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Abstract

The present study examines visual, spatial-sequential, and spatial-simultaneous working memory (WM) performance in children with mathematical learning disability (MLD) and low mathematics achievement (LMA) compared with typically developing (TD) children. Groups were matched on reading decoding performance and verbal intelligence. Besides statistical significance testing, we used bootstrap confidence interval estimation and computed effect sizes. Children were individually tested with six computerized tasks, two for each visuospatial WM subcomponent. We found that both MLD and LMA children had low visuospatial WM function in both simultaneous and sequential spatial WM tasks. The WM deficit was most expressed in MLD children and less in LMA children. This suggests that WM scores are distributed along a continuum with TD children achieving top scores and MLD children achieving low scores. The theoretical and practical significance of findings are discussed.

Keywords: Visuospatial working memory; Mathematical learning disability; Developmental dyscalculia; Low mathematics achievement

The underlying structure of visuospatial working memory in children with mathematical learning disability

Mathematical learning disability (MLD), referred to also by the term Developmental Dyscalculia by some researchers (e.g. Butterworth, 2005; 2008), is characterized by weaker than average mathematical achievement due to severe impairments in the acquisition of mathematical skills. Different terms and criteria are currently used in reference to MLD (see, Devine, Soltész, Nobes, Goswami, & Szűcs, 2013; Szűcs, 2016 for reviews). However, severe and more compromised clinical profiles of MLD (with typical performances under 10th percentile) should be distinguished from mild mathematical learning difficulties, in which children are usually scoring above the 16th percentile on standardized mathematics achievement tests (Mazzocco, Devlin, & McKenney 2008; Murphy, Mazzocco, Hanich, & Early, 2007). In fact, the lack of uniform criteria may be responsible for the variability in measuring the prevalence of MLD, as suggested by Devine, and co-authors (2013).

The association of MLD with impairments in working memory (WM) has been demonstrated in a number of studies (see Raghubar, Barnes, & Hecht, 2010 for a review, and Szűcs, 2016 for a meta-analysis). The majority of studies rely on the WM model proposed by Baddeley (1986; 2000) distinguishing between a central executive (responsible for a range of regulatory functions, such as attention, control of action, and problem-solving) and two slave systems: the phonological loop and the visuospatial sketchpad, which holds and manipulates material respectively in a phonological code or in a visuospatial code. However, in recent decades, alternative models of WM - emphasising the complex architecture of the system and its limits in the amount of information that can be stored and processed - have been proposed (see Conway, Jarrold, Kane, Miyake, & Towse, 2007, for a review). Although there are many studies on WM, there is relatively little research available regarding visuospatial working memory (VSWM). As of yet, there is no consensus on how VSWM is organized. Nonetheless, some interesting and useful information has emerged on this topic.

Different VSWM models have been proposed. According to Logie (1995), VSWM involves a visual store, (i.e, *visual cache*) which provides a temporary store for visual information (i.e., colour and shape), and a rehearsal mechanism, (i.e., *inner scribe*), which handles information about movement sequences and provides a mechanism through which visuospatial information can be rehearsed into this system. Others argued that VSWM tasks differ in terms of how the memory content is presented, i.e., static as opposed to dynamic (Gathercole, & Pickering, 2000; Pickering, Gathercole, Hall & Lloyd, 2001; Pickering, Gathercole & Peaker, 1998). In particular, Lecerf and de Ribaupierre (2005) proposed a distinction between three VSWM components rather than only two systems. This model includes an extra-figural encoding system, responsible for anchoring objects to an external frame of reference, and two intra-figural encoding system components, based on the relations each item presents within a pattern: pattern encoding (leading to a global visual image), and path encoding (leading to sequential-spatial locations). In a similar vein, other authors proposed a distinction between visual WM tasks, requiring memorisation of shapes and colours, and two kinds of spatial tasks: simultaneous and sequential (Cornoldi & Vecchi, 2003; Mammarella, Pazzaglia & Cornoldi, 2008). According to this model, both spatial tasks involve the storage of spatial pattern, but they differ in presentation format and type of spatial processes involved: simultaneous in one case and sequential in the other (Mammarella, Pazzaglia & Cornoldi, 2008; Mammarella, Borella, Pastore, & Pazzaglia, 2013a). Evidence collected with various groups of children supported the distinction between visual and spatial-simultaneous processes (Mammarella, Cornoldi & Donadello, 2003) and between spatial-simultaneous and spatial-sequential processes (Mammarella, Cornoldi, Pazzaglia, Toso, Grimoldi & Vio, 2006; Carretti, Lanfranchi, & Mammarella, 2013). The present study has been based on this model.

Visuospatial working memory and mathematics

Spatial processing plays an important role during the execution of different mathematical tasks. For example, solving a mental arithmetic problem requires following a spatial procedural sequence of steps and, according to the complexity of the problem itself, detecting all the relevant information to achieve the solution (Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Li & Geary, 2017). It is noteworthy that only few studies analysed the relationship between mathematical learning and the specific sub-component of VSWM in typically developing children (Holmes, Adams, & Hamilton, 2008; Reuhkala, 2001; Kyttälä & Lehto, 2008; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014) or in children with other learning difficulties (Mammarella, Lucangeli, & Cornoldi, 2010). In a very interesting study, Holmes et al., (2008) investigated age-related changes between math achievement and VSWM sub-components in primary school children, using two different tests requiring the simultaneous (i.e., static) or sequential (i.e., dynamic) processing of spatial information. The results of this study highlighted that spatial-sequential processing predicted algebra scores in younger children, whereas spatial-simultaneous processing predicted algebra performance in older children.

An increasing number of studies revealed specific WM impairments in individuals with MLD (e.g., Krajewski, & Schneider, 2009; Mammarella, Caviola, Lucangeli, & Cornoldi, 2013b; Passolunghi & Siegel, 2001; Schuchardt, Maehler, & Hasselhorn, 2008). However, conflicting results have been reported concerning the role of VSWM. Specifically, some authors found VSWM impairments in children with MLD (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Passolunghi & Mammarella, 2010; 2012; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013; van der Sluis, van der Leij, & de Jong, 2005), while others showed no differences between MLD and control children in VSWM tasks (Bull, Johnston, & Roy, 1999: Geary, Hamson & Hoard, 2000). Among these studies, only few compared the performances of children with MLD and typical development in visual vs. spatial WM tasks, showing that children with MLD were specifically impaired in the latter (e.g., Passolunghi & Mammarella, 2010; 2012).

To the best of our knowledge, only one meta-analysis investigated the role of verbal and VSWM on MLD (Szűcs, 2016). Results from this meta-analysis showed that studies in which reading decoding abilities were controlled for had lower effect sizes on verbal WM tasks and higher effect sizes on VSWM tasks, while the opposite was found in studies not controlling for reading. This finding seems to indicate that reading achievement should always be considered and controlled when interpreting WM discrepancies between children with MLD and typical development.

The present study

Given the conflicting findings observed in VSWM tasks in children with MLD, in the present study we aimed at testing visual, spatial-simultaneous and –sequential WM tasks in these children. Starting from a large sample, we examined VSWM abilities in three groups of children: i) with severe MLD, ii) with low math achievement (LMA; see for example, Mazzocco et al., 2008; Murphy et al., 2007), and iii) with typical development (TD). The distinction between children with severe MLD and LMA was included in the study to shed further light on the notion of mathematic disability. In particular, our objective was to shed light on whether MLD should be represented as a quantitative extreme of the cognitive skills associated with mathematical achievement, or as a discontinuous qualitative difference between MLD and TD children. In addition, as suggested by Szűcs (2016), we decided to match our groups for reading, decoding and verbal intelligence.

To test VSWM, two visual tasks (i.e., houses and balloons) derived from Mammarella, et al. (2008; 2013a) and Passolunghi and Mammarella, (2010; 2012), two spatial-simultaneous (i.e., spatial-simultaneous matrices, with and without grid) and two spatial-sequential tasks (i.e., spatial-sequential matrices, with and without grid) derived from Giofrè, Mammarella, and Cornoldi, (2013a; 2014) were administered. According to previous results (Passolunghi & Mammarella, 2010; 2012), we expected to find an impaired performance in children with MLD more in spatial than in visual WM tasks. To our knowledge, relatively few studies have distinguished between simultaneous and sequential spatial tasks. However, being sequential and simultaneous, tasks are part of the spatial WM subcomponent. In light of this, we hypothesized that children with MLD would present deficits in both types of tasks. Finally, we aimed to test whether children with MLD or LMA matched for their reading decoding ability differed to some extent in their performances on VSWM or whether their performances were distributed among a continuum, going from children with TD to children with severe MLD, with LMA in the middle.

Method

Participants

In the first phase, a large sample of 581 children (309 boys; 272 girls) aged 9 to 10 years old (292 in the fourth grade and 289 in fifth grade) were screened for mathematical ability. Mathematical achievement was evaluated using the AC-MT 11-14 standardized arithmetic battery (Cornoldi & Cazzola, 2004) and the AC-FL (Caviola, Gerotto, Lucangeli, & Mammarella, 2016), measuring math fluency in addition, subtraction and multiplication, both involving a group administration lasting about one hour.

In the second phase, 94 children (52 boys; 42 girls) with mathematics difficulties were identified and other 91 children (46 boys; 45 girls) without math difficulties were randomly selected from our sample. These children were individually tested in two further sessions lasting about 40 minutes each. Additional standardized measures of mathematics ability were administered by using five subtests of the BDE-2 battery (i.e., approximate calculation, quick calculation, number facts, multiplication, addition and subtraction, number judgments; Biancardi, Bachmann, & Nicoletti, 2016). This was done in order to confirm the presence of mathematics difficulties. In addition reading decoding was tested by using lists of pseudo-words (DDE-2; Sartori, Job, & Tressoldi, 2007), and finally block design and vocabulary of the WISC IV (Wechsler, 2004) were administered in order to control for general cognitive skills.

Children were defined as having MLD if they obtained: (1) a very low performance in standardized measures of arithmetic achievement (i.e., $\leq 10^{th}$ percentile in at least two specific aspects of mathematical learning and a total mathematical score of $\leq 16^{th}$ percentile); (2) a significant discrepancy between verbal intelligence and overall performance on arithmetic academic achievement testing (see Schuchardt et al., 2008); and (3) an average score in reading decoding, measured by using lists of pseudo-words. Children were defined as having LMA if they obtained: (1) a low performance in standardized measures of arithmetic achievement (i.e., ≤ 20th percentile in at least two specific aspects of mathematical learning and a total mathematical score of ≤ 30th percentile); (2) a significant discrepancy between verbal intelligence and overall performance on arithmetic academic achievement testing (see Schuchardt et al., 2008); and (3) an average score in reading decoding. Finally, the TD group included children who performed on average in standardized measures of arithmetic achievement, matched for age, gender, vocabulary and reading decoding with the other two groups.

For all children, parental consent was obtained prior to testing. Children were included in the study if they did not belong to disadvantaged sociocultural or linguistic groups.

Based on the previously mentioned criteria, 24 children for each group were selected: the MLD group (M_{age} = 117.42 [7.35] months, 14 M), the LMA group (M_{age} = 116.79 [6.48] months, 10 M), and the typically developing group (M_{age} = 117.58 [6.97] months, 10 M). The three groups were similar in gender, age, reading decoding and verbal intelligence, but differed in terms of math abilities (Table 1).

Table 1 about here

Materials

Visuospatial Working memory battery. Participants were presented with six visuospatial WM tasks mainly derived from Mammarella, et al (2008; 2013a). Several tasks included in the battery have been used in other studies (e.g., Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Giofrè, Mammarella, Ronconi, & Cornoldi, 2013b; Giofrè, et al., 2013a; 2014; Passolunghi & Mammarella, 2010; 2012; Mammarella, Hill, Devine, Caviola, & Szűcs, 2015) and revealed good psychometric properties. All tasks were of increasing difficulty. As for the scoring, we used the partial credit score, expressing the mean proportion of elements within an item that were recalled correctly, as this approach was shown to be more reliable and to increase the predictive validity of WM tasks (see Conway et al., 2005), particularly the visuospatial ones (Giofrè & Mammarella, 2014).

Visual working memory tasks (derived from Mammarella, et al., 2008; Passolunghi & Mammarella, 2010; 2012). The stimuli were schematic drawings seen from the front. Initially, a set of two drawings is shown for 4 seconds (Figure 1). Immediately after presentation, the participant has to recognize the target drawings within a set comprising three stimuli. Then a set of three drawings was presented for the same length of time and the participant must recognize them among a total of five drawings. From there, three larger sets of drawings were also used. The set of four, five, and six target drawings were placed in groups of six, eight and nine drawings, respectively. There were two different tests, in one pictures of houses were presented (*visual working memory, houses*) and in the other, images of balloons were presented (*visual working memory, balloons*). The level of complexity was defined as the number of houses/balloons to be recognized (from 2 to 6) (Figure 1). Cronbach's α = .72 for houses and .86 for balloons.

Spatial-simultaneous matrices tasks (derived from Giofrè, et al., 2013a; 2014). Participants were presented for 1.5s with a 5 × 5 grid. The number of black dots presented in each grid ranged from 2 to 8. After 3s the initial stimulus was removed and participants were presented with a blank test matrix in which they had to indicate the previously filled squares. There were two different conditions: in the first, the targets appeared and disappeared on a visible (5×5) grid in the center of the screen (*spatial-simultaneous matrices, grid)*; in the second, the targets appeared and disappeared on a plain white screen with no grid (*spatial-simultaneous matrices, no grid*) (Figure 1). Cronbach's α = .89 for spatial-simultaneous matrices, grid and .84 for no-grid.

Spatial-sequential matrices tasks (derived from Giofrè, et al., 2013a; 2014). Short-term visuospatial storage capacity was assessed by means of two location span tasks. The children had to memorize and recall the positions of black cells that appeared briefly (for 1 second) in different positions on the screen. After a series of black cells had been presented, the children clicked on the locations where they had seen a black cell appear. The number of black cells presented in each series ranged from 2 to 6. There were two different conditions: in the first, the targets appeared and disappeared on a visible (5×5) grid in the center of the screen (*spatial-sequential matrices, grid)*; in the second, the targets appeared and disappeared on a plain white screen with no grid (*spatial-sequential matrices, no grid*) (Figure 1). Cronbach's α = .83 for both spatial-sequential matrices, grid and no grid.

Figure 1 about here

Procedure

Participants were tested in an individual session lasting approximately one hour in a quiet room outside the classroom. All the WM tasks were presented on a 15" laptop and were programmed using E-prime II (Psychology Software Tools, Inc., Pittsburgh, PA, USA). Each task began with two training trials and the administration order was counterbalanced. Performance on the VSWM tasks was measured using partial credit scores (Conway et al., 2005) by considering the proportion of items accurately reproduced for each series length.

Statistical analyses

To be consistent with previous studies we computed statistical significance by performing "traditional" parametric tests on our data. We also computed standardized effect sizes following the the criteria of Cohen (1988): .01, .09, and .25 for the partial eta square (η^2_p), and .20, .50, and .80 for the Cohen's *d* were considered to be small, medium, and large effects, respectively.

In addition, we employed robust, distribution independent, bootstrap statistics. According to these, 'significant' differences appear if appropriate 95% bootstrap confidence intervals do not overlap. Hence, the term ‘significant’ will refer to such differences in confidence intervals. All bootstrap confidence interval estimations used 100,000 permutations with replacement (Chihara & Hesterberg, 2011) and computed bias-corrected and accelerated (BCa) confidence intervals (Efron, 1987). We assessed group differences by computing 95% BCa bootstrap confidence intervals for the main measures, which provide a better statistical solution than simply reporting *p*-values (Cumming, 2014).

Group differences.

Visual working memory. We performed a MANOVA comparing visual working memory tasks (houses and balloons) by group. We found a significant effect of group, $F(4,136) = 3.11$, $p = .017$, $\eta^2_p = .084$, with a small effect size. Univariate ANOVAs showed statistically significant differences in both visual working memory tasks with houses, $F(2,69) = 3.21$, $p = .046$, $\eta^2_p = .085$, and with balloons, $F(2,69) = 4.51$, $p = .014$, $\eta^2_p = .116$). Post hoc pair-wise comparisons, made by using the Tukey's HSD test, showed statistically significant differences only between TD and MLD groups. Bootstrap analyses confirmed the previously mentioned effects. In terms of magnitude of effect sizes, differences between TD and LMA, and between LMA and MLD, were small, while moderate differences between TD and MLD were found (Figure 2).

Spatial-simultaneous matrices. We performed a MANOVA comparing spatial-simultaneous matrices tasks (with grid and with no grid) by group. We found a significant effect of group, $F(4,136) = 5.06$, $p = .001$, $\eta^2_p = .129$, with a medium effect size. Univariate ANOVAs showed statistically significant differences in both spatial-simultaneous tasks with grid, $F(2,69) = 9.46$, $p < .001$, $\eta^2_p = .215$, and with no grid, $F(2,69) = 6.38$, $p = .003$, $\eta^2_p = .156$. Post hoc pair-wise comparisons, using the Tukey's HSD test, showed statistically significant differences only between TD and MLD and between TD and LMA groups. Bootstrap analyses confirmed the effects of post hoc tests (see Table 2). In terms of magnitude of effect sizes, differences between LMA and MLD were small, while differences between TD and MLD and between TD and LMA ranged between moderate to large (Figure 2).

Spatial-sequential matrices. We performed a MANOVA comparing spatial-sequential matrices tasks (with grid and with no grid) by group. We found a significant effect of group, $F(4,136) = 4.84$, $p = .001$, $\eta^2_p = .125$, with a medium effect size. Univariate ANOVAs showed statistically significant differences in both spatial-sequential tasks with grid, $F(2,69) = 8.17$, $p = .001$, $\eta^2_p = .191$, and with no grid, $F(2,69) = 7.91$, $p = .001$, $\eta^2_p = .187$. Post hoc pair-wise comparisons, using the Tukey's HSD test, showed statistically significant differences only between TD and MLD and between TD and LMA groups. Bootstrap analyses confirmed the effects of post hoc tests (Table 2). In terms of magnitude of effect sizes, differences were small between LMA and MLD, moderate between TD and LMA, and large between TD and MLD (Figure 2).

Table 2 and Figure 2 about here

Discussion

The main aim of the present study was to further elucidate the cognitive profiles of children with MLD and LMA by comparing their VSWM performance. The distinction between visual, spatial-sequential and spatial-simultaneous tasks was analysed in children with MLD and LMA and compared to a TD group matched for reading decoding and vocabulary. Additionally, the distinction between children with severe MLD and LMA was tested in order to analyze the presence of different performances in these VSWM subcomponents. In fact, previous studies showed conflicting results regarding VSWM impairments in children with MLD. To our knowledge, there is very little previous research testing VSWM subcomponents in children with MLD.

In agreement with previous results, we found that children with MLD were more impaired than TD children in spatial than in visual WM tasks, and large and comparable effect sizes were observed both for spatial-simultaneous and sequential WM tasks (Passolunghi & Mammarella, 2010; 2012; Szűcs, et al. 2013). In contrast, children with LMA differed more from TD children on spatial-simultaneous tasks than on spatial-sequential ones. There were no statistically significant differences between MLD and LMA either on spatial-simultaneous or -sequential tasks, as is clearly shown by the absence of differences within the two conditions of each task. This likely demonstrates that VSWM scores were distributed along a continuum ranging from TD children to children with severe MLD, with children with LMA in the middle.

Our data suggests that a robust dysfunction in children with MLD, without reading decoding difficulties and with normal verbal intelligence, is related to difficulties in spatial WM tasks. Specifically, children with MLD performed weakly both in tasks where the presentation order was crucial (i.e., sequential tasks), and in tasks where visuospatial information had to be maintained simultaneously. It is worth noting that multi-digit mental arithmetic problems, involving more than one single step, require several resources of VSWM, not only because of the task's demands on place value concepts (i.e., simultaneous), but also because it is necessary to process several steps (i.e., sequential) while keeping track of partial results (Caviola, et al., 2014; Kyttälä & Lehto, 2008; Li & Geary, 2017). In fact, during the execution of complex mental calculations it is important to correctly apply the right strategy, which usually involves the temporary maintenance of both operands and intermediate results (e.g., 84 – 12 = 80 – 10 = 70, and 4 – 2 = 2).

Visuospatial WM plays a role during the implementation of written calculation procedures (e.g., complex multiplications, or divisions), starting from the correct alignment of the operands, where the visuospatial position of each digit is critical to define the whole number. Within this perspective, previous studies showed that performance in ordinal processing abilities, in which the number comes before/after another in the number sequence, predicted the development of numerical abilities (see Sury & Rubinsten, 2012 for a review). Similarly, Lyons and Beilock (2011) showed that performance in symbolic number-ordering tasks predicted performance on complex mental-arithmetic tasks in young adults. Previous studies also suggested a possible link between ordinal processing in numerical and WM domains (e.g., Van Dijck & Fias, 2011). In a recent study, Attout, Noël and Majerus (2014) revealed that children with developmental dyscalculia performed poorer than control subjects in WM tasks that required the maintenance of the presentation order. In the present study, considering VSWM subcomponents, larger effect sizes were observed on both spatial-sequential and spatial-simultaneous tasks, in research designed to differentiate between children with MLD and TD children. Moreover, as previously mentioned, large effect sizes were observed in the comparison between children with LMA and controls in spatial-simultaneous tasks. Our simultaneous tasks required children to recall an increasing number of dots simultaneously presented in 5 × 5 matrices (the matrices were visible in once condition, but not in the other). It is important to note that the most commonly used WM test batteries (e.g., AWMA, Alloway, 2007; WMTB–C, Pickering & Gathercole, 2001) do not include spatial-simultaneous tasks, although, such tasks seem to be crucial not only for studying children with mathematical difficulties, but also for studying the underlying cognitive processes in children with specific visuospatial difficulties, such as those with nonverbal learning disability (Mammarella, Lucangeli, & Cornoldi, 2010), and with Williams syndrome (Lanfranchi, De Mori, Mammarella, Carretti & Vianello, 2015).

To sum up, our findings indicate that children with MLD are specifically impaired in spatial WM tasks, both simultaneous and sequential, and that MLD and LMA as defined by Mazzocco et al., (2008) and Murphy et al., (2007) had similar large effect sizes in spatial-simultaneous WM tasks when compared to controls, with Cohen’s d ranging from .92 to 1.23 in our sample. In other words, spatial-sequential WM tasks seem most useful for distinguishing MLD from LMA and controls, whereas spatial-simultaneous tasks seem to better discriminate children with LMA from TD children.

Important implications can be draw from the present study. Clarifying the role of VSWM in children with MLD or LMA is important for theoretical reasons, since it may help to cast light on the structure of VSWM. In particular, our findings showed that considering the presentation format of VSWM tasks might be useful in explaining specific impairments of children with mathematics difficulties. In particular, further studies should consider simple vs. complex visuospatial WM tasks by distinguishing among visual spatial-sequential, and simultaneous components, in order to better understand the involvement of simple storage or executive processing in mathematics achievement. Educational and clinical implications can also be inferred from our results, showing that children's difficulties in mathematics are clearly related with VSWM limited resources. For this reason, particular attention should be devoted to support children, especially when mental computation increases complexity by involving greater WM resources.

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Figure Captions

Figure 1. Examples of visual, spatial-simultaneous and spatial-sequential tasks included in the visuospatial working memory battery.

Figure 2. Group differences (expressed in Cohen's d) for each visuospatial working memory test (visual tasks in the upper panel, spatial-simultaneous in the middle, and spatial-sequential in the bottom), by comparing MLD vs. TD; LMA vs TD; and LMA vs. MLD.

Note: MLD = mathematical learning disabilities, LMA = low mathematics achievement, TD = typical development. Error bars represent 95% CI (only Lower Level shown). Differences are statistically significant if error bars do not include 0.

* $p < .05$

Table 1

Characteristics (M= means; SD=standard deviations) of the children with mathematical learning disabilities (MLD), low mathematics achievement (LMA) and typical development (TD) and ANOVAs results

	MLD M (SD)	LMA M (SD)	TD M (SD)	*F(df)=*	*p*	η^2
Maths achievement (AC-MT)						
Written calculation	-2.35 (1.53)	-1.26 (0.86)	0.65 (0.87)	$F(2,71) = 43.31$	.0001[a]	.56
Number words	-0.95 (1.12)	-0.34 (0.53)	-0.04 (0.64)	$F(2,71) = 7.71$	.001[b]	.18
Number ordering	-1.60 (1.88)	-0.72 (0.60)	0.18 (0.60)	$F(2,71) = 13.41$	.0001[a]	.28
Fluency additions	-1.48 (0.92)	-0.73 (0.47)	0.47 (0.32)	$F(2,71) = 59.51$	.0001[a]	.63
Fluency subtractions	-1.47 (0.57)	-0.93 (0.48)	0.60 (0.43)	$F(2,71) = 112.11$	.0001[a]	.77
Fluency multiplications	-1.81 (0.78)	-1.18 (0.79)	0.44 (0.42)	$F(2,71) = 68.58$	.0001[a]	.66
Maths achievement (BDE-2)						
Approximate calculation	-1.32 (0.72)	-0.67 (0.66)	0.51 (0.69)	$F(2,71) = 43.64$	.0001[a]	.56
Quick calculation	-1.06 (0.58)	-0.49 (0.43)	0.63 (0.60)	$F(2,71) = 59.54$	.0001[a]	.63
Number facts, multiplication	-1.92 (1.43)	-0.99 (0.95)	0.06 (0.84)	$F(2,71) = 19.22$	.0001[a]	.36
Number facts, add. & subtr.	-1.20 (1.05)	-0.51 (1.04)	0.43 (0.84)	$F(2,71) = 16.55$	.0001[a]	.32
Number judgments	-1.08 (1.59)	-0.13 (0.85)	0.52 (0.45)	$F(2,71) = 13.46$	.0001[b]	.28
Reading decoding (DDE-2)						
Pseudo-words times	.74 (1.38)	.49 (1.25)	-.03 (.75)	$F(2,71) = 2.77$	.07	.07
Pseudo-words errors	.12 (.99)	.08 (1.18)	-.44 (.69)	$F(2,71) = 2.47$	.09	.06
Intelligence (WISC-IV)						
Vocabulary	11.58 (2.70)	12.17 (2.24)	12.54 (2.45)	$F(2,71) = .92$	.40	.03
Block Design	10.71 (3.07)	12.33 (2.48)	14.50 (2.47)	$F(2,71) = 12.02$	.0001[b]	.26

Note.
[a] = MLD < TD; LMA < TD; MLD<TD
[b] = MLD < TD; MLD < LMA; LMA = TD

Table 2

Means and mean differences, with 95% bootstrapped confidence intervals in brackets, for each VSWM task

	Means		
Means	MLD	LMA	TD
Visual WM tasks			
Houses	0.69 [0.65, 0.73]	0.73 [0.70, 0.76]	0.75 [0.71, 0.78]
Balloons	0.76 [0.74, 0.78]	0.78 [0.76, 0.80]	0.80 [0.78, 0.82]
Spatial-simultaneous WM tasks			
Grid	0.58 [0.53, 0.62]	0.59 [0.55, 0.64]	0.70 [0.67, 0.73]
No grid	0.54 [0.47, 0.60]	0.57 [0.53, 0.62]	0.66 [0.63, 0.69]
Spatial-sequential WM tasks			
Grid	0.58 [0.54, 0.63]	0.63 [0.58, 0.68]	0.73 [0.67, 0.77]
No grid	0.43 [0.40, 0.48]	0.46 [0.43, 0.50]	0.55 [0.50, 0.59]
	Contrasts		
Mean Differences	TD vs. MLD	TD vs. LMA	LMA vs. MLD
Visual WM tasks			
Houses	0.06 [0.02, 0.11]	0.02 [-0.02, 0.06]	0.04 [0.00, 0.09]
Balloons	0.04 [0.01, 0.08]	0.02 [0.00, 0.05]	0.02 [-0.01, 0.05]
Spatial-simultaneous WM tasks			
Grid	0.12 [0.06, 0.17]	0.11 [0.05, 0.17]	0.01 [-0.05, 0.07]
No grid	0.13 [0.06, 0.20]	0.10 [0.02, 0.16]	0.03 [-0.04, 0.10]
Spatial-sequential WM tasks			
Grid	0.14 [0.07, 0.21]	0.09 [0.01, 0.17]	0.05 [-0.02, 0.11]
No grid	0.11 [0.06, 0.17]	0.08 [0.02, 0.14]	0.03 [-0.03, 0.07]

Note. MLD = mathematical learning disabilities, LMA = low mathematics achievement, TD = typical development. Contrasts are statistically significant if confidence intervals do not include zero.

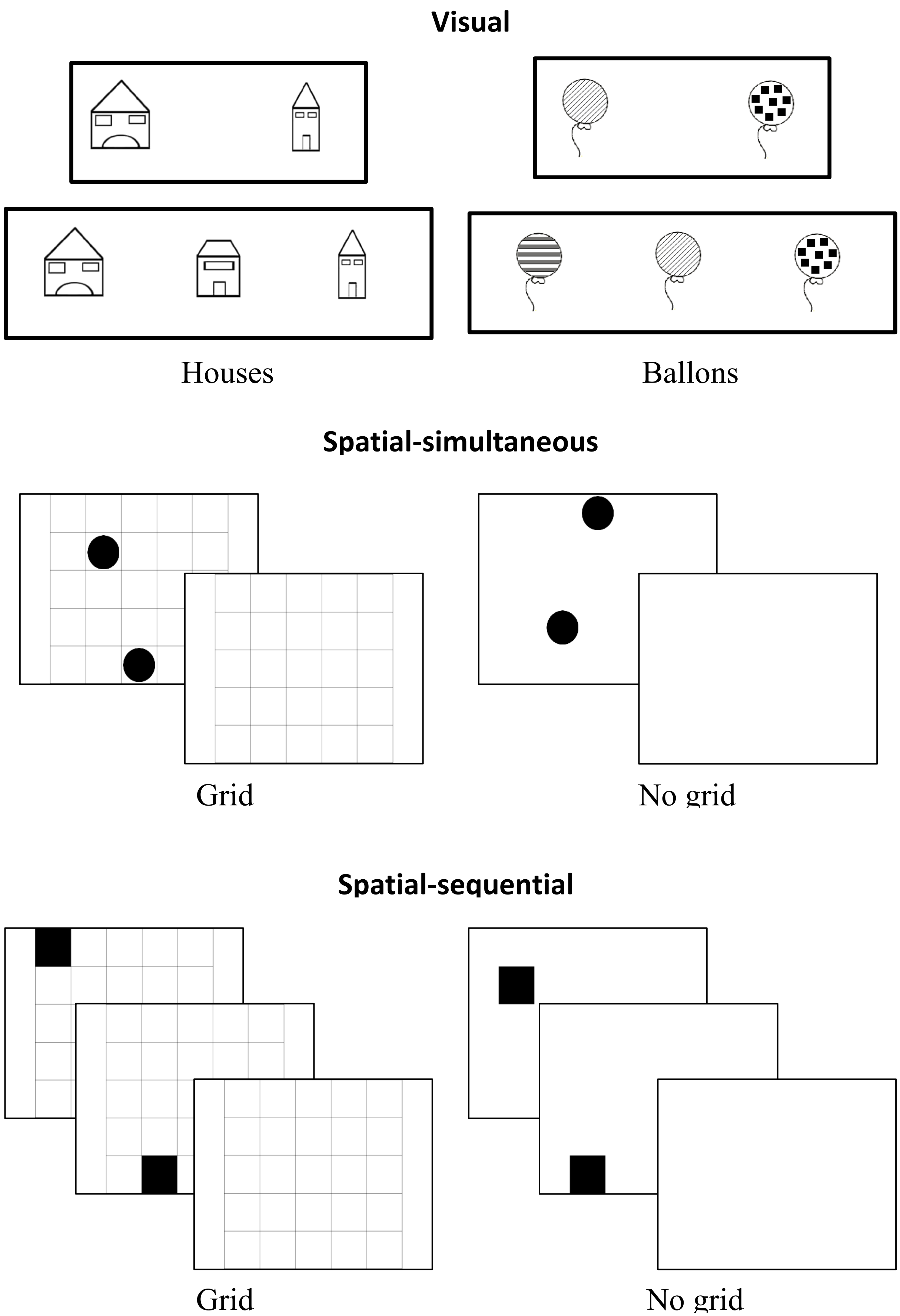
Visual
Houses
Ballons
Spatial-simultaneous
Grid
No grid
Spatial-sequential
Grid
No grid

Figure 1.

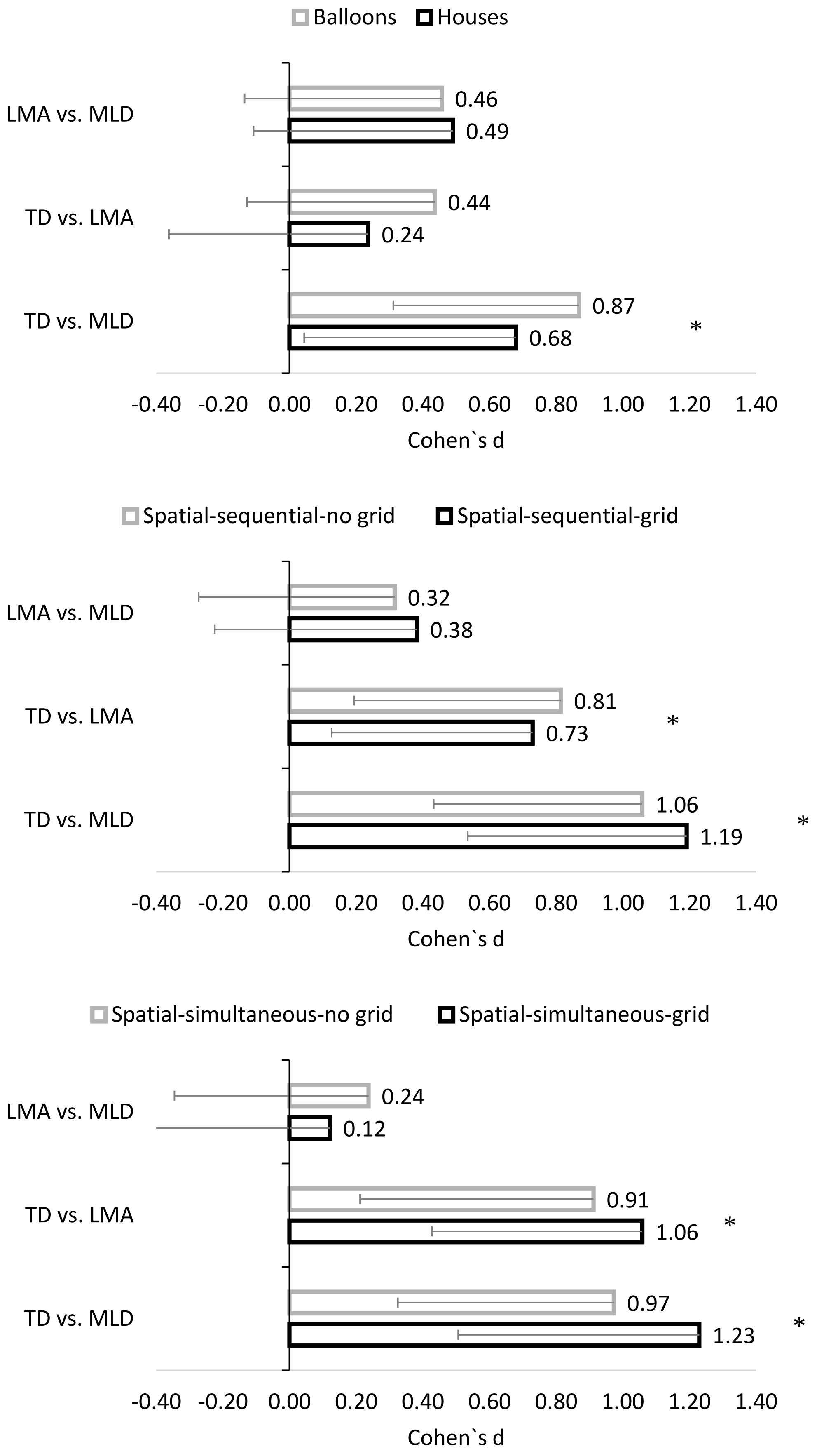
Balloons
Houses
LMA vs. MLD
TD vs. LMA
TD vs. MLD
0.46
0.49
0.44
0.24
0.87
0.68
*
-0.40 -0.20 0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40
Cohen`s d
Spatial-sequential-no grid
Spatial-sequential-grid
LMA vs. MLD
TD vs. LMA
TD vs. MLD
0.32
0.38
0.81
0.73
*
1.06
1.19
*
-0.40 -0.20 0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40
Cohen`s d
Spatial-simultaneous-no grid
Spatial-simultaneous-grid
LMA vs. MLD
TD vs. LMA
TD vs. MLD
0.24
0.12
0.91
1.06
*
0.97
1.23
*
-0.40 -0.20 0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40
Cohen`s d

Figure 2.