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A Review on Light Shafts Rendering for Indoor Scenes

A Review on Light Shafts Rendering for Indoor Scenes

Hatam H. Ali, Mohd Shahrizal Sunar, Hoshang Kolivand, Mohd Azhar Bin M. Arsad

***Abstract*—**Rendering light shafts is one of the important topics in computer gaming and interactive applications. The methods and models that are used to generate light shafts play crucial role to make a scene more realistic in computer graphics. This article discusses the image-based shadows and geometric-based shadows that contribute in generating volumetric shadows and light shafts, depending on ray tracing, radiosity, and ray marching technique. The main aim of this study is to provide researchers with background on a progress of light scattering methods so as to make it available for them to determine the technique best suited to their goals. It is also hoped that our classification helps researchers find solutions to the shortcomings of each method.

***Keywords*—**Shaft of lights, realistic images, image-based, and geometric-based.

I. Introduction

THE indoor rendering contains many elements; most important ones are light sources, lighting effects, shadows, and objects in various shapes and materials. This paper presents practically some definitions for more important parameters of light transport equation for rendering indoor atmospheres. The light sources, light scattering, particles in participating media, and shadows are the highlighted components indoor scenes. This survey addresses the methods and techniques to simulate light shafts and volumetric shadow in the indoor environments. The popular techniques are described according of features, limitations, and scope of utilization for each one of them. Usually, the realistic rendering of the indoor scenes requires solving the problems relevance in rendering volumetric lighting and volumetric shadows such as smooth transition at their edges. Therefore, it is also hoped that after reading this paper, the researcher will be aware of the latest and most widely used techniques, and will have the best information on each of them.

II. Global Illumination

In general, global illumination includes a direct lighting that comes from sunlight, in addition to indirect lighting occurs as a result of reflections of light between the surfaces of various objects in the scene. Therefore, an object's appearance depends on reflectance properties of its material that can be expressed by using bidirectional reflectance distribution function (BRDF). BRDF function represents the relationship between the incoming radiance and the outgoing radiance as illustrated in Fig. 1.

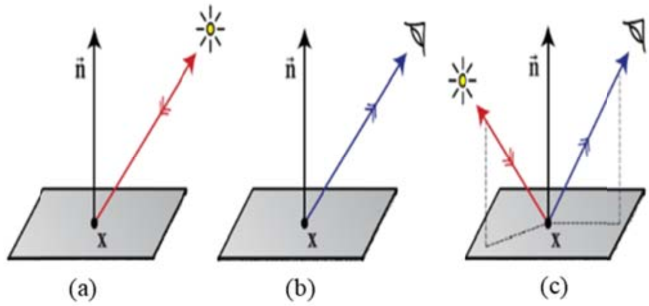

Fig. 1 (a) Radiance at a point x represents incoming radiance, (b) Radiance outgoing at x in direction, and (c) BRDF

The reflection mapping efficiently used indirect lighting to approximate the appearance of a reflective object based on texture image [1]. This texture is used for mapping from image of the distant environment surrounding into the rendered surface. The reflection maps are developed by using diffuse and specular reflection for corresponding components of reflection [2]. The method is rendered to appear illuminate in real-time of the atmosphere effects based on multiple anisotropic scattering and sky light as proposed by [3]. Preetham [4] presented analytic model for approximating the aerial perspective of atmosphere. The daylight includes direct and indirect sunlight and sky color [3]. The main resources of the illumination are artificial and natural lightings, which represent for example, lamps and sunlight respectively [4]. The ray tracing and radiosity are classic methods to implement global illumination [8]. Ray-tracing method is a traced rays emanating through their paths from light source until they hit the viewpoint, then it determines intersection points along paths to calculate the radiance [6]. On other hand, radiosity method depends on average illumination on surface elements. Ray tracing method is able to simulate various types of effects as the rays are traced from point to point [13]. Usually ray tracing is not compatible approach for creating a scene with a moving camera while the radiosity method is successful with a moving camera [7].

A. Ray Tracing

Whitted [5] is the first researcher who introduced the ray tracing to create some basic lighting effects perfectly, but the shortcomings of this method is prohibitively expensive for rendered scenes. The ray tracing is classified based on acceleration to three major categories which are faster

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intersections, fewer rays and generalize the rays [6]. Heckbert [7] proposed a method to use beams through a scene instead of ray tracing, which uses polygonal to form frustum boundaries. This method is efficient to get high quality scenes. The disadvantage in this method is that it fits only the polygonal objects. Kajiya [8] proposed a method to develop single scattering model of Blinn for ray tracing, eliminating all limitations of lighting and viewing due to derivate of an analytic solution is ineffective. Representing the participating media using a voxel model, the rendering conduct is separated into two stages: (1) calculation of the radiance arriving at each voxel from all light sources and (2) solving the eye radiance using the intermediate results of the previous step, by an illumination model [17]. Cook [9] proposed an approach for image synthesis based on ray tracing, where rays are distributed in time so that rays at different spatial locations are traced at different instants of time. Mark [10] presented cone tracing; the method traces circular frustum. This method is efficient to generate glossy reflections and shadow; moreover, it can get rid of artifact problem. This method is more expensive than previous methods. Jensen [11] presented method for rendering of sky at night involving illumination reaching from the Moon. The atmosphere model is applied using spherical model and approximation light scattering using phase functions, where use ray marching combined with distribution ray tracing.

B. Radiosity

In the 1950s, radiosity was originally developed in the field of heat transfer in engineering. After more than 30 years, radiosity was refined for using in computer graphics, specifically as an application to the problem of rendering by researchers at Cornell University [12]. Radiosity is a finite-element approach to the problem of global illumination [13]. In order to deal with large-area elements it is required to integrate over the area of the elements explicitly [13]. Radiosity is computed according to (1):

$$B_i = E_i \rho_i \sum B_j F_{ij} \tag{1}$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\varphi_i \cos\varphi_j}{\pi |r|^2} dA_i dA_j \tag{2}$$

where Bi: the radiosity leaving a surface I, Ei: the self-emitted radiosity, Bj: the radiosity outgoing surface j. Fij: the fraction of energy emitted from surface j and reaching the surface i. pi: reflectivity of the surface i. ∑ Bj Fij: the total amount of energy received by the surface i from other surfaces. pi ∑ Bj Fij: the total energy reflected by the surface i. dAt, dAj: the differential areas of the surface i and j respectively, r: the vector from dAt to dAj. Φi: the angle between r and the normal Nt of the surface i. φj: the angle between r and the normal Nj of the surface j.

III. Light Scattering

Kajiya [6] considered any environment divided into small volumes. Fig. 2 represents anisotropic media that contributes in light scattering process.

Fig. 2 The participating media as a collection of microscopic scattering particles

In order to provide the realistic appearance for objects that generate using rendering techniques should be used suitable techniques of lighting. Lighting techniques are used to solve this problem by simulating physical nature of light. As radiance travels through a participating media, it yields three types of phenomena: Absorption, scattering and emitting [14], [15] as illustrated in Fig. 3.

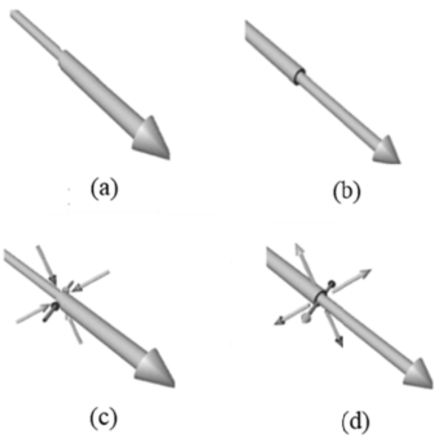

Fig. 3 Types of phenomena: (a) Emitting, (b) Absorption, (c) In-scattering, and (d) Out-scattering

The light nature requires understanding progression growingly complete; yet, complex models of light, optical phenomena can be represented according to successive models. These phenomena can be divided to quantum optics, electromagnetic optics, wave optics, and ray optics [16]. The ray optics is simplest model and used more. The ray optics is simplest model and used more than other to appear effects such as emission, reflection, and transmittance. However, there are effects completely ignored in computer graphics such as diffraction, interference, polarization, fluorescence, phosphorescence. These effects involve a transport of energy from one wavelength to another which considers thin along visible range of the light spectral [17]. The no scattering and single scattering models are simpler compared to the models mentioned above.

C. No Scattering

This situation arises when participating medium does not scatter (scattering = 0). For example, in the case of an explosion associated with high emission light that leads to the neglect of all types of scattering [18].

The rendering equation become as in (3):

$$L(x) = t_a(x_0, x)L(x_0) + \int_{x_0}^{x} t_0(u, x)k_a(u)L_e(u)du \quad (3)$$

where transmittance:

$$t_a(x_0, x) = e^{-\int_{x_0}^{x} k_a(u)du}$$

When homogeneous non-emitting medium ka = constant and Le = 0 rendering equation reduces to:

$$L(x) = e^{-k_a||x_0 - x||}L(x_0)$$

D.Single Scattering

The single scattering describes only scattered radiation event once along its path from a surface to the viewer [14].

According to nature of single light scattering that dominate visual effect is assumed that all light scattering occurred at surfaces, this means radiance which leaving a surface preserves the energy unchanged until it strikes another surface. In real life, this assumption does not work because participating media typically occupied the space between all objects by Infinitesimal particles [19]. Therefore, when the light travels from one point to another in space, the photons can change their paths and transform their powers due to interact with a participating media such as fog, dust, smoke and the particles in the air. Even in case clear air, the photons collide with small microscopic particles, they will either be absorbed or scattered in another direction (so sky is bluish) [20]. The change of radiation based on the properties media, leads to increase the challenging of rendering of participating media due to spectral dependence of medium and the complex interaction between radiance and media. The phenomena that occur in the media to participate involve of many difference effects of scattering such as absorption and emission. Fig. 4 illustrates single light scattering.

Fig. 4 Single light scattering

IV. Ray Marching

Ray marching is one of the popular methods for calculating single scattering in isotropic medium. In addition, it used to calculating airlight contribution. This method can be done on the Graphical Processing Unit along view rays as illustrated in Fig. 5. [21], [19], they calculate the scattering integral by putting planes at various depths, drawing blended slices, by clearly looping in a pixel shader [22]-[24], or by using completely ray marching in OpenGL [25]. In order to achieve some optimizations of ray marching due to cost computational [26] proposed used shadow volume to put bounding planes along view ray. Toth, [23] proposed reduced number samples based on relationship pixel and nearby pixels that allow taking results from each other.

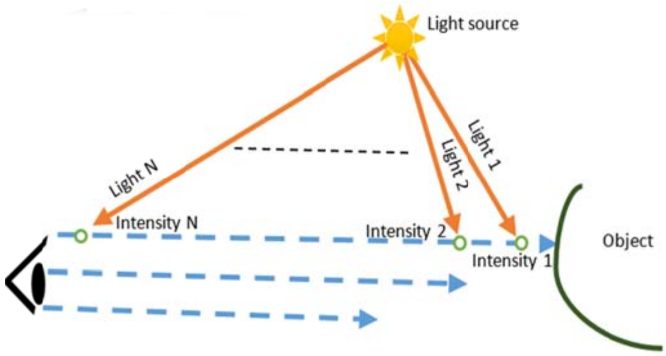

Fig. 5 Ray marching method

Engelhardet [24] proposed epipolar sampling technique to place samples in screen space. The technique determines the sample points based on changes of intensity abruptly at in-scattering position and used filtering between epipolar lines. Then, the in-scattering is calculated using ray marching for each point samples along epipolar lines. This technique reduced the computational cost of rendering equation but it gives crisper appearance of render image. In order to improve the above technique [25] presented method to compute in-scattering which is derived from a ray increase smoothly along same epipolar line that toward the light source. This method based on partial sum tree that made cost for computing the in-scattering only logarithmical along a view ray. Chen [27] proposed method dimensional min-max mipmap to speed up calculation of the shadow mapping but suffers of artifacts resulting from the discretization low-resolution.

V. Volumetric Shadows Effect

Volumetric shadows provide us information to uncover about relevance between objects in the scene. Furthermore, volumetric shadows are main part for many applications of the computer graphics to appear realistic phenomena [28]-[30]. Max [31] introduced method to use scan-line-based to determine the regions of illuminated and unilluminated atmosphere by contributed the shadow volume.

There are many researchers have tried to combines shadow with participating media to generate volumetric shadow effect. Jensen [32] introduced fog model as simplest volumetric effect based on fog density varies with respect to altitude. Mech [33] presents a method of capturing multiple areas of illuminated media. However, this approximation does not give accurate results due to complexly of shadow geometry, where used polygonal boundaries to determine volumes of fog. Biri [34] presented mathematical formulations to simulate shadow and fog. In fact, these methods suffered inability to appear true volumetric effects due to need variation in participating media density corresponding to variation in scattered light. Dobashi [19] proposed solution to create volumetric kinds effect using subsampling. In despite, the results of volumetric effect are acceptable for global illumination, while for local illumination leads to appear artifacts by the sub-planes. Instead, use virtual planes of the scene using arbitrary sample.

E. Image-Based Volumetric Shadows

Wyman [26] proposed method based on geometric-based shadows to add polygons along a view ray. The front polygon and farthest polygon clamps segment for which ray marching with visibility testing against the image-based shadows.

T´oth [23] proposed that only a downsamples are computed for each pixel, and neighboring pixels may take results from each other. Interleaved sampling is used to evaluate if the sample position along the view-ray is in scattering or if in shadow occurs). The technique is implemented in a pixel shader, which executed using the image-based shadows.

Engelhardt [24] presented method to put little sample along the lines of epipolar in screen space. The along lines of epipolar is linearly searched to get discontinuities in the Z-buffer and put samples only after and before such positions. Then, bilateral filtering is used along and between lines of epipolar to preserve the downsamples for the single scattering. Ali [35] proposed a method based on bilateral filtering to generate soft shadow maps using multiple image-based algorithms.

Baran [25] building on radiance for a ray can be evaluated incrementally form a ray along the same line of epipolar. This method based on rectification of shadow maps, that executes by eye rays and radiance rays are transformed into a new coordinate system. On the same approach of Baran, Chen [27] proposed method to accelerate partial sum tree method by replace a one-dimensional min-max mipmap method calculation of the image-based shadows.

Instead of ray marching method, Wyman [36] presented method to voxelize image-based shadow casters in epipolar geometry.

F. Geometrical-Based Volumetric Shadows

James [37] proposed method to use depth peeling instead of the shadow polygons. Biri [34] used sorting of geometric-based shadows quads in back-to-front order from the viewer.

Billeter [38] introduced method to guarantee that geometric-based shadows do not overlap is to create them from a image-based shadows.

VI. Light Shaft Effect

The basic definition of light shaft is information provided about the regions lit in space. The extra intensity is computed from information that represent light scattering, where leads to appear columns of scattered radiance visible with space is partly in shadow [23], [26].

G. Image-Based Light Shaft

Dobashi [21] proposed method to display light shafts with shadow map using texture-mapping hardware. The method includes rendering light shafts by placing of parallel slices in front of screen to compute light scattering induced of atmospheric particles. The principle fundamental of participating media to interact with light is based on rendering light transport equation for generating effects of atmospheric phenomena such as light shaft. Therefore, when light travels from light source through space in variety directions hits particles of atmosphere, based on physical properties of particles yield effects such as scattering and absorption of light. The integration of the in-scattering along view ray and light attenuation that reaches to viewpoint will appear light volume according to the density of particles in atmosphere. Moreover, the presence of objects inside light volume leads to generate shadow with light shaft. This method has been exploited graphics hardware rendering light shaft and shadow map. However, this method tried avoiding aliasing problem, but it needs multiple passes for number virtual planes that must proportional to the number of passes which lead to error sampling. Dobashi [19] later modified the method by adding sub-planes to overcome the sampling error, but this method requires more computation which time-consuming.

Mitchell [39] presented a method which based on work by Dobashi [19] for simulating light shafts. This method renders a traditional shadow map from opaque objects merely and then fills the scene with hundreds of translucent fog planes that receive the shadowing. This provides convincing light shafts, which can have color if the light has a projective texture or fog planes have colored texture. But this method, translucent objects cannot cast shadows, however, then Morgan [40] method proposed based on develop technique presented by [41] stochastic transparency, for shadow cast between any aggregation of colored transmissive, opaque, and partial coverage surfaces which it consistent with hardware shadow mapping techniques. This method used number infinity of surfaces that apply on GPUs and consoles. However it use depth buffer to store a depth of opaque surfaces and use to blending RGB channels to emulate a depth test for translucent surfaces [22] presented technique to use color buffer to light received by visible objects to calculate light frustum form shadow map and average extinction of participating media. The intersection of an eye ray is calculated in order to determine length of the light path through frustum. Bruneton [42] proposed method to render sunlight and illumination effects induced of attenuated light until reach to viewpoint and at different altitude from level ground for atmosphere. However, it based on compute the lit segments from view ray and ignore the dark parts through shadow test that reduce cost of integral equation of light.

H. Geometrically-Based Light Shaft

Imagire [43] proposed method practically consistent with the scenes that contain on bounding box (volumetric objects) like mist. This method based on virtual planes to rendering light scattering effects, through using sampling hull in order to reduce number alpha blending operations that leads to get rid of aliasing which appear at intersection between virtual planes and bounding box of volumetric object.

Billeter [38] proposed algorithm based on construct mesh of shadow-volume from shadow map in order to appear effect radial blur around light sources, light shafts also volumetric shadows without used ray marching. This algorithm gives speed up and quality images but it suffer from quite large overhead. Wyman [26] proposed approach to rendering atmospheric effects induced scattering of light due to particles air. This method based on restore intensity points in space which used as texture put in array. Moreover, the sampling is determined of shadow to calculate the intensity for create images at interactive rates. However, this method is used in widely range to render the effects from space also. This method has lack to calculate multiple scattering. Max [44] presented method to rendering light shafts through exploited the single-scattering assumption based on scan-line algorithm. In this method is calculated the radiance which reaching to the viewer by adding the contributions of the scattered light in the lit volumes only. However, the perception is considered a light source and participating media of isotropic scattering and constant density. Li [45] proposed consolidated volumes representation for shaft of light and shadow volume to simulate shadows and light shafts due to effect light scattering in atmosphere. Furthermore, in this method introduced analytical approach to solving scattering instead of airlight integral and exploited the capabilities of graphics hardware to execute integral calculation on each volume surface for scattering.

VII. Discussion and Future Research

In this article, we reviewed most important methods that used within participating media for rendering natural phenomena such as volumetric shadows and volumetric lighting. Ray tracing is a good method for rendering realistic static scenes, while radiosity is a good method for rendering of realistic dynamic scenes. However, these methods both still infeasible for rendering scenes in real-time.

Wyman [36] has presented a method in high perform to generate volumetric shadows. Chen [27] have demonstrated a very fast method to render shadow-volume-based approach. Also well, Billeter [38] proposed the simpler one to execute and has comparable perform as the ray-marching techniques.

Morgan [38] presented a method to create light shafts based on stochastic transparency using shadow maps to give color effect correctly in real-time. Gautron [22] proposed approach to apply color buffer for radiance received by visible objects to compute light frustum form image-based shadows and average extinction of participating media [38]. Table I shows summary a classification of the image-based shadows and geometric-based shadows method to render volumetric lighting and volumetric shadows.

The methods that used in generating both the volumetric shadows volumetric lighting not meet the realism of the rendered scenes. We hope that an incorporated between volumetric shadows and volumetric lighting to obtain more realistic [46], [47], [27].

TABLE I
Classification of the Image-Based Shadows and Geometric Based Shadows

Model	Shadow	
	Image-based	Geometric-based
Volumetric shadows	[23]-[27]	[36], [34], [37]
Light shafts	[21], [19], [38]-[41]	[37], [26], [42]-[44]

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