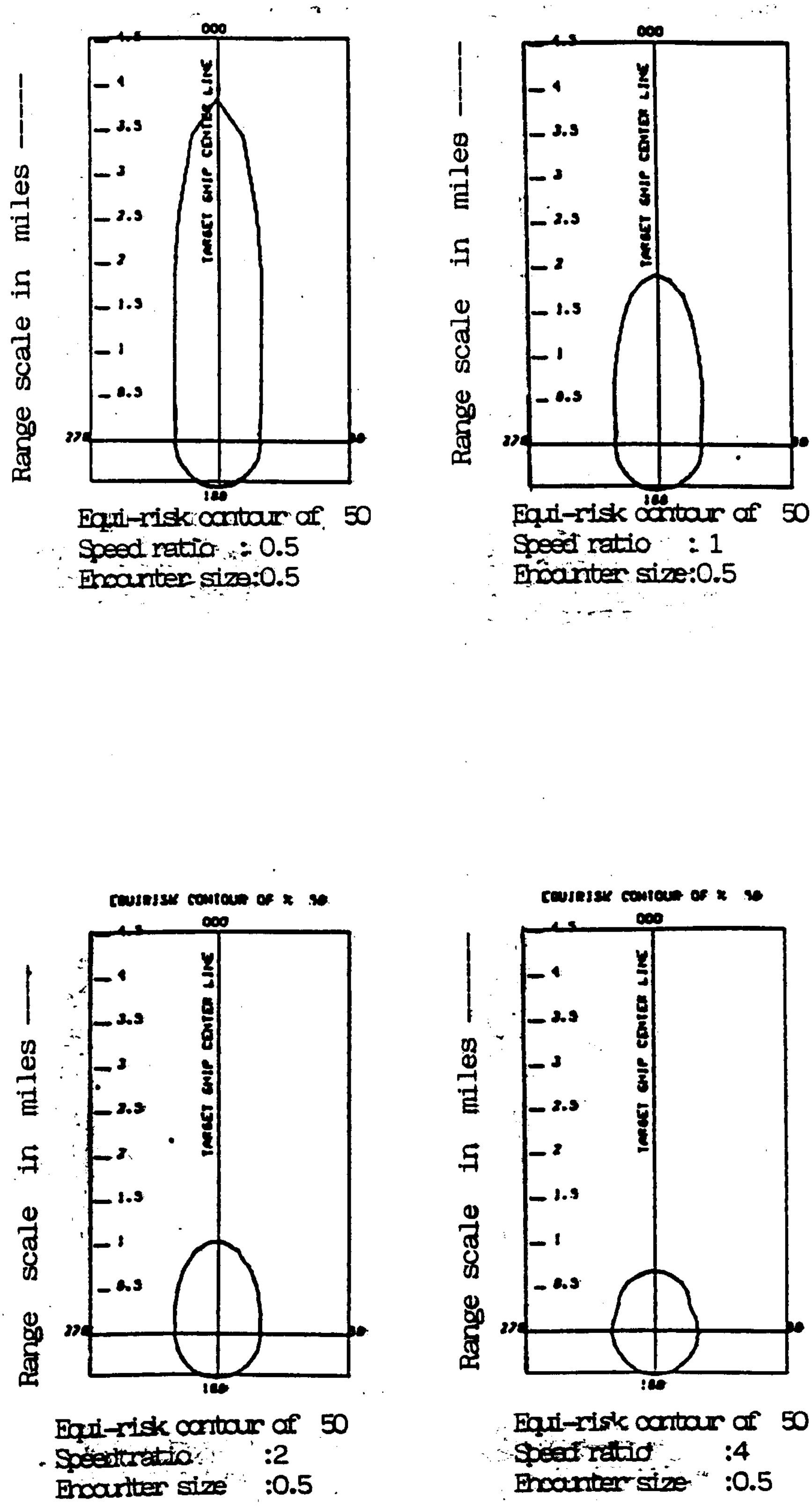
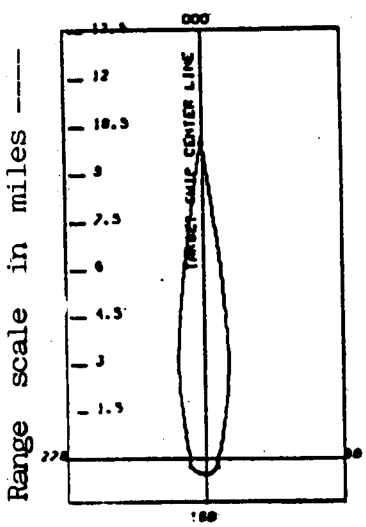
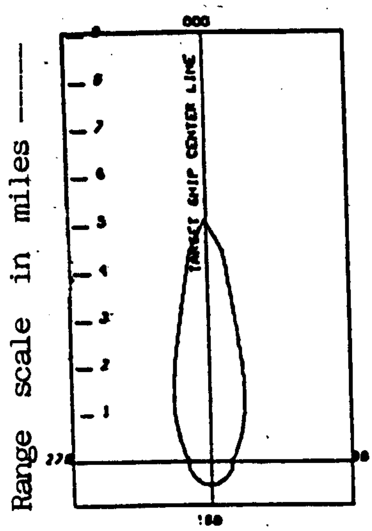


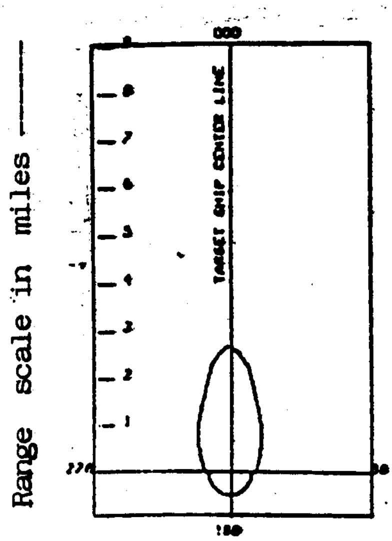
Figure (4-2) Generated iso-risk contours of (50%) risk level which surround target ship.



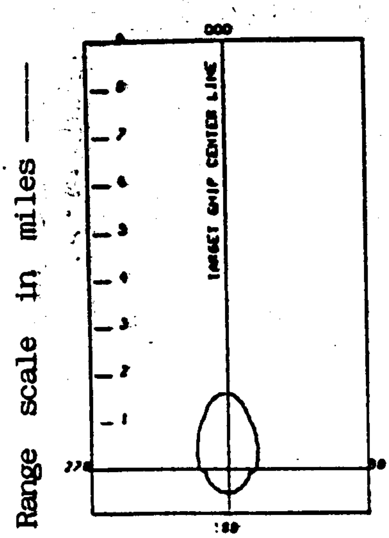
Equi-risk contour of 25
Speed ratio : 0.5
Encounter size : 0.5



Equi-risk contour of 25
Speed ratio : 1
Encounter size : 0.5



Equi-risk contour of 25
Speed ratio : 2
Encounter size : 0.5



Equi-risk contour of 25
Speed ratio : 4
Encounter size : 0.5

Figure (4-3) Generated iso-risk contours of (25%) risk level computed for different speed ratios.

Accordingly, the interpolation process has to be initiated at a range greater than the value specified by the above expression. There is also a second problem in the case of $E \leq 1$, when the risk curve switches abruptly from high risk to zero level, and hence the risk curve exhibits a steep slope. This condition occurs in the region of $Qs = 90 + \sin^{-1}E$. In this respect, all levels of risk defined in these specified regions which are greater than zero level, are assumed to exist on the same circle which represets the encounter dimension; i.e. at a range of (Ra) from target ship. The filtering of these situations can be traced by the algorithm shown in Figure (4-1).

Thus the program begins by classifying the solution in accordance with above mentioned constraints. Once a valid interval of range is determined the program continues computation by dividing the given range into smaller distances. Afterwards the corresponding values of risk of these distances are computed for an initial aspect with a given speed ratio. This is performed by "C - Risk Subroutine" illustrated in Appendix (C-2). If the value of risk to be interpolated is not found as near as possible to the centre of the range (P_o) to (P_n), the computed values (R_i, P_i) are rejected and new values of R_i are calculated by increasing or decreasing the range interval, depending on which end of the interval is the nearest point.
The same procedure is repeated till the condition is satisfied, then these values are used to interpolate inversely the non-tabular distance for which risk value has a prescribed magnitude using the Lagranian formula. A simplied flowchart of this program is shown in figure (4-1) and for more details one is referred to appendix (B-4) listing (ISO - Risk contour). The procedure is then repeated for different values of aspect with the same speed ratio. It is found that 36 values of equally spaced aspects are reasonably sufficient to produce a smooth equirisk contour of a specified risk value and a given speed ratio.

In order to provide insights into equi-risk functions and to compare the effects of parameter values on the size and shape of equirisk graphs, contours of the equirisk function, for levels of risk of 25% and 50% are computed and provided in figures (4-2) and (4-3). For each level of risk values of speed ratio (0.5), (1.5), (2) and (4) are investigated. In general the various equi-risk contours were found to bear a close relationship to an oval shape, with slight sidewise concavity.

4.3 SIMPLIFIED EQUI-RISK MODEL

While there is no fundamental objection to the development of the equirisk contour, the objective of this work involves not only the analysis of C/A problem but also the practical implementation of the risk control model, and it was accordingly a worthwhile simplification to consider the applicability of the solution, where computation time is a crucial factor. It is obvious that the need to derive the exact shape of equi-risk is the only concern in risk control. This will be seen in the following application illustrated in the next section (4-4). It is sufficient to keep the error below reasonable values in this respect.

In view of the fact that the resulting Equi-risk patterns as seen in Figures (4-2) and (4-3), are closed curves of nealy oval shapes around which an ellipse appears to fit. To determine the parameters of the corresponding ellipse, the following expressions for the ellipse parameters are proposed:

$$a_e = \tfrac{1}{2}(R_o + R_{180})$$

$$b_e = K_e R_o$$

$$e = (a_e^2 - b_e^2)^{\frac{1}{2}} / a \qquad 4.2$$

Where,

a_e = the major axis of the ellipse

b_e = the minor axis of the ellipse

K_e = coefficient of proportionality

e = the eccentricity of the ellipse

R_o = the argument of distance at zero aspect of equirisk contour.

R_{180} = the argument of distance of 180^o aspect of equirisk contour.

Having determined the parameters (a_e) , (b_e) and (e), then the corresponding ellipse can be defined in the target ship coordinate system as follows:

For a central ellipse with Y-axis oriented upward as shown in Figure (4-4) the polar form will be;

$$R_e^2 = a^2 . b^2 / (a^2.\sin^2\theta + b^2.\cos^2\theta)$$

$$= b^2 / (1 - \cos^2\theta + (b^2/ a^2).\cos^2 \theta)$$

$$= b^2 / 1 - \cos^2 \theta \cdot (1 - b^2/ a^2)$$

$$= b^2/ (1 - e^2.\cos^2 \theta)$$

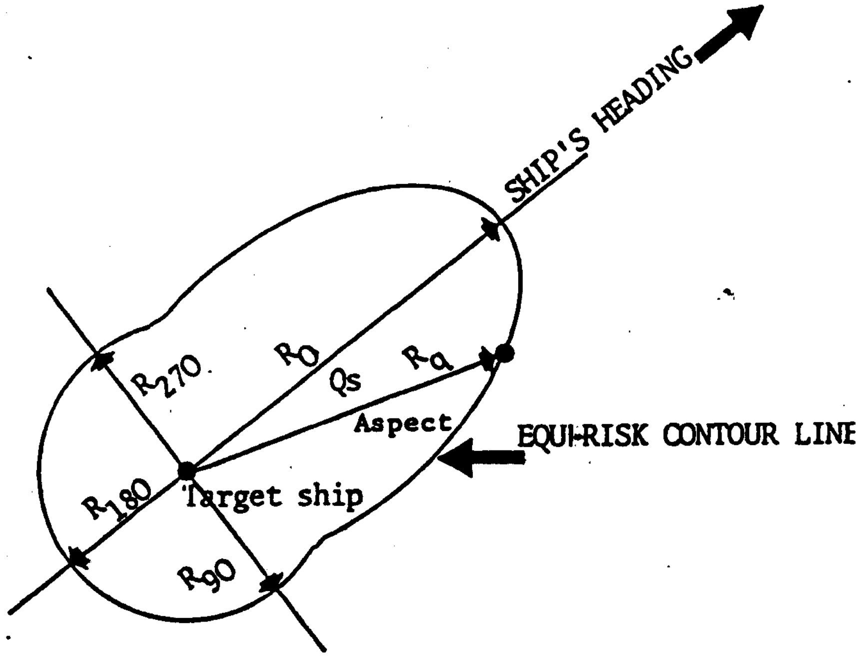
then $$R_e = b/ (1-e^2 . \cos^2 \theta)^{\frac{1}{2}} \qquad (4\text{-}3)$$

Transformation from polar coordinates related to ellipse centre point to cartesian coordinates related to target NCS can be deduced from figure (4-4). The associated transformation equations are:-

$$X = (a_e - R_{180}).\mathrm{COS}\ (Ca) + R_e . \mathrm{COS}\ (Ca + \theta)$$

$$Y = (a_e - R_{180}).\mathrm{COS}\ (Ca) + R_e . \mathrm{COS}\ (Ca + \theta) \qquad (4\text{-}4)$$

The computed values of R_o and R_{180} incorporated with equations (4-2), (4-3) and (4-4) are used for the calculation of the equivalent ellipse of equi-risk of collision. Detailed algebraic expressions for computational purposes are appended in the computer program. The resulting ellipses superimposed on the corresponding equi-risk contour, with the same set of parameter values as specified in section (4.3) and numerically indicated on Figures (4.2), are provided in Figure (4-5).



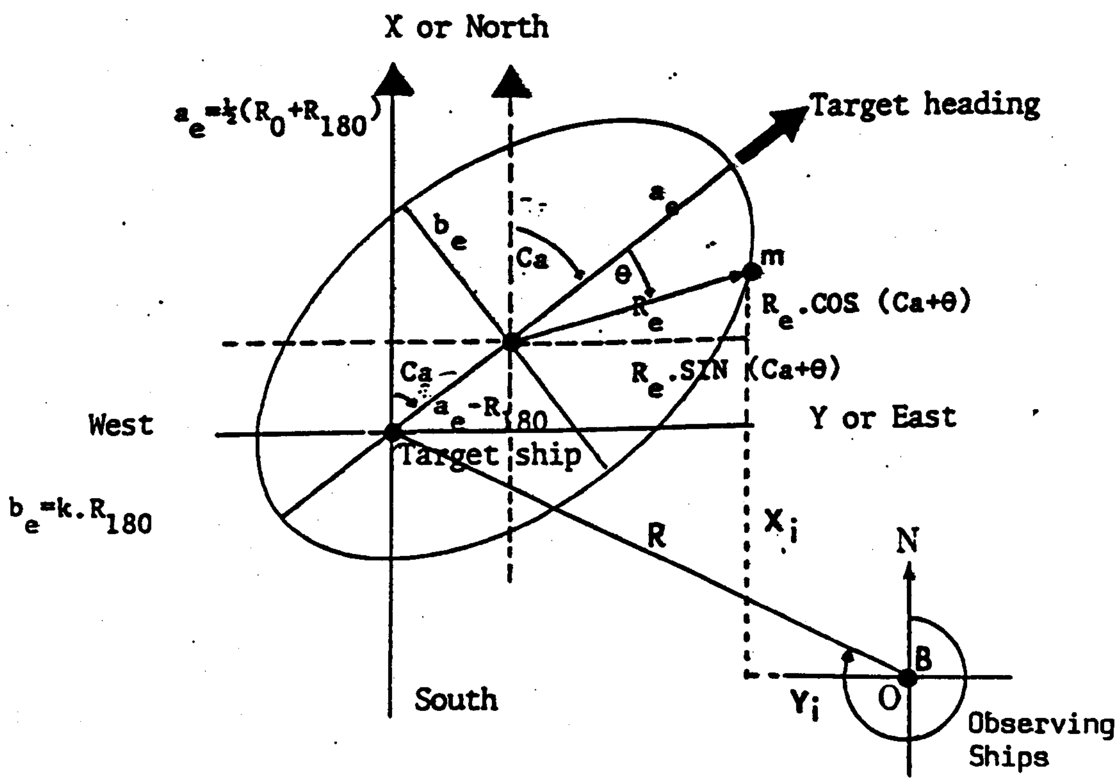
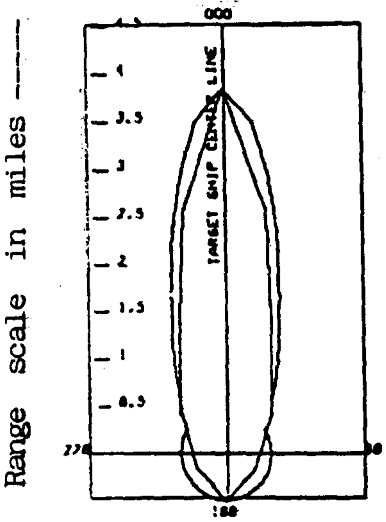
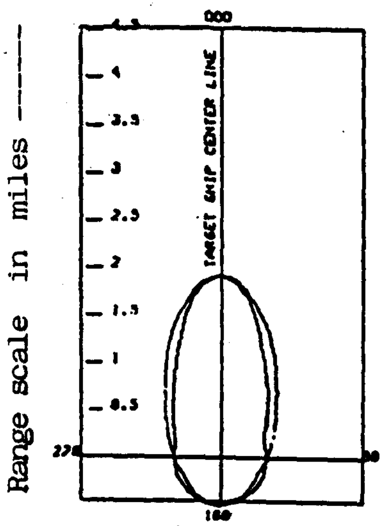


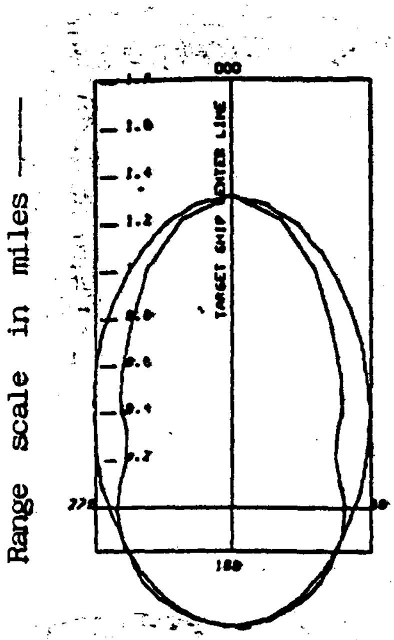
Figure (4-4) Relation between iso-risk contours and the corresponding iso-risk ellipse.



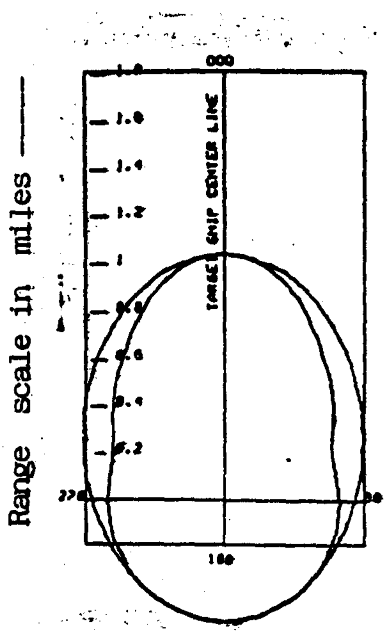
Equi-risk contour of 50
Speed ratio : 0.5
Encounter size : 0.5



Equi-risk contour of 50
Speed ratio : 1
Encounter size : 0.5



Equi-risk contour of 50
Speed ratio : 1.5
Encounter size : 0.5



Equi-risk contour of 50
Speed ratio : 2
Encounter size : 0.5

Figure (4-5) Comparison between iso-risk contours and the corresponding iso-risk ellipse.

4.4 DEVELOPMENT OF EQUIPOTENTIAL AREA OF RISK

In the preceding sections the problem is outlined by choosing a simple closed curve which has the property to enclose an area of collision risk greater than a specified level. On choosing (50%) risk level the curve is an equirisk contour of (50%). Such a curve is the envelope of the equi-potential area of risk (EPAR), and in representing it on the sea surface it defines an equi-potential area of risk to be avoided by the observing ship. A complete solution can be found to define the envelope of the (EPAR) by mapping the pre-set ellipse into the co-domain of the collision situation.

It follows from the above definition of the (EPAR), that the concept of equirisk contour criterion is quite logical to define the (EPAR). However, the particular interpretation of risk level in consideration is of no consequence to the following results and analysis, which can be carried out for any numerical value of risk. If the observing ship is at the origin of the (NCS) and target ship is considered as an ellipse of a specified dimension corresponding to a specific level of risk, then the relative positions of all points on the ellipse are given by a set of coordinates (Σ (X_i , Y_i)), as illustrated in figure (4-4).

Thus $X_i = R.\mathrm{COS}\ B + (a_e - R_{180}).\mathrm{COS}\ Ca + R_e.\mathrm{COS}\ (Ca + \theta_i)$

$Y_i = R.\mathrm{SIN}\ B + (a_e - R_{180}).\mathrm{SIN}\ Ca + R_e.\mathrm{SIN}\ (Ca + \theta_i)$

Where θ_i = is the vectorial angle of a point on the equirisk ellipse measured from target heading direction in a clockwise direction to this point.

R,B = are the range and bearing of target ship. (R_e) , (a_e), (R_{180}) and (Ca) are defined in the preceding sections.

Having defined the ellipse of the equi-risk contour by a set of coordinates, related to the observing ship, the envelope of the (EPAR) can be calculated by a similar expression derived in section (1.6) of unit (1.) These equations (1.23, 1.24 and 1.26) are used to determine the interceptable future point or points for each specific point on the ellipse in consideration. Generation of a set or sets of the interceptable future points based on at least (36) points on the ellipse can shape, when connnected, a smooth curve. This curve pictorially represents the envelope of the equi-potential area of risk (EPAR) (See Figure (4-6)).

4.5 INFLUENCE OF SPEED VARIATION ON EPAR

This section outlines the general aspects of risk control by means of changing the locations of the EPARs on the sea surface. Although risk control via speed variation could be reached by speed trials and subsequently observing the resulting motion of EPARs relative to the observing ship track, an insight into the general behaviour of the EPAR would be desirable. Present understanding of this behaviour has resulted from analytical studies and examining the effect of the E parameter and associated boundary conditions. The basis is the mathematical formulation developed in the preceding unit (1), section (1.6), which, however, was made without any regard to the question of this specific solution.

EPARs prediction involves some peculiarities which depend on speed ratio. In case of slower target there always exists one and only one EPAR , but in case of a faster target three possibilities arise:-

(a) - Existence of two EPARs

(b) - Existence of one EPAR

(c) - Existence of no EPAR.

Regarding case (a), some of the future interceptable points may not always be genrated, as can be seen in figures (4-6.c), (4-7.a), (4-7.c) and (4-9.b). However the resulting curve has to be closed as far as the risk control is concerned, because finding the observing ship's track orientated in the sector determined by the broken part of this curve will involve the interception of the higher risk area.

Should an EPAR exist, it will always be located ahead or around the target's present position. It is noticed that when the target ship is faster than the observing ship, double EPARs exist. They will be found on either side of the normal line to the line of sight at observing ship centre point.

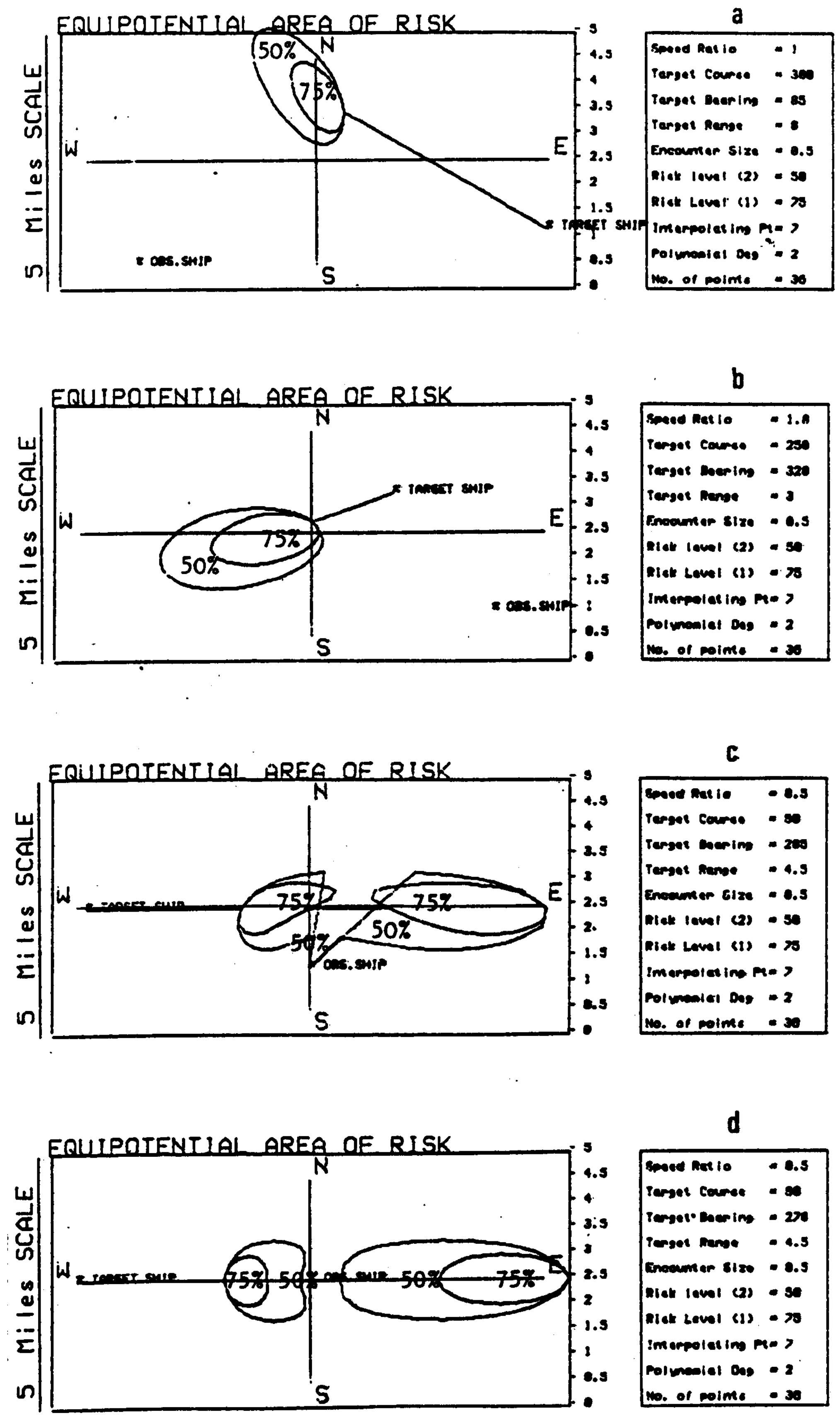


Figure (4-6) Presentation of two risk levels in a form of equipotential areas of risk mapped on the two-dimensional horizontal plane.

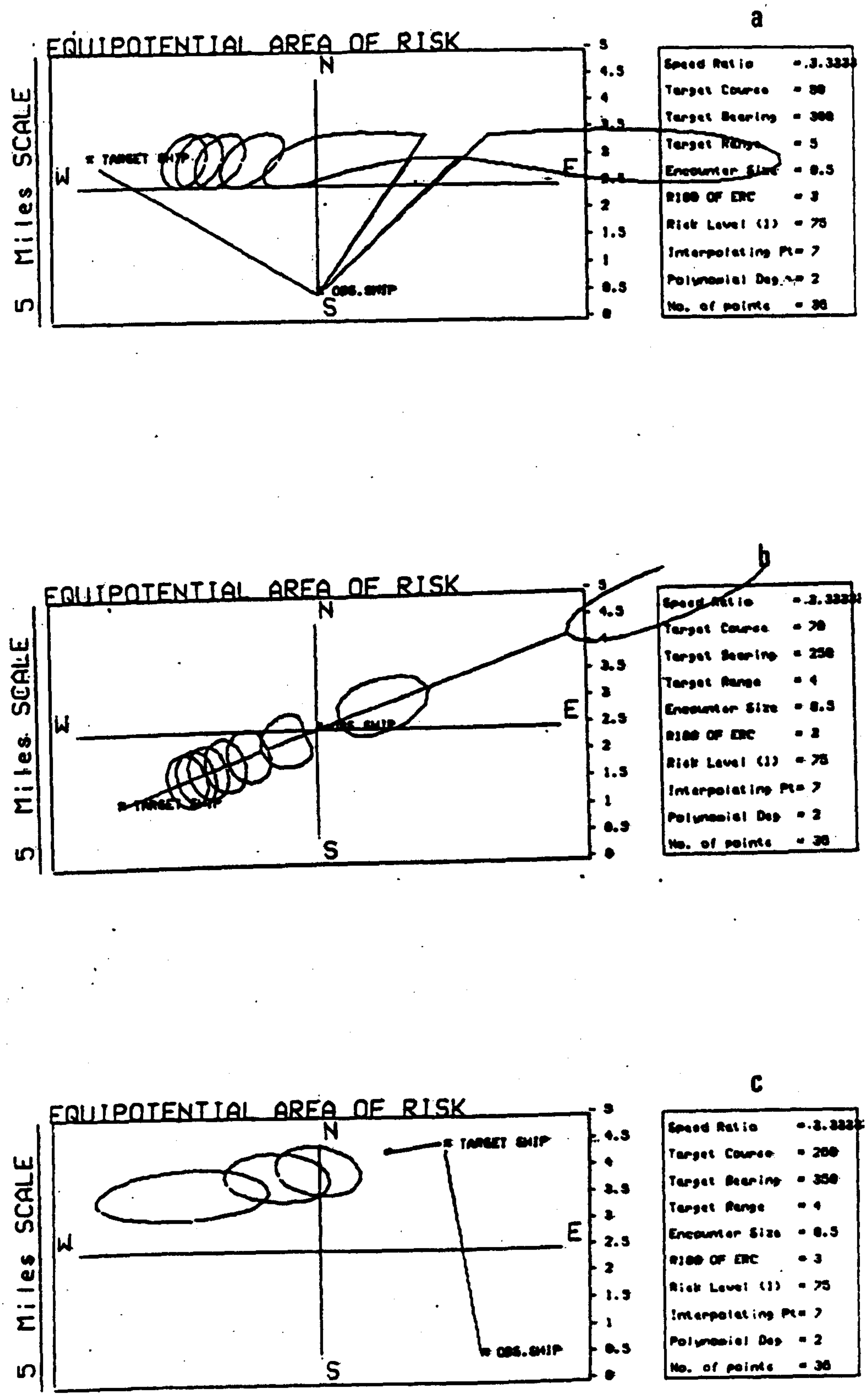


Figure (4-7) Effect of speed variation on the generated equipotential area of risk.

This can be seen by the dual EPARs in figures (4-6,d). Meanwhile, the particular condition of one EPAR in case of the equal speeds is characterised by the positioning of the EPAR around the intersection of target track and the normal line to the sight line at observing ship centre point (see figure (4-6,a)).

When considering the condition of the head-on situation with a slower target the EPAR is located between target initial position and the bisector normal to the sight line (see figure 4-7b). When observing ship decreases her speed the EPAR will move away from the target position until it reaches a midway position when both speeds are equal, then a slight decrease of the speed will cause a second EPAR to be generated at a great distance away on the other side of the sight line. With further decrease of speed ratio the two EPARs will move toward each other till they merge enclosing own ship point inside. At this stage risk reduction below the specified risk level is not feasible in spite of any manoeuvre by observing ship involving course alteration and/or speed reduction (see figure 4-7,b). In considering an initial situation, for small values of aspect a similar sequence will occur in the range of ($1 > E > 0$). The dual EPARs will move toward each other due to speed reduction until they merge around a point positioned on the normal line to the sight line at observing ship centre point. A further decrease of the speed will contract the EPAR until it disappears. At this stage risk increase will be impossible in spite of any manoeuvre by observing ship involving course alteration and/or speed reduction (see figure 4-7,a).

4.6 INFLUENCE OF ASPECT CHANGE ON EPAR

The motion of EPAR due to target aspect change is controlled by the dynamics of the situation. Similarly the observing ship may influence the behaviour by changes in her motion, the target ship can also abruptly introduce greater effect by altering her heading. Irrespective of the causes of aspect changes, the following discussion is basic to the general behaviour of EPAR's motion due to the aspect changes. This behaviour is best described in accordance with the class of the problem. These classes are $(E < 1)$, $(E = 1)$ and $(E > 1)$.

When $(E = 1)$, EPAR will move along the bisector normal to the line of sight until it disappears at infinity when the aspect becomes greater than (90^{o}) (See figure 4-8,b). When $(E < 1)$, the dual EPAR will move from the symmetrical position of zero aspect until it reaches a merging position,after which the EPAR will disappear. At this stage the stated level of such an encounter is declared non-existent (see figure(4-8,c) for illustration). If the target is the slower ship the EPAR will move around target's position. Although a possible EPAR always exists for every course taken by the target, such EPARs are always confined in a sector round the line of sight. If the observing ship takes up headings outside this sector, risk level is kept below the stated value inspite of any manoeuvre by target ship involving course alteration and/ or speed reduction. (see figure (4.7,a) for illustration).

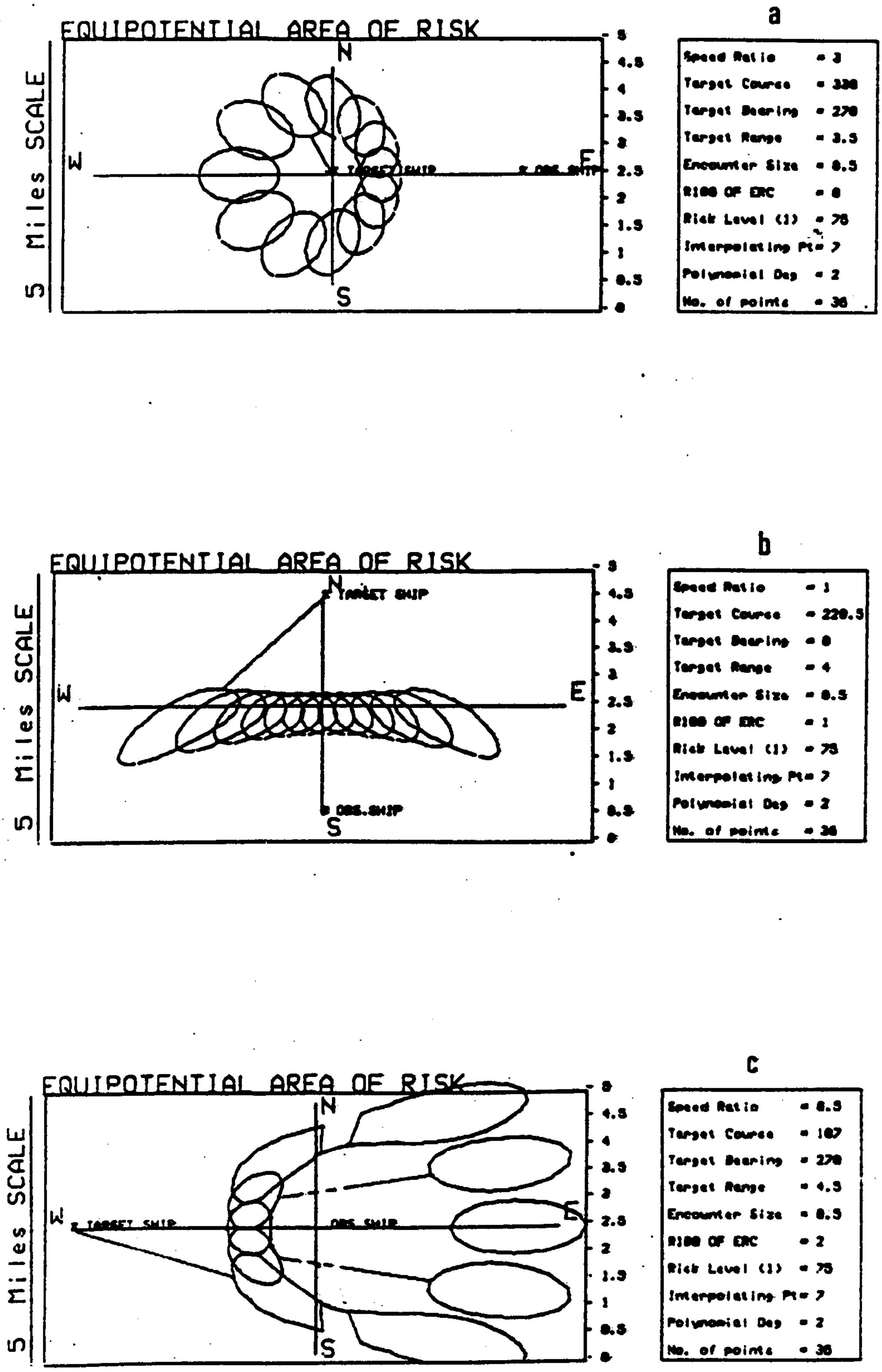


Figure (4-8) Effect of aspect change on the generated equipotential area of risk.

4.7 INFLUENCE OF SITUATION DYNAMICS

In the preceding sections the influence of each individual parameter is considered wherein no dynamical effects were considered. Here the consequences of the combined effects in a dynamically changing situation will be examined. The interest is confind to the prediction of the response due to the instantaneous manoeuvres.

Regarding a dynamically changing situation, when the parameters of ships size, speed, and stated risk level are considered unchangable, the variation of resulting EPAR depends on the changes in range and aspect angle. Since these changes are functions of time, the EPAR-time relationship can be established for a specific velocity of observing ship. A program that performs these calculations is obtained by inclusion of the equations of relative motion derived in unit (1), into the main program used to generate EPARs. Although the program can be used to examine the EPAR-time relationship in a fast varying situation, the following examples are chosen to illustrate the general features of the associated behaviour.

If the heading of observing ship is intersecting the EPAR, then the EPAR will be directed toward the observing ship position, and the effect is solely due to decrease of range, (see figure (4-9,c)). In the case of dual EPARs, each one will follow a straight line path towards the observing ship till they merge around observing ship's centre point.

If the oberving ship's heading is directed ahead of the EPAR then the relative motion of the EPAR will follow a curved line which is orientated between the relative motion direction and the observing ship true motion direction, as can be seen in figure (4-9,a). Regarding the dual EPAR case, where observing ship heading is directed between the two EPARs, the dual EPAR will follow a curved path, and there is an apparent approach of both EPARs on either side fo the observing ship. By the time the target has reached a very close situation, the two EPARs will merge, and then disappear (see figure(4-9,b)).

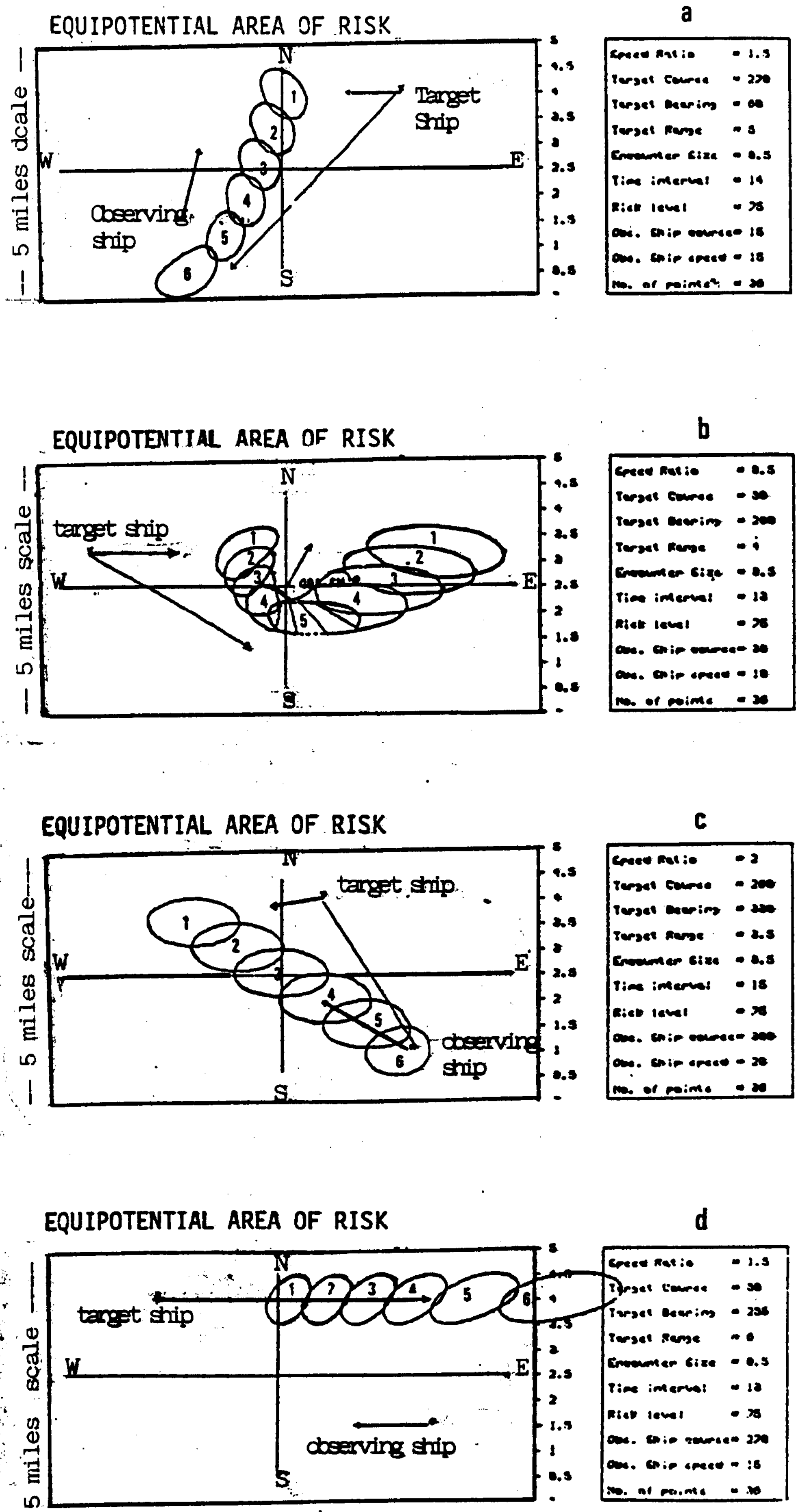


Figure (4-9) Effect of situation dynamics on the shape and location of the equipotential area of risk.

If observing ship heading is directed outside the dual EPAR, so the observing ship will clear the target by passing astern. The two EPARs will again follow a curved patch down one side of observing ship until they have merged and disappeared.

If the two ships are steering along parallel courses and (V_o) is greater than (V_a) then EPAR will follow a straight line path parallel to observing ship course and continue to move abaft the beam (see figure (4-9.d)). However, in case of $(V_o < V_a)$, the dual EPAR will merge and disappear near the beam position.

4.8 SUMMARY AND CONCLUSION

This unit has been devoted to one of the more important aspects of risk of collision by means of plane figure with equi-risk contours, where the control variable "Co" is found independent of the solution. It is theoretically and technically possible to obtain an accurate assessment of the situation and hence a complete control of risk of collision is also achievable. The equirisk contours are used to generate equi-potential areas of risk displayed on the horizontal plane of ship's motion. The prediction of such EPARs assumes that velocities are regular. The solution necessitates a pre-knowledge of a stated level of risk, encounter size, relative position of target ship, elements of target motion and observing ship's speed.

The EPAR is defined as the interior of a computed envelope of inter-ceptable points. It should be understood that the generated envelope is a general method of traversing a set of points specified by a collection of equivalent parametric representations of relative positions around target centre, which bear equal probability of collision

However, the far side of the envelope cannot be reached while avoiding higher risk levels inside the envelope.

The solution, introduced by displaying the EPAR on a graphic surface, has the advantage of applying equally, to a multiple-ship situation as well as the two-ship encounter. On keeping the track of the observ-ing ship away from the EPARs the risk of collision is maintained below a stated level for every ship in the encounter. The study of the influence of trial speed variation on the shapes, sizes and locations taken by the generated envelopes not only shows the application of risk control via speed variation, but also provides a measure of the degree of the involvement of the observing ship in the encounter. For example overlapping EPARs indicate that the two targets are involved in a risky situation. It is therefore to be expected that one or both will initiate a risk control manoeuvre. However, the inverse, the non-overlapping or non-existence of EPARs does not necessarily imply the existence of a lower level of risk between targets. If the overlapping of EPARs occurs due to a speed increase, the probable collision will occur before the time of the maximum risk with these targets, but if they overlap due to speed decrease, the probable collision will occur after the time of the maximum risk.

UNIT_5

Three Dimensional Equipotential Risk Matrix

5.1 INTRODUCTION

What has been discussed so far includes the general principle of dealing with the collision avoidance problem by the concept of risk assessment and control. Special rigid techniques for practising these principles and applying them have been developed in units (1), (2), (3) and (4). The various techniques described so far have all been concerned with developing and improving new concepts which convert collision-avoidance sense in a two-ship encounter into a definite statement. Unless the relevant data applying to the situation can be converted into a definite pattern, it is extremely difficult to determine rational weightings between the outcomes of the available options so that an effective decision can be made. The need for a more comprehensive way to look at the situation arises from some limitations of the developed techniques. It is, therefore the aim of this unit to move to a new arrangement of information which will bring together a more definite pattern of risk assessment & control. In this unit it is intended to include and enhance the effectiveness of the previously developed concepts in this work. The new criteria is now termed the 'Equipotential Risk Matrix' or (EPRM).

5.2 DEFINITION OF THE 3-D EPR MATRIX

To complete the discussion of risk assessment and control, it is intended to extend the discrete risk models which have been introduced to the stage where all control variables are presented. There is no basic reason why the alternatives of course and speed variations cannot be considered simultaneously. A rigid model will be presented that is based on the same factors as previously used. These factors are target velocity (Va, Ca), initial position (B_0, R_0) and a specified level of risk below which risk is accepted, (P).

On investigating these factors the observing ship elements of motion (Ca, Va) and the relevant run (So) are taken as the control variables. Meanwhile the other factors are taken as constant parameters. Hence a varsatile risk model can be formulated.

An EPR Matrix can be sensibly defined as a discrete set of coordinates representing ship's course, speed and run mapped in the three dimensional space. Each combination of the represented variables indicates a specific level of risk of collision. When the risk sets of coordinates are plotted an iso-risk locus is determined and consequently a weighted surface area is enveloped by all points where the rectangular coordinates satisfy the equation of the form:

$$f\ (Co\ ,\ Vo\ ,\ So\ ) = P \text{ const.}$$

It should be understood that the generated envelope is a general method of traversing a set of points ($\Sigma\ C_i$, V_i, S_i) specified by a collection of equivalent para-metric representations of relative positions around the target centre, which bear a constant prescribed level of risk. On keeping Ship (O) velocity outside the surface area, risk of collision will be maintained below the prescribed value, provided that ship (A) maintains its velocity.

5.3 FORMULATION

In the previous unit (4), it is found that the solution of the equipotential area of risk could be simplified by fitting an equivalent ellipse. This ellipse will also be used in taking the solution one step forward. Should the positions of the limiting aspects (Q1 , Q2) of the generated ellipse which surrounds target ship (A) be determined, then the calculation of both the initial risk of collision and sectors of stated level of risk are possible.

These two positions are the tangent points P (x , y) and D (x , y) of the tangents (t_p) and (t_d) drawn from the position of the observing ship (O) (see fig. (5.1)). To determine the tangent points the central equation of the ellipse together with its associated polar equation, these two equations must be solved. The real solutions give the coordinates of these points.

Given the central equation of the ellipse with:

$$b^2 . X^2 + a^2 . Y^2 - a^2 . b^2 = 0 \qquad 5.1$$

and the polar equation of the ellipse at the tangent points is given as:

$$b^2 . X_o . X + a^2 . Y_o . Y - a^2 . b^2 = 0 \qquad 5.2$$

With suitable manipulation of the two equations one of the variables X or Y can be eliminated as follows:-

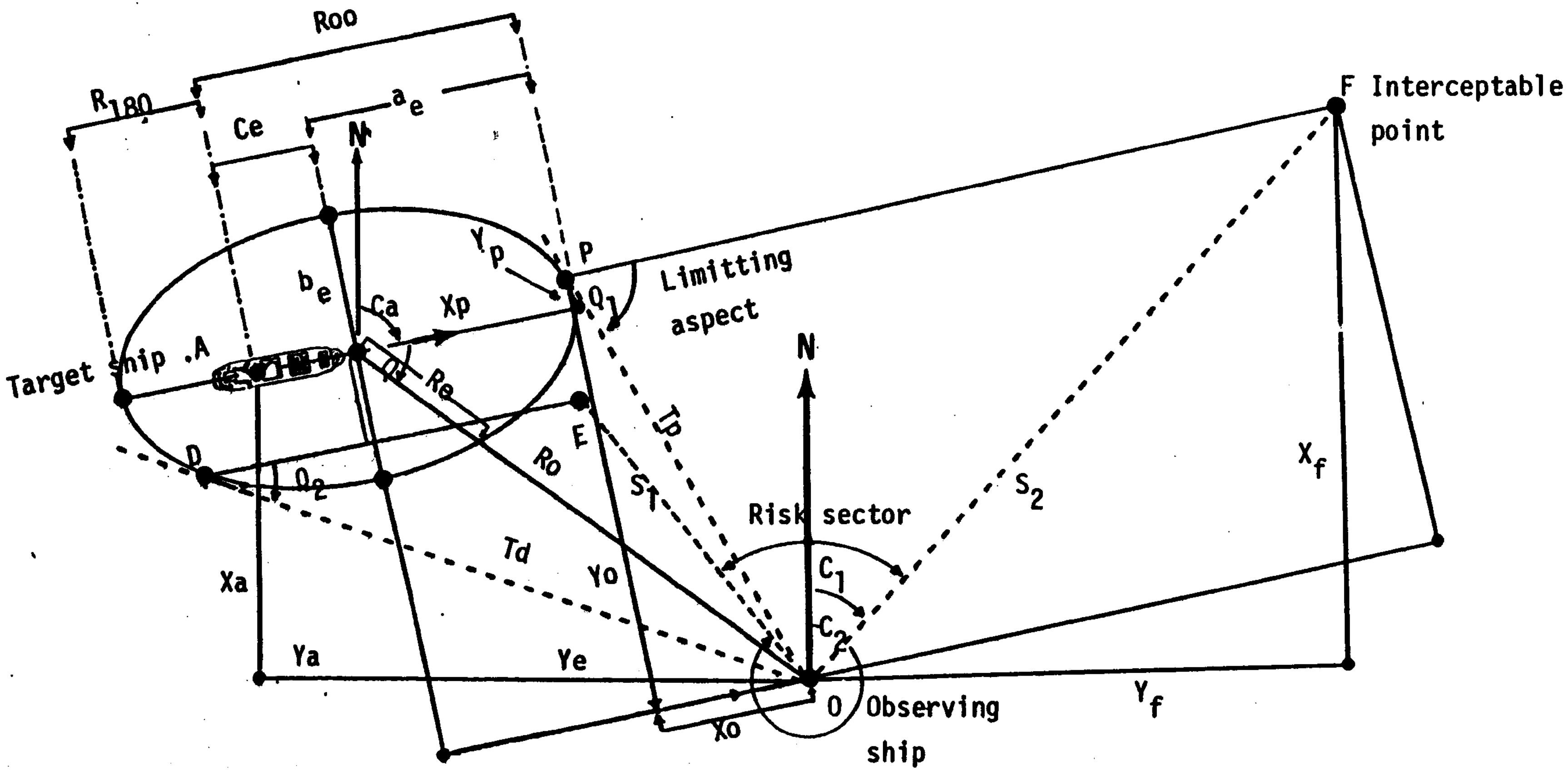
From relation (5.2)

$$Y = (a^2 . b^2 - b^2 . X_o . X) / a^2 . Y_o \qquad 5.3$$

Substituting in relation (5.1)

$$b^2 . X^2 + a^2 (a^2 . b^2 - b^2 . X_o . X)^2 / a^4 . Y^{o}_{2} - a^2 . b^2 = 0$$

$$(a^2 . b^2 . Y_o^2) X^2 + a^4 . b^4 - (2.a^2 . b^4 . X_o) X + a^4 . b^4 - a^4 . b^2 . Y_o^2) = 0$$



Figure(5-1)
Definition of the tangent points on the EPR-ellipse and the associated courses and runs to the projected interceptable points.

$$(a^2 . b^2 . Y_o^2 + b^4 . X_o^2) X^2 - (2.a^2 . b^4 . X_o) X +$$
$$(a^4 . b^4 - a^4 . b^2 . Y_o^2) = 0 \qquad 5.4$$

Let

$$A = b^2 (a^2 . Y_o^2 + b^2 . X_o^2) , \quad B = -2.a^2.b^4. X_o,$$
$$C = a^4. b^2 (b^2 - Y_o^2) \qquad 5.5$$

then:

$$X_p = (B + (B^2 - 4.A.C.)^{\frac{1}{2}}/2.A,$$

$$Y_p = (a^2. b^2 - b^2. X_o. X_p) a^2. Y_o$$

$$X_d = (B - (B^2 - 4A.C.)^{\frac{1}{2}} 2.A,$$

$$Y_d = (a^2. b^2 - b^2. X_o. X_d)/ a^2. Y_o \qquad 5.6$$

Provided that $Yo \neq 0$

It is clear that the parameters (a) and (b) always exist. However, one of the ordinates (X_o) or (Y_o), and only one, may be zero, and hence two cases are possible as follow:

Case of ($Y_o = 0$)

Substituting in relation (5.4)

$$A = b^4 . X_o^2$$

$$B = -2.a^2. b^4. X_o$$

$$C = a^4. b^4$$

Hence $B^2 = 4.A.C. = 4.a^4. n^8. X_o^2$

Then $X_p = X_d = B/2.A = -2^2 /X_o$

Substituting in relation (5.1)

$$Y_p = - Y_d = b.(1 - (X_p/a)^2)^{\frac{1}{2}}$$

<u>Case of $(X_o = 0)$</u>

Substituting in relation (5.4)

$$A = (a.\ b.\ Y_o)^2$$

$$B = 0$$

$$C = a^4.\ b^2.\ (b^2 - Y_o^2\ )$$

Substituting in relation (5.5)

$$X_d = - X_p = a\ (1 - (b/Y_o)^2\ )^{\frac{1}{2}}$$

$$Y_d = Y_p = b^2\ /Yo \qquad 5.8$$

Having defined the tangent points (X_p, Y_p) and (X_d, Y_d) on the EPR-ellipse relative to ship (O) centre point, then determination of the possible intercept... positions with these points are obtainable by using the equations (1-27) (developed in (art.1-7)).

The headings $(C_{1}, {}_{2})$ and runs $(S_{1,2})$ to the interceptable points are found by applying Equations (1-7) and (1-9) and hence the unsafe sector of courses is defined. On following the same procedure other interceptable points mapped from the EPR-ellipse, if necessary, can also be found and hence the corresponding headings and runs to bring the observing ship to the boundary of the equi-risk ellipse will be obvious.

5,4 FLOWCHART AND PROGRAM FOR EQUIPOTENTIAL RISK MATRIX

The computer program was written in BASIC as shown in appendix (B-6) listing (3D -(EPR-MATRIX)). A simplified flowchart of this program is shown in (Fig. (5.2)). The program provides an equipotential risk matrix which is simply a cross-tabulation of the alternatives (Co_i) and (Vo_i) and the dependant run (S_r). The cell in the matrix which represent the value of the prescribed risk outcomes are determined and then used to construct an iso-risk contour in the (V_o , C_o , S_o) space.

It can be seen that it requires as input, the initial position (Ro , Bo), target velocity (C_a, V_a), encounter dimension (R_a), and the prescribed level of risk (P). Program routines require four subroutines given in the listing, (RISK FUNCTION), (LAGRAN INT), (COMB-VECTOR) and (QUADRANT). The program is sequentially executed and is controlled by a step increase in the speed of Ship (O) by one knot at a time, passing through the speed range (V_a) from (1) to (18) knots. The first step after setting the speed parameter concerns the computation of the two distance ($D_{(0)}$) and ($D_{(180)}$) at which risk values have prescribed magnitude. Based on these two distances the parameters of the equivalent equipotential risk ellipse are then computed following a similar procedure as previously described (in art. 4-2).

A conditional branching causes a jump if the position of ship(O) is found inside the EPR ellipse of Ship (A). In this case the situation is declared hazardous for the given speed which is then stepped one knot and the sequence is repeated again. If this condition is not met, then the next instruction to be executed will be the determination of the tangent points on the ellipse from Ship (O) centre point, using the expression (5-6). From the directions of these tangent points together with the ship (A) heading the limiting aspects are determined.

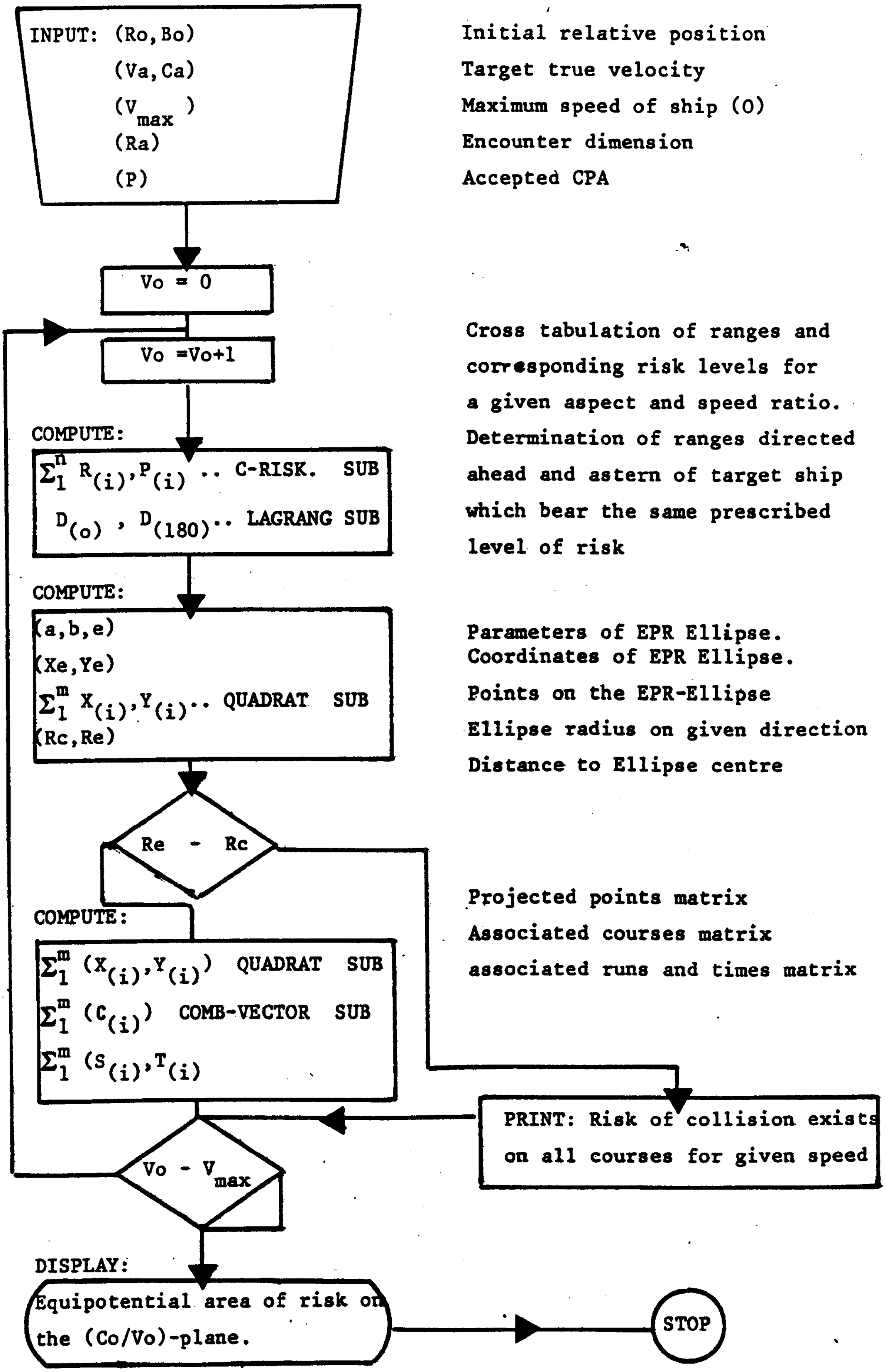


Figure (5-2). A simplified flowchart program for computing the equipotential area of risk on the (Co/Vo)-plane.

At this point a second conditional branching is instructed to declare that the encounter is not hazardous when steaming at the speed under consideration.

This condition is recognized in case of $(V_o / V_a \leqslant 1)$ and the limiting aspects (Q_1) and (Q_2) are greater than the value taken by the expression $\sin^{-1} (V_o / V_a)$. If this condition is not met a set of an arbitrary (say 36) computable points defined all around the circumference of the EPR ellipse are executed in accordance with the expressions (4-3).

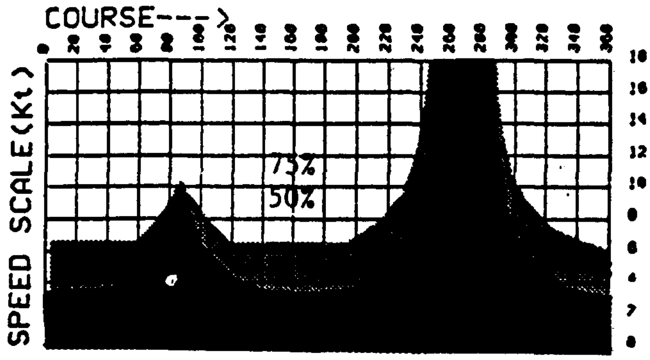
Using values of point coordinates previously determined the relevant interceptable points can be computed with the aid of the relations that have been established (in art. 1-7). Due to the quadratic feature of these relations then the (QUADRANT) subroutine is called to solve the roots of the equation. The returned values are the positions of the feasible interceptable points. Having defined the set of the interceptable points then the corresponding courses and distances directed from the observing ship centre point to these points are computed. These values $(\Sigma_1^{36} C_i , R_i)$ constitute the two dimensional cells in the risk matrix. Through a repetitive process, new cells are determined for new values of the observing ship speed (V_o). The speed variable (V_o) is increasingly stepped until the value of the maximum available speed is reached. Once the total number of cells $(\Sigma_1^{36}, V_i, C_i, S_i)$ of the given binary encounter have been generated a graphical risk presentation is possible.

5,5 GRAPHING EPR- MATRIX DISTRIBUTION

Figures (5-3a) - (5-3b) and (5-3c) illustrate three discrete binary encounters. Each EPR-locus is projected on the (Co/Vo) plane where (Co) is the abscissa and (Vo) is the ordinate. The generated graphs of the various EPR distributions look like the figures of the (E/H) diagram. In assessing the findings the dominating aspect (Qo) is varied while the other parameters are maintained unchanged for the three cases. Values of (Ra = 1),(Ro = 10) ,(Va = 10),(Vo_{max} = 18) and (P= 75%) are taken as the common parameters. Figure (5.3.a) displays the risk distribution for the case of an exact possible meeting-end-on situation where (Qo = zero). The distribution shows a bimodal with respect to speed variability. One mode is found at(V_o/V_a = 1) and the other at (V_{max}/ V_a = 1.8). The second important feature is the extending of risk all around the 360^o courses for a range of speed ratios less than 0.35; in other words risk of collision cannot be avoided by any course alteration of ship (O), if the speed (V_o) is maintained in a range of less than 3.5 knots. It is also worth noticing that a relatively large range of unsafe courses and speeds are found comprising of nearly one third of all the possible ways that can be considered by the observing ship.

A look at Fig. (5-3.b) input data shows that the binary encounter is characterised as a possible crossing situation, featuring a small angle of aspect (Q_o = 15^o) . The generated graph shows a similar bimodal feature to that of Fig. (5-3.a). However, there is a distinct difference in the lower range of speed (V_o / V_a $<$ 0.2). In this region there is no risk of collision and safe courses are extended all around the horizon circle. A second additional feature is worthy of attention. The area shown defining the corresponding courses and speeds of the risk of collision has been substantially reduced relative to case (a). In figure (5-3.c) the binary encounter is also a crossing situation but featuring a wide angle of aspect (Qo = 90^o). The generated risk region is clearly different and greatly reduced. Safe courses now extend nearly all around the horizon circle and have been obtained in a large speed range (V_o / V_a $<$ 1). Compared to the range of (V_o / V_a $>$ 1) of the other two cases, the unsafe courses and speeds are squeezed into a smaller sector.

a

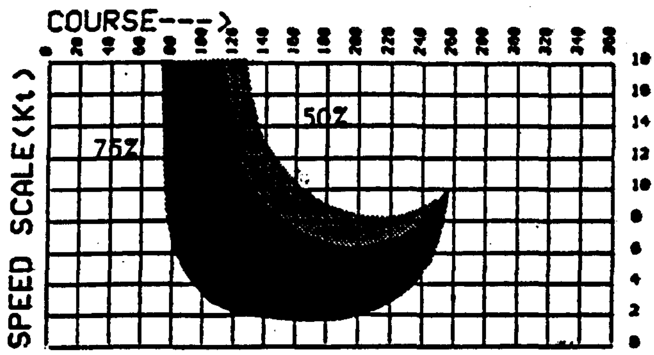


INPUT OF EPR MATRIX

Target Speed	=	10
Target Course	=	90
Target Bearing	=	270
Target Range	=	6
Encounter Size	=	1
Risk Level	=	75
Polynomial Deg	=	2
Polynomial Coe	=	8
No. of points	=	36

A) Possible head-on or overtaking situation.

b

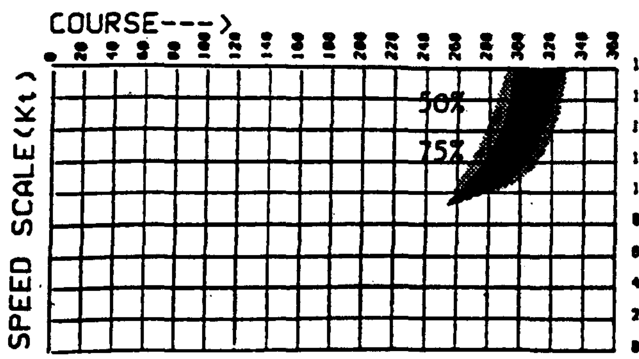


INPUT OF EPR MATRIX

Target Speed	=	10
Target Course	=	255
Target Bearing	=	90
Target Range	=	6
Encounter Size	=	1
Risk Level	=	75
Polynomial Deg	=	2
Polynomial Coe	=	8
No. of points	=	36

B) Possible crossing situation at fine angle of aspect.

b



EPR MATRIX

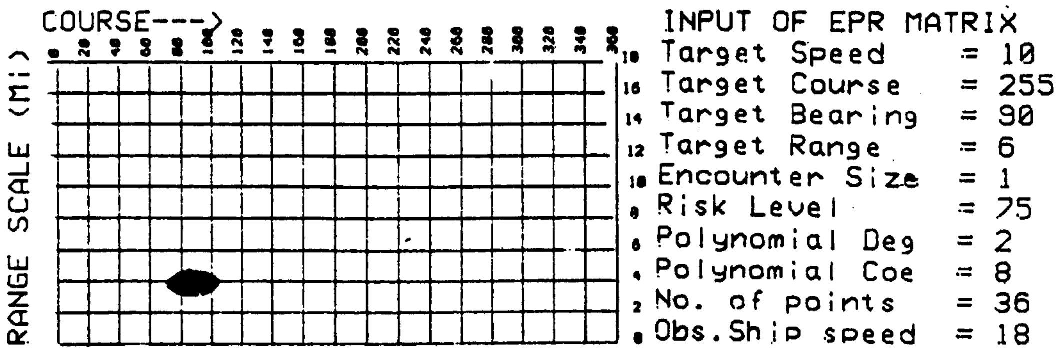
Target Speed	=	10
Target Course	=	260
Target Bearing	=	350
Target Range	=	6
Encounter Size	=	1
Risk Level	=	75
Polynomial Deg	=	2
Polynomial Coe	=	8
No. of points	=	36

C) Crossing situation at large angle

Figure(5-3)

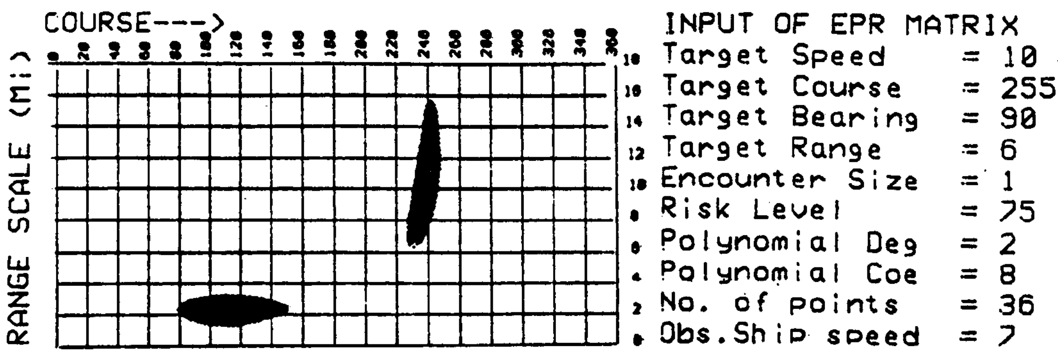
Generation of equipotential risk areas on the Co/Vo-plane for three different situations.

a



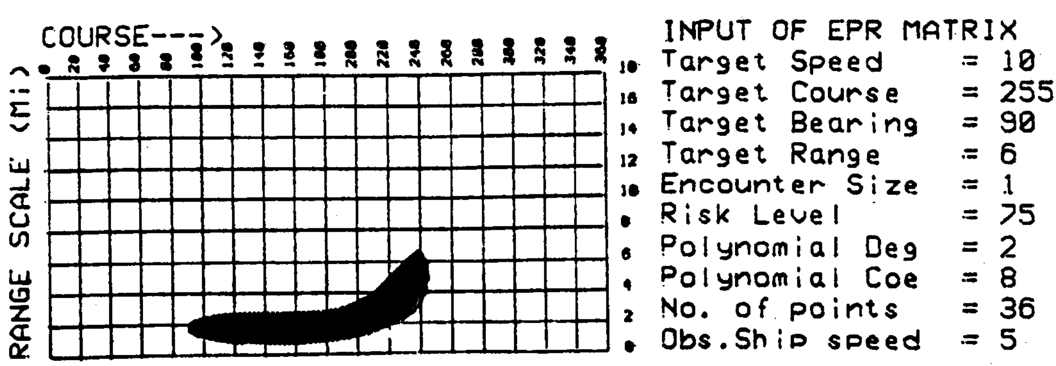
A) Presentation at a level of ship's speed of 18 knots.

b



B) Presentation at a level of ship's speed of 7 knots.

c



C) presentation at a level of ship's speed of 5 knots.

Figure(5-4)

Generation of equipotential area of risk on the Co/To-plane for one situation of three speed alternatives.

In order to illustrate the time factor in the assessment of the risk the computed runs $(\sum_1^n S_{(i)})$ along unsafe courses are shown in Figs. (5-4a), (5-4b) and (5-4c). The graphs are based on the initial data of the example shown in Fig. (5-3b). The conditions are presented corresponding to three different speed ratios, (V_o / V_a = 1.8, 0.7 and 0.5). Graphing of the runs $(\sum_1^n S_{(i)})$ for a specific speed (V_o) is actually an indication of the corresponding time intervals$(\sum_1^n \Delta T_{(i)})$ from the present time (T_o) to the interception of the prescribed risk domain of target ship. Each time interval means how much time allowance a mariner has on the given track. Moreover, the usefulness of illustrating or plotting these data lies in the comparative importance attached to the assessment of risk in multiple binary encounters.

The investigation of the generated graphs shows a close resemblance with graphs presented by the (E/H) diagram. However, there is still an observable distinction between the two figures. Obviously for a given initial situation the area of risk on the (V_o / C_a) - plane is relatively larger than that of the (E/H) diagram, especially in the region of ($V_o/V_a < 1$). The difference lies in the function used to generate the envelope of the area. The boundary envelope of (V_o/C_o) graphs is an equirisk contour based on the resulting aspects of the tangent points of the EPR ellipse. The boundary envelope of the (E/H) graph is an iso-aspect contour based on the limiting aspects resulting from the tangent points of the critical circle (R_a). It has been shown in unit (4) that the size of the ellipse is speed ratio dependant. The difference between the two aspects increases as the speed ratio decreases resulting in a wider risk area on the (V_o/C_o) diagram. However, as speed ratio increases, the ellipse will be squeezed to nearly the size of the critical circle. The resulting effect on both areas in this range of speed ratios is to cause them to coincide.

In general, presentation of the EPR marrix on the (V_o / C_o) and (T_o / C_o) planes at any moment enables the user to immediately obtain the answers to the following important questions:

1) Will any proposed course and/or speed changes increase risk of collision with any ship in the vicinity ?

2) Will the present elements of ship's motion risk collision with any ship in the vicinity ?

3) Is it impossible to avoid risk with course alteration only ?

4) Will risk be best dealt with the early coordination of both course and speed changes ?

5) Is it too time consuming to avoid risk of collision ?

6) Will the chosen action prolong the engagement period with other targets ?

7) Can the individual risks be separately ranked ?

8) Will the risk with other ship be reduced by taking this action ?

9) Is it necessary to retain a certain level of risk ?

10) Will the proposed control satisfy legal requirements?

...

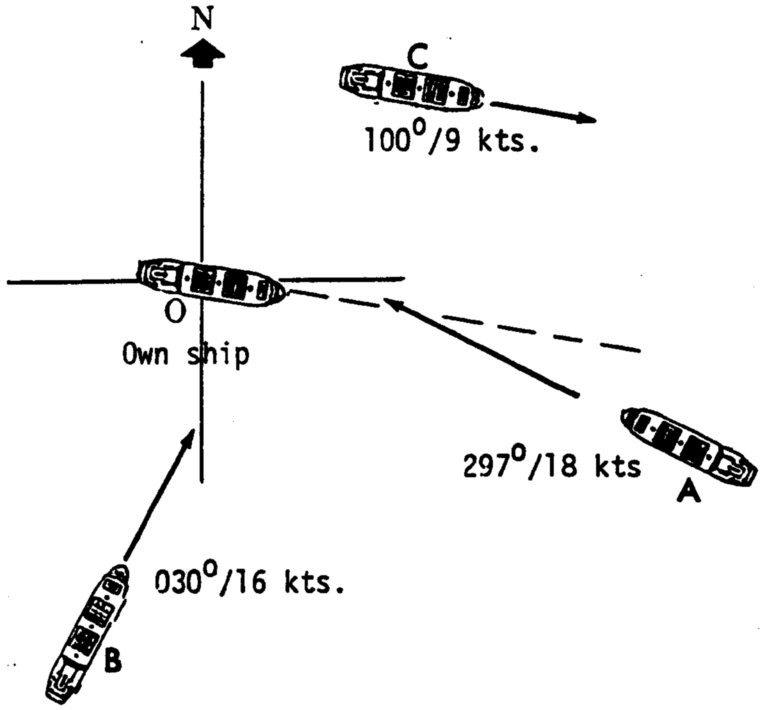
5.6 RESULTS OF CASE STUDY

Here is an example of what might be expected of a complex situation consisting of three binary encounters as observed from one ship. The observing ship (O) is sailing on course (100^{O}T) with a speed of (12) knots. Figure (5-5) illustrates that ship (O) is overtaking a ship (C) which is about 5 miles distant, bearing (045^{O}), steering on a parallel course with a speed of (9) knots. A second target ship (A) is located at (10) miles distant and (17^{O}) on the starboard bow. It is steering on a course of (297^{O}) with a speed of (18) knots.

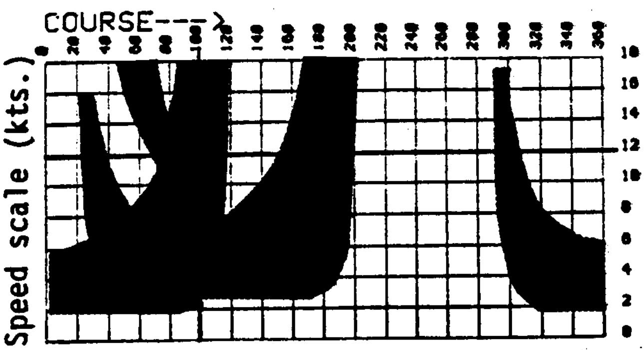
A third target ship (B) is crossing on a course of (030^{O}T) from the starboard beam and is located (10^{O}) abaft the beam at a distance of (8) miles. For greater safety, it is obviously to the advantage of ship (O) to have prescribed a level of risk of (50%). However, to demand a lower risk may result in a large off-course deviation, which may not be feasible when navigating in confined waters and / or congested areas. It is assumed that this is the situation under consideration and consequently a higher risk level of(75%) is taken as the upper limit boundary.

On board the observing ship (O), the situations, primary information are processed and the resulting EPR patterns on the (C_O / V_O) plane and (C_O/T) plane are generated in the form shown in Fig. (5-5). Looking at the risk areas gives the impression that the situation is somewhat difficult! However, it is neither that hopeless nor that simple. Examination of the risk patterns confirms at a glance that there is a risk of collision with ship (A) which will start to build up above the (75%) level after (5) minutes. In order to reduce the risk of collision with target ship (A) the following control actions are examined:

(a) On turning to port, the safe sector of courses are limited due to the presence of target ship (C) together with the second unsafe sector of target ship (B). A mid course of (80^{O}T) will reduce the maximum risk below the prescribed level.



Ā complex multible ship encounter.



E.P.R.Matrix	A	B	C
Target speed	=18	16	9
Target course	=297	30	100
Target bearing	=110	200	45
Target range	=10	8	5
Encounter size	=1	1	1
Risk level	=75%	75%	75%
Polynomial deg.	=2	2	2
Polynomila coe.	=8	8	8
No. of points	=36	36	36

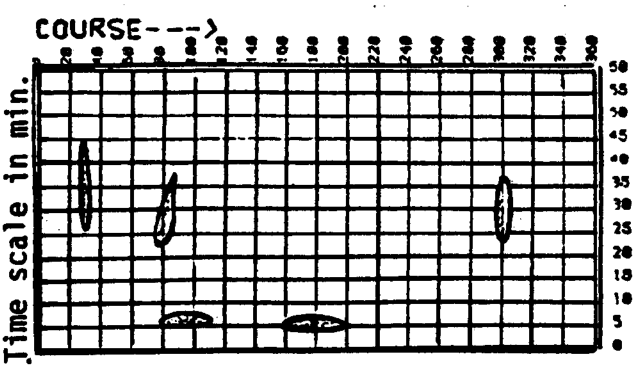


Figure (5-5). Equipotential risk presentation of three binary encounters on the (Co/Vo) and (Co/To) planes.

However, this control action may not be favoured due to a corresponding long time of engagement and the requirements to cross ahead of both targets (A) and (B)

(b) A reduction of speed is a second control action to be considered. However, on doing so the risk of collision with other target ship (B) will be increased above the prescribed level, unless ship (O) is nearly stopped, which will also extend the time of engagement.

(c) A third option is a starboard turn to steer either (140^{o}T) or (220^{o}T). On following the course (140^{o}T) the main threat will be cleared by passing astern of target ship (A), yet on doing so the risk with target ship (B) will be increased in trying to cross ahead of it at a smaller distance. On altering the course to steer (220^{o}T), all risks will be greatly reduced by passing astern of targets (A) and (B) while increasing the distance with target (C).

Having steered the chosen course and/or speed the situation is monitored through the risk diagram which is continuously updated. As the shaded risk areas are altered, due to the changing in the relative positions between the ships, then when the original course and/or speed is clear of the risk area, the course and/or speed resuming manoeuvre can be initiated.

5.7 SUMMARY

In this unit a unique approach for handling C/A problem has been advocated. Obviously to do risk evaluation properly a resort to the application of a suitable risk criteria is essential. Handling risk of collision must feature three basic objectives; identification, assessment and control. Identification of risk is achieved by defining a risk domain around target ship with various levels. Each level is represented by a closed curve of equal collision probability which embraces all the contributing factors relevant to situation geometry. These curves are found to be nearly elliptical, and hence for practical purposes the equivalent ellipses are determined and used.

The assessment and control of risk are effectively made possible by transforming the EPR ellipse into interceptable equivalent areas of risk mapped on (C_o / V_o) and (C_o / T_o) planes, and hence the coordination of the two control variables (C_o) and (V_o) are presented on the same risk pattern. The risk model presented in this unit has been developed to deal with compound encounters. However, the user must be involved throughout the risk analysis and to arrive at an action for risk control he has to set down priorities for acting on individual risks if he cannot reduce the risks from all other ships by one manoeuvre. In general all alternatives should be evaluated for their susceptibility to reduction or avoidance of risk and appropriate action taken, leaving only the residual risk which must be retained.

One question which has not been answered is - will the proposed control strictly satisfy legal requirements ? The answer to the basic requirement by law will be the subject of the following discussion included in unit (6).

UNIT_6

Legal Constraints on the Control of Risk of Collision

6.1 INTRODUCTION

This unit is concerned with the analysis of control of collision risk in a two-ship encounter which is subject to legal constraints imposed by the International Rules for Preventing Collision at Sea. Since the level of risk is influenced by the control variables, R the range, Q the aspect and E and speed ratio, the controlled motion of both ships in an encounter will change the levels of risk associated with each ship separately. Thus, if the risk controls of both ships are so loosely related that there is equal probability of any control action negating the effect of the other ship's action, it would lead to a state of "chaos" or complete randomness, and hence, lack of "constraints". A strong risk controllability can be achieved if it is characterised by a relatively high degree of constraints in the form of regulations. These regulations should acquire features that will permit them to discriminate, act upon, and respond to aspects of the situation variety. The regulations will have mapped parts of the situation variety into their structure and/ or information. Thus, the existence of such regulations and the compliance with them by all parties in the encounter should increase the risk controllability ; through avoiding conflicting control actions.

In these terms, then, existing Regulations for Preventing Collision at sea are based on the situation geometry and the prevailing visibility condition. The point of distinguishing between the two parties is so that they may be assigned different but complementary roles in taking avoiding actions. These rules specify, for any binary encounter involving risk of collision, a single action for each ship to be carried out in ample time.

Regarding the condition of unrestricted visibility the following constraints are imposed in accordance with the geometry for the encounter:

1 - One ship is constrained by one option to "STAND ON" allowing the other the choice of risk control. This is the case of crossing and overtaking encounters.

2 - Both ships are constrained by a single control action. This is the case of head-on situations.

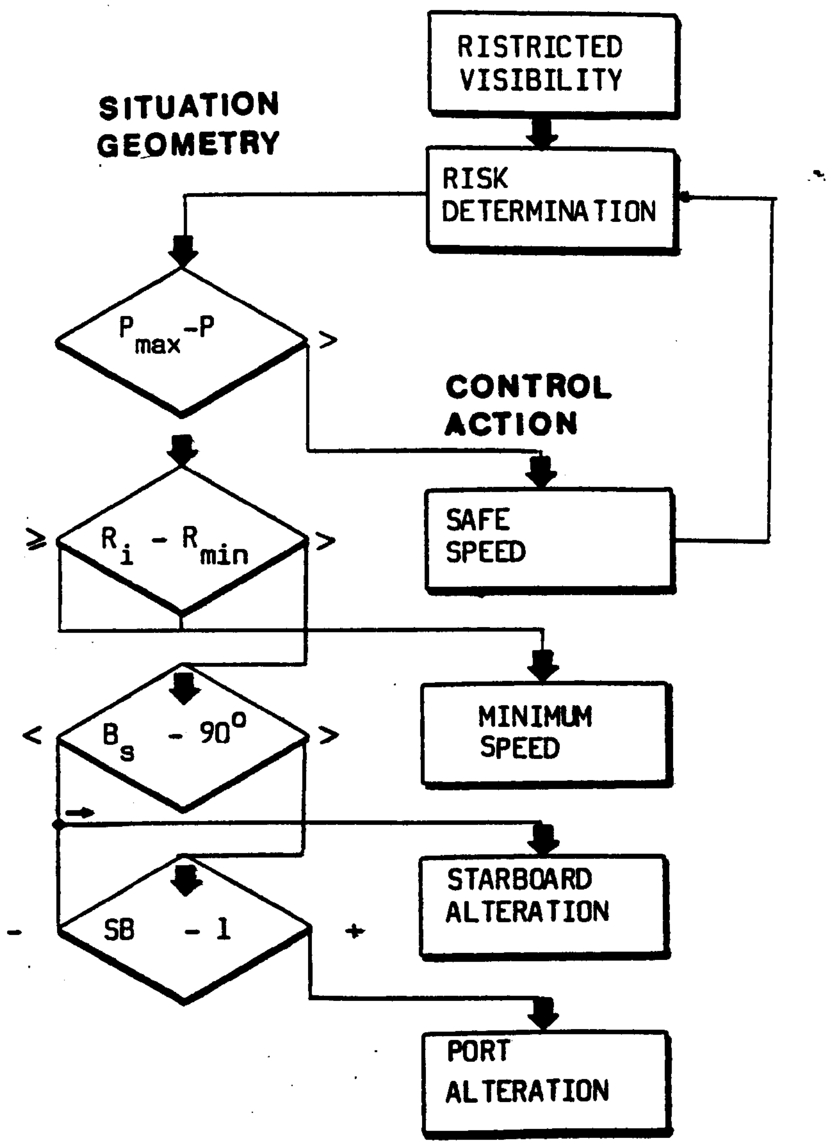
3 - One ship is constrained by one option to "STAND ON" until a certain range, after which she is allowed to take control action provided that the other ship is seen not to be taking sufficient action.

If the condition of visibility is restricted, then both ships are free to take control action but constrained by the following as far as possible:-

1 - An alteration of course to port for a vessel forward of the beam is to be avoided.

2 - An alteration of course towards a vessel abeam or abaft the beam is to be avoided.

The analyses of the constrained options subject to the International Regulations for Prevention of Collision at Sea is summarized in the logic flowchart as illustrated in figures (6.1a) and (6.1b).



NOTATION:

B = Relative bearing
B_s = Relative bearing semicircular
Q = Aspect
Q_s = Aspect semicircular
SB = Signum (sin(B))
SQ = Signum (sin(Q))
P_{max} = Maximum risk
P = Potential risk
R_i = Initial range
R_{min} = Close quarter range
$>$ greater than
$<$ less than

Figure.(6-1a) Analyses of risk control action subject to legal constraints in case of ristricted visibility.

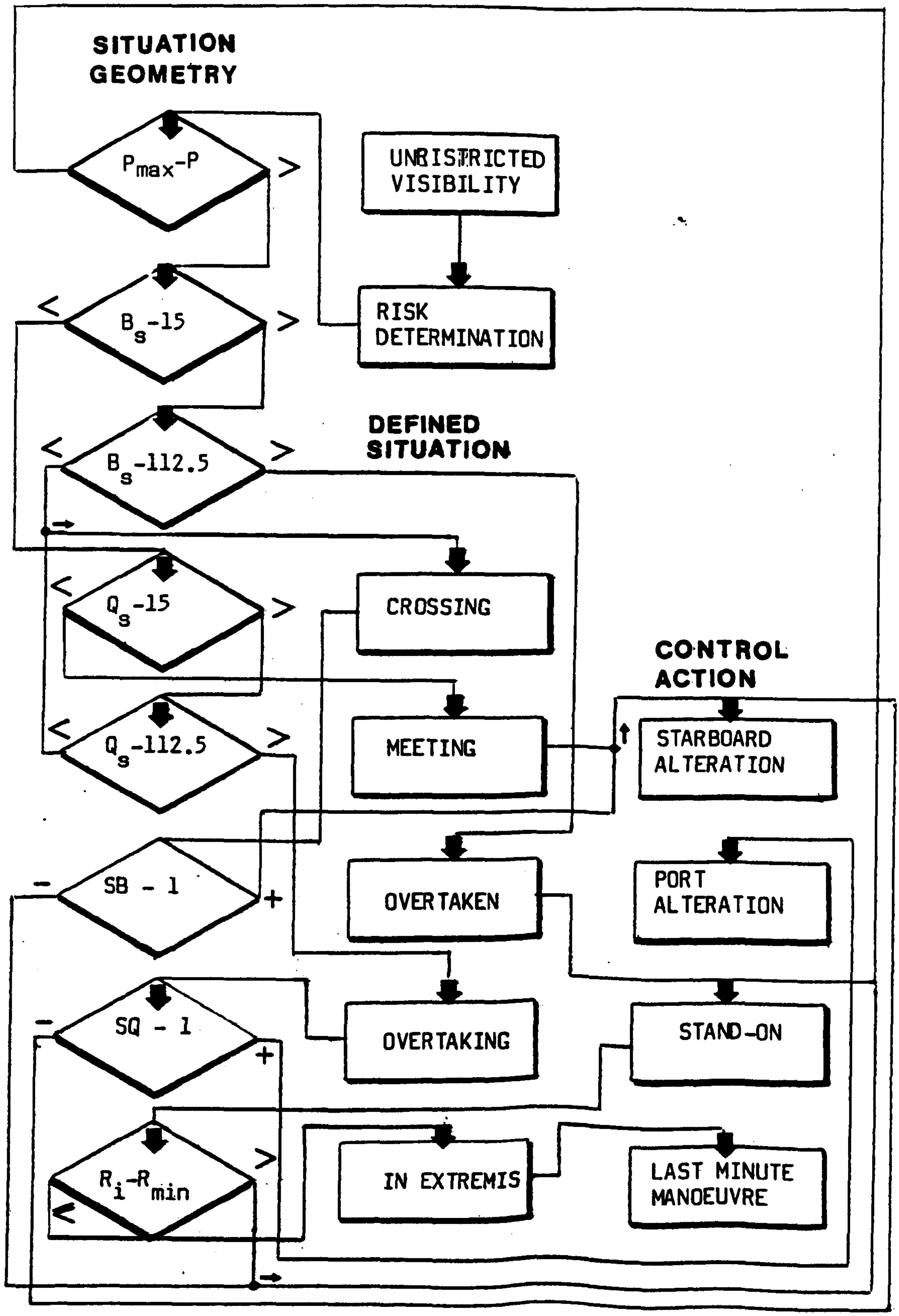


Figure. (6-1b) Analyses of risk control action subject to legal constraints in case of un-ristricted visibility.

6.2 SIGNIFICANT COURT DECISION

The application of the International Regulation for Prevention of Collision at sea over a long period of time by the mariners has resulted in the development and refinement of the Regulations to their present wording. However, their application depends upon the ability to assess when risk of collision exists and accurate definition of the geometry of the situation. In this section, two particularly significant cases of collision will be examined in order to appreciate:

(a) That the Regulations should be understood and applied equally by all mariners.

(b) How the Regulations are interpreted by the courts.

(c) How court decisions can change when a case is taken from a lower to a higher court.

(d) The importance of the role of the expert assessor, who has to rely on his long marine experience and has at present no mathematical means of defining risk criteria which take into account the relevant parameters contributing to the risk of collision.

The "Auriga" case

This compound event of a binary encounter arose between the Spanish ship " Manuel Compos" , owned by the plaintiffs, and the Italian ship "AOriga" owned by the defendants, which took place in the Atlantic Ocean off the west coast of Spain on the evening of January 11, 1973.

The collision occured after dark in fine weather and with good visibility. At 1908 "Manuel Compos" altered course from 239° to 205°. Being steady on the new course, with a speed of 11.5 knots the master noticed the existence of "Auriga" bearing about 2 points abaft the beam and 3 miles distant. It was assumed that "Auriga" was overtaking.

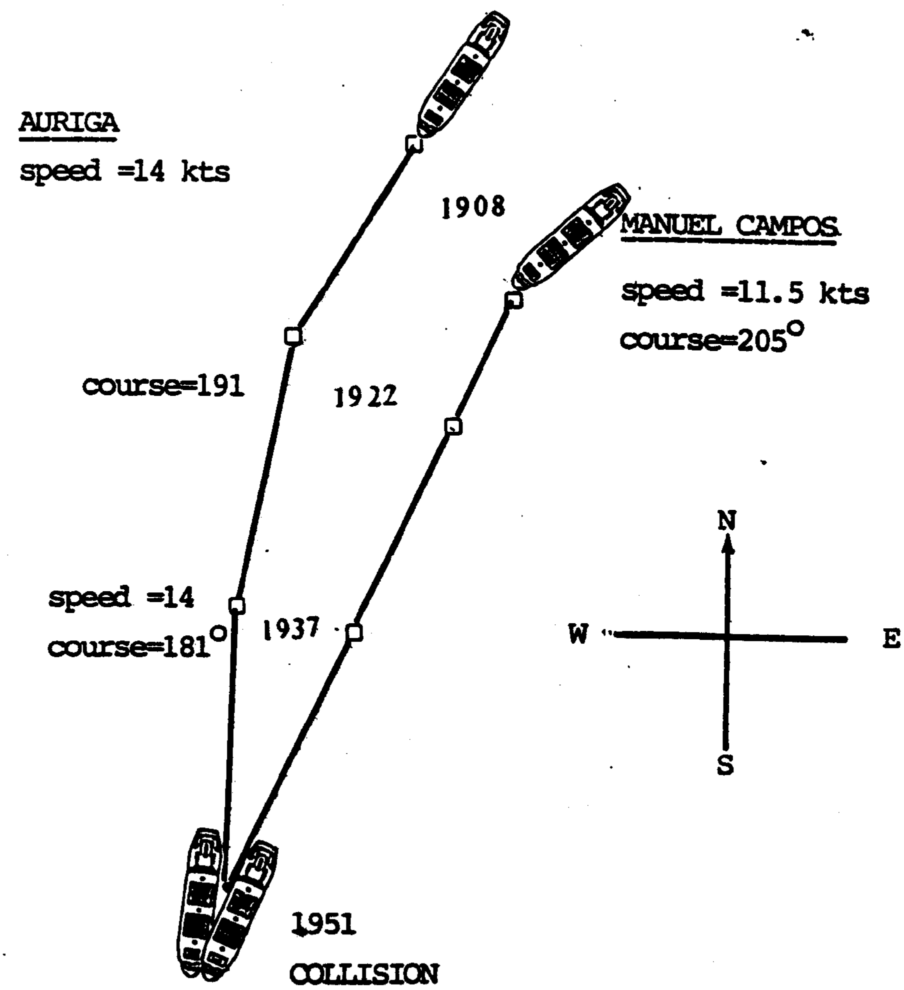


Figure. (6-2) The geometry of the binary encounter of "AURIGA" case

"Auriga" was proceeding at her full speed of 14 knots on a course of 212° and the courses of the two ships were diverging at a relative heading of (7°) while "Auriga" was coming up with "Manuel Compos" at a relative speed of (2 - 2.5) knots. At about 1922 "Auriga" manoeuvred to steer an average course of 191° until 1937, when she steadied on to a new course of 181°. The courses of the ships were now converging at relative heading of (24°) with a risk of collision if the courses were maintained. Neither of the ships took any effective avoiding action nor did they keep a continual and careful observation of each other. At about 1952 the collision occurred. Based on the information given in the Lloyd's report (1977 Vol.1) the situation geometry is re-constructed as shown in figure (6-2).

One of the principal matters in dispute between the parties is whether the "Auriga" was overtaking the "Manuel Campos" so that the overtaking rules applied, as the plaintiffs claimed, or whether the two ships were on crossing courses involving risk of collision, so that the crossing rules applied as the defendants claimed.

Held, by Q.B. (Adm.Ct) Brandon); that-

(1) The overtaking rules only applied when the relationship between the two vessels concerned was such that risk of collision between them existed.

(2) There never was an overtaking situation to which r. 24 applied in that during the period when "Auriga" was bearing more than two points abaft the beam of "Manuel Campos", there never was at any time a risk of collision between them since the courses of the two vessels were diverging and the distance between them was too great for risk of collision to exist.

(3) r. 19 applied to the crossing situation which occurred when the courses of the two vessels were converging at an angle of 24°, with "Manuel Campos" as the give-way ship and "Auriga" as the stand-on ship.

(4) Both vessels were at fault in several respect. "Manuel Campos" was at fault in respect of keeping a bad look out and was in breach of r.19 in failing, as the give-way ship in a crossing

situation, to keep out of the way. "Auriga" was fault in respect of keeping a bad lookout, altering her course to 181°at an improper time in relation to "Manuel Campos" and was in breach of r. 21 in failing, as the stand-on vessel in a crossing situation, to take sufficient avoiding action in sufficient time. All of these faults contributed to the collision.

(5) Although the fault of "Manuel Campos" in failing to give way was greater than the fault of "Auriga" in failing to act sufficiently early, the fact that a crossing situation involving risk of collision came into being was entirely the fault of "Auriga", and liability would be apportioned 60 percent to "Auriga" and 40 percent to "Manuel Campos".

The "Nowy Sacz" case:

This collision arose out of a binary encounter of the Cypriot ship "Olympian" and the Polish ship " Nowy Sacz " which took place in the Atlantic Ocean to the south of Cape. St.Vincent just few minutes before (0400) on February 14, 1972. The event occured at night in fine weather and with good visibility. Using the best reconstruction of the event, it is concluded that the relative heading was of order of 10°, giving a course for the "Olympian" of about 331° T and for the "Nowy Sacz" of about 341°T. It is believed that both ships must have been on these courses since at least as early as (0245). It was further found that the headings of both ships were altered to starboard by about the same amount before the collision, that of the "Olympian" as a result of putting her wheel hard to starboard and that of the "Nowy Sacz" either due to interaction or as a result of working her engines astern. It was also found that the "Nowzy Sacz", as a result of her engine action, took off some of her speed before the collision, as a result of which the "Olympian" drew ahead of her more quickly at the last minute than she would otherwise have done. Based on the information given in the Lloyds report (1976) Vol.2, the situation geometry is reconstructed as shown in Figure (6-3).

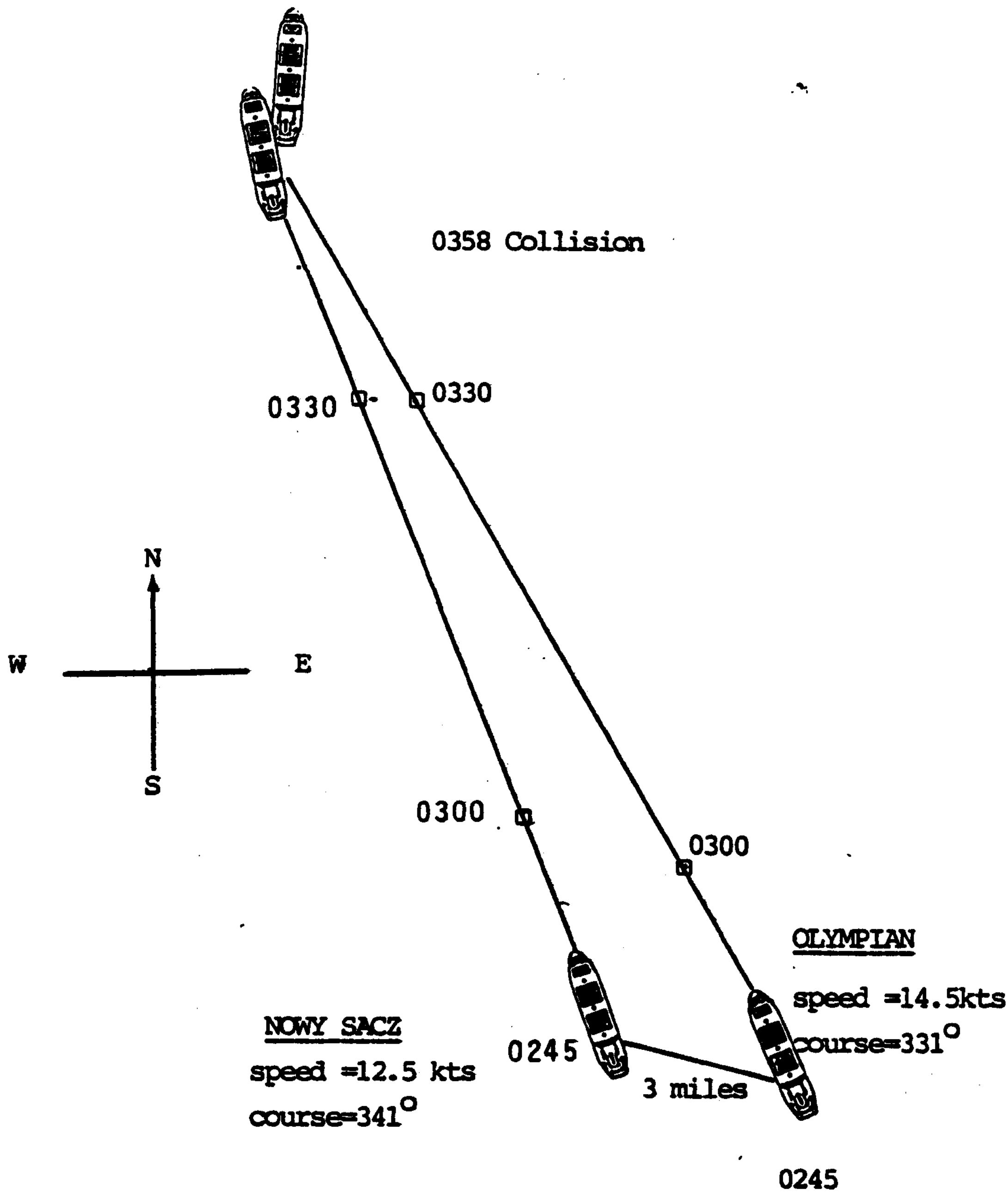


Figure (6-3). The geometry of the binary encounter of the " NOWY SACZ " case.

The plaintiffs brought an action for damages contending that the Crossing Rules, specifically part D of the Steering and Sailing Rules of the Collision Regulations, 1960 applied, and that under them the "Nowy Sacz" was under duty, as one of two crossing ships having the other on her own starboard side, to keep out of the way of the "Olympian". The defendants argued that the overtaking rules applied, in that it was the duty of the "Olympian", as the overtaking ship to keep out of the way of the "Nowy Sacz", the ship being overtaken.

Held, by Q.B. (Adm. Ct) (Brandon, j). that -

(1) The overtaking rules were only applicable if before 0300 when the "Olympian" was still veering more than two points abaft the beam of the "Nowy Sacz", two conditions were fulfilled:
First, that the two ships were in sight of each other, and second, that the risk of collision between them had by then already arisen.

(2) It was not possible to say with certainty that the stern light of the "Nowy Sacz" became visible to those on board the "Olympian" at some time after 0245 and before 0300, but as her green light was visible by about 0300, it was likely that the two ships were in sight of each other before 0300, thus fulfilling the first condition; but not the second condition.

(3) The situation, if the Steering and Sailing Rules applied at all, was a crossing situation and not an overtaking situation; and "Nowy Sacz" was fault in failing to take early and positive action to keep out of the way of the "Olympian" and in failing to reduce her speed in ample time so as to allow the "Olympian" to pass ahead of her; or failing to make an early and substantial alteration of course to port.

(4) With regard to the "Olympian", the proper action for her to take would have been to alter course more slowly to starboard at an appreciably earlier stage, thereby avoiding the danger of swinging the stern of the "Olympian" into the "Nowy Sacz". The Master of the "Olympian" was in fault in that he waited too long before taking starboard wheel action, and therefore, found himself obliged to take the more drastic wheel action by going

hard to starboard than would otherwise have been desirable.

(5) Both ships were to blame for the collision, the "Nowy Sacz" for breach of r.19 is not keeping out of the way, and the "Olympian" for breach of the proviso to r.21 in leaving it too late to take avoiding action.

(6) As to apportionment of blame, the "Nowy Sacz" was more to blame than the "Olympian", in that it was her fault which allowed a dangerous close-quarters situation to arise at all, whereas the fault of the "Olympian" lay only in her failure to take the right emergency action at the right time in the situation so created and the division of blame should be divided, as to three-quarters to the "Nowy Sacz" and as to one quarter to the "Olympian".

On appeal by the defendants the issue for determination being; "Does r.24 operate only when the positions, courses and speeds of the ships are such as to involve risk of collision, and if so was a risk of collision involved when the "Olympian" was more than two points abaft the beam of "Nowy Sacz" ?"

-- Held by C.A (STEPHENSON & SHAW and Sir DAVID CAIRNS), that -

(1) Rule 24 was applicable when vessels were proceeding so as to involve risk of collision.

(2) The words "Coming up with another vessel" in r. 24 imported the concept of proximity in space and time which might be before the time when there was risk of collision.

(3) Rule 24 was applicable before there was risk of collision.

(4) The learned Judge's conclusion that the risk of collision arose at about 0330 was clearly correct and by the time the vessels were in sight of each other and less than 3 miles apart, the "Olympian" was coming up with the "Nowy Sacz" as the stand-on ship and the "Olympian" was the give-way vessel.

(5) The apportionment of blame would be varied to the effect that the "Olympian" should be held three-quarters to blame and the "Nowy Sacz" one quarter.

Reassessment

In order to investigate the risk of collision involved in these cases the risk function introduced in the previous work is used to construct four risk patterns as illustrated in the figures (6-4a) , (6-4b),(6-5a) and (6-5b). These four risk transition profiles are based on the information given in the LLOYDS reports. Profiles of the corresponding range and aspect changes are also included. Each diagram shows the risk transition involved by one ship as indicated by the figures.

On comparing the four risk profiles, they show nearly a similar behaviour in the transition distribution. However, the encounters differ in the time interval of the approaching phase. At the initial stage the trend exhibits a regular transition of a very low level of risk with very low rate of increase, where the range is steadily decreasing with nearly constant aspect. The effect of the aspect at the initial stage of the norrowly converging situations shows very low risk values due to the large angle of aspect, while the range is the only parameter which contributes to the increase of risk, especially at the final stage of the encounter at which the risk increases more rapidly from a very low level of risk upto 100% risk.

Although the two ships in each encounter were in such a dangerous situation, those on board either ship were not aware of it. This was, as concluded, because the lookout on board both ships was seriously defective. However, on inspecting the risk profiles, it is proved that such a situation would lead to a deceptive perception of risk involved due to the very low level of risk, which lasts for a long period of time before the sudden increase. It is clear that the current knowledge of risk assessment used by both ships in each binary encounter had proven insufficient for formulating the details of causal chains and hence effective risk control strategies. Further, this situation may not change in the foreseeable future, for many of the encounters, unless a detailed and correct risk criteria has been introduced to the mariners and is fully understood by them.

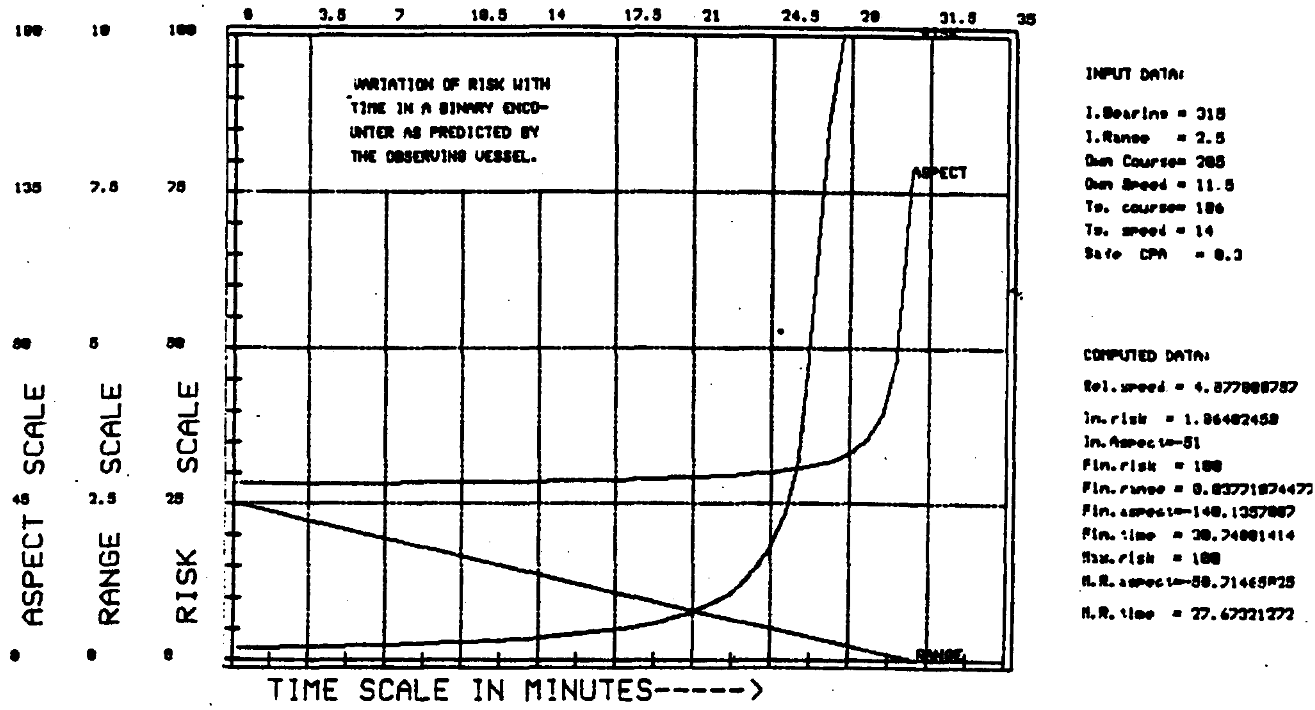


Figure (6-4a). Risk transition distribution along the track of " MANUEL CAMPOS ".

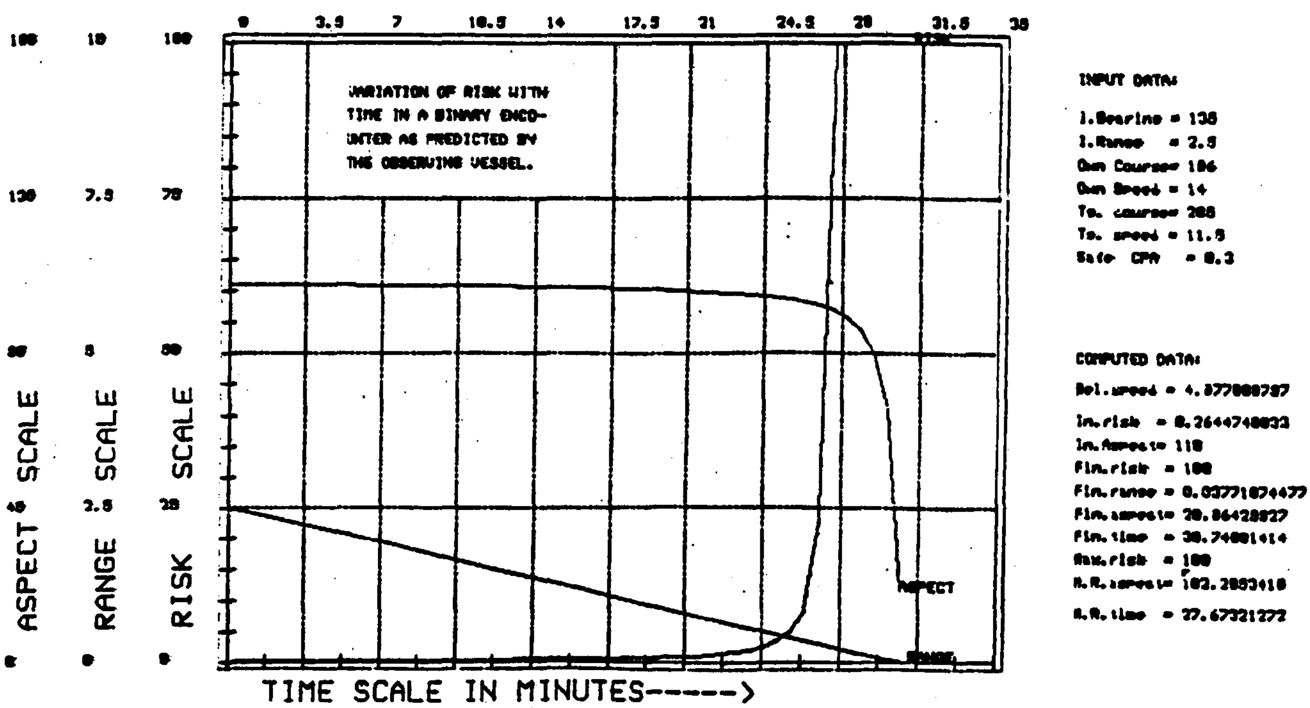


Figure (6-4b). Risk transition distribution along the track of " AURIGA ".

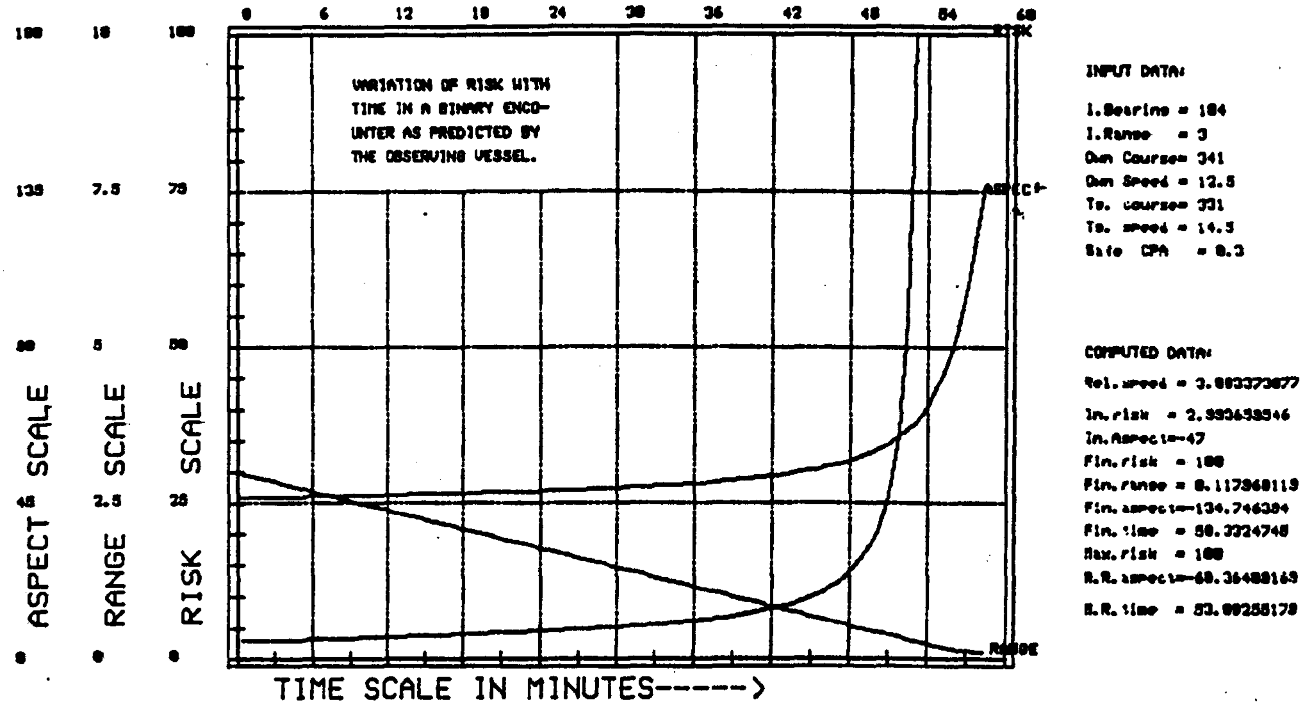


Figure (6-5a). Risk transition distribution along the track of " NOWY SACZ ".

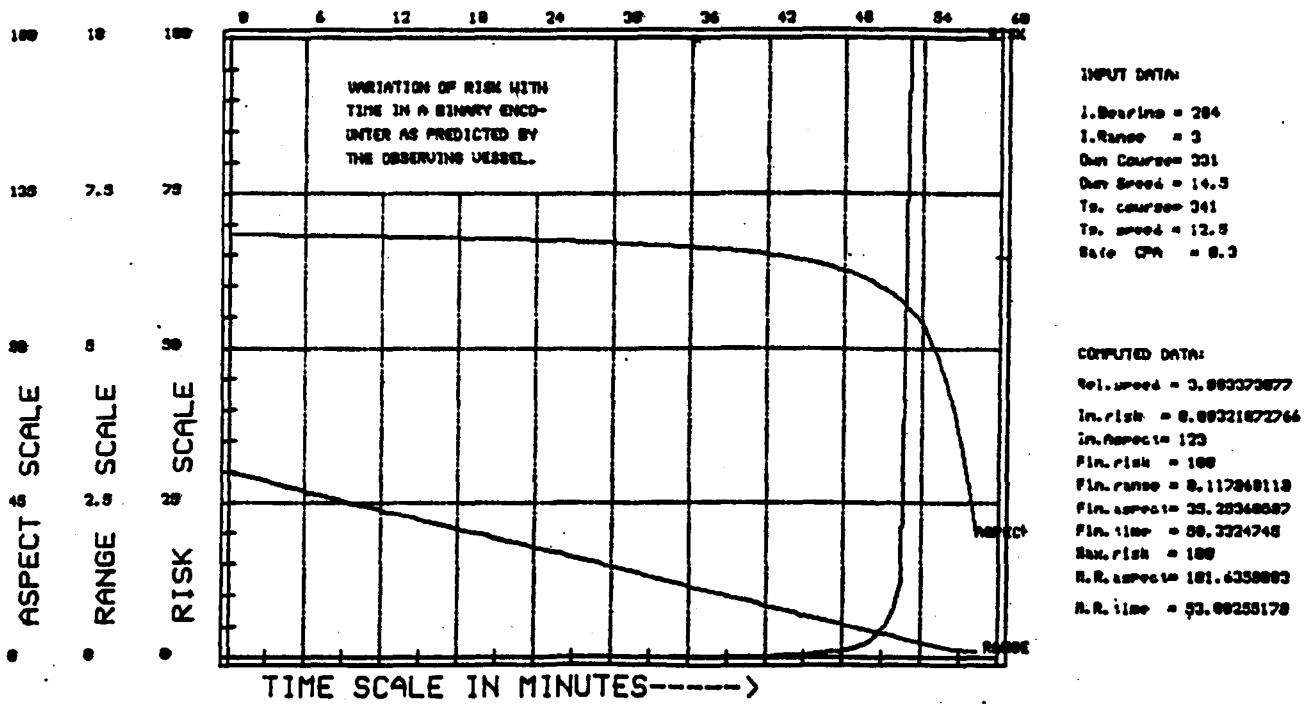


Figure (6-5b). Risk transition distribution along the track of " OLYMPIAN ".

The words "involving risk of collision" may mean different things to mariners, and it is quite possible that the purpose which the law-maker had in mind when they passed the acts is difficult to evaluate in practice. what really matters is the ability to interpret these words in the same way by an other court, mariner or assessor.

Section 6.3 uses the previous work to establish a mathematically accurate method of stating when risk of collision exists, which will make a consistent interpretation of these words universally available.

6.3. RISK OF COLLISION

The assessment of risk of collision aims to determine how serious a risk is apparent and whether a ship can be exposed to it. The decisions eventually taken by the mariner on a specific control action will reflect the analytic weighing of the predicted risk of alternative courses of action. Determination of risk of collision has been given greater concern in the 1972 regulations by the introduction of a specific rule which deals with this aspect of the C/A problem. In order to determine if risk of collision exists, proper use of radar equipment, radar plotting or equivalent systematic method should be made. However there is no quantifiable evaluation as to how and when risk of collision should be considered to exist. This important question has been left to the mariner's personal assessment of risk. In consequence and due to lack of a complete definition of risk of collision the mariner in many collision cases fails to exert the control action even when control was possible and would have been effective.

Characteristically, control of risk of collision assumes that risk is, or will soon be defined. It is increasingly clear, however, that the current method of risk assessment which is based on the prediction of CPA and TCPA is insufficient for formulating an adequate risk criteria and hence effective control Fortunately, it has been proved from the findings introduced in the previous units, that this situation may change and an adequate risk assessment based on a complete definition of risk of collision is possible.

On using the conventional set of chronological observations of bearing and range together with given parameters, any two ship encounter can be usefully formulated as a risk matrix enclosing all the possible outcomes, each outcome designated with a value that is in turn weighted by the probability of collision. In formulating these risk models it has been found that the following factors have to be among those taken into account:-

(a) The geometry of the encounter in terms of initial range , initial aspect, final range, final aspect, relative heading and speed ratio.

(b) The physical dimension of the ship in the encounter.

(c) The relative speed of approach and the related time interval of the encounter.

(d) The manoeuvrability of the ship.

(e) The observational errors in the computed information.

Now the methods used for assessment and control of risk of collision which have been devloped in units (2) to (5) and based on the theory introduced and presented in unit (1), have brought these rational factors to the C/A problem in terms of a quantified risk value as:

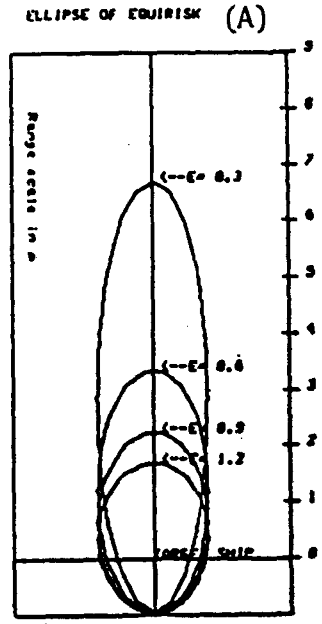
> *" The ratio of the different ways for a collision to occur to the total number of the possible ways that could be considered by the observing vessel."*

Thus it is now possible to rank the alternatives unambiguously and the choice is obvious. However this choice must be made with some level of risk in mind. In theory risk of collision may be defined at any range, but for "zero tolerance", it indicates that the risk value is to be reduced to the vanishing point. This vanishing point does not imply "absolute Zero". but a level below which no risk can be measured. On considering the two-ship encounter it is limited by a standard maximum detectable range at which reliable information can be deduced. A maximum figure of (25%) is taken as a limit, which covers practical parameters of a two-ship encounter at zero aspect. Consequently, the range at which the level of risk is found below (25%) may be used to define the zero in which the legal constraints are relaxed, or in other words the zone in which the rules do not apply.

When risk control is to be undertaken, the strategy relvolves around selecting risks having various probabilities to find an appropriate action, which, when taken, will reduce the risk to a retainable level. This implies that some level of risk above zero tolerance is to be accepted by balancing the off-course penalty against the level of risk introduced. A probabilistic figure of (50%) is thought to be a fair criteria. Based on this hypothesis the following definition of risk of collision is given :-

> " *If an observing vessel has been a partner in a compound event of a binary encounter involved in a possible collision, in which the levels of risk are recognised to be higher than (50%) along the projected track of her motion, then risk of collision shall be deemed to exist.* "

It is worth noting that the rate at which the risk level changes in the interval of the encounter has no direct impact on this definition. It is an important parameter to be included in the stimulus to the required response. It is hoped that this definition would provide a fairly concise common language for characterizing risk of collision between ships at sea. In order to appreciate this definition appropriate examples are given in figure (6-6). The generated diagrams are based on the concept of the equi-risk ellipse developed in unit.(4). Each ellipse defines on the sea surface all the positions related to the other ship's centre point. The diagram (6-6b) and 6-6c) show (50%) level of risk defined by the isorisk contours. Each diagram comprises four ellipses, covering speed ratio parameter of range (0.5) to (2) computed for the same speed ratios but for (75%) level of risk. A maximum range of (2) to (12,5) miles corresponding to speed ratios of (2) to (0.3) respectively represent the ranges at which risk of collision shall be deemed to exist. These Figures fall from (12.5) down to a range of between (6.5) to (1) miles depending upon the speed ratio.



INPUT DATA
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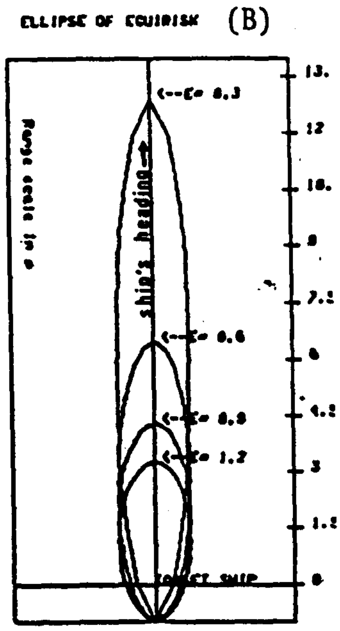
Encounter size=1 n.m



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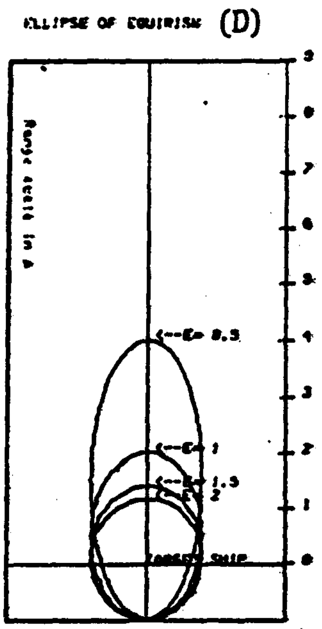

INPUT DATA
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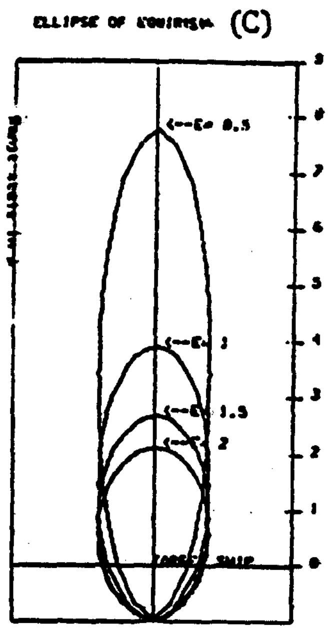
Encounter size=1 n.m



INPUT DATA
Level of risk =50%
Encounter size=1 n.m

Figure (6-6) Definition of the isorisk contours around target ship.

6.4 CONSTRAINED CONTROL ACTIONS IN RISK ZONE

To introduce a large pattern of situation concepts from the point of view of the observing ship, it is important to develop a higher order explanation that links the transition distribution of risk levels around the ship and the other relevant parameters together. A pattern of action of zone referred to the observing ship coordinate system might be visualized in a concentric contours diagram, wherein the central zone represents the collision domain enveloped in an action zone of several risk levels.The outer boundary of the action zone is characterised by risk levels of less than (25%). risk levels are defined for each point relative to observing ship on the two dimensional plane.

On the basis of the collision situation all the points B_i, R_i at which the risk has the same level (P) are joined. The equirisk contour ($U(B_i,R_i) = P$) is drawn. By varying (P) the given figure represents a 1-parameter family of curves, See figure (6-7).

Given a specified speed ratio ($E= V_o / V_a$), the strategy for calculating the points of the equirisk contour under consideration is therefore as follows. For values of (H_i),in the range of (00^o) to (180^o), the collision equation (1-30), is applied directly to determine the corresponding collision aspect (Q_i). Hereby, the associated relative bearing (B_i) for the given situation is determined in accordance with the following relationship:-

$$B_i = 180 - (H_i + Q_i)$$

(See appendix A-2)

It is expected that the resulting values of the relative bearings will not be equally spaced, which will affect the smoothness of the equirisk contour. In order to overcome this effect the increment of (H_i) has to be varied in such a way as to produce equal intervals of relative (ΔB=constant). The variation of (H_i) which satisfies this requirement is determined by the following relationships;-

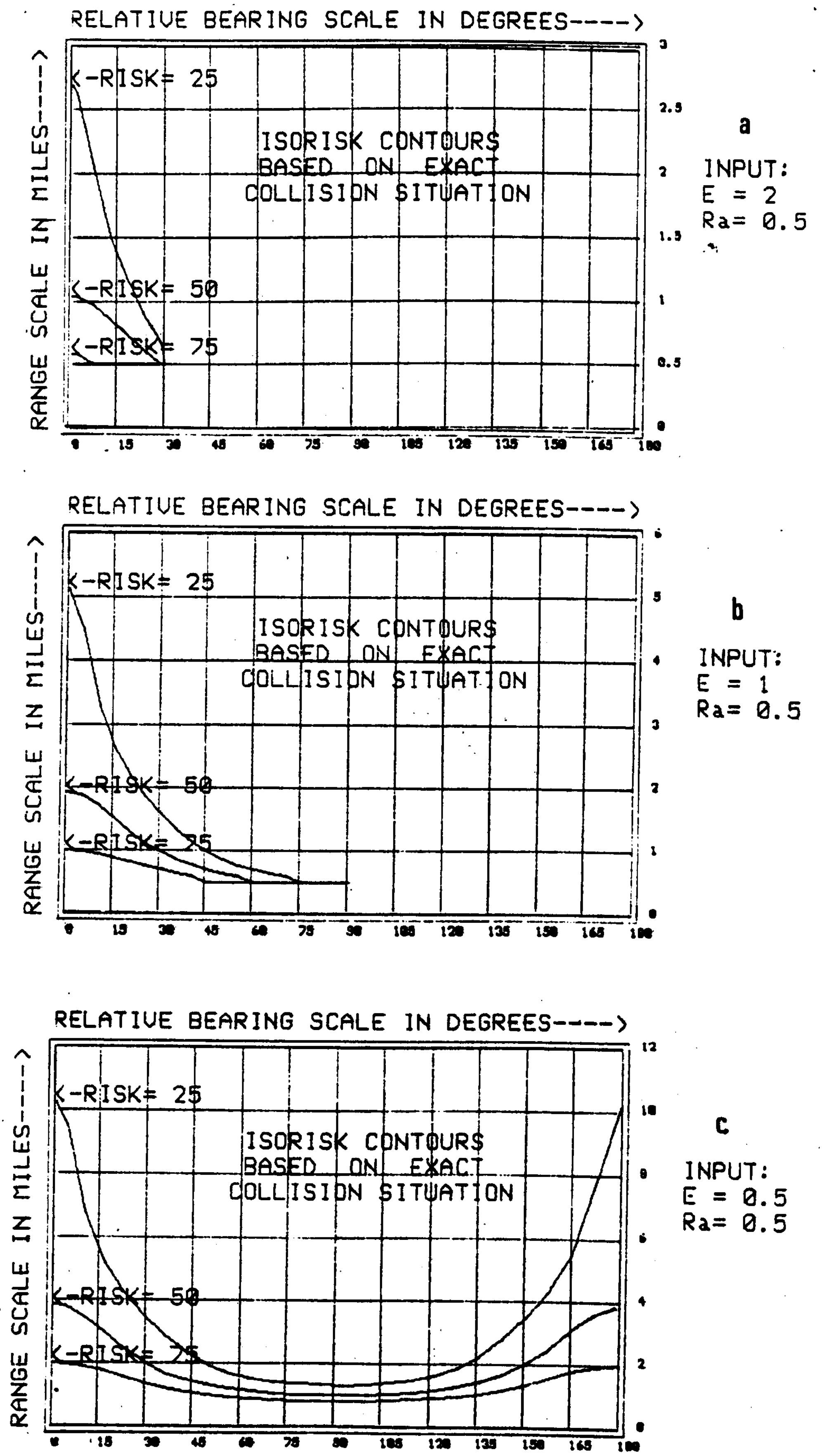


Figure.(6-7). Action zones defined relative to observing ship.

$$\Delta H = \Delta B.((1-2.E.COS(H_{i-1})) + E^2) / (1-E.COS(H_{i+1})))$$

Where ΔH = Incremental value of the relative heading

E = Speed ratio

ΔB = An arbitrary constant increment of relative bearing.

Derivation of the above relationship which satisfies the collision condition is given in the appendix (A-2).

Having determined the required increment of (ΔH_i), then the corresponding values of (H_i) and (Q_i), can be calculated by the known relationships:-

$$H_i = H_{i-1} + \Delta H_i$$

$$Q_i = COS^{-1}((1-E.COS(H_i))/(1-2.E.COS(H_i)+E^2)^{\frac{1}{2}}).$$

The next step now is to calculate for each bearing the corresponding ranges at which risk levels of (25%), (50%) and (75%) are recognised. The solution is based on the risk function and Lagrange interpolation technique used in unit (4). A computer program written in BASIC and shown in the Appendix (B-4) listing (ISO - risk contour) is used to generate the isorisk contours of the diagrams (6-7a), (6-7b), (6-7c) (6-8a), (6-8b), and (6-8c).

Based on accepted (CPA) of (0.5) mile, three diagrams are computed and plotted as shown in figure (6-7). The iso-risk levels of (25%) (50%) and (75%) which are related to speed ratio parameter of (0.5) (1) and (2) are plotted in terms of other ship's relative position (B,R). Inspection of three diagrams shows in the case of head-on situations three groups of ranges; (2.5 : 1.1 : 0.6), (5.1 : 1.9 :1) and (10 : 3.9 : 2). These ranges define the levels of risk just specified for the corresponding speed ratios (2), (1) and (0.5).

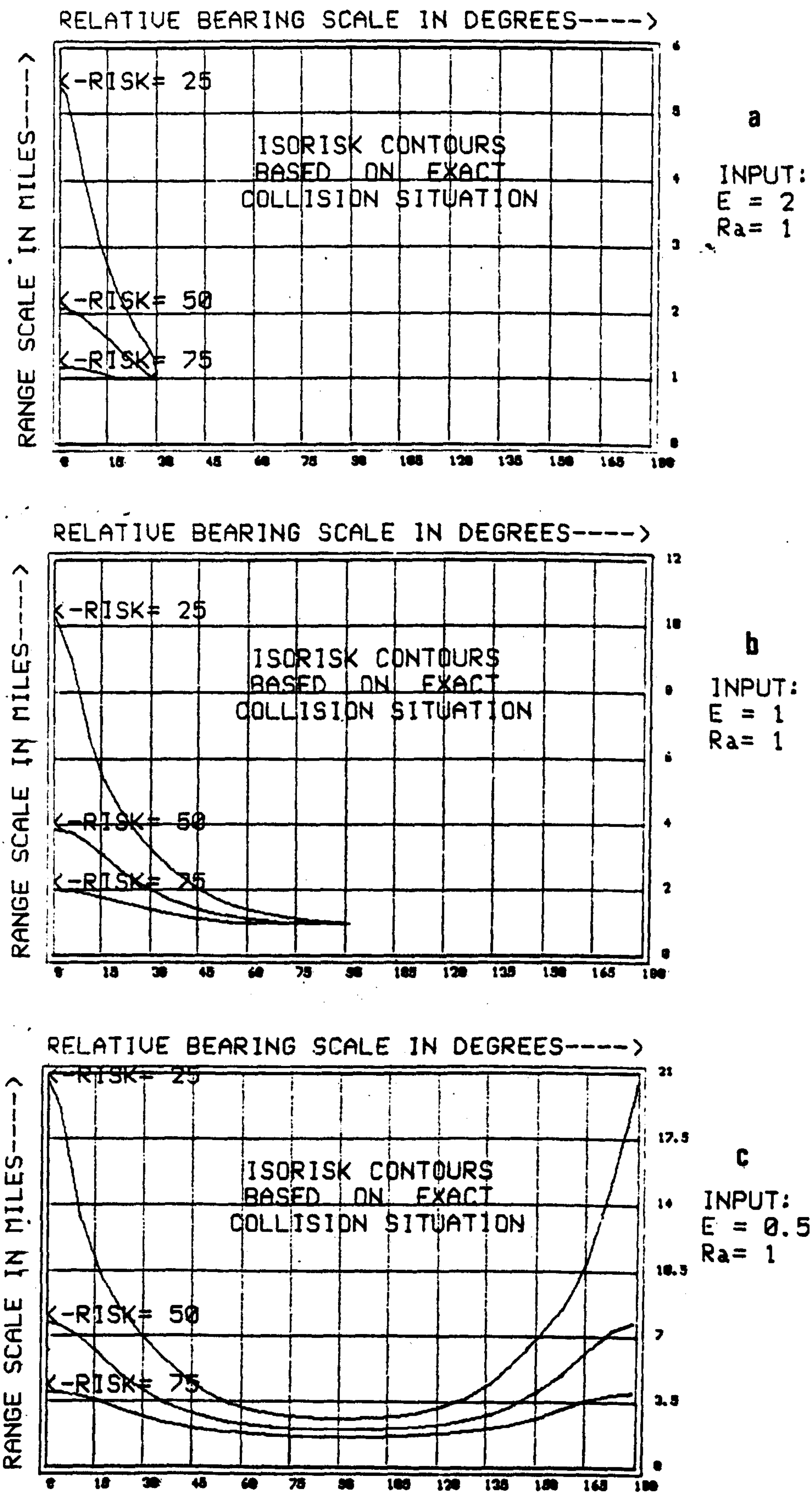


Figure.(6-8) Action zones defined relative to observing ship.

These ranges and their action zones indicated by the isorisk profiles are greately influenced by the speed ratio parameter. It is shown as expected, that the action zone becomes relatively smaller for the case of a slower observing ship. Consequently, the associated small action time necessitates an early stage control action. Because of the high rate of increasing risk which is a direct consequence of smaller aspect angles and high speed of approcach, the situation may not be resolved by one ship action. This fact imposes a great difficulty especially for the slower ship, and hence a co-ordinate action must be contemplated. Based on this fact Rule 14 is formulated which implies consistent co-ordinate action. It is required that when two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other. Because of the symmetry of the situation both control actions will contribute in decreasing the speed of approcach and increasing aspect angles by large amounts and hence a small level of risk is maintained and a slow rate of risk change is achieved and the maximum risk level is reduced, as recognised from each ship.

For being overtaken, as shown in diagram (6-3d), the isorisk contours show indentical behaviour to that of the head on situation. However, a relatively low rate of increasing risk is usually recognised in this case as there is unlikely to be a high speed of approach. On the other hand, the other ship which is overtaking and obviously the faster will usually have little difficulty in keeping control of the relatively lower risk, by either course alteration and/or speed variation. It is also evident from the study of the risk function that any action taken by an overtaken ship will impose a temporary increase of the value and rate of risk recognised by the other ship. Thus an action taken by the overtaken ship could confuse the situation. Based on these facts it would be wise to allocate the responsibility for taking action to the overtaking ship, while requiring the overtaken ship to maintain her course and speed. The constraint is imposed by Rule (13), which requires that any vessel overtaking any other shall keep out of the way of the vessel being overtaken.

One of the most widely complicated two-ship encounters in the context of risk assessment and control is the crossing situation. Diagram (6-7 c) shows the transition distribution of levels of risk of collision when recognized by a slower observing ship. Consequently the case will be characterized by the spreading of the possible risk over the full range of the relative bearings. Tracing of the crossing situation range of bearings indicates that low levels and rates of risk show over the early stages of the situation. Afterwards a (25%) level of risk begins to be recognised between a range of (4-2) miles and then increases up to (75%) between (2-1) miles.

This unique behaviour characterized by a long period of low levels and rates of risk followed by a small period of sudden increase of risk may lead to a deceptive perception with serious implications for decision making. In order to avoid such complications, the control action must be initiated at an early stage. Considering the coordinate action and its importance in resolving fast developing situations, it has been found that on turning towards each other the levels and rates of risk will be increased to higher levels due to the combination of decreasing aspect and increasing approach speed. In seeking to investigate the other alternative, when the two ships turn away from each other the levels of risk and their rates will be reduced due to decreasing approach speed and increase of both aspects angles. However, this action will prolong the encounter and in some cases does not resolve the situation at all when both ships resume their original courses. To avoid these difficulties it is believed that if only one of them is put into action, the situation can be controlled effectively.

Rule(14) is put to regulate such a situation which requires that when two power driven vessels are crossing so as to involve risk of collision the vessel which has the other one on her starboard side shall keep out of the way and the vessel which has the other on her own port side shall maintain her course and speed.

Provided that the give-way vessel is taking early control action by following a starboard evasive course, a unique case of balancing effects between a decreasing range and increasing aspect is obvious. The result is a condition of low level of risk being maintained, throughout the interval of the closing range. Following this stage there usually will be a complete disengagement, even after resumption of the original course.

In both the crossing and overtaking situations, control of actions is constrained by an additional parts which require that the give-way ship should avoid crossing ahead of the other ship. The ground upon which this rule is substantiated is the relatively high level of increasing risk at the stage of crossing ahead, especially at a small (CPA), as clearly shown in (Fig. 1.11) when small aspects show greater risk. However, this rule need not be stated if the proposed definition of risk is adopted and the give-way vessel is permitted to cross ahead if, and only if, the maximum risk level being recognised along the initial track or the evasive trajectory does not exceed the (50%) level.

The outer boundary corresponding to starboard and port turns are illustrated in figure (6-9) as the two curves "MFC" and "LFG" respectively. Each curve may intersect one of the boundary lines of the risk zone forming four sectors inside the risk zone.

The first of these sectors, is the sector (M' GFC) which is located to the outside of the two generated curved lines and inside the zone of risk. Inside such an area a (CPA) larger than the radius of the critical circle can be obtained by either starboard turn or port turn of the observing ship. In contrast to the first zone, the area laid inside both curves shown as (MFL) in the figure is the collision zone in the sense that, for initial position of ship (O) to be inside such a sector, a (CPA) greater than (R_a) cannot be achieved by either starboard or port turns; provided that course alteration is the only permitted action.

The possible third zone is determined by the sector (LFC) which lies to the outside of the curve (LFG) and inside the curve (MFC). Being inside such an area a (CPA) greater than (R_a) can only be achieved by a starboard turn.

Lastly among these four risk sectors is the sector (MFG) which determines the area inside which a (CPA) greater than (R_a) can be achieved by a port turn only. It is now of interest to know how these sectors can be generated and on what mathematical principles they are based. Understanding of these principles is important, so that the limitations of the proposed solution can thoroughly be perceived. These principles will be the subject of the following section.

6.5 DEFINITION OF EXTREME RISK ZONE

Turning to the previous sections dealing with risk assessment, it is noted that where it is assumed that course alteration or speed variation are instanteneous, the consequences are insignificant on the outcome. This assumption is no longer reasonable when ships are located near to each other, and ship's manoeuvrability has to be considered in formulating risk models. It is intuitively obvious that contours specifying these critical ranges at which the manoeuvre characteristics are important must exist. The question is how to determine these contours in accordance with collision avoidance related to ship's manoeuvrability.

At such close ranges, the effect of observational error on the relative position of target ship can be considered proportionally insignificant and hence the conceptual critical circle will only represent the physical dimensions of the two ships. Thus, on the assumption that both ships are taken as circular discs of radii L_o and L_a, then the critical circle has radius (R_a) equal to the sum of their radii. Accordingly a (CPA) less than (R_a) would correspond to a physical collision.

Suppose that the other ship (A) is located on the horizon plane at point (A) as shown in figure (6-9), and encircled by the critical circle of radius (R_a). Then if the velocity triangle defining the encounter is obtained as the triangle (abc), the relative motion of the critical disc can be determined on the horizontal plane with reference to the (NCS) by constructing a band formed by the tangents (LL') and (MM') to the critical circle and parallel to the relative motion direction (C_r). The inside area of the formed band, extended infinitely from the critical disc in the direction of the relative motion is the risk zone, and the outside the no-risk zone. If the observing ship is initially found inside this band, then the risk of collision can in general be avoided by course alteration of either ship.

If the observing ship has initiated a starboard or port turn, then the relative motion will change during time of turning and the resulting path will be a curved line. Hence the critical circle will slide inside a curved band.

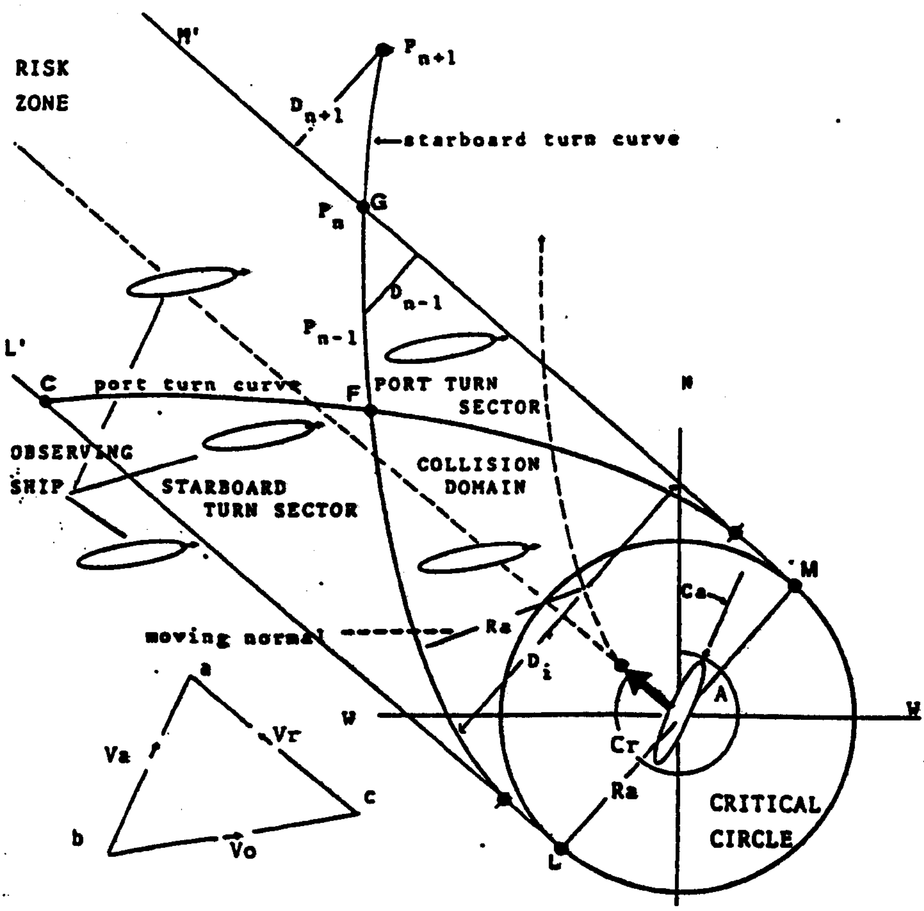


Figure (6-9). Graphical definition of risk sectors and the collision domain.

6.6 FORMULATION

Considering a possible application to multiple-ship encounters it is more convenient to express the relative motion with reference to the (NCS) whose origin concides with the other ship's centre point. On the assumption that the path of the relative motion to be composed of several simple addible parts, each of which is a function of the other ship's linear motion and observing ship's circular motion, the following relations are established. The following procedure arriving at the stated equations is based on equations (1.10) of unit (1), so a small part of relative motion can be expressed as:

$$\Delta X_r = Sa.\cos(ca) - So_i \cos(Co_i)$$

$$\Delta Y_r = sa.\sin(Ca) - So_i \sin(Co_i) \qquad 6.1$$

Where

ΔX_r , ΔY_r are the components of the small part of the relative motion.

Sa target ship true displacement in the small interval of time.

So_i observing ship segment of motion on the turning circle

Ca target ship true course

Co_i the true direction of the circular segment after small interval of time (Δt_i)

However, when (So) is taken as a very small part, it will ultimately be represented by the magnitude and the direction of the corresponding chord of the turning circle.

Then:

$$So_i = 2.Rt.\sin(\Delta\phi_i /2)$$

$$Co_i = Co_{i-1} + (\Delta\phi_i /2)$$

$$\Delta\phi^o = 57,3\ \Delta T_i .\ V_o /Rt. \qquad 6.2$$

Where

Rt	is the turning radius
$\Delta\phi$	amount of course change in the interval (ΔT_i)
Co_{i-1}	the true heading of observing ship just at the beginning of the time interval.
ΔT_i	the incremental time interval.
V_o	the tangential velocity of ship (O) along the turning circle.

Now if the coordinates of an initial position on the relative motion path are known as $(Xr_i\ ,\ Yr_i)$ then the coordinates of the successive points on the trajectory, when taken at small intervals of time can be given as:-

$$Xr_{i+1} = Xr_i + V_a.\Delta T_{i+1}.\cos(Ca) - 2.Rt.\sin(\tfrac{1}{2}\cdot\Delta\phi_i).\cos(Co_i + \tfrac{1}{2}\cdot\Delta\phi_i)$$

$$Yr_{i+1} = Yr_i + V_a.\Delta T_{i+1}.\sin(Ca) - 2.Rt.\sin(\tfrac{1}{2}\cdot\Delta\phi_i).\sin(Co_i + \tfrac{1}{2}\cdot\Delta\phi_i) \qquad 6.3$$

Where

$$\Delta\phi_i = 57{,}3.\ \Delta T_{i+1}.\ V_o/Rt.$$

The equations describe the curved motion of the centre of the critical circle, point (A) in the figure. The outer boundary of the motion, curve (LFG) in the figure, is formed by the moving normal attached to circle centre when it moves in accordance to equations (6.3). These equations can be modified by addition of two simple terms so as to express the curve in consideration:-

$$\Delta X_r = Ra.\cos(Cr_i \pm 90)$$

$$\Delta Y_r = Ra.\sin(Cr_i \pm 90) \qquad 6.4$$

Where

Cr_i is the direction of the segment between two sucessive points, and can be determined by the following relation

$$Cr_i = tg^{-1} (Yr_i - Yr_{i-1})/(Xr_i - Xr_{i-1})) \qquad 6.5$$

In order to determine the different sectors of the risk zone, it is necessary to take the points of intersection of the outer boundary curves with the limiting straight lines of the risk zone points (G) and (C) as shown in the figure. Determination of these points is possible by using the HESSIAN normal form of the equation of a line which is the border-line of the risk zone in this respect, the two end points of the relative motion segment on the curve in consideration (Xr_i, Yr_i) and (Xr_{i-1}, Yr_{i-1}) can be checked for the intersection in accordance to the following relation:-

$$D_{i+1} = X_{i+1}.\cos (C_r + U) + Y_{i+1}.\sin (C_r + U) - Ra \qquad 6.6$$

Where $U = 270^o$ for right-hand side relative motion.

$U = 90^o$ for left-hand side relative motion.

For points (X_i, Y_i) in the negative half-plane (risk zone) distance (D_{i+1}) from the limiting line (LL') is negative, and for points in the positive half-plane (non-risk zone) the distance is positive. During the process of extending the curve part by part the corresponding decreasing distances $(D_i)$are continuously computed and checked for the following conditions:-

(a) If $D_i = 0$ then the last point lies on the line

(b) If $D_i > 0$ and $D_{i-1} < D_i$, then the curve is moving away from the line, and hence the turning should be stopped at this stage and the new course is taken as (Co_{i-1}). The curve is to be extended from this point on the basis of completely linear motion.

(c) If $D_i < 0$, then the turning should be terminated at this stage, and the intersection point (Xr_n , Yr_n) can be determined in accordance to the ratio of the last two distances, as:

$$X_n = X_{n-1} - Sr.\ Cos\ (Cr_n)$$

$$Y_n = Y_{n-1} + Sr.\ sin\ (Cr_n)$$

Where

$$Sr = [\,|D_{i-1}|.\sqrt{(Xr_{i+1} - Xr_{i-1})^2 + (Yr_{i+1} - Yr_{i-1})^2}\,] / (\,|D_{i-1}| + |D_{i+1}|\,) \qquad 6.7$$

The heading of the observing ship at the intersection point represents the new course to be followed until clear disengagement is achieved. This new course (Co_n) and its assocated time interval of terminating turning action is simply given by the following expressions.

$$Co_n = Co + \sum_{i=1}^{i=n} \tfrac{1}{2} \cdot \Delta\phi \,, \qquad T_n = \sum_{i=1}^{i=n} \Delta T \qquad 6.8$$

In considering the effect of turning on ship's tangential velocity, the empirical exponential formula advised by the (D.O.T) Specification for A Marine Navigational Equipment Simulator (1980) has been used to define the tangential velocity (V_o) at (T_n) times after alteration.

$$V_n = (V_{max} - V_{min}).e^{-T_n/K} + V_{min}$$

Where;

V_n = Speed at T_n minutes after alteration has been made

K = A constant which simulates the chosen type of ship and is either a fixed value between (2) and (25) or continuously variable over this range.

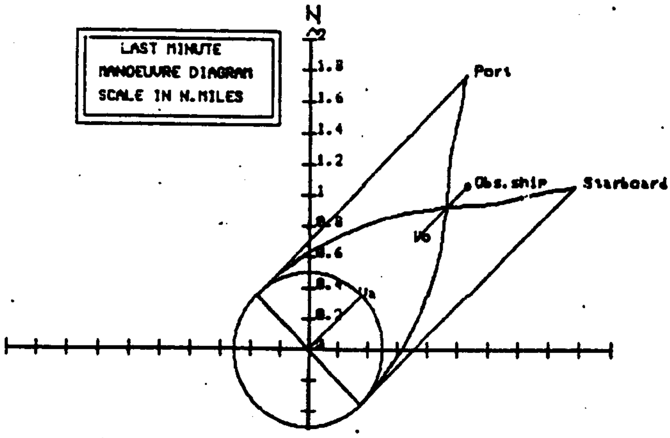
6.7 CASE DUTY

The solution described in section (6.6) has been tested on the computer. The calculated extreme risk patterns which define the range of a last minute action are graphically presented on the horizontal plane and can be seen in figures (6-10) (6.11) and (6.12). To demonstrate the performance and effectiveness of the solution the following examples are taken for one_ship manoeuvres. The observing ship (O) is chosen as a VLCC of the following characteristics:-

- Speed of approach	= 17.7 knots
- Time interval of approach	= 40 sec
- Tangential speed of steady turn	= 7.5 knots
- Turning index	= 3.5
- Length of observing ship	= 300 meters
- Turning radius of 20^{o} rudder	= 600 meters
- Maximum turn rate	= 722^{o}/sec.
- Steady turn rate	= 0.368^{o}/sec.

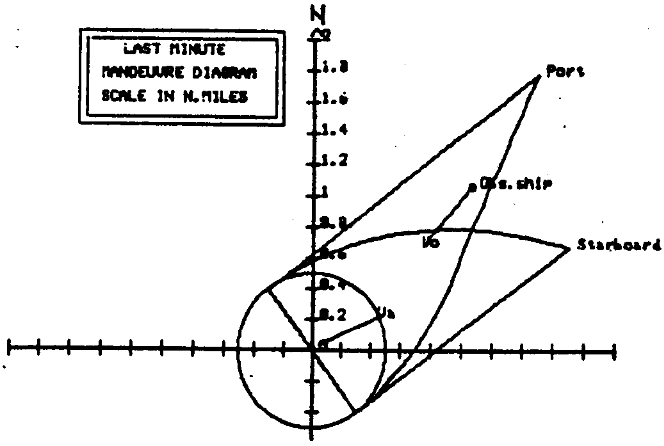
A critical circle of radius (0.5) miles is taken as a parameter. The significance of the generated curves and the associated risk sectors has already been explained in section (6.5)

Regarding figure (6-10a) for a typical one-ship manoeuvre with the initial parameters (V_o / V_a = 1) and (H = 180), it is a very simple situation and optimal last-minute action fully conforms to intuition. Ship (A) is on reciprocal course and ship (O) has taken three different locations with respect to the risk sectors. The range at which a last minute action is to be taken varies between (2.15) and (1.2) miles, while considering the two possible options of starboard turn or port turn. If the observing ship is initially located in the sector (S) or (P), there is only one option permitted which is either a hard-starboard turn or a hard-port turn respectively. If the turning is initiated at a longer distance than specified by the generated curves, then a (CPA) greater than the specified critical circle can be attained.



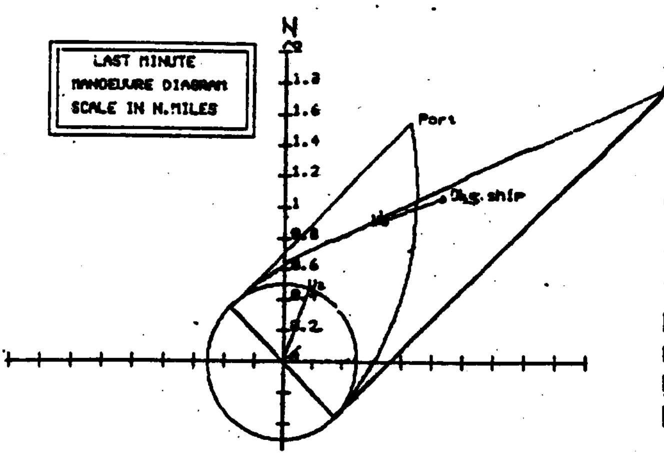
a

Targt course = 45
Target speed = 14.7
Obs.ship course = 225
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 16
New course(port)= 73



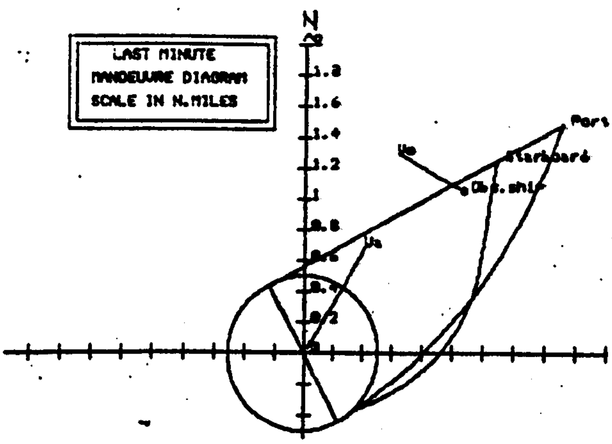
b

Targt course = 65
Target speed = 14.7
Obs.ship course = 220
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 18
New course(port)= 97



c

Targt course = 20
Target speed = 14.7
Obs.ship course = 250
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)=-110
New course(port)= 112



d

Targt course = 30
Target speed = 24
Obs.ship course = 300
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 20
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 122
New course(port)= 163

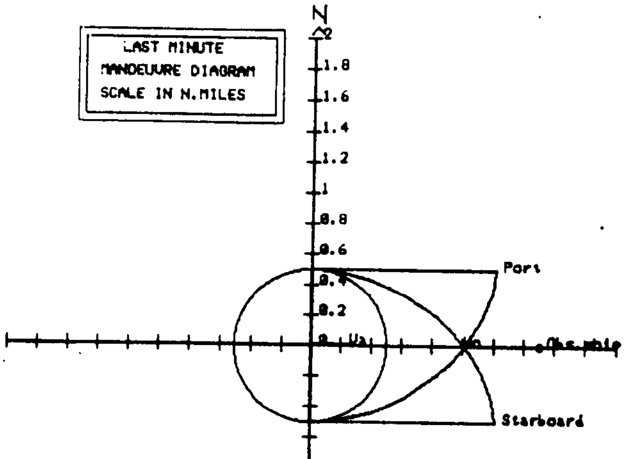
Figure (6-10). Extreme Risk Zone defined relative to target ship.

The amount of course alteration required is about (163^{o}), followed by a straight course of the last heading. When a complete disengagement is attained then course resuming is permitted. It is obvious from the figure that in case of green aspect and starboard turn compliance, the action should be taken at an early stage.

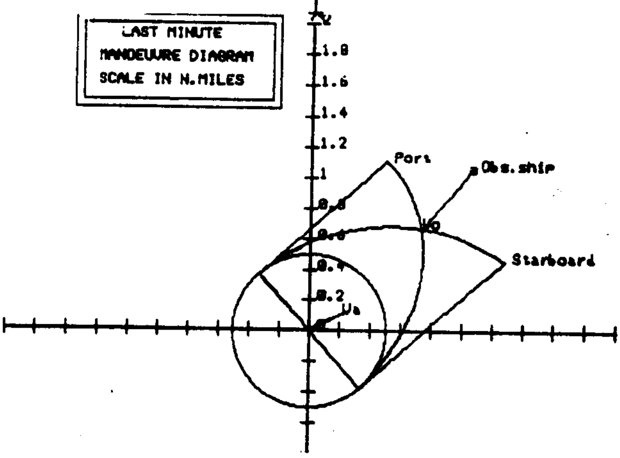
Figure (6-10c) shows a crossing situation with initial parameters ($V_o/V_a = 1$) and ($H = 155^{o}$). The investigation of the numerical data attached to the diagram reveals that the amount of the starboard course alteration is smaller than that of the port turn. It is also noticed that the maximum reaction range of the last minute action is reduced from (2.15) to (1.75).

The diagram shown in figures (6-11, a,b,c and d) are computed for a faster observing ship ($V_o/V_a = 1.9$). On comparing with the previous diagrams the influence of a faster manoeuvring ship on the shape and size of risk sectors and collision domain is evident. However, the picture changes when moving to figure (6-12d) which shows the effect of speed ratio smaller than 1. The initial parameters are (V_o / $V_a = 0.7$) and ($H = 45^{o}$). Regarding this case the two generated curves show similar concavities and hence two risk sectors only are formed, the starboard turn sector (S) and the starboard or port sector. On performing any type of turn the action will be terminated by passing astern of the other ship. The reaction range of starboard turn manoeuvre is nearly half that of port turn manoeuvre. If the observing ship is initially found at a range of two miles or less, then the only successful action is turning away from other ship so as to avoid collision, but if port turn is to be considered the action should be taken at early stage.

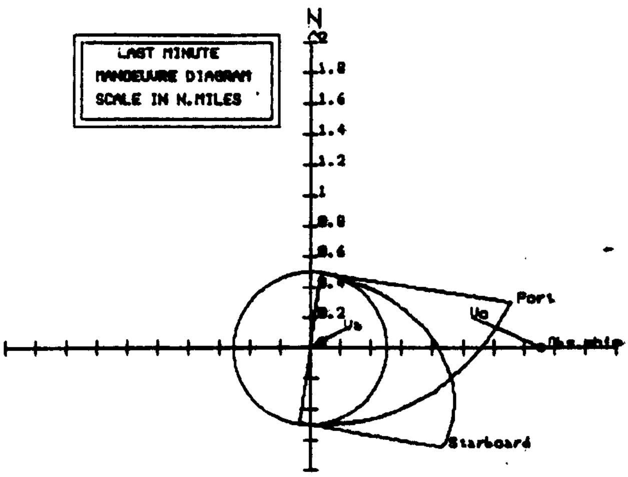
Figure (6-12a) decribes a situation in which the observing ship is overtaking other ship with initial parameters ($V_o/V_a = 2.9$) and ($H= 05^{o}$). This situation is characterized by relatively smaller sectors of risk and a collision domain which approximates the turning circle. It is also shown that the maximum reaction range of last minute manoeuvre is reduced to (0.7) miles with a relatively smaller



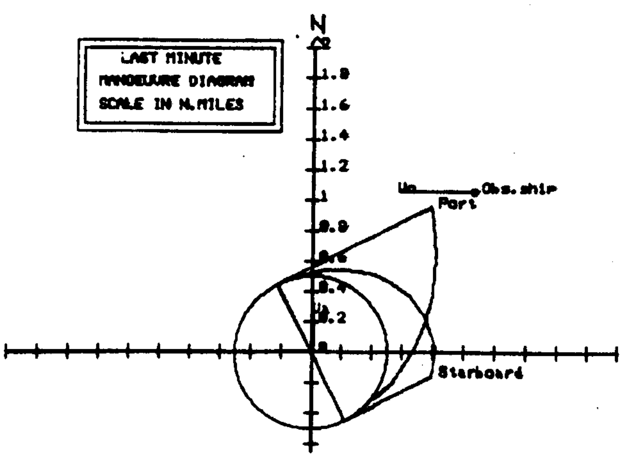
Targt course = 90
Target speed = 7.5
Obs.ship course = 270
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 47
New course(port)= 132



Targt course = 65
Target speed = 7.5
Obs.ship course = 220
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)=-10
New course(port)= 82

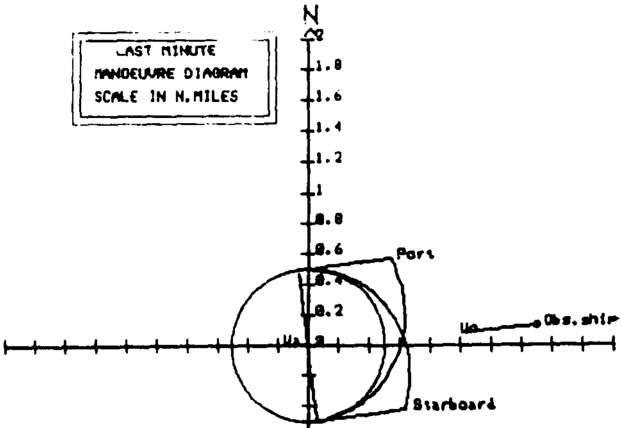


Targt course = 65
Target speed = 7.5
Obs.ship course = 295
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 65
New course(port)= 172

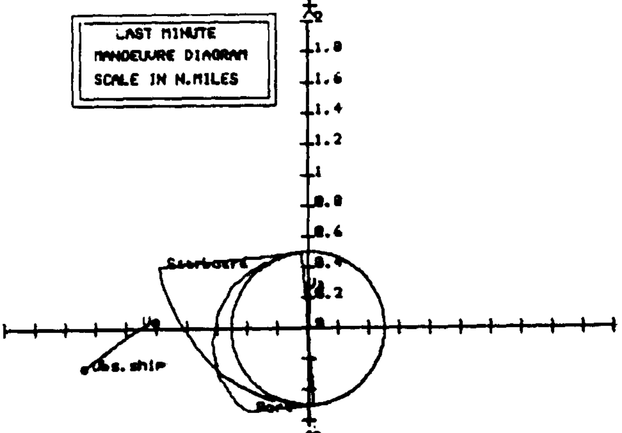


Targt course = 0
Target speed = 7.5
Obs.ship course = 270
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 9
New course(port)= 154

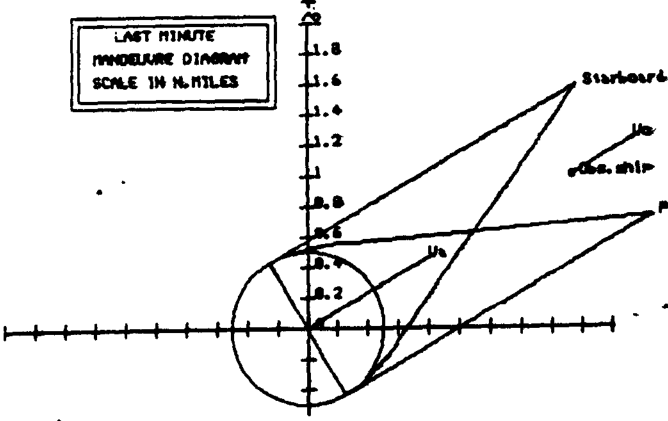
Figure.(6-11).Extreme Risk Zone defined relative to target ship.



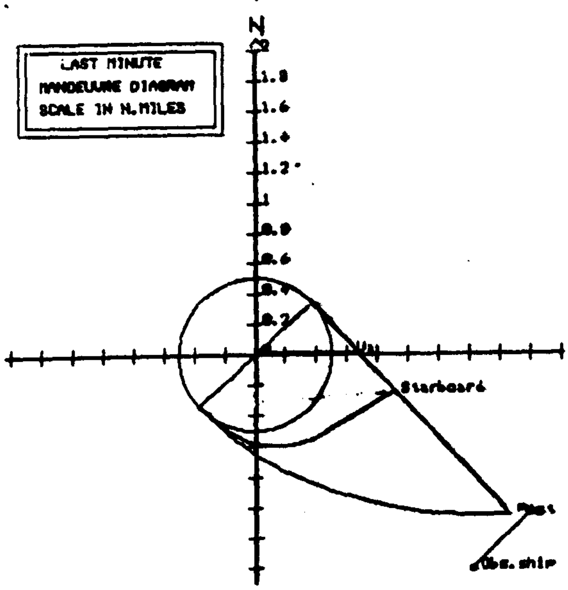
Targt course = 270
Target speed = 5
Obs.ship course = 265
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 10
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)=-3
New course(port)= 172



Targt course = 0
Target speed = 7.5
Obs.ship course = 54
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)=-199
New course(port)= 330



Targt course = 60
Target speed = 28
Obs.ship course = 60
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 15
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)= 118
New course(port)= 1



Targt course = 90
Target speed = 20
Obs.ship course = 45
Obs.ship speed = 14.7
O.ship low speed= 7.5
Time increment S= 8
Turning index = 3.5
Tactical radius = 0.324
Encounter size = 0.5
COMPUTED DATA :
New course(star)=-185
New course(port)= 263

Figure (6-12). Extreme Risk Zone defined relative to target ship.

angle of course alteration. The situation does not change much when moving to figure (6-12b) with the parameters (V_o / V_a= 1.9) and (H = 45^o) except that the reaction range of starboard turning is relatively larger than of port turn, which will be reversed in case of starboard aspect.

The last minute actions of a typical overtaken situation are illustrated in figure (6-12c), Both observing ship and other ship are on parallel tracks. The situation is characterized by longer risk sector and collision domain. It is noticed that the amount of course alteration is minimal, yet the straight course part is longer.

6.8 GENERAL FEATURES

The definition of short-range collision domain and the associated risk sectors together with the previous case study reflect the following special features of the solution:-

(a) It is a solution based on one-ship manoeuvres . However the solution can easily be modified to perform coordinated action. on the same basis.

(b) The basic input data consists of elements of ships' motions and the manoeuvrability characteristics.

(c) The solution is independent of ships' relative positions (range and bearing).

(d) The solution provides a presentation of risk pattern in a real dimensional profile and hence a direct assessment and quick perception of the situation is possible. Consequently a correct decision is obvious.

(e) Relating the risk sectors to target's centre point provides the possibility of multiple-ship application in a relative motion mode presentation.

(f) The solution is optimised in the sense that minimum turning angle and minimum relative path are considered.

(g) The measures taken by the manoeuvring ship generally turn out to be first a specific approach path, second a circular path with a decreasing angular turn rate, third a continued circular path with steady turn rate, finally followed by an occasional straight course which may be initiated at earlier stages in accordance with situation parameters.

(h) In case of ($V_o/V_a < 1$) and ($H = \cos^{-1} (V_o/V_a)$), then both a starboard or port turn will cause the relative motion to rotate in the same direction.

6.8 SUMMARY

The intent of this unit was to introduce a conceptual frame work for risk criteria which was set up as a reasonable starting point to define risk of collision, to assess the transition distribution of risk in a two ship encounter and to compare the outcome of the alternatives of risk control. The investigation shows that the assessment of risk has to include not only consideration of sight line behaviour but also pays attention to many other factors influencing risk of collision. Preliminary analyses based on the introduced iso-risk hypothesis suggest that the existing rules which regulate the control actions may not be far from being effective if and only if a complete definition of risk is reached which should acquire features that permit it to describe, act upon and respond to aspects of situation variety. Formulation of these features in terms of quantitative risk criteria has been achieved. It is now possible to describe accurately from a risk point of view the geometry of a binary encounter action zone, related ample time and appropriate action, which are all important for practical decision making. The effort to clarify these points is rewarded by achievement.

In addition to the above fulfilment, there is one more problem the stand on ship must encounter when the give-way ship fails to take the appropriate control action and a solution is required. To resolve this problem an extreme risk pattern is introduced which defines the last-minute control action. The representation of such an area on the sea surface provides a numerical index of the relationship between the critical range and encounter parameters together with ship's manoeuvrability.

The basic finding in this unit, as expected, is that, in spite of the feasibility of defining any stage of the encounter, there is no one simple rule to be followed at any stage. The practical determination of the necessary quantitative information of all stages of the encounter provided in an uninterrupted flow of relevant data cannot be attained by raw radar data or even by pre-computed diagrams. The fully automated computer based radar system with the exact software is the only effective solution possible.

Appendix_A.1

Collision Function

The forms of the Collision Function

In the first unit, the collision situation in a binary encounter has been defined in terms of three quantities, Q, H, and E in accordance with the following expression :-

$$\tan(Q) = (E.SIN(H))/(1-E.COS(H)) \quad .... \text{A.1.1}$$

squaring

$$\tan^2(Q) = (E.SIN(H))^2/(1-E.COS(H))^2$$

$$(1+\tan^2(Q)) = (1-2.E.COS(H)+E^2.COS^2(H)+E^2.SIN^2(H)) \; /$$

$$(-;2.E.COS(H)-E.^2COS^2(H))$$

$$SEC^2(Q) = (1-2.E.COS(H)+E^2) \; / \; (1-2.E.COS(H)-E^2.COS^2(H))$$

$$COS^2(Q) = (1-E.COS(H))^2 \; / \; (1-2.E.COS(H)+E^2)$$

$$COS\,(Q) = (1-E.COS(H)) \; / \; (1-2.E.COS(H)+E^2)^{\frac{1}{2}} \quad .... \text{A.1.2}$$

multiplying by tan(Q), then

$$SIN\,(Q) = ((E.SIN(H)) \; /(1-E.COS(H))).(1-E.COS(H)) \; /$$

$$(1-2.E.COS(H))+E^2)^{\frac{1}{2}})$$

$$= (E.SIN(H)) \; / \; (1-2.E.COS(H)+E^2)^{\frac{1}{2}} \quad .... \text{A.1.3}$$

Appendix_A.2

Derivatives of C.F

Derivatives of the Collision Function

On differentiating the collision function with respect to (H);

$$\tan(Q) = E.SIN(H)/(1-E.COS(H))$$

$$SEC^2(Q).(dH/dH)=((1-E.COS(H)).E.COS(H)-E^2.SIN^2(H))/(1-E.COS(H))$$

$$1+\tan^2(Q).(dQ/dH)=(E.COS(H)-E^2)/(1-E.COS(H))^2$$

$$dQ/dH=(1/(1+\tan^2(Q)).(E.COS(H)-E^2)/(1-E.COS(H))^2$$

Substituting from the general formula;

$$dQ/dH=(1-E.COS(H))^2.(E.COS(H)-E^2)/(((1-E.COS(H))^2+ E^2.SIN^2(H)).(1-E.COS(H))^2)$$

$$dQ/dH=(E.COS(H)-E^2)/(1-2.E.COS(H)+E^2.COS^2(H)+E^2.SIN^2(H))$$

$$dQ/dH=(E.COS(H)-E^2)/(1-2.E.COS(H)+E^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad A.2.1$$

On differentiating the general formula with respect to (E);

$$SEC^2(Q).(dQ/dE)=(SIN(H).(1-E.COS(H))+E.SIN(H).COS(H))/(1-E.COS(H))^2$$

$$(1+\tan^2(Q)).(dQ/dH)=(SIN(H))/(1-E.COS(H))^2$$

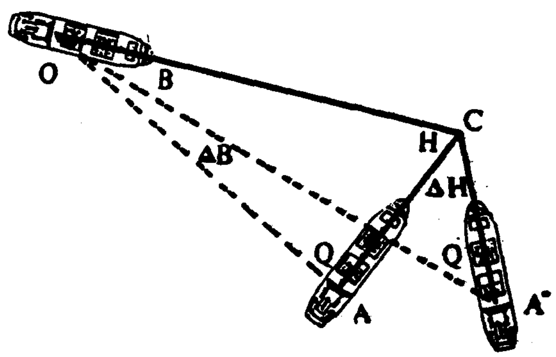
$$dQ/dE=(SIN(H))/((1+\tan^2(Q)).(1-E.COS(H))^2)$$

$$=((1-E.COS(H))^2.SIN(H))/((1-E.COS(H))^2+ E^2.SIN^2(H)).(1-E.COS(H))^2)$$

$$= SIN(H)/(1-2.E.COS(H)+E^2.COS^2(H)+E^2.SIN^2(H))$$

$$= SIN(H)/(1-2.E.COS(H)+E^2) \quad \ldots\ldots\ldots\ldots\ldots \quad A.2.2$$

The incremental relation between heading and bearing changes in a collision situation can be established as follows ;



From relation (A.1.2) then

$$(\Delta H+\Delta Q)/\Delta H=(E.COS(H)-E^2+1-2.E.COS(H)+E^2)/(1-2.E.COS(H)+E^2)$$

$$=(1-E.COS(H))/(1-2.E.COS(H)+E^2) \quad \ldots\ldots\ldots\ldots \quad A.2.3$$

From the triangle OCA in the above figure

$$B=180-(H+Q)$$

By differentiation

$$\Delta B=-(\Delta H+\Delta Q) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad A.2.4$$

From (A.1.4) and (A.1.5) then

$$\Delta H=-\Delta B.(1-2.E.COS(H)+E^2)3(1-E.COS(H)) \quad \ldots\ldots\ldots\ldots \quad A.2.5$$

Appendix_A.3

Extreme values of C.F

Sufficient Condition for a Minimum of Collision Function

The graphical inspection of the Collision Function when it is mapped on the (H/E)-plane shows the possible existence of a minima . The corresponding point of extremum can be determined by manipulating the partial derivative of the Collision Function :

Solving the equation $f'(H) = 0$

then

$$E.(COS(H)-E)/(1-2.E.COS(H)+E^2) = 0$$

when

$$H = COS^{-1}(E) \quad \text{and} \quad (1 \geq E \geq 0)$$

then, at the point (COS(H)=E) ; the Collision function is reduced to;

$$\begin{aligned} SIN(Q) &= E.SIN(H)/(1-2.E.COS(H)+E^2)^{\frac{1}{2}} \\ &= E.(1-COS(H)^2)^{\frac{1}{2}}/(1-2.E.COS(H)+E^2) \\ &= E.(1-E^2)^{\frac{1}{2}}/(1-E^2)^{\frac{1}{2}} \\ SIN(Q) &= E \end{aligned}$$

Hence the coordinates of the point of extremum on the (E/H)-plane are given as ;

$$E = SIN(Q) \quad , \quad H = COS^{-1}(SIN(Q)) \quad \ldots\ldots\ldots\ldots\ldots \quad A.3.1$$

Provided that $(Q \leq \frac{1}{2}\Pi)$ and $(1 \geq E \geq 0)$

Appendix_A.4

Area under the C.F

<u>Determination of the area bounded by the iso-aspect contour and the (H) axis in range of headings defined by speed ratio.</u>

The figure shows the area ($H_1C_1C_2H_2$) sought, a representative strip (R-S-T-U) and its approximating rectangular (R-V-W-U). For this rectangular, the base is (H), and the altitude is (E_p) .

$$E_p = \tan(Q)/(\mathrm{SIN}(H_p)+\tan(Q).\mathrm{COS}(H_p))$$

and the area as (E_p. H), then

$$N = \mathrm{Lim} \sum_{H_k=H_1}^{H_k=H_2} E_p.\ H$$

$$= \int_{H_1}^{H_2} (\tan(Q).dH)/(\mathrm{SIN}(H_p)+\tan(Q).\mathrm{COS}(H_p))$$

$$=\tan(Q)\int_{H_1}^{H_2} dH/(\mathrm{SIN}(H)+\tan(Q).\mathrm{COS}(H))$$

Referring to the special integral form :

$$\int dX/(P.\mathrm{SIN}(a.X)+q.\mathrm{COS}(a.X)=$$

$$(1/(a,(p^2+q^2)^{\frac{1}{2}}.\mathrm{Ln}(\tan\tfrac{1}{2}.(a.X+\tan^{-1}(q/p)))$$

then

$$N= \left((\tan(Q)/(1+\tan^2(Q))^{\frac{1}{2}}).(\mathrm{Ln}(\tan(\tfrac{1}{2}.(H+\tan^{-1}(\tan(Q))))))\right)_{H_1}^{H_2}$$

$$= \mathrm{SIN}(Q).(\mathrm{Ln}(\tan\tfrac{1}{2}(H_2+Q))/(\tan\tfrac{1}{2}(H_1+Q))))$$

$$= \mathrm{SIN}(Q).\mathrm{Ln}((\tan\tfrac{1}{2}(H_2+Q))/(\tan\tfrac{1}{2}(H_1+Q))) \quad \ldots\ldots\ldots\ldots \text{ A.4.1}$$

But in case of $(E>1)$, we have $(H_1=0)$, then

$$N = \mathrm{SIN}(Q).\mathrm{Ln}((\tan\tfrac{1}{2}(H+Q))/(\tan\tfrac{1}{2}Q))$$

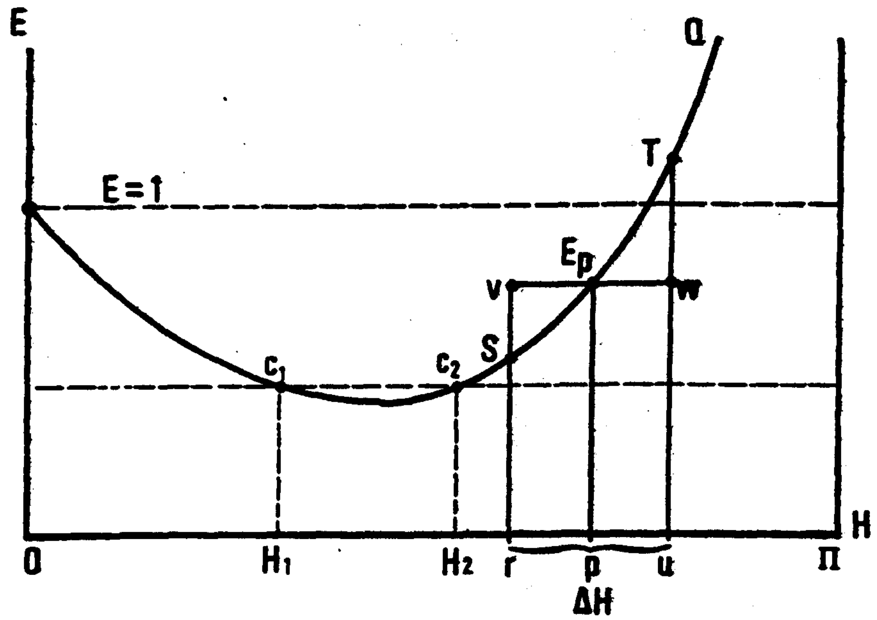
and in case of $(E=1)$ we have $(H_1=0)$ and $(H_2= 180-2.Q)$, then

$$N = \mathrm{SIN}(Q).\mathrm{Ln}(\tan\tfrac{1}{2}(180-2.Q+Q)/\tan(\tfrac{1}{2}Q))$$

$$= \mathrm{SIN}(Q)\ \mathrm{Ln}(\mathrm{cotan}(\tfrac{1}{2}Q)/\tan(\tfrac{1}{2}Q))$$

$$= \mathrm{SIN}(Q).\mathrm{Ln}(\mathrm{cotan}^2(\tfrac{1}{2}Q))$$

$$= 2.\mathrm{SIN}(Q).\mathrm{Ln}(\mathrm{cotan}^2(\tfrac{1}{2}Q))$$



Appendix_A.5

Lagrange interpolation

LAGRANGE interpolation method

The process of interpolation may be regarded as a special case of the general process of curve fitting. A function, y=f(x), is known to us only to the extent that we have some set of values (X_1,Y_1), (X_2,Y_2), (X_3,Y_3),... and it is required to infer reasonable values of (Y) for values of (X) intermediate between the given ones. The major difference between this problem and the general one of curve-fitting is that there is no interest in having the functional expression f(X) to have any particular form, or even having the same form for the entire range of values for (X). For interpolation, then, the polynomials for f(X) are nearly always used, since these are so easy to calculate. Different polynomials may be used for different values of (X). For example the first-degree polynomial "straight line" passing through (X_1,Y_1) and (X_2,Y_2) may be used to interpolate between the first two points, the first- degree polynomial passing through (X_2,Y_2) and (X_3,Y_3) to interpolate between the second and third points, and so forth. This is the ordinary method of linear interpolation commonly used in obtaining the values between those tabulated in trigonometric or other tables. If more accurate interpolation were desired, it would be necessary to fit second-, or third-, or higher-, degree polynomials through the nearby points in order to obtain values intermediate between two points.

The **Lagrange**'s interpolation formula is the equation for an nth-degree polynomial through (n+1) points (X_o,Y_o), (X_1,Y_1), ... (X_n,Y_n) which are not necessarily equally spaced. It is :-

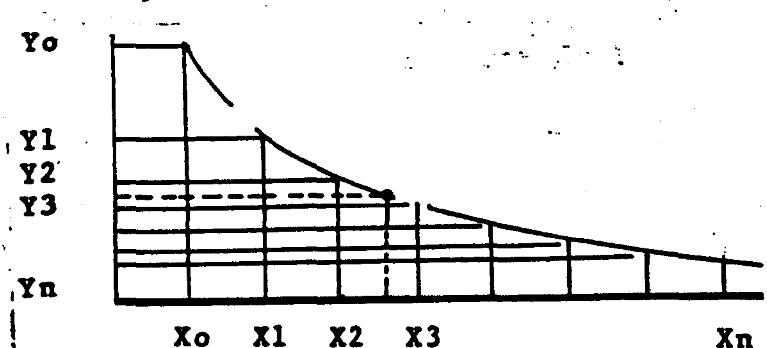
$$Y = \frac{(X-X_1)\ (X-X_2)\ldots\ldots.(X-X_n)}{(X_o-X_1)(X_o-X_2)\ldots..(X_o-X_n)}\ Y_o$$

$$+ \frac{(X-X_o)(X-X_2)\ldots\ldots(X-X_n)}{(X_1-X_o)(X_1-X_2)\ldots.(X_1-X_n)}\ Y_1$$

$$+ \ldots\ldots\ldots\ldots.$$

$$+ \frac{(X-X_o)(X-X_1)\ldots\ldots(X-X_{n-1})}{(X_n-X_o)(X_n-X_1)\ \ldots(X_n-X_{n-1})}\ Y_n$$

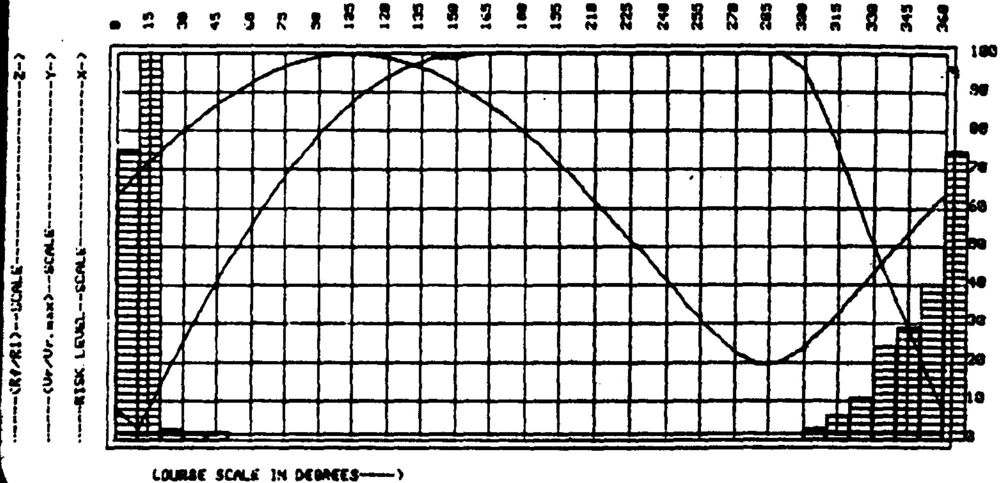
It can be seen that this formula involves large numbers of multiplications and hence becomes quite slow if (*n*) is large. Since the other classical interpolation formulas do not work for unequal intervals, this one is used.



0	1	2	3	4	5	6				n-1	n
Xo	X1	X2	X3	X4							Xn
Yo	Y1	Y2	Y3	Y4							Yn

Appendix_B.1

VARIATION OF MAXIMUM
RISK WITH COURSE.



INPUT:
R1= 9
R2= 7
B1= 45
B2= 43
T1= 0
T2= 6
Td= 0
Ra= 0.5
Vo= 16
Co= 20
OUTPUT:
Ca= 283
Va= 10
=IP 1

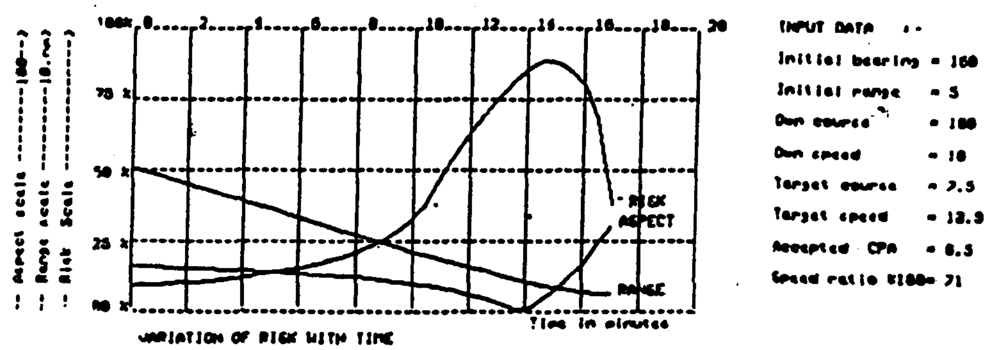
```
   10 PRINT "Variation of max.risk with
course."
   11 CLEAR :LPRINT "S";0
   12 LPRINT CHR$(28);CHR$(37):LPRINT "F
";5
   14 LPRINT "M5,0":LPRINT "O";5;",";0
   15 LPRINT "A";-1;",";1;",";51;",";-10
9
   16 LPRINT "A";0;",";0;",";50;",";-108
   17 FOR I=1 TO 23
   18 Y=-I*4.5:X=50
   24 LPRINT "D";0;",";Y;",";X;",";Y
   25 NEXT I
   26 FOR I=1 TO 10
   28 X=I*5
   30 LPRINT "D";X;",";0;",";X;",";-108
   32 NEXT I
   33 LPRINT "S";0
   34 FOR I=0 TO 24
   36 Y=-I*4.5:X=52
   38 LPRINT "M";X;",";Y
   40 LPRINT "P";15*I
   42 NEXT I
   44 LPRINT "Q1"
   46 FOR I=0 TO 10
   48 X=I*5:Y=-110
   50 LPRINT "M";X;",";Y
   52 LPRINT "P";I*10
   54 NEXT I
   56 LPRINT "M-5,-5":LPRINT "S";0
   58 LPRINT "P";"COURSE SCALE IN DEGREE
S---->"
```

```
   60 LPRINT "M0,4":LPRINT "Q0":LPRINT "
S0"
   62 LPRINT "P";"----RISK LEVEL--SCALE-
-----------------X->"
   64 LPRINT "M0,8"
   66 LPRINT "P";"-----(Vr/Vr.max)--SCAL
E----------------Y->"
   68 LPRINT "M0,12"
   70 LPRINT "P";"-----(Rf/Ri)--SCALE---
-----------------Z->"
   72 LPRINT "M63,0":LPRINT "S";1:LPRINT
 "Q1"
   74 LPRINT "P";"VARIATION OF MAXIMUM"
   76 LPRINT "M59,0"
   78 LPRINT "P";"RISK WITH COURSE."
   98 LPRINT "M0,0":CLEAR
  100 DIM R(40),B(3),T(3),Q(3),P(40),X(4
0),Y(40)
  110 INPUT "R(1)=";R(1)
  112 INPUT "R(2)=";R(2)
  114 INPUT "B(1)=";B(1)
  116 INPUT "B(2)=";B(2)
  118 INPUT "T(1)=";T(1)
  120 INPUT "T(2)=";T(2)
  122 INPUT "Vo=";VO
  124 INPUT "Co=";CO
  126 INPUT "Ra=";RA
  128 INPUT "Td=";T(0)
  130 KO=CO : T(3)=T(2)+T(0)
  132 X(1)=R(1)*COSB(1)
  134 Y(1)=R(1)*SINB(1)
  136 X(2)=R(2)*COSB(2)
  138 Y(2)=R(2)*SINB(2)
  140 DX=X(2)-X(1)
  142 DY=Y(2)-Y(1)
  144 DT=T(2)-T(1)
  146 X=DX:Y=DY
  148 GOSUB PROG 9
  150 VR=60*Z/DT:CR=U
  152 X=VO*COSCO+VR*COSCR
  154 Y=VO*SINCO+VR*SINCR
  156 GOSUB PROG 9
  158 VA=Z:CA=U
  160 J=B(2):C=CA
  162 GOSUB PROG 7
  164 Q=J:R=R(2):E=VO/VA
  166 GOSUB PROG 8
  168 PP=100*P
  170 X(3)=(VR*T(0)/60)*COSCR
  172 Y(3)=(VR*T(0)/60)*SINCR
  174 X(3)=X(3)+X(2)
  176 Y(3)=Y(3)+Y(2)
  177 PRINT "X3=";X(3),"Y3=";Y(3)
```

```
178 X=X(3):Y=Y(3)
180 GOSUB PROG 9
182 R(3)=Z:B(3)=U
184 J=B(3):C=CA
186 GOSUB PROG 7
188 Q(3)=J:R=R(3):Q=Q(3)
200 GOSUB PROG 8
202 IP=100*P:BEEP 1:BEEP 1
204 LPRINT "D";IP/2;",";0;",";IP/2;","
;-108
206 FOR I=4 TO 40
208 CO=(I-4)*10:BEEP 1:BEEP 0:BEEP 0:B
EEP 1
210 XR=VA*COSCA-VO*COSCO:PRINT "CO=";C
O
212 YR=VA*SINCA-VO*SINCO
214 RR=X(3)*XR+Y(3)*YR:PRINT "RR=";SGN
RR
216 X=XR:Y=YR
218 GOSUB PROG 9
220 VR=Z:CR=U
222 X(I)=(VR/(VO+VA))*100
224 IF RR<0 THEN 232
226 IF RR>0 THEN 230
228 RM=R(3):PM=IP:GOTO 274
230 RM=R(3):PM=0:GOTO 274
232 J=B(3):C=CR:GOSUB PROG 7
234 QR=J:RM=ABS(R(3)*SINQR)
236 SM=ABS(R(3)*COSQR)
238 X(0)=X(3):Y(0)=Y(3)
240 FOR D=1 TO 10
242 Y(0)=Y(0)+(SM/10)*SINCR:X(0)=X(0)+
(SM/10)*COSCR
244 X=X(0):Y=Y(0)
246.GOSUB PROG 9
248 J=U:R=Z:C=CA
250 GOSUB PROG 7
252 Q=J
254 GOSUB PROG 8
256 P(D)=P
258 NEXT D
260 D=1
262 IF P(D+1)<P(D) THEN 272
264 D=D+1
266 IF D>10 THEN 270
268 GOTO 262
270 D=10
272 PM=INT(100*P(D))
274 R(I)=INT(100*(RM/R(3))):CO=-(I-4)*
3
276 IF PM<2 THEN 282
```

```
278 LPRINT "A";0;",";CO;",";PM/2;",";C
O-3
280 LPRINT "G";2;",";PM/2;",";-3;",";1
282 NEXT I
284 FOR I=4 TO 40
286 IF I>39 THEN 294
288 Y(I)=-(I-4)*3
290 Y(I+1)=-(I-3)*3
292 LPRINT "D";R(I)/2;",";Y(I);",";R(I
+1)/2;",";Y(I+1)
294 NEXT I
296 FOR I=4 TO 40
298 IF I>39 THEN 302
300 LPRINT "D";X(I)/2;",";Y(I);",";X(I
+1)/2;",";Y(I+1)
302 NEXT I
304 LPRINT "M55,-116"
306 LPRINT "Q1"
308 LPRINT "P";"INPUT:"
310 LPRINT "M50,-116"
312 LPRINT "P";"R1=";R(1)
314 LPRINT "M46,-116"
316 LPRINT "P";"R2=";R(2)
318 LPRINT "M42,-116"
320 LPRINT "P";"B1=";B(1)
322 LPRINT "M38,-116"
324 LPRINT "P";"B2=";B(2)
326 LPRINT "M34,-116"
328 LPRINT "P";"T1=";T(1)
330 LPRINT "M30,-116"
332 LPRINT "P";"T2=";T(2)
334 LPRINT "M26,-116"
336 LPRINT "P";"Td=";T(3)
338 LPRINT "M22,-116"
340 LPRINT "P";"Ra=";RA
342 LPRINT "M18,-116"
344 LPRINT "P";"Vo=";VO
346 LPRINT "M14,-116"
348 LPRINT "P";"Co=";KO
350 LPRINT "M10,-116"
352 LPRINT "P";"OUTPUT:"
354 LPRINT "M6,-116"
356 LPRINT "P";"Ca=";INTCA
358 LPRINT "M2,-116"
360 LPRINT "P";"Va=";INTVA
362 LPRINT "M-2,-116"
364 LPRINT "P";"IP=";INTIP
366 END
```

Appendix_B.2




```
10 REM RISK/TIME
11 CLEAR :LPRINT "S";0
12 LPRINT CHR$(28);CHR$(37)
14 LPRINT "M5,0":LPRINT "O";5;",";0
15 LPRINT "A";-1;",";1;",";81;",";-10
1
16 LPRINT "A";0;",";0;",";80;",";-100
17 FOR I=1 TO 9
18 Y=-I*10
19 IF ABS(Y)>10 THEN IF ABS(Y)<50 THE
N 21
20 X=80:GOTO 23
21 X=60
23 LPRINT "L";3
24 LPRINT "D";0;",";Y;",";X;",";Y
25 NEXT I
26 FOR I=1 TO 3
28 X=I*20
30 LPRINT "D";X;",";0;",";X;",";-100
32 NEXT I
33 LPRINT "L";0
34 LPRINT "O";5;",";0
36 INPUT "No.of Ponts";M
40 LPRINT "X1,4,20"
42 LPRINT "X2,5,20"
44 LPRINT "Q1":LPRINT "M5,10"
45 LPRINT "S";0
46 FOR I=0 TO 4
48 X=I*20
50 LPRINT "M";X;",";10
52 LPRINT "P";I*25
54 NEXT I
```

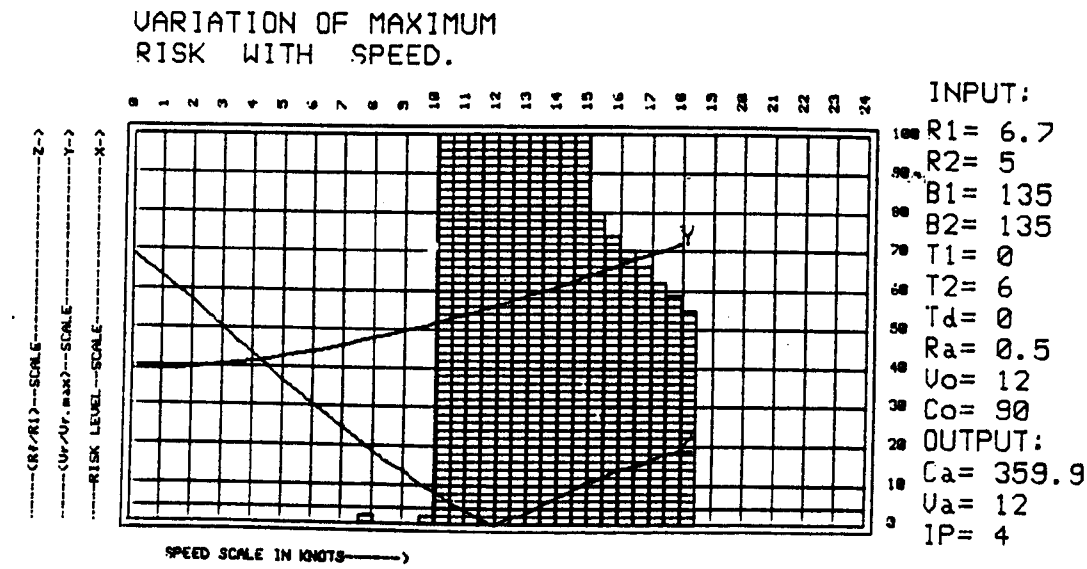
```
 55 LPRINT "S";1
 56 LPRINT "M-5,-5"
 58 LPRINT "P";"TIME SCALE IN MINUTES-
---->"
 60 LPRINT "Q0":LPRINT "M5,5"
 62 LPRINT "P";"RISK    SCALE"
 64 LPRINT "M0,0":LPRINT "Q";0;",";0
 66 LPRINT "M5,15":LPRINT "Q1"
 67 LPRINT "S";0
 68 FOR I=0 TO 4
 70 X=I*20+5
 72 LPRINT "M";X;",";20
 74 LPRINT "P";I*2.5
 76 NEXT I
 77 LPRINT "S";1
 78 LPRINT "M5,15":LPRINT "Q0"
 80 LPRINT "P";"  RANGE   SCALE"
 82 LPRINT "M5,30":LPRINT "Q1"
 83 LPRINT "S";0
 84 FOR I=0 TO 4
 86 X=I*20+5
 88 LPRINT "M";X;",";30
 90 LPRINT "P";I*45
 92 NEXT I
 93 LPRINT "S";1
 94 LPRINT "M5,25":LPRINT "Q0"
 96 LPRINT "P";"  ASPECT  SCALE"
 97 LPRINT "S";0
 98 DIM X(M),Y(M),Q(M),R(M),P(M),T(M)
100 INPUT "I.Bs =";BI
102 INPUT "I.Rs =";RI
104 INPUT "T.SP =";VA
106 INPUT "T.CO.=";CA
108 INPUT "A.CPA=";RA
110 INPUT "O.SP.=";VO
115 SC=5
120 INPUT "O.CO.=";CO
122 INPUT "M.T.S=";MT
130 X1=RI*COS(BI)
132 Y1=RI*SIN(BI)
150 YR=VA*SIN(CA)-VO*SIN(CO)
152 XR=VA*COS(CA)-VO*COS(CO)
160 X=XR:Y=YR
162 GOSUB PROG 9
170 VR=Z:CR=U
180 RR=(X1*XR+Y1*YR)/SQR(X1*X1+Y1*Y1)
200 J=BI:C=CR
202 GOSUB PROG 7
204 QR=J
210 J=BI:C=CA
212 GOSUB PROG 7
220 Q=J:E=VO/VA
```

```
222 R=RI:Q(0)=Q:R(0)=RI:TR=0:Y(0)=0
224 GOSUB PROG 8
230 P(0)=P*100
232 IF Q(0)>180 THEN 236
234 GOTO 240
236 Q(0)=Q-360
240 IF RR<0 THEN 250
242 IF RR=0 THEN 246
244 TM=SC:SM=VR*TM/60:GOTO 300
246 SM=0:TM=0:GOTO 300
250 RM=ABS(RI*SIN(QR))
260 SM=ABS(RI*COS(QR))
262 TM=60*SM/VR
270 IF TM<=SC THEN 300
280 SC=SC+5
290 IF SC<MT THEN 270
300 LPRINT "Q";1:LPRINT "S";0:LPRINT "
J";0
302 FOR I=0 TO 10
303 Y=(-10*I)
304 LPRINT "M";87;",";Y
305 X=SC*I/10
306 LPRINT "P";X
308 NEXT I
309 LPRINT "M5,0"
310 FOR I=1 TO M
320 X1=X1+(SM/M)*COS(CR)
330 Y1=Y1+(SM/M)*SIN(CR)
340 TR=TR+TM/M
342 T(I)=TR
350 X=X1:Y=Y1:GOSUB PROG 9
360 BG=U:R(I)=Z
362 J=BG:C=CA:GOSUB PROG 7
370 Q(I)=J:Q=Q(I):R=R(I)
372 GOSUB PROG 8
374 P(I)=P*100
376 Y(I)=TR*100/SC
378 IF Q(I)>180 THEN 382
380 GOTO 384
382 Q(I)=Q(I)-360
384 PRINT "I=";I
385 PRINT "R=";R(I)
386 PRINT "Q=";Q(I)
390 NEXT I
395 LPRINT "M5,0"
400 FOR I=1 TO M
402 X(I-1)=5+P(I-1)*0.8:X(I)=5+P(I)*0.
8
410 LPRINT "D";X(I-1);",";-Y(I-1);",";
X(I);",";-Y(I)
420 NEXT I
```

```
  421 LPRINT "P";" RISK":LPRINT "M5,0"
  424 FOR I=1 TO M
  425 X(I-1)=5+R(I-1)*8:X(I)=5+R(I)*8
  426 LPRINT "D";X(I-1);",";-Y(I-1);",";
X(I);",";-Y(I)
  428 NEXT I
  429 LPRINT "P";" RANGE":LPRINT "M5,0"
  431 FOR I=1 TO M
  432 X(I-1)=5+ABS(Q(I-1)*4/9):X(I)=5+AB
S(Q(I)*4/9)
  434 LPRINT "D";X(I-1);",";-Y(I-1);",";
X(I);",";-Y(I)
  436 NEXT I
  438 LPRINT "P";"ASPECT"
  440 FOR I=0 TO M
  442 IF P(I+1)<=P(I) THEN 446
  444 NEXT I
  446 N=I
  450 LPRINT "Q1":LPRINT "M80,-110"
  452 LPRINT "P";"INPUT DATA:"
  454 LPRINT "M75,-110"
  456 LPRINT "P";"I.Bearing =";BI
  458 LPRINT "M72,-110"
  460 LPRINT "P";"I.Range   =";RI
  462 LPRINT "M69,-110"
  464 LPRINT "P";"Own Course=";CO
  466 LPRINT "M66,-110"
  468 LPRINT "P";"Own Speed =";VO
  470 LPRINT "M63,-110"
  472 LPRINT "P";"Tg. course=";CA
  474 LPRINT "M60,-110"
  476 LPRINT "P";"Tg. speed =";VA
  478 LPRINT "M57,-110"
  480 LPRINT "P";"Safe  CPA   =";RA
  482 LPRINT "M44,-110"
  484 LPRINT "P";"COMPUTED DATA:"
  486 LPRINT "M40,-110"
  488 LPRINT "P";"Rel.speed =";VR
  502 LPRINT "M36,-110"
  504 LPRINT "P";"In.risk  =";P(0)
  506 LPRINT "M33,-110"
  508 LPRINT "P";"In.Aspect=";Q(0)
  510 LPRINT "M30,-110"
  512 LPRINT "P";"Fin.risk  =";P(M)
  514 LPRINT "M27,-110"
  516 LPRINT "P";"Fin.range =";R(M)
  518 LPRINT "M24,-110"
  520 LPRINT "P";"Fin.aspect=";Q(M)
  522 LPRINT "M21,-110"
  524 LPRINT "P";"Fin.time  =";TM
  526 LPRINT "M18,-110"
  528 LPRINT "P";"Max.risk  =";P(N)
```

```
530 LPRINT "M15,-110"
532 LPRINT "P";"M.R.aspect=";Q(N)
534 LPRINT "M11,-110"
536 LPRINT "P";"M.R.time  =";T(N)
538 LPRINT "M78,-15"
540 LPRINT "P";"VARIATION OF RISK WITH
"
542 LPRINT "M75,-15"
544 LPRINT "P";"TIME IN A BINARY ENCO-
"
546 LPRINT "M72,-15"
548 LPRINT "P";"UNTER AS PREDICTED BY"
550 LPRINT "M69,-15"
552 LPRINT "P";"THE OBSERVING VESSEL."
560
```

Appendix_B.3




```
 1 PRINT "Variation of max.risk with
Speed."
 2 CLEAR :LPRINT "S";0:INPUT "Vo.max=
";VM
 4 FOR I=1 TO 3
 6 IF VM>I*12 THEN 10
 8 SC=I/2:GOTO 12
10 NEXT I
12 LPRINT CHR$(28);CHR$(37):LPRINT "F
";5
14 LPRINT "M5,0":LPRINT "O";5;",";0
15 LPRINT "A";-1;",";1;",";51;",";-97
16 LPRINT "A";0;",";0;",";50;",";-96
17 FOR I=1 TO 23
18 Y=-I*4:X=50
24 LPRINT "D";0;",";Y;",";X;",";Y
25 NEXT I
26 FOR I=1 TO 10
28 X=I*5
30 LPRINT "D";X;",";0;",";X;",";-96
32 NEXT I
33 LPRINT "S";0
34 FOR I=0 TO 24
36 Y=-I*4:X=52
38 LPRINT "M";X;",";Y
40 LPRINT "P";I*SC
42 NEXT I
44 LPRINT "Q1"
46 FOR I=0 TO 10
48 X=I*5:Y=-98
50 LPRINT "M";X;",";Y
```

```
   52 LPRINT "P";I*10
   54 NEXT I
   56 LPRINT "M-5,-5":LPRINT "S";0
   58 LPRINT "P";"SPEED SCALE IN KNOTS--
---->"
   60 LPRINT "M0,4":LPRINT "Q0":LPRINT "
S0"
   62 LPRINT "P";"----RISK LEVEL--SCALE-
-----------------X->"
   64 LPRINT "M0,8"
   66 LPRINT "P";"-----(Vr/Vr.max)--SCAL
E----------------Y->"
   68 LPRINT "M0,12"
   70 LPRINT "P";"-----(Rf/Ri)--SCALE---
------------------Z->"
   72 LPRINT "M63,0":LPRINT "S";1:LPRINT
 "Q1"
   74 LPRINT "P";"VARIATION OF MAXIMUM"
   76 LPRINT "M59,0"
   78 LPRINT "P";"RISK  WITH  SPEED."
   98 LPRINT "M0,0"
  100 DIM R(52),B(3),T(3),Q(3),P(52),X(5
2),Y(52)
  110 INPUT "R(1)=";R(1)
  112 INPUT "R(2)=";R(2)
  114 INPUT "B(1)=";B(1)
  116 INPUT "B(2)=";B(2)
  118 INPUT "T(1)=";T(1)
  120 INPUT "T(2)=";T(2)
  122 INPUT "Vo=";VO:INPUT "VOmax=";VM
  124 INPUT "Co=";CO
  126 INPUT "Ra=";RA
  128 INPUT "Td=";T(0)
  130 KO=VO :T(3)=T(2)+ T(0)
  132 X(1)=R(1)*COSB(1)
  134 Y(1)=R(1)*SINB(1)
  136 X(2)=R(2)*COSB(2)
  138 Y(2)=R(2)*SINB(2)
  140 DX=X(2)-X(1)
  142 DY=Y(2)-Y(1)
  144 DT=T(2)-T(1)
  146 X=DX:Y=DY
  148 GOSUB PROG 9
  150 VR=60*Z/DT:CR=U
  152 X=VO*COSCO+VR*COSCR
  154 Y=VO*SINCO+VR*SINCR
  156 GOSUB PROG 9
  158 VA=Z:CA=U
  160 J=B(2):C=CA
  162 GOSUB PROG 7
  164 Q=J:R=R(2):E=VO/VA
```

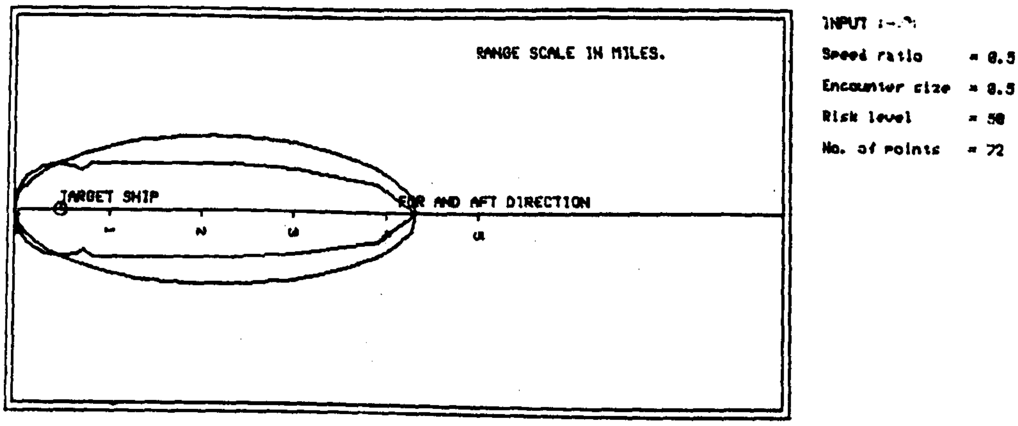
```
166 GOSUB PROG 8
168 PP=100*P
170 X(3)=(UR*T(0)/60)*COSCR
172 Y(3)=(UR*T(0)/60)*SINCR
174 X(3)=X(3)+X(2)
176 Y(3)=Y(3)+Y(2)
177 PRINT "X3=";X(3),"Y3=";Y(3)
178 X=X(3):Y=Y(3)
180 GOSUB PROG 9
182 R(3)=Z:B(3)=U
184 J=B(3):C=CA
186 GOSUB PROG 7
188 Q(3)=J:R=R(3):Q=Q(3)
200 GOSUB PROG 8
202 IP=100*P:BEEP 1:BEEP 1
204 LPRINT "D";IP/2;",";0;",";IP/2;","
;-96
206 FOR I=4 TO 52
208 VO=(I-4)/2:E=VO/VA:PRINT "VO=";VO:
BEEP 1:BEEP 0:BEEP 1:BEEP 1
210 IF VO>VM THEN 282
212 XR=VA*COSCA-VO*COSCO:YR=VA*SINCA-V
O*SINCO
214 RR=X(3)*XR+Y(3)*YR:PRINT "RR=";SGN
RR
216 X=XR:Y=YR
218 GOSUB PROG 9
220 VR=Z:CR=U
222 X(I)=(VR/(VM+VA))*100
224 IF RR<0 THEN 232
226 IF RR>0 THEN 230
228 RM=R(3):PM=IP:GOTO 274
230 RM=R(3):PM=0:GOTO 274
232 J=B(3):C=CR:GOSUB PROG 7
234 QR=J:RM=ABS(R(3)*SINQR)
236 SM=ABS(R(3)*COSQR)
238 X(0)=X(3):Y(0)=Y(3)
240 FOR D=1 TO 10
242 Y(0)=Y(0)+(SM/10)*SINCR:X(0)=X(0)+
(SM/10)*COSCR
244 X=X(0):Y=Y(0)
246 GOSUB PROG 9
248 J=U:R=Z:C=CA
250 GOSUB PROG 7
252 Q=J
254 GOSUB PROG 8
256 P(D)=P
258 NEXT D
260 D=1
262 IF P(D+1)<P(D) THEN 272
264 D=D+1
```

```
  266 IF D>10 THEN 270
  268 GOTO 262
  270 D=10
  272 PM=INT(100*P(D)):PRINT "Pmax=";PM
  274 R(I)=INT(100*(RM/R(3)))
  276 IF PM<2 THEN 282
  278 LPRINT "A";0;",";-(I-4)*2;",";PM/2
;",";-(I-3)*2
  280 LPRINT "G";2;",";PM/2;",";-2;",";1
  282 NEXT I
  284 FOR I=4 TO 52
  286 IF (I-4)/2>=UM THEN 292
  288 Y(I)=-(I-4)*2:Y(I+1)=-(I-3)*2
  290 LPRINT "D";R(I)/2;",";Y(I);",";R(I
+1)/2;",";Y(I+1)
  292 NEXT I
  294 LPRINT "P";"Z"
  296 FOR I=4 TO 52
  298 IF (I-4)/2>=UM THEN 302
  300 LPRINT "D";X(I)/2;",";Y(I);",";X(I
+1)/2;",";Y(I+1)
  302 NEXT I
  304 LPRINT "P";"Y"
  306 LPRINT "Q1":LPRINT "M55,-104"
  308 LPRINT "P";"INPUT:"
  310 LPRINT "M50,-104"
  312 LPRINT "P";"R1=";R(1)
  314 LPRINT "M46,-104"
  316 LPRINT "P";"R2=";R(2)
  318 LPRINT "M42,-104"
  320 LPRINT "P";"B1=";B(1)
  322 LPRINT "M38,-104"
  324 LPRINT "P";"B2=";B(2)
  326 LPRINT "M34,-104"
  328 LPRINT "P";"T1=";T(1)
  330 LPRINT "M30,-104"
  332 LPRINT "P";"T2=";T(2)
  334 LPRINT "M26,-104"
  336 LPRINT "P";"Td=";T(3)
  338 LPRINT "M22,-104"
  340 LPRINT "P";"Ra=";RA
  342 LPRINT "M18,-104"
  344 LPRINT "P";"Uo=";KO
  346 LPRINT "M14,-104"
  348 LPRINT "P";"Co=";CO
  350 LPRINT "M10,-104"
  352 LPRINT "P";"OUTPUT:"
  354 LPRINT "M6,-104"
  356 LPRINT "P";"Ca=";(INT(CA*10))/10
  358 LPRINT "M2,-104"
  360 LPRINT "P";"Ua=";(INT(UA*10))/10
```

```
362 LPRINT "M-2,-104"
364 LPRINT "P";"IP=";(INT(IP*10))/10
366 END
```

Appendix_B.4

EQUI-RISK CONTOUR
& EQUI-RISK ELLIPSE

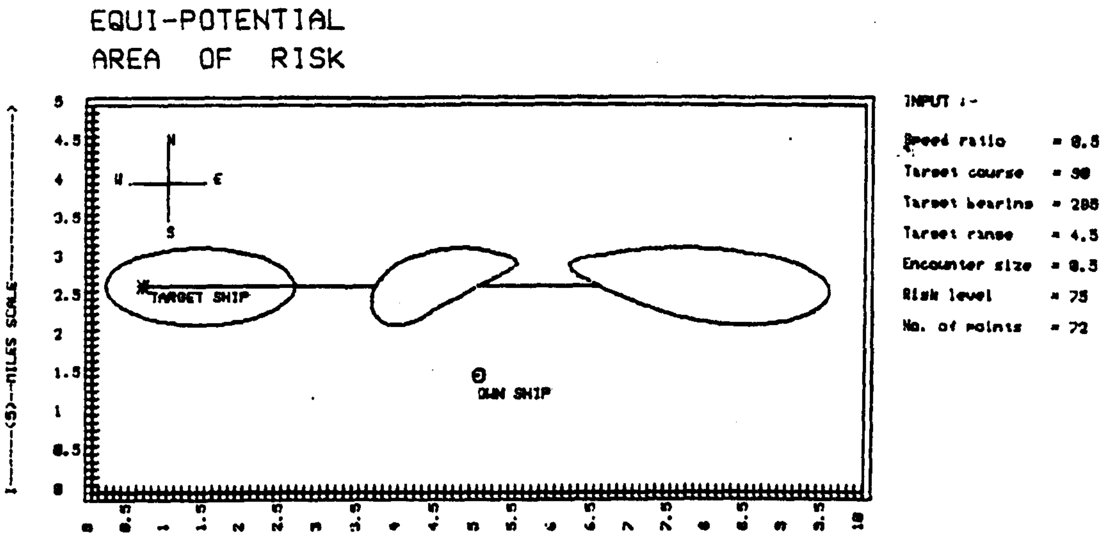



```
10 PRINT "EQUI-RISK CONTOUR"
12 CLEAR :LPRINT CHR$(28);CHR$(37):LP
RINT "S0"
14 INPUT "RISK LEVEL=";PO
16 LPRINT "M5,0"
18 LPRINT "A";5;",";0;",";55;",";-100
20 LPRINT "D";30;",";0;",";30;",";-10
0
22 PRINT "Input no. of point=2,if EQ
conoure is not required"
24 LPRINT "M31,-50":LPRINT "Q1"
26 LPRINT "P";"FOR AND AFT DIRECTION"
28 LPRINT "M50,-60"
30 LPRINT "P";"RANGE SCALE IN MILES."
32 LPRINT "M65,0":LPRINT "S1"
34 LPRINT "P";"EQUI-RISK CONTOUR"
36 LPRINT "M60,0"
38 LPRINT "P";"& EQUI-RISK ELLIPSE"
90 LPRINT "A";4;",";1;",";56;",";-101
100 INPUT "Speed ratio=";E
102 INPUT "Degree of polynomial=";NN
104 INPUT "Interpolation No.=";M
106 INPUT "No.of points=";WW
108 INPUT "Encounter size=";RA
112 DIM X(2*36+1),Y(2*36+1),B(M+N)
114 DIM R(M+N),P(M+N),D(2*WW+1)
116 LL=1:R=0
120 FOR ZZ=0 TO WW/2
130 Q=ZZ*360/WW
320 IF E>1 THEN 370
330 IF Q<=ACSE THEN 380
340 QQ=90+ASNE
350 IF Q<QQ THEN 370
```

```
360 D(ZZ)=RA:GOTO 590
370 RQ=RA:GOTO 390
380 RQ=RA/SIN(Q+ASNE)
390 FOR I=1 TO M
391 BEEP 1:BEEP 0
400 R=RQ+LL*(I-1)
401 PRINT "R=";R,"LL=";LL
410 R(M-I+1)=R
420 GOSUB PROG 8
430 P(M-I+1)=P*10C
440 NEXT I
450 IF PO<P(M-2) THEN 500
460 IF PO>P(M-2) THEN 480
470 D(ZZ)=R(M-2):GOTO 590
480 IF LL>.05 THEN 570
490 D(ZZ)=RA:GOTO 590
500 IF PO<P(M-4) THEN 550
510 IF PO>P(M-4) THEN 530
520 D(ZZ)=R(M-4):GOTO 590
530 GOSUB PROG 6
540 D(ZZ)=R:GOTO 590
550 IF LL>=25 THEN 580
570 LL=.7*LL:GOTO 390
580 D(ZZ)=D(ZZ-1)
590 NEXT ZZ
592 FOR I=0 TO WW/2
594 D(WW-I)=D(I)
596 NEXT I
600 I=0
602 I=I+4
604 IF (D(0)+D(WW/2))>I THEN 602
606 MM=96/I:PRINT "MM=";MM
610 FOR I=0 TO WW
612 Q=I*360/WW
614 X(I)=MM*(D(I)*COS(90+Q))+30
616 Y(I)=MM*(D(WW/2)+D(I)*SIN(90+Q))
618 X(I)=INT(10*X(I))/10
620 Y(I)=INT(10*Y(I))/10
622 NEXT I
624 FOR I=1 TO WW
626 LPRINT "D";X(I-1);",";-Y(I-1);",";
X(I);",";-Y(I)
628 NEXT I
646 REM"DETERMINATION OF ISORISK ELLIP
SE.
648 PRINT "ELLIPSE"
650 IF PO>50 THEN 654
652 KE=1.6:GOTO 656
654 KE=1.2
656 AE=(D(0)+D(WW/2))/2
658 BE=KE*D(WW/2)
```

```
  660 IF BE<=AE THEN 664
  662 BE=D(WW/2)
  664 CC=AE-D(WW/2)
  666 EE=(SQR(AE*AE-BE*BE))/AE
  668 FOR I=0 TO 72
  670 Q=I*360/72:PRINT "Q=";Q:PRINT "i="
;I
  672 RE=BE/SQR(1-(EE*COS(Q))^2)
  674 X(I)=MM*RE*COS(Q+90)+30
  676 Y(I)=MM*(RE*SIN(Q+90)+AE)
  678 X(I)=INT(10*X(I))/10:Y(I)=INT(10*Y
(I))/10
  682 NEXT I
  684 FOR I=1 TO 72
  686 LPRINT "D";X(I-1);",";-Y(I-1);",";
X(I);",";-Y(I):PRINT "i=";I
  688 NEXT I
  690 LPRINT "S0":LPRINT "Q2"
  692 FOR I=0 TO 20
  694 X=30:Y=I*MM
  696 LPRINT "M";X;",";-Y
  698 LPRINT "P";"-";I
  700 IF Y>=2*MM*AE THEN 704
  702 NEXT I
  704 LPRINT "O30,0"
  708 LPRINT "M";0;",";-MM*D(WW/2)
  710 LPRINT "S1":LPRINT "Q1"
  712 LPRINT "N";6:LPRINT "M";1;",";-MM*
D(WW/2)
  714 LPRINT "S0":LPRINT "P";"TARGET SHI
P"
  716 LPRINT "M";24;",";-105
  718 LPRINT "P";"INPUT :-"
  720 LPRINT "M";20;",";-105
  722 LPRINT "P";"Speed ratio      =";E
  724 LPRINT "M";16;",";-105
  726 LPRINT "P";"Encounter size   =";RA
  728 LPRINT "M";12;",";-105
  730 LPRINT "P";"Risk level       =";PO
  732 LPRINT "M";8;",";-105
  734 LPRINT "P";"No. of points    =";WW
  736 END
```

Appendix_B.5




```
   10 PRINT "EPAR"
   12 CLEAR :LPRINT CHR$(28);CHR$(37)
   14 LPRINT "F5"
   18 LPRINT "S0":LPRINT "Q1"
   20 LPRINT "M5,0":LPRINT "Q";5;",";0
   32 LPRINT "A";0;",";0;",";50;",";-100
   34 LPRINT "D";40;",";-5;",";40;",";-1
5
   36 LPRINT "P";" E"
   38 LPRINT "D";35;",";-10;",";45;",";-
10
   40 LPRINT "P";"N"
   42 LPRINT "M33,-10":LPRINT "P";"S"
   44 LPRINT "M40,-3":LPRINT "P";"W"
   46 LPRINT "A";-1;",";1;",";51;",";-10
1
   48 LPRINT "M0,10":LPRINT "Q0"
   50 LPRINT "P";"I------(5)--MILES SCAL
E------------------>"
   52 LPRINT "Q1":LPRINT "S1":LPRINT "M6
0,0"
   54 LPRINT "P";"EQUI-POTENTIAL"
   56 LPRINT "M55,0"
   68 LPRINT "P";"AREA  OF  RISK"
   70 LPRINT "S0"
   72 FOR I=0 TO 10
   74 X=I*5:Y=6
   76 LPRINT "M";X;",";Y
   78 LPRINT "P";X/10
   80 NEXT I
   82 LPRINT "M0,0"
```

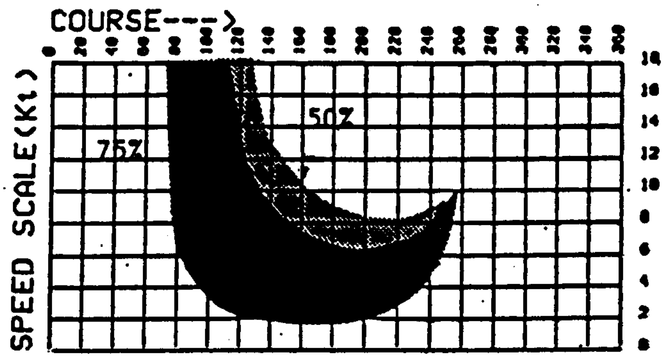
```
84 LPRINT "X1,1,50"
86 LPRINT "X2,1,100":LPRINT "Q0"
88 FOR I=0 TO 20
90 X=-6:Y=-I*5:Z=-Y/10
92 LPRINT "M";X;",";Y:LPRINT "P";Z
94 NEXT I
96 INPUT "Int.pola.No 8=";M
97 INPUT "EPAR points =";WW
98 PRINT "Input coordinates of ship(o
)"
100 DIM X(2*WW+1),Y(2*WW+1),H(2*WW+1)
102 DIM R(M+N),P(M+N),B(M+N),D(4),Q(2)
,K(2)
104 INPUT "North ordinat=";D(3):D(3)=1
0*D(3)+5
106 INPUT "East ordinate=";D(4):D(4)=-
10*D(4)
108 LPRINT "O";D(3);",";D(4)
110 LPRINT "M0,0":LPRINT "S1"
112 LPRINT "N";6:LPRINT "M";-3;",";0:L
PRINT "S0"
114 LPRINT "Q1":LPRINT "P";"OWN SHIP"
210 INPUT "Ship(o)speed=";VO
212 INPUT "Bearing of A =";BG
214 INPUT "Range of   A =";O
216 INPUT "Course of  A =";CA
219 INPUT "Target speed =";VA
220 INPUT "Deg.of polynomial 2=";NN
226 INPUT "Risk level =";PO
228 INPUT "Encounter size =";RA
235 IM=1000
236 IF PO>=50 THEN 240
238 KE=1.6:GOTO 245
240 KE=1
245 XA=O*COSBG
250 YA=O*SINBG
252 LPRINT "M";(XA*10);",";-(YA*10)
254 LPRINT "S1":LPRINT "N";3:LPRINT "S
0"
256 LPRINT "M";(XA*10-2);",";-(YA*10):
LPRINT "P";" TARGET SHIP"
258 LL=1:R=O
260 E=VO/VA
288 REM DETERMINATION OF Ro AND R180 O
F SPECIFIC RISK."
290 FOR ZZ=0 TO 1
300 Q=ZZ*180
320 IF E>1 THEN 370
330 IF Q<=ACSE THEN 380
340 QQ=90+ASNE
350 IF Q<QQ THEN 370
360 D(ZZ)=RA:GOTO 590
```

```
370 RQ=RA:GOTO 390
380 RQ=RA/SIN(Q+ASNE)
390 FOR I=1 TO M
391 BEEP 1:BEEP 0
400 R=RQ+LL*(I-1)
401 PRINT "R=";R,"LL=";LL
410 R(M-I+1)=R
420 GOSUB PROG 8
430 P(M-I+1)=P*100
440 NEXT I
450 IF PO<P(M-2) THEN 500
460 IF PO>P(M-2) THEN 480
470 D(ZZ)=R(M-2):GOTO 590
480 IF LL>.05 THEN 570
490 D(ZZ)=RA:GOTO 590
500 IF PO<P(M-4) THEN 550
510 IF PO>P(M-4) THEN 530
520 D(ZZ)=R(M-4):GOTO 590
530 GOSUB PROG 6
540 D(ZZ)=R:GOTO 590
550 IF LL>=25 THEN 580
570 LL=.7*LL:GOTO 390
580 D(ZZ)=D(ZZ-1)
590 NEXT ZZ
594 Q=0:GOSUB PROG 8
596 PRINT "RISK=";100*P
646 REM"DETERMINATION OF ISORISK ELLIP
SE.
647 PRINT "ELLIPSE"
648 AE=(D(0)+D(1))/2
650 BE=KE*D(1)
652 CE=AE-D(1)
654 XE=XA+CE*COSCA
656 YE=YA+CE*SINCA
658 EE=(SQR(AE*AE-BE*BE))/AE
660 XO=-YE*SINCA-XE*COSCA
662 YO=-YE*COSCA+XE*SINCA
664 X=XO:Y=YO:GOSUB PROG 9
666 RO=Z:Q(0)=U
668 RE=BE/SQR(1-(EE*COSQ(0))^2)
858 REM" DETERMINATION OF POINTS ON TH
E EP ELLIPSE.
860 FOR I=0 TO WW
862 Q=I*360/WW
864 RE=BE/SQR(1-(EE*COSQ)^2)
866 X(I)=XE+RE*COS(CA+Q)
868 Y(I)=YE+RE*SIN(CA+Q)
870 NEXT I
872 FOR I=1 TO WW
874 LPRINT "D";10*X(I-1);",";-10*Y(I-1
);",";10*X(I);",";-10*Y(I)
876 NEXT I
```

```
 880 REM" DETERMINATON OF THE INTERCEPT
ABLE FUTURE POINTS."
 890 FOR I=0 TO WW
 900 Q=I*360/WW:PRINT "POINT=";I
 940 X=X(I):Y=Y(I)
 950 XO=-Y*SINCA-X*COSCA:YO=-Y*COSCA+X*
SINCA
 960 A=1-E^2
 970 B=-2*XO
 980 C=XO*XO+YO*YO
 990 GOSUB PROG 5
 998 IF E>1 THEN 1014
1000 IF E<1 THEN 1006
1002 IF B>=0 THEN 1024
1004 IF B<0 THEN 1014
1006 IF B*B<4*A*C THEN 1020
1008 IF B*B=4*A*C THEN 1014
1010 X(I+WW+1)=10*(X+X2*COSCA)
1012 Y(I+WW+1)=-10*(Y+X2*SINCA)
1014 X(I)=10*(X+X1*COSCA)
1016 Y(I)=-10*(Y+X1*SINCA)
1018 GOTO 1024
1020 X(I+WW+1)=0:Y(I+WW+1)=0
1022 Y(I)=0:Y(I)=0
1024 NEXT I
1028 REM" PLOTTING OF EPAR."
1030 LPRINT "M";X(0);",";Y(0)
1032 FOR I=1 TO WW
1034 IF X(I-1)=0 THEN 1038
1036 LPRINT "D";X(I-1);",";Y(I-1);",";X
(I);",";Y(I)
1038 NEXT I
1040 LPRINT "D";10*XA;",";-10*YA;",";X(
WW/2);",";Y(WW/2)
1042 IF E>=1 THEN 1056
1044 FOR I=WW+2 TO 2*WW
1046 IF X(I-1)=0 THEN 1050
1048 LPRINT "D";X(I-1);",";Y(I-1);",";X
(I);",";Y(I)
1050 NEXT I
1052 IF X(WW+1)=0 THEN 1058
1054 LPRINT "D";X(0);",";Y(0);",";X(WW+
1);",";Y(WW+1)
1058 LPRINT "M";(55-D(3));",";(-105-D(4
))
1060 LPRINT "P";"INPUT :-"
1062 LPRINT "M";(50-D(3));",";(-105-D(4
))
1064 LPRINT "P";"Speed ratio     =";E
1066 LPRINT "M";(46-D(3));",";(-105-D(4
))
1068 LPRINT "P";"Target course   =";CA
```

```
1070 LPRINT "M";(42-D(3));",";(-105-D(4
))
1072 LPRINT "P";"Target bearing  =";BG
1074 LPRINT "M";(38-D(3));",";(-105-D(4
))
1076 LPRINT "P";"Target range    =";O
1078 LPRINT "M";(34-D(3));",";(-105-D(4
))
1080 LPRINT "P";"Encounter size  =";RA
1082 LPRINT "M";(30-D(3)):",";(-105-D(4
))
1084 LPRINT "P";"Risk level      =";PO
1086 LPRINT "M";(26-D(3));",";(-105-D(4
))
1088 LPRINT "P";"No. of points   =";WW
1090 END
```

Appendix_B.6



INPUT OF EPR MATRIX

Target Speed	=	10
Target Course	=	255
Target Bearing	=	90
Target Range	=	6
Encounter Size	=	1
Risk Level	=	75
Polynomial Deg	=	2
Polynomial Coe	=	8
No. of points	=	36

```
10 PRINT "RISK MATRIX BASED ON VOmax=
16 KtS"
11 CLEAR :LPRINT "S";0
12 LPRINT CHR$(28);CHR$(37)
14 LPRINT "M5,0":LPRINT "O";5;",";0
15 LPRINT "A";-1;",";1;",";81;",";-12
1
16 LPRINT "A";0;",";0;",";80;",";-120
17 FOR I=1 TO 23
18 Y=-I*5:X=80
24 LPRINT "D";0;",";Y;",";X;",";Y
25 NEXT I
26 FOR I=1 TO 15
28 X=I*5
30 LPRINT "D";X;",";0;",";X;",";-120
32 NEXT I
33 LPRINT "S";1
34 FOR I=0 TO 12
36 Y=-I*10:X=82
38 LPRINT "M";X;",";Y
40 LPRINT "P";30*I
42 NEXT I
44 LPRINT "Q1"
46 FOR I=0 TO 16
48 X=I*5:Y=-122
50 LPRINT "M";X;",";Y
52 LPRINT "P";I
54 NEXT I
56 LPRINT "M-5,-5":LPRINT "S";1
58 LPRINT "P";"COURSE SCALE IN DEGREE
S---->"
```

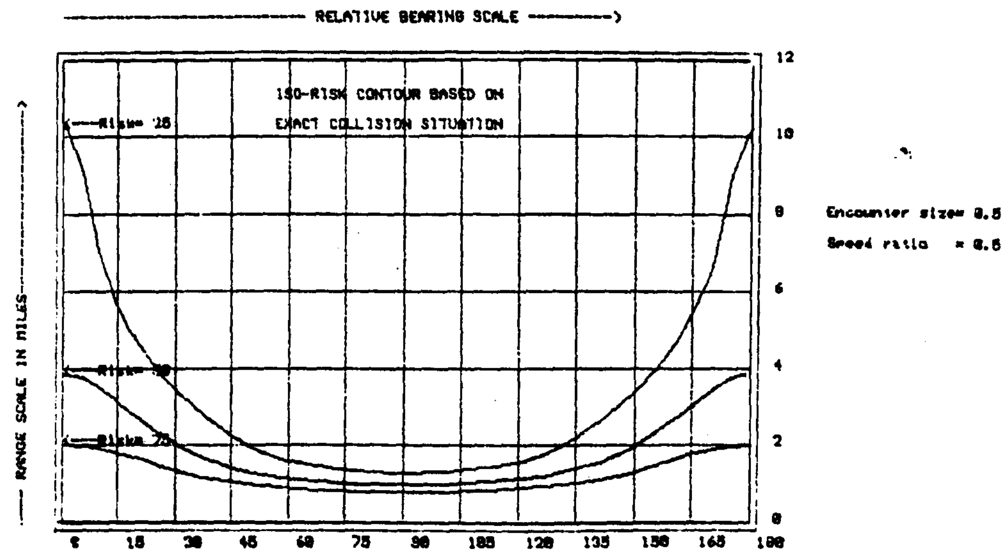
```
  60 LPRINT "Q0":LPRINT "M5,5"
  62 LPRINT "P";"SPEED SCALE IN KNOTS -
------>"
  64 LPRINT "M0,0"
 212 INPUT "Bearing of A =";BG
 214 INPUT "Range of   A =";O
 216 INPUT "Course of  A =";CA
 219 INPUT "Target speed =";VA
 220 INPUT "Deg.of polynomial 2=";NN
 222 INPUT "Int.pola.Nc 8=";M
 226 INPUT "Risk level =";PO
 228 INPUT "Encounter size =";RA
 230 INPUT "EPAR points =";WW
 232 DIM X(2*WW+1),Y(2*WW+1),H(2*WW+1)
 233 DIM R(M+N),P(M+N),B(M+N),D(2),Q(2)
,K(2)
 235 IM=1000
 236 IF PO>=50 THEN 240
 238 KE=1.6:GOTO 245
 240 KE=1
 245 XA=O*COSBG
 250 YA=O*SINBG
 255 LL=1:R=0
 260 FOR V=16 TO 1 STEP -1
 261 E=V/VA
 262 REM DETERMINATION OF Ro AND R180 O
F SPECIFIC RISK."
 290 ZZ=0
 300 Q=ZZ*180
 320 IF E>1 THEN 370
 330 IF Q<=ACSE THEN 380
 340 QQ=90+ASNE
 350 IF Q<QQ THEN 370
 360 D(ZZ)=RA:GOTO 590
 370 RQ=RA:GOTO 390
 380 RQ=RA/SIN(Q+ASNE)
 390 FOR I=1 TO M
 391 BEEP 1:BEEP 0
 400 R=RQ+LL*(I-1)
 401 PRINT "R=";R,"LL=";LL
 410 R(M-I+1)=R
 420 GOSUB PROG 8
 430 P(M-I+1)=P*100
 440 NEXT I
 450 IF PO<P(M-2) THEN 500
 460 IF PO>P(M-2) THEN 480
 470 D(ZZ)=R(M-2):GOTO 590
 480 IF LL>.05 THEN 570
 490 D(ZZ)=RA:GOTO 590
 500 IF PO<P(M-4) THEN 550
 510 IF PO>P(M-4) THEN 530
 520 D(ZZ)=R(M-4):GOTO 590
```

```
530 GOSUB PROG 6
540 D(ZZ)=R:GOTO 590
550 IF LL>=25 THEN 580
560 LL=1.5*LL:GOTO 390
570 LL=.7*LL:GOTO 390
580 D(ZZ)=LL*M
590 ZZ=ZZ+1
592 IF ZZ<2 THEN 300
646 REM"DETERMINATION OF ISORISK ELLIP
SE.
647 PRINT "ELLIPSE"
648 AE=(D(0)+D(1))/2
650 BE=KE*D(1)
652 CE=AE-D(1)
654 XE=XA+CE*COSCA
656 YE=YA+CE*SINCA
658 EE=(SQR(AE*AE-BE*BE))/AE
660 XO=-YE*SINCA-XE*COSCA
662 YO=-YE*COSCA+XE*SINCA
664 X=XO:Y=YO:GOSUB PROG 9
666 RO=Z:Q(0)=U
668 RE=BE/SQR(1-(EE*COSQ(0))^2)
669 REM"DETERMINATION OF TANGENT POINT
S OON THE ISORISKELLIPSE.
670 IF RO<=RE THEN 1046
768 IF XO=0 THEN 790
770 IF YO=0 THEN 800
772 A=BE^2*((AE*YO)^2+(BE*XO)^2)
774 B=-2*(AE*BE)^2*BE^2*XO
776 C=AE^4*BE^2*(BE^2-YO^2)
778 GOSUB PROG 5
780 X(1)=X1
782 X(2)=X2
784 Y(1)=((AE*BE)^2-BE^2*XO*X(1))/(AE^
2*YO)
786 Y(2)=((AE*BE)^2-BE^2*XO*X(2))/(AE^
2*YO)
788 GOTO 804
790 X(1)=AE*SQR(1-(BE/YO)^2)
792 X(2)=-X(1)
794 Y(1)=BE^2/YO
796 Y(2)=Y(1)
798 GOTO 804
800 X(1)=-AE^2/XO:X(2)=X(1)
802 Y(1)=BE*SQR(1-(X(1)/AE)^2):Y(2)=-Y
(1)
804 X(1)=XO-X(1):Y(1)=YO-Y(1):X(2)=XO-
X(2):Y(2)=YO-Y(2)
805 REM"DETERMINATION OF ASPECTS AT TA
NGENT POINTS.
808 FOR I=0 TO 2
810 IF I=0 THEN 813
```

```
811 X=X(I):Y=Y(I):GOSUB PROG 9
812 Q(I)=U
813 K(I)=SGN(SINQ(I))
814 IF Q(I)<=180 THEN 816
815 Q(I)=360-Q(I)
816 NEXT I
817 PRINT "Q(0)=";Q(0)
818 PRINT "Q(1)=";Q(1)
819 PRINT "Q(2)=";Q(2)
820 IF E>1 THEN 860
821 IF BE<RO*SINQ(0) THEN 860
822 IF Q(1)<ASNE THEN 860
824 IF Q(2)<ASNE THEN 860
826 IF Q(0)<90 THEN 1046 ELSE 1070
858 REM" DETERMINATION OF POINTS ON TH
E EP ELLIPSE.
860 FOR I=0 TO WW
862 Q=I*360/WW
864 RE=BE/SQR(1-(EE*COSQ)^2)
866 X(I)=XE+RE*COS(CA+Q)
868 Y(I)=YE+RE*SIN(CA+Q)
870 NEXT I
880 REM" DETERMINATON OF THE INTERCEPT
ABLE FUTURE POINTS."
890 FOR I=0 TO WW
891 PRINT "POINT=";I
900 Q=I*360/WW
940 XO=-Y(I)*SINCA-X(I)*COSCA
950 YO=-Y(I)*COSCA+X(I)*SINCA
960 A=1-E^2
970 B=-2*XO
980 C=XO*XO+YO*YO
990 GOSUB PROG 5
995 IF X1<0 THEN 1011
1000 IF E>=1 THEN 1005
1001 IF B*B>4*A*C THEN 1016
1003 IF B*B<4*A*C THEN 1011
1005 X(I)=X(I)+X1*COSCA
1006 Y(I)=Y(I)+X1*SINCA
1007 X=X(I):Y=Y(I)
1008 GOSUB PROG 9
1009 H(I)=U
1010 H(I+WW+1)=IM:GOTO 1029
1011 H(I)=IM
1012 H(I+WW+1)=IM:GOTO 1029
1016 X(I)=X(I)+X1*COSCA
1017 Y(I)=Y(I)+X1*SINCA
1018 X(I+WW+1)=X(I)+X2*COSCA
1019 Y(I+WW+1)=Y(I)+X2*SINCA
1020 X=X(I):Y=Y(I)
1021 GOSUB PROG 9
1022 H(I)=U
```

```
1023 X=X(I+WW+1):Y=Y(I+WW+1)
1024 GOSUB PROG 9
1025 H(I+WW+1)=U
1029 NEXT I
1030 LPRINT "S";5
1031 REM" PLOTTING OF RISK MATRIX POINT
S."
1032 FOR I=0 TO 2*WW
1034 IF H(I)=IM THEN 1042
1036 VO=5*V:H(I)=H(I)/3
1038 LPRINT "M";VO;",";-H(I):LPRINT "N"
;3
1040 IF E>=1 THEN IF I>=WW THEN 1060
1042 NEXT I
1044 GOTO 1060
1046 FOR I=0 TO 23
1048 VO=5*V:H(I)=(7.5/3)+(5*I):LPRINT "
S";5
1050 LPRINT "M";VO;",";-H(I):LPRINT "N"
:3
1052 NEXT I
1053 V=V-1
1054 IF V=0 THEN 1070 ELSE 1046
1059 BEEP 1:BEEP 0:BEEP 1:BEEP 1:BEEP 1
:BEEP 1
1060 NEXT V
1070 LPRINT "M";0;",";0
1072 LPRINT "S";0
1080 LPRINT "Q1"
1210 LPRINT "M";65;",";-135
1212 LPRINT "P";"INPUT DATA:-"
1214 LPRINT "M";60;",";-135
1216 LPRINT "P";"TARGET SPEED  =";VA
1218 LPRINT "M";55;",";-135
1220 LPRINT "P";"TARGET COURSE =";CA
1222 LPRINT "M";50;",";-135
1224 LPRINT "P";"TARGET BEARING=";BG
1226 LPRINT "M";45;",";-135
1228 LPRINT "P";"TARGET RANGE  =";O
1230 LPRINT "M";40;",";-135
1232 LPRINT "P";"ENCOUNTER SIZE=";RA
1234 LPRINT "M";35;",";-135
1236 LPRINT "P";"RISK LEVEL    =";PO
1238 LPRINT "M";30;",";-135
1240 LPRINT "P";"POLYNOMIAL    =";NN
1242 LPRINT "M";25;",";-135
1246 LPRINT "P";"INTER.POINTS  =";M
1248 LPRINT "M";20;",";-135
1250 LPRINT "P";"No. OF POINTS =";WW
1252 LPRINT "S";1
1254 LPRINT "M";80;",";-135
1256 LPRINT "P";"RISK MATRIX:"
1260 END
```

Appendix_B.7

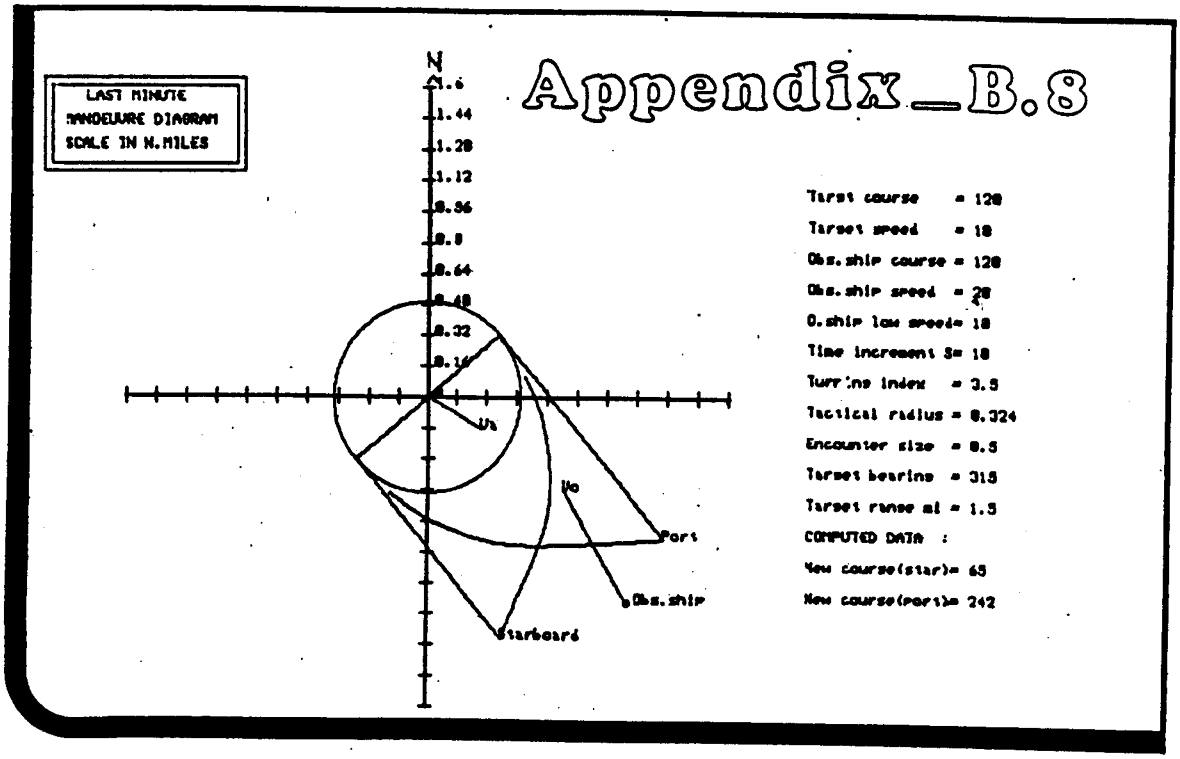



```
    5 LPRINT "F3"
   10 PRINT "ACTION ZONE"
   12 CLEAR :LPRINT CHR$(28);CHR$(37):LP
RINT "S0"
   16 LPRINT "M5,0"
   18 LPRINT "A";5;",";0;",";65;",";-90
   20 LPRINT "A";4;",";1;",";66;",";-91
   22 FOR I=1 TO 11
   23 Y=I*7.5
   24 LPRINT "D";5;",";-Y;",";65;",";-Y
   25 NEXT I
   26 LPRINT "M70,0":LPRINT "Q1"
   27 LPRINT "P";"-----------------------
----- RELATIVE BEARING SCALE ---------->"
   28 LPRINT "M60,-28"
   30 LPRINT "P";"ISO-RISK CONTOUR BASED
 ON"
   32 LPRINT "M56,-28"
   34 LPRINT "P";"EXACT COLLISION SITUAT
ION"
   36 LPRINT "M5,5":LPRINT "Q0"
   38 LPRINT "P";"---- RANGE SCALE IN MI
LES-------------------->"
   90 LPRINT "M5,0":LPRINT "O5,0"
   92 FOR I=0 TO 6
   94 X=I*10
   96 LPRINT "D";X;",";0;",";X;",";-90
   98 NEXT I
  100 INPUT "Speed ratio=";E
  102 INPUT "Degree of polynomial(2)=";N
N
```

```
 104 INPUT "Interpolation No.(8)=";M
 106 INPUT "No.of points=";WW
 108 INPUT "Encounter size=";RA
 112 DIM X(2*WW+1),Y(2*WW+1),B(M+N)
 114 DIM R(M+N),P(M+N),D(2*WW+1),T(2*WW
+1)
 116 LL=1
 220 FOR W1=1 TO 3
 230 PO=25*W1
 240 HH=0:ZZ=0:DH=0
 250 IF E>1 THEN 280
 255 IF E<1 THEN 270
 260 DB=90/WW:GOTO 290
 270 DB=180/WW:GOTO 300
 280 DB=2*(ASN(1/E))/WW:GOTO 300
 290 Q=(180-HH)/2:GOTO 315
 300 Q=ACS((1-E*COSHH)/SQR(1-2*E*COSHH+
E*E))
 315 IF HH<=180 THEN 330
 320 HH=180:GOTO 250
 330 BG=180-(HH+Q)
 332 IF E>1 THEN 370
 335 IF Q<=ACSE THEN 380
 340 QQ=90+ASNE
 350 IF Q<QQ THEN 370
 360 D(ZZ)=RA:GOTO 585
 370 RQ=RA:GOTO 390
 380 RQ=RA/SIN(Q+ASNE)
 390 FOR I=1 TO M
 391 BEEP 1:BEEP 0
 400 R=RQ+LL*(I-1)
 401 PRINT "R=";R,"LL=";LL
 410 R(M-I+1)=R
 420 GOSUB PROG 8
 430 P(M-I+1)=P*100
 440 NEXT I
 450 IF PO<P(M-2) THEN 500
 460 IF PO>P(M-2) THEN 480
 470 D(ZZ)=R(M-2):GOTO 585
 480 IF LL>.05 THEN 570
 490 D(ZZ)=RA:GOTO 585
 500 IF PO<P(M-4) THEN 550
 510 IF PO>P(M-4) THEN 530
 520 D(ZZ)=R(M-4):GOTO 585
 530 GOSUB PROG 6
 540 GOTO 585
 550 IF LL>=25 THEN 580
 560 LL=1.3*LL:GOTO 390
 570 LL=.7*LL:GOTO 390
 580 D(ZZ)=D(ZZ-1)
```

```
585 IF W1>1 THEN 780
590 IF HH<>180 THEN 780
591 I=3
592 IF D(ZZ)>=42 THEN 596
593 IF D(ZZ)>I THEN 595
594 SC=I:GOTO 604
595 I=I+3:GOTO 592
596 SC=42
604 KK=60/SC:T(ZZ)=BG
612 LPRINT "Q1"
614 FOR I=0 TO 12
616 Y=I*7.5
618 LPRINT "M";-3;",";-Y
620 LPRINT "P";I*15
622 NEXT I
624 FOR I=0 TO 6
626 X=I*10
628 Y=X/KK
630 LPRINT "M";X;",";-92
632 LPRINT "P";Y
634 NEXT I
780 T(ZZ)=BG
819 IF HH>=180 THEN 925
840 IF (1/E)=COSHH THEN 870
860 DH=ABS(DB*(1-2*E*COSHH+E*E)/(1-E*C
OSHH))
865 IF DH<=DB*(1+E*E) THEN 880
870 DH=DB*(1+E*E)
880 IF HH>=180 THEN 925
890 ZZ=ZZ+1
895 PRINT "ZZ=";ZZ
900 HH=HH+DH:GOTO 250
925 LPRINT "M";-KK*D(0);",";-T(0)/2
926 FOR Z=1 TO ZZ
927 LPRINT "D";KK*D(Z-1);",";-T(Z-1)/2
;",";KK*D(Z);",";-T(Z)/2
928 NEXT Z
929 LPRINT "S0":LPRINT "Q1"
930 LPRINT "P";"<---Risk=";PO
931 LPRINT "M0,0"
932 NEXT W1
950 LPRINT "M40,-100"
960 LPRINT "P";"Encounter size=";RA
965 LPRINT "M36,-100"
970 LPRINT "P";"Speed ratio   =";E
1000 END
```

Appendix_B.8




```
   10 REM"EXTREME RISK ZONE"
   11 LPRINT CHR$(28);CHR$(37):CLEAR
   12 LPRINT "F";20
   13 LPRINT "L";0
   14 LPRINT "Q50,50":LPRINT "Q1":LPRINT
 "S1"
   15 LPRINT "M";42;",";.5:LPRINT "P";"N
"

   16 LPRINT "M";39;",";.5:LPRINT "P";"^
"

   18 LPRINT "D";0;",";40;",";0;",";-40
   19 LPRINT "D";40;",";0;",";-40;",";0
   20 LPRINT "S0"
   22 LPRINT "A";40;",";30;",";30;",";5
   24 LPRINT "A";41;",";31;",";29;",";4
   26 LPRINT "M";38;",";25
   28 LPRINT "P";"LAST MINUTE"
   30 LPRINT "M";35;",";28
   32 LPRINT "P";"MANOEUVRE DIAGRAM"
   34 LPRINT "M";32;",";28
   36 LPRINT "P";"SCALE IN N.MILES"
   94 INPUT "Sc.mm/m=";SC
   95 INPUT "Time inc sec";DT:DT=DT/60
   96 INPUT "Turning Radius in meters=";
RT:RT=.00054*RT
   97 INPUT "Reach.T.sec";TR:TR=TR/60
  100 INPUT "Rg.A=";R1
  102 INPUT "Sp.A=";VA
  103 INPUT "H.Sp.O=";VH
  104 INPUT "L.Sp.O=";VL
  105 INPUT "Sp.Coe=";K
  106 INPUT "Co.ofA=";CA
```

```
107 INPUT "Co.ofO=";CO
112 INPUT "En.size=";RA
113 LPRINT "C";0;",";0;",";SC*RA
114 INPUT "No.points=";N
115 DIM B(N+2),X(N+2),Y(N+2),Z(N+2),W(
N+2),D(N+2),S(N+2)
116 E=VH/VA:SX=VA*DT*(COSCA)/60
117 SY=VA*DT*(SINCA)/60
118 H=CA-CO
119 IF H>=0 THEN 121
120 H=360+H
121 IF H<=180 THEN 123
122 H=360-H
123 Q=ACS((1-E*COSH)/SQR(1-2*E*COSH+E*
E))
124 B1=180-(Q+H):GOTO 140
140 IF B1>90 THEN 146
145 G=270:US=90:L=0:M=1:GOTO 147
146 G=90:US=270:L=0:M=1:BEEP 1:BEEP 0:
BEEP 1:BEEP 1:BEEP 1
147 FOR J=0 TO N
148 X(J)=0:Y(I)=0:Z(I)=0:W(J)=0:B(J)=0
:D(J)=0
149 NEXT J
150 X=VA*COSCA-VH*COSCO
155 Y=VA*SINCA-VH*SINCO
160 GOSUB PROG 9
162 VR=Z:CR=U
164 X1=SC*RA*COS(CR+90):Y1=SC*RA*SIN(C
R+90)
166 X2=SC*RA*COS(CR+270):Y2=SC*RA*SIN(
CR+270)
168 LPRINT "D";X2;",";-Y2;",";X1;",";-
Y1
170 X(0)=RA*COS(CR+G)
175 Y(0)=RA*SIN(CR+G)
180 Z(1)=(VR*TR*COSCR)/60
185 W(1)=(VR*TR*SINCR)/60
190 X(1)=Z(1)+X(0)
195 Y(1)=W(1)+Y(0)
196 B(1)=0
206 I=2
207 T=DT*(I-1)
208 U=(VH-VL)*EXP(-T/K)+VL
209 PRINT "I=";I,"U=";U
210 B(I)=B(I-1)+DT*U*57.3/(RT*60)
211 Z(I)=(I-1)*SX-2*RT*SIN(B(I)/2)*COS
(CO+(L+M*(B(I)/2)))
212 W(I)=(I-1)*SY-2*RT*SIN(B(I)/2)*SIN
(CO+(L+M*(B(I)/2)))
213 GOTO 233
214 X=5*(X(I-1)-X(I-2)):Y=5*(Y(I-1)-Y(
I-2))
```

```
 215 B(I)=B(I-1)
 216 X(I)=X(I-1)+X:BB=B(I)
 217 Y(I)=Y(I-1)+Y
 218 D(I)=X(I)*COS(CR+US)+Y(I)*SIN(CR+U
S)-RA
 219 IF D(I)>0 THEN 223
 220 IF D(I)=0 THEN 279
 221 I=I+1:BEEP 1:PRINT "I=":I
 222 IF I<N THEN 215 ELSE 279
 223 X=X(I)-X(I-1)
 224 Y=Y(I)-Y(I-1)
 225 GOSUB PROG 9
 226 S=(ABSD(I-1)*Z)/(ABSD(I-1)+ABSD(I)
)
 227 X(I)=X(I-1)+S*COSU
 228 Y(I)=Y(I-1)+S*SINU
 230 GOTO 279
 233 X=Z(I)-Z(I-1):Y=W(I)-W(I-1):GOSUB
PROG 9:U1=U
 234 X(I)=Z(I)+RA*COS(U1+G)
 235 Y(I)=W(I)+RA*SIN(U1+G)
 236 X=X(I):Y=Y(I):GOSUB PROG 9
 237 S(I)=Z
 238 D(I)=X(I)*COS(CR+US)+Y(I)*SIN(CR+U
S)-RA
 239 IF I<6 THEN 256
 242 PRINT "D4=";D(4):PRINT "D5=";D(5)
 243 IF ABS(D(5))<ABS(D(4)) THEN 245
 244 IF B1<=90 THEN 146 ELSE 145
 245 IF D(I)>0 THEN 260
 250 IF D(I)=0 THEN 279
 255 IF S(I)<S(I-1) THEN 214
 256 IF I>=8 THEN 214
 257 I=I+1:GOTO 207
 260 X=X(I)-X(I-1)
 265 Y=Y(I)-Y(I-1)
 270 GOSUB PROG 9
 275 S=(ABSD(I-1)*Z)/(ABSD(I-1)+ABSD(I)
)
 276 X(I)=X(I-1)+S*COSU:Y(I)=Y(I-1)+S*S
INU
 279 IF G=270 THEN 281
 280 X(I+1)=X2/SC:Y(I+1)=Y2/SC:GOTO 282
 281 X(I+1)=X1/SC:Y(I+1)=Y1/SC
 282 BB=B(I)
 283 FOR J=0 TO I+1
 284 Y(J)=INT(SC*Y(J)*10)/10:X(J)=INT(S
C*X(J)*10)/10
 285 NEXT J
 286 FOR J=4 TO I+1
 287 LPRINT "D";X(J-1);",";-Y(J-1);",";
X(J);",";-Y(J)
 288 NEXT J
```

```
 290 IF M<1 THEN 294
 291 CS=CO+BB
 292 IF CS<=360 THEN 297
 293 CS=CS-360:GOTO 297
 294 CP=CO-BB
 295 IF CP>=0 THEN 297
 296 CP=CP+360
 297 LPRINT "D";X(1);",";-Y(1);",";X(4)
;",";-Y(4)
 298 LPRINT "M";X(I);",";-Y(I)
 300 IF M<1 THEN 304
 302 LPRINT "P";" Starboard":GOTO 306
 304 LPRINT "P";" Port"
 306 LPRINT "S0"
 355 IF M<0 THEN 386
 356 IF B1>90 THEN 362
 360 G=90:US=270:L=360:M=-1:BB=B(I)
 361 GOTO 147
 362 G=270:US=90:L=360:M=-1:BB=B(I)
 363 GOTO 147
 386 X=SC*R1*COS(Q+CA):Y=SC*R1*SIN(Q+CA
)
 388 LPRINT "M";X;",";-Y:LPRINT "N";6:L
PRINT "P";" Obs.ship"
 400 FOR I=0 TO 3
 402 LPRINT "X";I;",4,10"
 404 NEXT I
 406 LPRINT "S0":LPRINT "Q1"
 408 FOR I=0 TO 10
 410 LPRINT "M";I*4;",";0
 412 LPRINT "P";I*4/SC
 414 NEXT I
 416 X1=X+SC*UH*COS(CO)/30:Y1=Y+SC*UH*S
IN(CO)/30
 418 LPRINT "D";X;",";-Y;",";X1;",";-Y1
:LPRINT "P";"Vo":LPRINT "S1"
 419 LPRINT "M";36;",";-50:LPRINT "P";"
Targt course    =";CA
 420 LPRINT "M";32;",";-50
 422 LPRINT "P";"Target speed    =";VA
 424 LPRINT "M";28;",";-50
 426 LPRINT "P";"Obs.ship course =";CO
 428 LPRINT "M";24;",";-50
 430 LPRINT "P";"Obs.ship speed  =";VH
 432 LPRINT "M";20;",";-50
 434 LPRINT "P";"O.ship low speed=";VL
 436 LPRINT "M";16;",";-50
 438 LPRINT "P";"Time increment S=";DT*
60
```

```
440 LPRINT "M";12;",";-50
442 LPRINT "P";"Turning index   =";K
444 LPRINT "M";8;",";-50
446 LPRINT "P";"Tactical radius =";RT
448 LPRINT "M";4;",";-50
450 LPRINT "P";"Encounter size  =";RA
452 LPRINT "M";0;",";-50
454 LPRINT "P";"COMPUTED DATA  :"
456 LPRINT "M";-4;",";-50
458 LPRINT "P";"New course(star)=";INT
CS
460 LPRINT "M";-8;",";-50
462 LPRINT "P";"New course(port)=";INT
CP
464 LPRINT "S0"
480 X=SC*UA*COS(CA)/30:Y=SC*UA*SIN(CA)
/30
482 LPRINT "D";0;",";0;",";X;",";-Y:LP
RINT "P";"Va"
484 LPRINT "M";-54;",";-45
500 END
```

Appendix_C.1

Subroutines

```
1299 REM"QUADRAT"
1300 IF A=0 THEN 1304
1302 GOTO 1314
1304 IF B<>0 THEN 1310
1306 X1=10^6
1308 GOTO 1344
1310 X1=-C/B
1312 GOTO 1344
1314 IF B*B>=4*A*C THEN 1324
1316 X1=0:X2=0
1320 GOTO 1344
1324 S=SQR(B*B-4*A*C)
1326 IF B>=0 THEN 1332
1328 X1=(-B-S)/(2*A)
1330 GOTO 1336
1332 X2=(-B+S)/(2*A)
1334 IF X2=0 THEN 1338
1335 GOTO 1343
1336 IF X1=0 THEN 1338
1337 GOTO 1342
1338 X1=0:X2=0
1340 GOTO 1344
1342 X2=C/(A*X1):GOTO 1344
1343 X1=C/(A*X2)
1344 RETURN
1346 END
```

```
1400 REM SUB.ASPECT
1404 IF J>180 THEN 1408
1406 J=J+180-C:GOTO 1410
1408 J=J-180-C
1410 IF J>=0 THEN 1414
1412 J=J+360
1414 RETURN
1416 END
```

```
1700 REM"LAGRANGE"
1702 I=0:BEEP 0:BEEP 1:BEEP 0:BEEP 1
1704 I=I+1
1706 IF PO>P(I) THEN 1704
1708 I=I-1
1710 FOR J=0 TO NN
1712 B(J)=1
1714 NEXT J
1716 D(ZZ)=0
1718 FOR K=0 TO NN
1720 FOR J=0 TO NN
1722 IF J=K THEN 1726
1724 B(K)=B(K)*(PO-P(J+I))/(P(I+K)-P(J+
I))
1726 NEXT J
1728 D(ZZ)=D(ZZ)+B(K)*R(I+K)
1730 NEXT K
1734 RETURN
1736 END
```

```
1600 REM"SUB COMP-VECTOR"
1602 IF X>0 THEN 1608
1604 IF X=0 THEN 1610
1606 U=ATN(Y/X):GOTO 1618
1608 U=ATN(Y/X):GOTO 1628
1610 IF Y>0 THEN 1626
1612 IF Y<0 THEN 1624
1614 U=0:GOTO 1632
1618 IF Y=0 THEN 1622
1620 U=U+180:GOTO 1632
1622 U=180:GOTO 1632
1624 U=270:GOTO 1632
1626 U=90:GOTO 1632
1628 IF Y>=0 THEN 1632
1630 U=U+360
1632 Z=SQR(X*X+Y*Y)
1634 RETURN
1636 END
```

Appendix_C.2

Subroutine

```
1500 REM RISK SUB.
1504 IF R<RA THEN 1590
1506 II=1:U=ASN(RA/R):C=(1-E)/(1+E)
1508 G=Q+U:GOTO 1512
1510 G=360+G
1512 IF G<360 THEN 1516
1514 G=G-360
1516 M2=SIN(G)
1518 IF G>180 THEN 1524
1520 IF G<180 THEN 1526
1522 K=0:H=0:L=0:GOTO 1564
1524 G=360-G
1526 IF G>90 THEN 1538
1528 IF G=90 THEN IF E<=1 THEN 1535
1529 IF G=90 THEN IF E>1 THEN 1536
1530 IF G>0 THEN 1538
1532 IF G<0 THEN 1510
1534 K=E*PI:GOTO 1564
1535 K=0:GOTO 1564
1536 B=0:GOTO 1540
1538 B=2*E/((TAN(G))*(1+E))
1540 S=B*B-4*C
1542 IF S<0 THEN 1550
1544 Y=SQR(S)
1545 IF B>0 THEN 1548
1546 Z=(B-Y)/2:W=2*C/(B-Y):GOTO 1552
1548 Z=2*C/(B+Y):W=(B+Y)/2:GOTO 1552
1550 Z=0:W=0
1552 IF E<1 THEN 1556
1554 Z=0
1556 H=2*ATN(Z):L=2*ATN(W)
```

```
1558 N=TAN((L+G)/2)/TAN((H+G)/2)
1560 N=(SIN(G))*(LOG(N))
1562 K=(PI/180)*(L-H)*E-N
1564 IF II>1 THEN 1571
1566 J=K:M1=M2:II=II+1:U=-U
1568 GOTO 1508
1571 IF M2<0 THEN 1576
1572 IF M1<0 THEN 1580
1574 GOTO 1578
1576 IF M1>0 THEN 1582
1578 A=ABS(J-K):GOTO 1584
1580 A=J+K:GOTO 1584
1582 A=2*E*PI-(J+K)
1584 IF A>0 THEN IF E=0 THEN 1590
1586 IF A=0 THEN IF E=0 THEN 1592
1588 P=A/(2*E*PI):GOTO 1594
1590 P=1:GOTO 1594
1592 P=0
1594 RETURN
1596 END
```

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Abbreviations and Symbols

The following list shows special abbreviations, symbols and notations which appear most frequently in this work ;-

Abbreviations

C.P.A	Closest point of approach.
C.A	Collision avoidance.
E.P.A.R	Equi-Potential Area of Risk.
E.P.R.M	Equi-Potential Risk Matrix.
N.C.S	Navigational Coordinate System.
T.C.P.A	Time to Closest Point of Approach.

Symbols

A	The designation of the target ship.
a_e	The major axis of the equirisk ellipse.
B	The designation of the second target ship.
	The bearing of the ship.
B_o	The present bearing.
B_m	The bearing at CPA.
B^o	The rate of the bearing change.
b_e	The minior axis of the equi-risk ellipse.
ΔB	The increment of the bearing change.
C_a	The course of ship (A).
C_o	The course of ship (O).

C_r	The relative course of the target ship.
D , d	A displacement , a distance.
E	Speed ratio of a binary encounter as observed by the refrence ship.
ΔE	An increment of the speed ratio.
E_{max}	The maximum speed ratio available.
e	The eccentricity of the equirisk ellipse.
F	An index of range rate.
$f(Q)$	Collision function.
H	The relative heading.
ΔH	An increment of the relative heading.
i , j	An integer from 1 to n used to subscript the variables.
K	The turning Index of speed deceleration.
K_e	A coefficient of proportionality.
Ln	The natural logarithm.
M	An area representing a set of the different ways for a collision to occur.
N	An area representing the total number of the possible ways that can be considered by the observing ship.
O	The designation of the refrence ship centre point (obsrving ship), the point of origin of the coordinate system.
P	A point in the co-ordinate system. The geometrical probability of collision. The risk function.
P_o	The present risk level.
P_m	The risk level at the CPA.
P_{max}	The maximum value of the risk function.
Q	The aspect angle.
Q_s	The collision aspect.
Q_o	The present aspect.
Q_m	The aspect angle at CPA.
R	A range.
R_o	The present range between ships in the encounter.

R_a	An accepted CPA.
R_m	The distance at CPA.
R_t	The turning radius of a ship.
R_e	The radius of an equi-risk ellipse.
R_{000}	A distance of a point ahead of target ship which bears a prescribed level of risk.
R_{180}	A distance of the point astern of target ship which bears a prescribed level of risk.
S_a	The run of target ship "displacement"
S_0	The run of the observing ship."displacement"
S_r	The relative run "displacement"
S_c	The distance run by a point 'C'.
T	Time.
T_n	The time interval after the alteration of course has been made.
ΔT	Time interval.
V	Speed.
V_a	The speed of target ship.
V_o	The speed of observing ship.
V_r	The relative speed of target ship.
Vx_a	The N-S speed component of target ship.
Vy_a	The E-W speed component of target ship.
Vx_o	The N-E speed component of observing ship.
Vy_o	The E-W speed component of observing ship.
Vx_r	The N-S relative speed component.
VY_r	The E-W relative speed component.
X_a	The abscissa of centre point of taget ship "A".
Y_a	The ordinate of centre point of target ship.
ΔX	The increment of displacement in the direction of the X-axis.

ΔY	The increment of displacement in the direction of the Y-axis.
ΔC	Thre amount of course alteration.

Notation

$E = 1$	E equal 1
$E < 1$	E is less than 1
$E > 1$	E is greater than 1
$\lvert X \rvert$	Absolute value of 'X'.
df/dH,df/dE	The partial derivatives of the collision function.
$\int_{H1}^{H2} f$	The definite integralof f in H1,H2.
Σ	A set , matrix.