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Photorealistic Rendering: A Survey on Evaluation

**Hoshang Kolivand ·
Mohd Shahrizal Sunar · Samira Y.
Kakh · Riyadh Al-Rousan · Ismahafezi
Ismail**

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Abstract This article is a systematic collection of existing methods and techniques for evaluating rendering category in the field of computer graphics. The motive for doing this study was the difficulty of selecting appropriate methods for evaluating and validating specific results reported by many researchers. This difficulty lies in the availability of numerous methods and lack of robust discussion of them. To approach such problems, the features of well-known methods are critically reviewed to provide researchers with backgrounds on evaluating different styles in photo-realistic rendering part of computer graphics. There are many ways to evaluating a research. For this article, classification and systemization method is use. After reviewing the features of different methods, their future is also discussed. Finally, dome pointers are proposed as to the likely future issues in evaluating the research on realistic rendering. It is expected that this analysis helps researchers to overcome the difficulties of evaluation not only in research, but also in application.

Keywords Evaluation, validation, benchmarks for rendering, real-time rendering, realistic rendering

H. Kolivand
Department of Computer Science, Liverpool John Moores University, Liverpool, L3 3AF, UK E-mail: H.Kolivand@ljmu.ac.uk

Mohd Shahrizal Sunar, Riyadh Al-Rousan
MaGIC-X (Media and Games Innovation Centre of Excellence) UTM-IRDA Digital Media Centre Universiti Teknologi Malaysia 81310 Skudai Johor Malaysia

Samira Y. Kakh
EDENZ Colleges, Auckland, New Zealand

Ismahafezi Ismail
Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Malaysia

1 Introduction

Evaluation as defined by Patton [1] is "the systematic collection of information about the activities, characteristic and outcomes of an activity or action, in order to determine its worth or merit". Patton [2][3] argues that a principal evaluation method in any specific research can stand on as a dynamic process in which both *quantitative* and *qualitative* measures play significant roles.

Evaluation is a critical argument. It assists and convicts investors to find out the comparison between different methods for an existing problem [3–5]. Evaluation may contribute to making correct decisions and coming up with an appropriate methodology to achieve goals.

Evaluation not only plays a crucial role in the quality of publication but also it contributes to the quality of product in the market. A critical literature review that provides appropriate validation and evaluation of the existing literature would be beneficial for immediate users of scientific articles. To this aim different types of evaluation in this area were investigated. Then, the major studies that have been a benchmark for researchers were considered.

Photorealistic rendering suffers from lack of systematic evaluation and validation of qualitative and quantitative studies. Commonly, Frame Per Second (FPS) and Big-O notation are used to evaluate quantitative studies. This is while validating qualitative research in computer graphics has remained problematic [6][7][8][9][10][11].

In evaluating and validating computer generated data particular considerations should be taken. Enhancing the quality of digitally generated data usually speeds down the performance of algorithms. Considering this feature is essential in evaluation process [12]. Some projects are highly outcome focussed and high visual image quality preferred. In this case, the number of frame per second is the secondary priority. In such cases off-line rendering is preferred, but there are cases in which real-time rendering is necessary. Therefore, considering the purpose of rendering is essential in evaluation. Nevertheless, when evaluation is done for a general conclusion, a method with a good balance of quality and speed is considered outstanding in photorealistic rendering research.

In this regards, knowing different types of evaluation can be beneficial as some evaluation methods may not be applicable in some situations. For instance, in analysing research methodologies careful selection of evaluation method is required. The first step towards this aim is considering qualitative and quantitative approaches in evaluation [5]. Choosing a suitable evaluation method in the field of rendering is critical. Categorization can be start with photorealistic rendering in both virtual reality and mixed reality. Speed-up studies is another main category which is also evaluated by diversity benchmarks. In this survey, different benchmarks in different areas of rendering techniques are analysed. Then, widely used benchmarks are categorized. Finally, the directions to which future rendering evaluation should proceed are discussed.

Evaluation and validation of the outcomes of photorealistic rendering mainly involve comparing the quality of digital images with real ones. The aim of realism in photorealistic rendering is to generate indistinguishable images form photographs [13]. It is sometimes necessary to integrate several evaluation methods to reach a meaningful conclusion.

This literature review paper focuses on four main keywords: *Photorealistic rendering evaluation*, *Photorealistic rendering techniques*, *Outdoor rendering*, *Indoor rendering* and *Realistic real-time rendering*. The corpus was constructed by searching ACM, IEEE, Springer, Science Direct, and Taylor and Francis databases for articles with the stated keywords. The corpus is made of articles that report some degrees of innovation. The corpus then analyzed critically by a particular focus on evaluation methodologies and strengths and shortcomings of the methods. This study also highlights the different characteristics of frequently used methods. Moreover, the evaluation sections of the selected articles were analyzed for their theoretical and practical benefits of the proposed techniques. This analysis critiques the primary goals of a method and its limitations that should be further improved [5]. This inclusive evaluation seems necessary as it could provide an overview of the features of available methods and improve the quality of decision making when it comes to choosing a method for a particular purpose.

In this survey, highly investigated methodologies or perspectives of evaluation are categorized. Taking an unbiased perspective, the information related to photorealistic rendering is systematically summarized, categorized, and evaluated. Unbiased evaluation reveals strengths and shortcomings of existing methods. This includes eliminating publication bias, which refers to evaluating the positive aspects that authors discuss and highlighting considerable negative aspects of methodologies. Such information would help users in decision-making.

Additionally, photorealistic rendering in Virtual Reality and Augmented Reality is taken into consideration. Some technical terminologies are playing key roles in understanding the content of this article. In what follows, these terms are defined and explained.

Rendering : The process of generating digital product form virtual scenes. The result of this process is called image synthesis or rendering.

$Real - time\ rendering$: This refers to the situations in which an image must be generated in incredibly rapid pace. They are mostly used in gaming, interactive virtual, and mixed environments.

$Offline\ rendering$: This refers to the cases that time is not a big issue or the process can be shared in multiple CPUs. It is mostly used in animation effect work.

Photorealistic rendering : This is rendering virtual scenes by adding elements such as colour, lighting, shading, shadows so that they appear more life-like.

Augmented Reality : It is a real-time integration of digital information with real environments.

The idea of this article came to mind when the authors were working on a project on photorealistic rendering. Photorealistic rendering is engaged with both indoor and outdoor rendering. As the parameters for indoor and outdoor rendering are different, identifying some main criteria for evaluating both indoor and outdoor environments was necessary. Moreover, since virtual reality and augmented reality are different in producing photorealistic images, the criteria for evaluation were selected based on the main resources of lighting. Accordingly, for outdoor illumination (the sun and the sky) and for indoor (interior lighting) were chosen.

2 Psychophysical and Questionnaire Methods

Questionnaires are made of sets of coherent questions. The questions should be logically related to the issue and responses from respondents. Questionnaires just like any other data collecting instruments have advantages and disadvantages. The main advantages of them are being practical, collecting a large amount of data in a short while, and having quick and straight forward analyses. The main disadvantage of questionnaires is unreliability. According to Ackroyd [14] and Popper [15] this unreliability is the result of users' lack of focus and critical thinking.

Although questionnaires are used for qualitative data which can be asked by discrete questions such as *how many*, *how often* and *how long*, they can be also employed for qualitative data such as *how satisfy* and *how realistic* using categorisation like *virtual*, *little realistic*, *realistic* and *more realistic* [16].

A widely-used technique for evaluation in many disciplines is questionnaire. In Computer Graphics, some researchers have employed questionnaire technique to validate their work. This technique asks expert and non-expert users to evaluate the results based on their expectations and satisfaction. Usually an evaluation done by an expert in the area, like a university professor, produces more meaningful information compared with that of an inexpert user. Although this technique is not as rebuts as it should be, it can present us with valuable information.

2.1 Virtual and Augmented Reality

2.1.1 Outdoor Rendering

Generating realistic objects in the case of virtual environments and Augmented Reality (AR) is the ultimate goal in Computer Graphics [17]. Many research studies have compared virtual environments with real ones to evaluate and validate studies. The comparison between the speed of rendering in FPS [18, 19], memory consuming [20,21], and data analysis [22] are the main qualitative factors to be considered. This is a more valuable virtual benchmark compared to other existing methods.

Sunar did sky colour modelling [6] based on Perez Model [23]. In order to evaluate the method a questionnaire was distributed among some expert and inexpert users to compare real sky colour with the model. This method was proved to be robust by asking several users during the software testing stage. Kolivand and Sunar [24] evaluated the trade-of between realistic AR outdoor environments and FPS using the two following aspects: 1) comparing real sky for the realistic parts and 2) comparing numerical evaluation for the qualitative parts. Kolivand and Sunar did not use a questionnaire to evaluate their work, but they validated the realism side by side with real sky in different locations.

2.1.2 Indoor Rendering

Shadow mapping is implemented in augmented reality systems by Sugano et al. [22]. They employed pre-creation of phantom to cast virtual shadows on real environments. The work focused on the ability of shadows in AR systems as the title desired. The technique was examined in two cases, changing the presence of virtual objects by representing the shadows and the effects of the shadow orientation by changing the position of light. A static and an animated object were considered to highlight the effects of shadows in AR. Twenty undergraduate and graduate students were chosen to complete questionnaires about the realistic-ness of AR system with and without shadows. They wore a binocular and a monocular COASTAR [25]. Number 1 shows the presence without shadows and number 5 shows more presence with shadows. Making use of the results of the data from the questionnaire, Sugano et al. proved how shadows would increase the realism of AR systems. The results are illustrated in some charts for reveal the Low/High and With/Without Shadow effects for the presence of virtual objects in the cases of binocular and monocular. There is no focus on of the time delay, when the virtual images are augmented in the real world. Although, the target was realistic rending, the produced images had low resolution. An issue which was ignored.

Some researchers used several methods to evaluate their work. For instance, Grasset et al. [26] employed three methods to evaluate their technique, which places labels in augmented reality systems. The first method was comparing visual quality to other works. The second method was studying the performance of the technique on a desktop and a handheld platform for six different operations such as resize, saliency, edges, thresholding, layout, and representation. The third method was distributing questionnaires. The aim of distributing questionnaires was collecting users' feedback on the proposed image-based layout and adaptive representation method: 4 conditions for the layout and 5 conditions for the adaptive representation. The researchers validated the technique for three criteria including: scene understanding, readability, aesthetics, which refer to identifying real and virtual, ability to read the labels, and subjective satisfaction of end-users respectively. The number of users were 7 students (4 females and 3 males) aged between 21 to 29. The results showed the percentage of each criteria.

Reflections, refractions and caustics are opened issues in augmented reality. Kan and Kaufmann [27] presented a high-quality rendering technique based on ray-tracing using Fresnel reflection [28]. The aim of this technique was to reveal the caustic, reflection and refraction of real environments on non-rigid objects such as glass. Caustics were created using a GPU implementation of photon mapping [29] for both virtual and real specular objects. They used a camera reprojection method similar to Grosch [30] to address the incoming radiance of environments reflected on glass.

3 Qualitative Methods

Some works must be evaluated for realism only. In this case, there is no need to take rendering speed, such as generating realist objects for non-real-time, into account. Since the only criteria is realism, numerical methods cannot evaluate these kinds of research. For evaluation therefore, some cases have to be evaluated by virtual environments [31,32] and some others must be evaluated by real environments [6,33,9,10,34,11,35].

3.1 Virtual Environments

3.1.1 Real Benchmarks

Sunar [6] modelled sky colour based on Perez Model [23]. He compared it with the real sky colour to evaluate and validate his work. Although this method is not robust in general, it is the only way to evaluate the work.

Applying shadow volumes in augmented reality to cast virtual shadows on virtual and real objects have been done by Haller et al. [36]. In this algorithm, a virtual object such as the real one must be modelled in advance using 3D software. These simulated objects are called Phantoms. The silhouette of both the virtual and the phantom objects must be detected. Phantoms must be placed in the location of the real objects. Generating phantoms is one of the issues with this technique. Phantoms cannot be constructed in real-time. The method is expensive due to employing shadow volumes. As the technique was innovative in a sense that it was the first method that casted virtual shadows on real environments, there were no other benchmarks to compare it with. They show the robustness and capability of the technique by illustrating many figures in different scenes to be compared with real shadows on other objects that users see in real environments. Kolivand et al. [37] employed HSM [38] based on shadow mapping [39] and figured out the problem of static phantoms in [36]. They generated an interactive mixed environment, in which virtual shadows can be cast on real objects in real-time.

Deep Shadow Maps was a technique proposed by Lokovic et al. [40]. This technique was aimed at generating shadows for hair, fur, and smoke. In contrast to traditional shadow maps that store one depth for each pixel, deep shadow maps store fractional visibility functions that represented the visibility through a given pixel at all different depth levels. The biggest shortcoming of deep shadow maps is that they are more expensive to render than traditional shadow maps with the same pixel resolution. The results were validated by comparing them with the shadows of real hair, fur, and smoke.

3.1.2 Virtual Benchmarks

To generate soft shadows, Mehta et al. [32] presented a technique based on adaptive image-space filtering. The accuracy of the method was because of using Monte Carlo sampling. Optix GPU raytracing was the reason for obtaining an acceptable speed. The shortcoming of this method was its expensive post-processing step which was based on sheared filtering reconstruction. They extended the axis-aligned filtering for soft shadows for developing the image-space filtering method for inter-reflections. Mehta et al. [41] proposed Axis-Aligned Filtering for interactive rendering which was a physical based illumination. The idea was to reveal the indirect illumination. It was based on Monte Carlo ray using GPU-accelerated raytracer and diffused indirect and specular lighting were also taken into account. They evaluated their work with Monte Carlo Ground Truth, because of the similarities between the techniques. Finding such a benchmark for evaluating the results have been a common way of validating works.

In order to reduce aliasing, Donnelly et al. [42] proposed an algorithm entitled "Variance Shadow Maps" (VSM) based on shadow mapping. This algorithm was suitable for generating semi-soft shadows. Diversity comparison in terms of different resolution with different shadow algorithms were the evaluation method in this paper. One of the main concerns with VSMs was light bleeding, especially in complex scenes. "Layer Variance Shadow Mapping" [43] had been proposed to solve the bleeding problem of VSM which suffered from overlapping. They evaluated LVSM with a deep comparison by conventional shadow maps[39], VSM [42], CSM[44] in different layers. This method is one of the strongest methods to evaluate and validate photorealistic rendering results.

To develop an effective arbitrary linear shadow filter Annen et al. [20] proposed a technique named "Convolution Shadow Maps". Convolution Shadow Maps replaced storing depth values at each pixel by encoding a binary visibility function. A comparison between shadow maps[39], PCF[45], VSM [42], and CSM[44] showed that validation was based on virtual benchmark. These methods of evaluation were employed in the other algorithm which called "Exponential Shadow Maps" [46].

In order to generate a volumetric soft shadows, Billeter et al. [19] used polygonal light volumes. They introduced a more appropriate method of computing the effects of scattering in homogeneous contributor media for real-time purposes rather than using the common old ray-marching based algorithms. A polygonal mesh was used to enclose the volume of space that was directly illuminated by a light source. The algorithm was based on Max's [47] algorithm which generated shadow volumes from a shadow map using the GPU raster to compute the lit segments. The algorithm sharpened the shadow map resolution and it was fast enough for low resolution shadow maps. However, it was not convenient when the shadow map resolution reached 4086 by 4096. They validated the algorithm in three variety light volumes. Two different direct light sources and one omni light source were considered. They proved that although the algorithm performance was not dependent to the number triangle, it was mostly dependent on shadow map resolution. For example, for 32*32*6 shadow maps performed at 230 FPS while 512*512*6 shadow maps ran at almost 25 FPS. They compared the algorithm to ray-marching based algorithms qualitatively and quantitatively. For qualitative section, they illustrated some figures and for the quantitative, they mentioned that the algorithm performed a scene with 400 samples with 96 FPS, while the ray-marching could run it slowly. The algorithm was almost fast in the same quality with others like [48] and the exact quality compare to [48] and [18]. In the same manner, Ali et al. [49] introduced an algorithm called Soft Bilateral Filtering Shadows for dynamic scenes to enhance the realism of soft shadows in real-time. They employed penumbra region to benefit from bilateral filtering and enhance the outline of soft shadows. A side by side comparison between previous works such as Bidirectional Penumbra , HSM [38] and GEARS [50] based on the FPS, along with implementation in the same scenes and same data are the strong aspects of this evaluation. The performance was checked by testing different data sizes to validate the algorithm.

To enhance the appearance of virtual world, significant parameters such as volumetric effects should be considered. Single scattering could also be considered, but it is prohibitively expensive. Baran et al. [51] proposed a hierarchical algorithm apart from regular ray-marching based on partial sum trees. They used an epipolar correction to reduce the scattering integration. They evaluated the algorithm by comparing epipolar sampling and ray-marching [18]. Time of rendering and quality of the light shaft were considered for quantitative and qualitative examination respectively. The time of rendering was computed and compared only for some parts of the algorithm which were not similar to the others. In other word, speed of rendering was calculated only for volumetric scattering integration, which was different from the algorithm with Brute force and epipolar sampling. Big-O notation was also calculated and mentioned as the benchmark algorithms. In addition to CPU basing, the algorithm was implemented on GPU with 33 FPS compare to Brute force, which ran the same scene in 24 FPS. Qualitative part of the evaluation was employed using some figure with description, but quantitative part was done in descriptively instead of being presented in the form of tables and graphs.

In order to illuminate rough surface transparency objects, Rousiers et al. [52] proposed an algorithm. The objects were illuminated from long distance lighting under all-frequency illumination. A spherical Gaussian approximation extended the pre-convolution for refractive materials. They evaluated the algorithm against the stochastic raytracing, which was a widely-used technique at the time. A noticeable point was that the researchers analysed their work from different aspects such as transport model and [53] specular materials [54].

The same was done by Zhou et al.[55] , but they focused only on qualitative part due to the not fully optimized proof of concepts. They compared their reconstructed transmittance with some other works such as A-buffer[56] and 8 nodes of [57]. Fourier series [58] was another part of their evaluation.

In order to simulate thin hype-realistic skin, Li et al. [59] developed an algorithm. The algorithm was suitable for any parts of body that stretches and slides such as muscles, hands and neck. It was used as a pre-processing stage in any animation pipeline. The method was not limited with texture pixels; it could be applied on normal maps too. The evaluation was employed compare to a Skinning method [60] [61]. This comparison can be seen in the video which is posted on YouTube: http://www.youtube.com/watch?v=kh7Bpe2h6Yk.

For fluid simulation, Macklin and Muller [62] proposed an iterative density solver technique. The method allowed the incompressibility and convergence like a modern SPH [63], in real-time. The method was tested by dropping a liquid bunny into a pool of liquid. The results were compared with [64] which had time-steps shortcomings, which made the methods impractical for real-time applications. To validate the algorithm, a graph was drawn which illustrated more than twice increasing in the time-steps.

In order to enhance the efficiency of liquid simulation, Ando et al. [65] proposed a method. This method focused on speed up rendering of fluid simulation for large scales such as Bojsen-Hansen and Wojtan [66] who focused on a combination of Eulerian liquid with a high-resolution surface tracker. Extensive series of tests were performed to evaluate the method in comparison to previous works. They evaluated their work using videos instead of static images.

3.2 Mixed Environments

3.2.1 Real Benchmarks

In a study on outdoor images, Liu et al. [8,9] used sunlight and skylight for live videos. This study showed that the effect of environment and sky colour on virtual objects enhanced the realism of virtual objects. This method could be applied in non-real-time rendering.

In another study on outdoor images, Xing et al. [10,11] employed similar images [8,9] for realistic augmented objects in live videos. They revealed an interaction between the environment and virtual objects in non-real-time rendering. For evaluation purposes comparing the results with real environments were used. Their benchmarks were real objects in the same situation. They also compared their work with previous works such as [67] and [68], which used three methods to estimate the light parameters of the scene. They also had an evaluation based on the required storage capacity compared to the same methods in [67] and [68].

Kolivand et al. [35] [69] focused on Generating realistic virtual objects in augmented reality. In these studies, the researchers worked on sky colour and semi-soft shadows with respect to the position of sun considering different locations, dates, and times. The final results were validated by comparing the similarities and differences of produced images and real environments. The highlighted parts in real and virtual environments were also compared to show the features of the technique and validate it. Similar to the techniques used by Xing et al. [10,11] and Liu et al. [8,9] , they compared the results with the colour of real sky and the effect of sky colour on real objects. A side-by-side comparison between real and virtual object with the same data for the qualitative evaluation along with FPS comparison between different situation of the environments for quantitative ones were the strengths of the evaluation that they did. This was while comparing results with the existing methods was the shortcoming of their evaluation [69]. In their later publications, nevertheless, they tackled this shortcoming [35,24].

In lighting studies, Nowrouzezahrai et al. [70] applied light factorization for mixed frequency shadows in AR to facilitate hard and soft shadows. To do so, these researchers used shadow mapping algorithm with surrounding scene lighting. Although, they emphasized direct and indirect lighting, they could generate hard and soft shadows for static and animated virtual objects in AR. The lighting was captured from the real-world, but only supports a single directional light. They combined marker-based and marker-less technique to place a virtual object in the real scene. There was not any especial focus on qualitative evaluation, but the supplement video shows that the evaluation was based on real lighting. That was while an accurate evaluation on the performance of the algorithm for different parts of methods was done. They computed the projection of the captured lighting by precomputing values in a texture and using graphics hardware to apply this precomputed texture. The evaluation is based on milliseconds for find the markers, LDR to HDR and the projection.

KinectFusion is a method introduced by Izadi et al. [71]. The aim of this method was reconstructing a real-time mesh of real environments. They employed Kinect camera to read deep data and track 3D pose of the Kinect sensor for creating 3D model of real scenes. The algorithm was based on Newcombe et al. [72] who presented a real-time mapping of indoor environments using a Kinect camera to reconstruct the real environments. As this work was completely novel the evaluation part was also different from others. They did not compare the proposed technique with others or even with real objects. Instead, they compared it with an attached video to validate the technique. They also indirectly compared it with real environments by illustrating the realistic reconstructed environments and studying the interaction between real objects and virtually reconstructed environments. In a similar work by Kolivand et al. [37] a reconstruction technique in mixed reality was used. This of this work was casting virtual shadows on real objects in real-time. They performed qualitative and quantitative evaluations. For the qualitative part, the work was compared with four highly used techniques such as Shadow Maps [39], PCF [73], CSMs [44] and HSMs [38]. They tried to show the strength of their algorithm by comparing it with that of Castro et al.[74] , which does not support self-shadowing. For quantitative evaluation, five case studies were done. The environments that were used had different complexities. This study did not compare its results with other works such as Newcombe et al. [72] and Keller et al. [75].

In order to enhance realism, Madsen et al. [76] considered sky illumination and volume shadows in AR. In this study, they did not consider FPS for the performance of their work. In other words, they just took realism into account. They did not use quantitative methods, since they were working on non-real-time rendering. They estimated the outdoor illumination conditions in AR systems by detecting dynamic shadows. Shadow volumes were employed for generating virtual shadows. They also considered direct sun and sky radiances from pixel values of dynamic shadows in life video. In a similar study, Alhajhamad et al. [77] detected the light sources in indoor environments; the light could be generated from a source either inside or outside of the room. The algorithm estimates the position, directions, and intensities properties of the light sources using the analysis of the saturation channel HSV data. They evaluated the performance of their algorithm for detecting single, two, and multiple light sources based on FPS. A side-by-side qualitative comparison with Wu et al. [78] was done to evaluate the results.

3.2.2 Virtual Benchmark

To generate soft shadows in AR, Aittala [79] applied "Convolution Shadow Maps" (CoSMs) [80] by employing both mip-map filtering and fast summed area tables [81]. The aim of this study was enhancing blurring with variable radius. Aittala proposed a solution for observing the results of real lighting conditions in AR based on L_1-regularization. A post-processing imitative image quality was performed after rendering to enhance the image quality. The qualitative evaluation was done by comparing the results with basic OpenGL rendering in AR. The evaluation of the technique was done by comparing the quality of digital images and basic lighting in OpenGL. A more robust evaluation could be done by comparing the results with advanced techniques.

Casting virtual shadows on real environments and vice versa has been a challenging issue in augmented reality. Gruber et al. [82] tried to improve lighting in AR by casting shadows on virtual objects. Revealing the interaction between virtual lighting and real ones was the main target of this rendering pipeline. Estimating the environment light using a fiducial marker for geometric registration and glossy black ball was employed by [83], [84] and [85]. In contrast, Aittala et al. [79] estimated the diffused lighting using a diffused light probe. Gruber et al. [82] tested the pipeline in three different scenes and four different lighting situations. This means that applying the approach in twelve different cases was the method, which validated the pipeline. This study also illustrated the lighting of virtual objects from the real light including shadows. A shortcoming of the presented technique was using a static camera. This was while, Kolivand et al. [37] proposed that this issue could be addressed by casting virtual shadows on realm objects in real-time using a depth camera to reconstruct environments.

In order to track and augment real objects, Zheng et al. [86] proposed a simultaneous model-based method in a close-loop fashion. In this method, real objects had to be tracked and virtual ones rendered. For evaluation, they used qualitative and quantitative methods separately. They utilized three qualitative experiences to show the feasibility of the proposed method, by using projector-based AR, video-based AR, and diminished reality. The projector-based was used by Audet et al. [87], but without colour calibration. The video-based AR was optimised for 3D pose which was similar to Xiao [88]. To remove an object and reconstruct an appropriate background, they used a simple proof-of-concept such as the one in [89].

For dynamic objects, Lensing and Broll [90] considered indirect illumination in AR system using RGB-D camera. This approach was an image-space global illumination which was based on reflective shadow maps to reveal diffuse indirect illumination. In this work, the real direct illumination onto virtual objects and shadows were not considered. For evaluation purposes, they focused on visualisation compare with virtual benchmark. The approach was implemented and compared with the same scene without the approach. The also carried out quantitative measurement for evaluation.

This approach had some limitations such as incomplete representation of the real scene due to lack of depth information for the whole field of view.

Keller et al. [75] extended KinectFusion [71] and worked on online 3D reconstruction in dynamic scenes using point-based fusion. They used a moving sensor to collect depth measurements using a single model which refined it continuously. They tested the technique in seven different types of scenes with 640*640 pixels. The parameters that they considered were i) the number of frames in cases of input and processed frame per second and average timings (ms). They qualitatively compared the proposed system with original KinectFusion [71].

4 Quantitative methods

Quantitative evaluations are simple and usually robust. These evaluations must be implemented in the same environment as the previous work and the results are calculated in numerical values. This kind of results are comparable with previous results. FPS is a widely-used criterion in rendering. This method is efficient, when the speed of rendering is important.

4.1 Virtual Environments

4.1.1 Image Based

It is common to validate a work using both qualitatively and quantitatively methods. Lauritzen et al. [43] firstly compared the proposed soft shadow generation algorithm with other widely used algorithms that mentioned in previous section. They also performed a deep comparison in different layers, resolutions, and FPS.

Convolution Shadow Maps (CoSMs) [20] has become an effective arbitrary linear shadow filter. CoSMs replaced storing depth values at each pixel by encoding a binary visibility function. CoSMs was supporting mip-mapping which helped to improve the quality of tri-linear and anisotropic filtering and decrease screen space aliasing. Another advantage of CoSMs was its ability to apply blur filtering to produce soft shadows directly. Moreover, CoSMs avoided non-planarity problems by using double-bounded for its shadow reconstruction function. However, CoSMs has some shortcomings. Most importantly, the quality depends on the truncation order. The higher the truncation order, the more memory required to save basis textures. Memory consuming is a highlighted parameter. For evaluation purposes, rendering time measurement and comparison with other works was employed. The performance measurements were summarized in terms of shadow map sizes, convolution sizes, and varying reconstruction.

To evaluate the impact of an interactive approach, Kan et al. [27] employed quantitative methods to evaluate their hypothesis. This hypothesis was designed on two criteria: i) the positive impact of realistic features and ii) the importance of the realistic effects. The users filled a linear visual analogue scale from -3 to 3 to measure the realism of the scene. Moreover, the users were asked to identify real objects and virtual ones. Two different scenes were designed to include a virtual non-rigid sphere and a virtual ring. The results were illustrated in some bar charts to show the robustness of the approach.

Aliasing has been a persistent problem in computer graphics for many years. Aliasing happens due to the regular structure of dots that making up images. Many researchers attempted to overcome this issue [91][92][93][94][95][96][97]. A comprehensive survey on this was done previously [98]. Many researchers tried to qualitatively evaluate their anti-aliasing approach using because the results are visual. However, Barringer and Akenine-Möller [21] employed a quantitative method for evaluation. They proposed an algorithm to overcome this issue by sharing memory be GPU and CPU. As usual the system renders the scene in GPU, but the algorithm executed asynchronously on the CPU, which ran sparse rasterizer and fragment shader. They tested the algorithm in four different scenes including: Sponza, Chess, Hairball, and Buddha. The first three scenes used one Phong shading, but the last one used four Phong shading light sources with shadow maps. They compared the algorithm against integrated GPU and WARP [99], which was the optimised software for Windows 7. For GPU and WARP 4 and 8 sample per pixel (spp) were used. The algorithm was tested with 64K, 262K, 1.1 M and 2.85 M triangles with three different resolutions of 1280 800, 2048 1280, and 2880 1800. Then the spent times in different stage of the proposed pipeline were compared.

4.1.2 Geometrically Based

Rousiers et al. [52] evaluated their work both qualitatively and quantitatively to ensure a robust validation. Rousiers et al. [52] compared their work with different models and results presented in the form of a table and a graph.

Nowrouzezahrai et al. [70] considered light factorization for augmented frequency shadows in augmented reality environments to enhance the realism. They focused on shadow generation as indoor rendering. Illustrating five cases for a single character included unshadowed, cast shadows, self-shadowing, direct shadowing, and sphere proxies show the step by step process of the algorithm. Light projection performance is the main quantitative parameter which is taken into consideration to validate the technique. Since the aim of this study was method optimization, the researchers did not evaluate their technique.

A study done on handling motion blur in augmented reality by Park et al. [100]. The paper introduced a method for object tracking by extending the Efficient Second-order Minimization (ESM) [101]. This algorithm is one of the most efficient algorithms that handles motion blur. They have also improved motion blur in augmented reality to enhance the realism of motion augmented objects. As the contribution was twofold, the evaluation part also was divided in two corresponded subsections. For the improved tracking algorithm a quantitative method was taken into consideration. Comparing the improved algorithm which was called ESM-Blur-SE(ESM-Blur with shutter speed estimation) with ESM[101] and ESM-Blur [102] in three types of motion consists: 3D translation, 3D rotation, and a combination of 3D translation and rotation was the first part of evaluation. These samples were the same dataset that had been used by Lieberknecht et al. [101]. This has been a common technique to implement current work in a dataset and it has been employed in the original algorithm. As they had added Gaussian noise to reveal the blur effect, to show the robustness of the improved algorithm they performed a same degradation rate as the original algorithm. This performance was illustrated in a bar chart to show that there was no weakness and shortcoming compare to the original one. They explained some of the results instead of presenting them in the form of tables and graphs. Most of the information which was presented in tables were discussed in detail. Computation times for most consuming steps of algorithm were considered and illustrated in many tables among descriptions in details. The number of the chosen polygons in the 3D model was moderate (31,144). A video was submitted to show the qualitative part of the evaluation. Moreover, some figures were presented to discuss the qualitative evaluation further.

Another group of researchers, Li et al. [59], presented an algorithm to simulate deformative realistic skin. They also implemented the method on different case studies such as head, hand, cloth, torus, and dinosaur. Different types of animation were used to validate and show the robustness of the proposed method. A short discussion along with a table were presented to prove that the overall computation time was linear in 7 different models.

A big part of Ando et al.'s [65] evaluation was qualitative. Other than this, the method with different resolution and a deep comparison of duration and FPS was tested and the results were presented in the form of a graph and a table in the evaluation section. The outstanding points of Bojsen-Hansen and Wojtan [66] were the ability to remove topological noise and high-resolution surface noise from a low-resolution simulation. Compared to other studies, this study considered the complexity of memory usage as an additional parameter.

Jiang et al. [103] presented an approach for photorealistic facial animation synthesis to control asynchronies between the articulatory features, such as lips, tongue and glottis-velum. The evaluation was done both qualitatively and quantitatively. For subjective evaluation on facial animations, they have considered matching degree, smoothness, and naturalness. Xie et al. [104] did same work but a statistical parametric approach for video-realistic text-driven talking avatar. Their evaluation was mostly quantitative.

4.2 Mixed Environments

In order to blend virtual objects into the real environments, Knecht et al. [105] applied a radiosity technique. This approach revealed the reflection of virtual objects into real environments and vice versa. The algorithm which was based on [106] and [107], was a combination of Instant Radiosity [108] and Differential Rendering. Some shortcomings such as light bleeding and double shadowing combined instant radiosity and differential rendering. It was feasible for real-time rendering which was the shortcoming of previous works such as [30] and [109]. Indirect illumination was captured using a fish-eye. The final work avoided inconsistent colour bleeding artifacts. The results were mainly validated on visualisation. The effects of virtual objects on real ones were visual as there was no virtual benchmark to be compared with. The results were robust enough with 256 VPL. Just like other works such as [109] and [90], the results were compared with the same scenes without indirect illumination.

To show the validation of an algorithm in AR, Lensing and Broll [90] performed the indirect illumination approach in different conditions such as 256,1024,4096 and 16384 VPL (Virtual Point Light). All performances were employed using Dachsbacher's Reflective Shadow Maps [110].

As mentioned earlier Zheng et al. [86] evaluated the proposed method qualitatively and quantitatively. For numerical experiences, two different cases were conducted, which were tracker error and collaboration error.

Lack of a fixed volume in space was the main shortcoming of KinectFusion [71] (discussed in real benchmark of qualitative part) that led Roth and Vona [111] to proposed a solution. They tried to address this gap by proposing a moving volume KinectFusion algorithm. The algorithm translated and rotated the volume, when the camera moved. A quantitative technique was used to validate the improved algorithm this was while the original KinectFusion [71] was evaluated by qualitative technique as the quality of the results was as good as the original KinectFusion. They validated the algorithm based on performance and accuracy. Six different datasets were taken into account to show enhancement, which increased the performance from 26ms per frame to 33ms per frame. Absolute and relative errors were computed in three datasets to show the robustness of the algorithm. Tian et al. [112] worked in the same area focusing on collision detection between real and virtual objects in augmented reality. Visual comparison was the main part of evaluation in this work. Average processing time and Total time for rendering were also measured quantitatively. A comparison between the proposed method and the previous methods was done based on some parameters like static and dynamic scene, viewpoint, and handling mutual real occlusion.

In summary, a full comparison between the evaluation methods discussed is provided in Table 1. This shows that qualitative evaluation is the most common form of evaluations within computer graphics, particularly within photorealistic rendering. A significant number of investigated research articles utilised qualitative evaluation with different characteristics including texture, lighting, shading and shadowing. Although fewer research articles utilise quantitative evaluation compared to qualitative evaluation, quantitative method is considered to be more accurate and reliable. However, the most robust method of evaluation within research is to combine both qualitative and quantitative methods, as can be seen in most of the investigated research presented in Table 1. Depending upon the field of study, a side-by-side evaluation is recommended by prominent researchers, for both virtual and real benchmarks. If there is a possibility to compare the computer generated image with the real image, this will provide a more effective evaluation. However, an accurate comparison is needed to show that the generated image is replicating the real image. Due to the variances in color perception by humans, those experienced with the fine tolerances in the field of computer graphics can provide greater support to specific validations.

On the other hand, a side-by-side comparison with the previous computer generated image with respect to the provided conditions, is a commonly used method of evaluation in the field of photorealistic rendering. This is the easiest way to evaluate research in this area, if the same conditions are being taken into consideration. Providing the same scenarios, as have been prepared in the existing research, will make the evaluation sufficiently fair and accurate for the readers to judge the outcomes of the current research.

Based on information provided in Table 1, in photorealistic area, Figure 1 delivers that 100% of the research apply qualitative methods, considering different factors. 53.125% considered both qualitative and quantitative, which majority of them measured FPS with factors such as resolution, layers and lights. Of all the research reviewed, 37.5% of the research employed real benchmarks in comparison to 68.75% research which employed virtual benchmarks. Only 12.5% compared their work in both cases with real and virtual benchmarks. Findings suggest that employing questioners are less recommended compare to other methods. On the contrary of many other research areas, questionnaires only employed in thesis rather than a research paper.

Table 1 A full comparison of evaluation based on different methodologies(Quan: Qualitative, Qual: Quantitative, RB: Real Benchmark, VB: Virtual Benchmark, Ques: Questionnaire)

Research(er)	Area	Qual	Quant	RB	VB	Ques
Sunar [6]	Sky colour generation	X		X	X	X
Sugano et al. [22]	Shadows in AR	X		X		X
Grasset et al. [26]	Labeling in AR	X	X	X	X	
Kan and Kaufmann [27]	Reflections AR	X	X	X		
Haller et al. [36]	Shadows in AR	X		X		
Liu et al. [8, 9]	Live video AR	X		X		
Xing et al. [10, 11]	Live video AR	X	X			
Kolivand et al. [35][69]	Outdoor AR	X	X	X	X	
Lokovic et al. [40]	Shadows of hair	X	X	X		
Nowrouzezahrai et al. [70]	Lighting in AR	X				
Izadi et al. [71]	Reconstructing	X		X		
Madsen et al. [76]	Sky illumination in AR	X		X		
Mehta et al. [32][41]	Soft shadows	X			X	
Donnelly et al. [42]	Soft shadows	X			X	
Lauritzen et al. [43]	Soft shadows	X	X		X	
Dimitrov [44]	Semi-soft shadows	X			X	
Annen et al. [20]	Soft shadows	X	X		X	
Billeter et al. [19]	Soft shadows	X	X		X	
Aittala [79]	Shadows in AR	X			X	
Baran et al. [51]	Scattering	X	X		X	
Rousiers et al. [52]	transparency illumination	X			X	
Gruber et al. [82]	Lighting in AR	X			X	
Zheng et al. [86]	Tracking in AR	X	X		X	
Li et al. [59]	hype-realistic skin	X			X	
Lensing and Broll [90]	indirect illumination in AR	X			X	
Macklin and Muller [62]	fluid simulation	X	X		X	
Ando et al. [65]	liquid simulation	X	X	X		
Keller et al. [75]	Reconstructing	X	X		X	
Knecht et al. [105]	Radiosity	X	X	X	X	
Park et al. [100]	Motion blur in AR	X	X		X	
Barringer et al. [21]	Anti-aliasing	X	X		X	
Levine et al. [113]	Directional locomotion	X	X		X	

In photorealistic area, especially in 3D rendering, a qualitative method for the evaluation purpose should be considered. However, in most cases, a qualitative evaluation is not sufficient. Absence of systematic technique, invalid generalizations, uncertain findings, lack of a variety of standard datasets, and inaccessibility of small and reginal datasets make a qualitative evaluation difficult or in some cases not feasible. Therefore, quantitative evaluations are used to weigh up the robustness of the technique. Table 2 and Table 3 present advantages and disadvantages of the presented methods with focus of each research in the case of both presentations and evaluations. The tables show how a quantitative method based on FPS, resolution, and number of resources (i.e., lights and layers) could support the qualitative evaluation. It is of significant to highlight that quantitative methods are generally insignificant in photorealistic research.

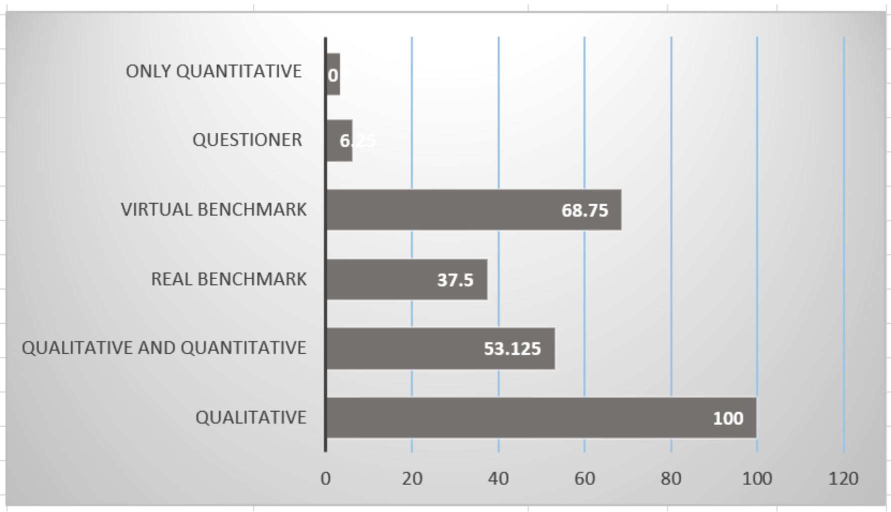

Fig. 1 Percentage of different methods applied in photorealistic rendering

Table 2 A comparison between advantages and disadvantages of qualitative photorealistic research with respect to real and virtual benchmarks

Table 2 shows that factors such as observation, description, negotiation interpretation, and evaluation based on a small dataset, offer significant advantages when evaluating qualitative photorealistic rendering research. Whilst Table 3 highlights factors that offer significant advantages when evaluating quantitative photorealistic rendering research i.e. considering standard benchmarks, measuring, statistical testing, direct addressing of the issue and the repeatability of an identical situation.

Mixed methods have been criticized for increasing complexity of implementation and evaluation as well as the need for involving researchers from multiple disciplines to conduct the research. Nevertheless, mixed techniques are particularly efficient in carrying out through evaluation which lead to better understanding of the logical inconsistencies between quantitative finding and qualitative outcomes. Mixed methods provide opportunities for multidisciplinary photorealistic research to evaluate the findings in both aspects. This is because, mixed method approaches are generally flexible and adaptable to be used in this field.

A side-by-side comparison of the previously computer-generated image with respect to the provided conditions, is a commonly used method of evaluation in the field of photorealistic rendering. This is the easiest way to evaluate research in this area, if the same conditions are being taken into consideration. Providing the same scenarios, as have been prepared in the existing research,

Table 3 A comparison between advantages and disadvantages of qualitative photorealistic research with respect to real and virtual benchmarks

	Real Benchmark	Virtual Benchmark
Advantages		
Available benchmarks[8][35][10][76]		
No need implementation of benchmarks[71][72] [76]		
Easy validation[40] [71][72][86]		
Individual experience[35][10][11] [76]		
Observable [36] [40] [71][72] [76][75]		
Image based[40][20] [75]		
Describable[11][8] [71][72] [76]		
Naturalistic approach [35] [40] [71][72]		
Time consuming due to fieldwork setting[35][10]		
Negotiable and interpretable[40] [71][72] [76] [75]		
Endlessly creative[11][8][9] [71][72] [75]		
Evaluation based on small datasets [35][8] [86]		
Disadvantages	lack of systematic evaluation [6] [71][72] [75]	
Requires experts? knowledge of the area[10][35][60] [76]		
Difficult to evaluate [6][35] [71][72] [75]		
Difficult to generalize[6] [86]		
No statistical tests[11][35] [71][72][75] [76]		
Different observation, different results[35][8]		
Long analysis time[71][72]		
Lack of standard dataset [6][37][35][10] [76] [86]		
Subjective nature [11] [76]		
More difficult to address the issue[35][10]		
Detail understanding of issue is needed[10][11][8][35]		
Not measurable[5][10] [71][72]		
Not fully inductive approach[71][72]		
Concerned with human understanding[6][35][69] [71][72] [75] [76]		
Uncertain findings [37] [40] [76]		
Inadequate validity and reliability [10][11][67] [15] [16]		
Difficulty in replication [40]		

Table 4 A comparison between advantages and disadvantages of quantitative photorealistic research with respect to real and virtual benchmarks

would make the evaluation sufficiently fair and accurate for the readers to judge the outcomes of the current research.

5 Conclusion

Different areas in computer graphics should be investigated to find more common factors for evaluating research. A combination of testing, validating and evaluating is the main requirement of evaluation. For each research, evaluation must be planned and designed using a systematic and well-organised procedures. Selecting factors for evaluation can be challenging for beginners but using the experiences of elite researchers is always beneficial.

Although in photorealistic rendering quality of generated images is highly demanded, quantitative evaluation is also needed to be done. The trade-off between qualitative and quantitative evaluation of proposed algorithm or method must be discussed along with some samples with different degrees of complexities. This is because considering the qualitative techniques for a photorealistic research, which has focused on quality of generated images may not be sufficient. Increasing the quality may reduce other features such as FPS. A qualitative technique is required to show whether an enhancement on quality is worth reducing the rendering speed or keeping FPS constant. In general, poverty and unfair division of revenue are highly interconnected [98]. This is much stronger in photorealistic research, in which, for a desirable result, an enhancement in quality produces a lower reduction in FPS. Therefore, any enhancement in quality is considered undesirable for rapid reduction in FPS and vice versa. However, in some developed techniques there is a trade-off between FPS and quality. Introducing a compromise technique to enhance quality, which keeps FPS constant, could be a desirable achievement for any research. This has been of interest of many researchers for evaluating and validating their studies.

The other point is that comparing the results with real environments and real objects is significant for achieving photorealistic results. While more than one third of the investigated research used this method to evaluate their works, this benchmark has rarely been used in other fields.

Distributing questionnaires between users is a technique, which can be employed in photorealistic results for judging the quality of computer generated images and comparing them with the expectation of users.

To sum up, a research paper without an appropriate evaluation cannot prove the strengths of the work. This final part of research must be highlighted in each photorealistic research by making comparison to existing techniques and algorithms or real images both qualitatively and quantitatively.

We hope this paper has shed light on most common evaluation techniques in this field.

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