Wake interference of two identical oscillating cylinders in tandem: An experimental study

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Nomenclature

Symbols		Symbols	
m	Mass of the system	D	Cylinder's diameter
l	Length of the cylinder	L_r	Length of the supporting rod
ξ	Damping Ratio	ν	Kinematic viscosity of water
L	Distance between centres of two cylinders	m^*	Mass ratio per unit length $\left(\frac{m}{\frac{\pi}{4}\rho D^2}\right)$
Re	Reynolds Number $\left(Re = \frac{VD}{\nu}\right)$	f_n	Natural frequency
U_r	Reduced Velocity $\left(U_r = \frac{V}{D \times f_n}\right)$	V	Undisturbed flow velocity
A_{X_1}	Root mean square of response amplitude of upstream cylinder in stream-wise direction over diameter	A_{X_2}	Root mean square of response amplitude of trailing cylinder in stream-wise direction over diameter
A_{Y_1}	Root mean square of response amplitude of leading cylinder in cross-flow direction over diameter	A_{Y_2}	Root mean square of response amplitude of trailing cylinder in cross-flow direction over diameter
X_1	Leading cylinder position from its original location in stream-wise direction	X_2	Trailing cylinder position from its original location in stream-wise direction
Y_1	Leading cylinder position from its original location in stream-wise direction	Y_2	Leading cylinder position from its original location in cross-flow direction

Abstract

This study aims to assess the impact of the spacing between cylinders on their dynamic behaviour. Observations during this experiment have helped to identify the effect of spacing on excitation mechanism of each cylinder as well as establishing the relationship between cylinders response and the spacing between them. Arrays of cylindrical Structures in close proximity are common in variety of engineering structures, particularly in the offshore industry. If the cylinders are flexible, and are subject to excitation from fluid flow, then the dynamics is influenced by complex interactions between the cylinders, dependent on a variety of parameters. Two cylinders were flexibly mounted in tandem at different spacings from 20D to 3.5D and were towed at different Reynolds numbers ranging from 8.7×10^3 to 5.2×10^4 to. The gap between cylinders was set to 20, 15, 10, 8, 5, 4, 3.5D respectively; the responses of the cylinders were measured and the mutual interactions were compared. The two cylinders were identical and free to oscillate in both cross-flow and stream-wise directions. The validity of results was examined by comparing to available data in literature and results attained by testing cylinders independently in the towing tank.

The interaction of two cylinders are studied through frequency analysis of both cylinders response motion to determine how upstream vortices impact the trailing cylinder response in stream-wise and cross-flow differently. It was observed that leading and trailing cylinders oscillate at different frequencies which is in contrast with results obtained from two fixed cylinders in tandem where both cylinders oscillate at the same frequency. Additionally, it is discussed how spacing controls the excitation mechanism.

Key words

Vortex Induced Vibration, Wake Induced Vibration, Wake Interference, Wake-Structure Interaction, Bluff Body Hydrodynamics.

1. Introduction

A cluster of bluff structures placed in a close proximity is a familiar scene to engineers in many industries. A group of risers coming to the side of a FPSO or tendons anchoring a tensioned leg platform (TLP) in Offshore Engineering, an array of suspension cables on a bridge or closely located chimney stacks in Civil Engineering and power transmission cables in Electrical Engineering, are instances of such an arrangement. The distance between these bodies, their diameters, masses and natural frequencies are not necessarily similar which results in a complex system, challenging to analyse.

Many studies have investigated an isolated single cylinder, focusing on Von Karman Vortex and the associated structural response, known as vortex induced vibration (VIV). VIV has been studied extensively via three methods: (i) experimental investigation, (ii) numerical simulation and (iii) mathematical modelling. Different approaches can be adopted in each method: The cylinder may be free to vibrate and the response amplitude and the frequency of motion are studied, or when the cylinder could be fixed and hydrodynamic forces and vortex shedding frequency are considered. Additionally, the cylinder may be forced to oscillate at different frequencies, with fixed amplitude at a specific Re which allows researchers to observe the change in flow regime around the structure and study the mechanism of vortex generation. Studies may focus on long flexible cylinders to examine three dimensional motions; alternatively rigid cylinders may be suspended on flexible mounts to examine two dimensional fluid-structure interactions; this will be the subject of this paper.

In a more general configuration where a flexibly mounted rigid cylinder is placed immediately in the wake of another identical cylinder, fluid-structure interaction of both structures can transform drastically. The flow regime behind each body becomes a function of the gap between them as well as the Reynolds number (*Re*). The two cylinders may be placed in three configurations: side by side, tandem (forward and aft) and staggered, i.e. the most general case. Zdravkovich (1987) adopted a systematic approach and proposed a classification of three regimes: “wake interference” when the trailing cylinder is immersed in the wake of the leading one partially or completely; “proximity interference” in which the cylinders are placed close together but neither of them enters the wake of the other one, however they still have effect on the Karman vortex street behind their counterpart; finally “no-interference” regime in which two cylinders are out of each other’s wake and have no effect on each other. Configurations with extremely large transverse or/and stream-wise gaps fall into the last category.

Sumner et al. (2005) conducted an experiment on two identical fixed cylinders in a staggered arrangement at subcritical *Re*. They studied the aerodynamic forces and Strouhal number for both cylinders in different settings and tried to classify them regarding the centre to centre “*pitch ratio”* and “*incidence angle”*. They studied the cylinders in three groups of closely spaced, moderately spaced and widely spaced arrangements, and changed the incidence angle. In the first group, aerodynamic forces on both cylinders were extremely sensitive to changes in the incidence angle. In the second group, they observed higher Strouhal numbers. The pair of cylinders in the third group behaved similar to an independent cylinder although, at small angles, the lift force acting on the trailing cylinder was at its highest value, pushing the cylinder away from the upstream wake. Side by side set-up falls into “proximity interference” flow regime described by Zdravkovich (1987). Sumner et al. (1999) and Wang et al. (2002) observed in their studies that when two cylinders were placed side-by-side extremely closely, they behaved as a single body and only one Karman Street was developed for both cylinders.

As the transverse distance between two cylinders increased each cylinder developed a distinct vortex street; however, these vortex streets were not independent from each other or symmetric but were biased towards one cylinder (Bearman and Wadcock, 1973; Ishigai et al., 1972; Williamson, 1985; Xu et al., 2003). At large spacings two cylinders behaved more independently and developed two parallel vortex streets however, they were still not independent: their wakes interacted with one another and went through synchronization.

The tandem arrangement typically falls into the "wake interference" flow regime. Igarashi (1981) considered two identical fixed cylinders in tandem and confirmed that flow patterns behind the cylinders depend on the gap between two cylinders as well as *Re*. This experimental investigation included very small spacing of 1.1*D* up to moderate spacing of 5*D*; during which he tried to identify different flow regimes by varying the *Re* from 8.7×10^3 to 5.2×10^4. Zhou and Yiu (2006) carried out a similar experiment and discovered, for a specific *Re*, that the flow pattern around two tandem cylinders could be divided into three categories based on the gap between them. These were designated as follow (i) "Extended-body regime" where two cylinders were placed extremely close to each other and acted as a single cylinder. In this regime the shear layer separated from surface of the leading cylinder could not roll behind the leading cylinder so passed over the trailing cylinder and generated vortex in its wake. There was no separation on the surface of the trailing cylinder in this regime. (ii) "Reattachment regime" could be observed in moderate spacings. Shear layers separated from the surface of the leading cylinder still did not roll behind the cylinder but reattached on the surface of the trailing cylinder and partially moved towards the stagnation point in front of the cylinder; the other part joined the shear layer separating from the trailing cylinder's surface (Alam et al., 2003a). This region was subdivided into two smaller groups based on the position of reattachment points. When the gap was relatively small the attachment points were located on the leeside of the trailing cylinder, however, as the trailing cylinder shifted further downstream and the gap grew larger, the reattachment points fell on the upstream half of the trailing cylinder. (iii) In "Co-shedding region" the first cylinder developed its own vortex street because the gap grew sufficiently large for the shear layer to roll into a vortex. The spacing threshold for each flow regime were susceptible to *Re*. Furthermore, the transition from reattachment to co-shedding regime was usually indicated by a sudden jump in values of properties such as drag and lift coefficients, base pressure and Strouhal number (Xu and Zhou, 2004). The boundary between "Reattachment regime" and "Co-shedding region" is referred to as *critical spacing* in literature.

Alam et al. (2003b) investigated the behaviour of drag coefficient as a function of spacing and *Re*; they found that the drag coefficient experienced a sudden jump at the critical spacing too. They suggested referring to critical spacing as “drag inversion” at which drag value changes from negative to positive. In addition, the Strouhal number experienced a similar sudden jump after critical spacing (Okajima, 1979). The Strouhal number decreased at farther distances in the two regions of “Extended-body regime” and “Reattachment regime”. In contrast, the Strouhal number increased in the “Co-shedding region” until it reached the value equal to that of a single cylinder.

There are also studies on an oscillating cylinder in the wake of a rigidly fixed cylinder which explored the influence of upstream wake on the trailing cylinder (Hover and Triantafyllou, 2001). Assi et al. (2010) conducted an extensive experiment to study the effect of spacing on two cylinders in tandem where leading cylinder was fixed. They discovered that the trailing cylinder acted independently for centre to centre spacing greater than 20*D*. The trailing cylinder (with low mass-damping parameter) had a galloping like behaviour at smaller spacings which calmed down at larger spacings. They suggested that the response of the trailing cylinder consisted of three regions depending on *Re*. The cylinder response was similar to an isolated cylinder in the first region. As *Re* rose the effect of the wake from the fixed leading cylinder increased which established the second region, in which the cylinder response was a combination of vortex induced vibration and wake induced vibration. In the third region, vortices from the cylinder itself did not excite it, nevertheless, the response amplitude of the cylinder increased due to buffeting vortices coming from the upstream cylinder.

As this brief review is trying to demonstrate, studies on tandem arrangement in which both rigid cylinders are flexibly mounted are limited; most of the ones with such a focus have used computational fluid dynamics. Moreover, *Re* in these studies is typically very low, around 100 to 200. In practice the *Re* value is much larger than this and closer to the region considered in this study so results from this study offer a more realistic simulation. None the less, these studies confirmed that trailing cylinder had higher response amplitude in comparison to its upstream counterpart (Borazjani and Sotiropoulos, 2009; Papaioannou et al., 2008; Prasanth and Mittal, 2009). Unlike the study of Assi et al. (2010) with a fixed upstream cylinder, in which the trailing cylinder response increased with velocity, in these studies with both cylinders flexibly-mounted, the amplitude reduced at higher Re. It shows the different impact of a weak wake from a stationary leading cylinder in comparison to a strong wake from an oscillating leading structure.

Kim et al. (2009) also conducted experiments on a pair of flexibly mounted rigid cylinders which could oscillate in transverse direction only. They observed a strong influence of the leading cylinder on the trailing one. However, due to low Re number ($Re = 4365 - 74200$) one of the cylinders needed a jump start; then its vibration would excite the other cylinder as well. They considered spacings $L/D = 0.1$ to 3.2, which is out of this paper scope, and studied lift coefficient and cylinders vibration amplitude. Huera-Huarte and Gharib (2011) conducted their experiment at a similar range of spacings. They focused on two flexible cylinders and varied the top tension, spacing and reduced Re. No Karman vortex street was observed in the gap between cylinders up to $L/D = 4$ and then trailing cylinder demonstrated large amplitudes at high reduced velocities which was considered as wake induced galloping. It was observed that upstream vortices interacted with downstream ones and create a chaotic wake behind the trailing cylinder. This current paper is looking at the spacings beyond this range to improve the understanding of wake interaction.

Huang and Herfjord (2013) focused on two flexibly-mounted cylinders in staggered and tandem configurations. They initially observed that variation in reduced velocity did not affect the motion trajectory of either cylinder which is in contrast with observations made on a single cylinder. Although Zdravkovich claimed that the effect of the trailing cylinder on the leading one is insignificant in spacings larger than 4D for the case of fixed cylinders, Huang and Herfjord (2013) observed that for a system with both cylinders free to oscillate this threshold reduced to 3D. To the authors' knowledge, other similar experiments are non-existent.

It is clear that having two cylinders in tandem creates a complex system where many parameter control the system dynamics and small change in any of them could project a significant change in system response. In most of existing studies either cylinders or the leading one are fixed; however, when the leading cylinder is free to oscillate, it generates a strong wake which increases its impact on the trailing body. Strong vortices, detached from the leading cylinder, impact the body in the wake at random position on its trajectory, and it results in an irregular response from the trailing cylinder which has not been discussed previously. Consequently the response vs velocity graph of the trailing cylinder exhibits a zig zag behaviour even though it follows a clear trend overall. This paper is trying to explain such a behaviour by considering two known excitation mechanism Vortex induced vibration (VIV) and Wake induce vibration (WIV). While other papers have considered configurations with one cylinder oscillating and/or free to vibrate in cross-flow direction only, this paper is considering a broader condition and focusing on both cylinders free to oscillate in stream-wise and cross-flow directions.

This experimental study on the fluid-structure interaction where both cylinders are flexibly mounted and free to oscillate in both transverse and stream-wise directions. The *Re* in this study was in subcritical region varying between 7.79×10^3 to 10^5. Both cylinders were firmly placed at 3.5, 4, 5, 8, 10, 15 and 20D apart. The arrangement used falls into “wake interference” region.

2. Experiment set-up

This experiment was conducted in the towing tank at Kelvin Hydrodynamics Laboratory, University of Strathclyde. The tank is 76m long and 4.6m wide with the water depth set to approximately 1.4m for this experiment. The rig supporting two cylinders was securely mounted on a carriage located above the water tank.

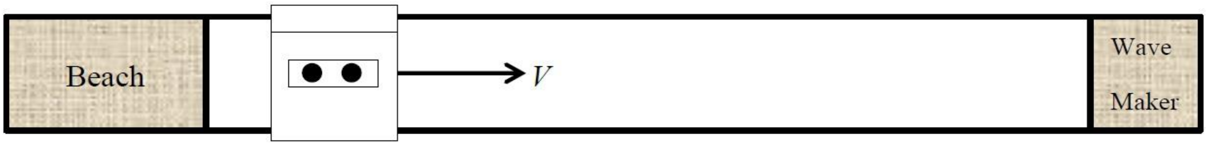

Figure 1. Kelvin Hydrodynamic Laboratory view from the top

The cylinders were 0.1m in diameter and 1.01m in length, yielding an aspect ratio of 10:1. The cylinders were fabricated from nylon; the surfaces of both cylinders were machined prior to the experiment and considered hydrodynamically smooth.

Each cylinder was supported by a solid aluminium rod of length 1.8m, rigidly attached to the rig at the upper end with two clamps. This supporting method provided equal damping and mass ratio in both in-line and cross flow directions, as well as equal natural frequencies. Figure 2 is a schematic view of the experiment rig (note that the figure is not to scale). The supporting rods allowed the cylinders to oscillate in both stream-wise and transverse directions; the rods were designed to have minimal deflection under the drag force so that collision between two cylinders could be prevented in small spacings, and they remained relatively vertical throughout the experiment. In this particular set-up a displacement of one diameter at the bottom of the cylinder resulted in a slope of less than four degrees from the vertical position.

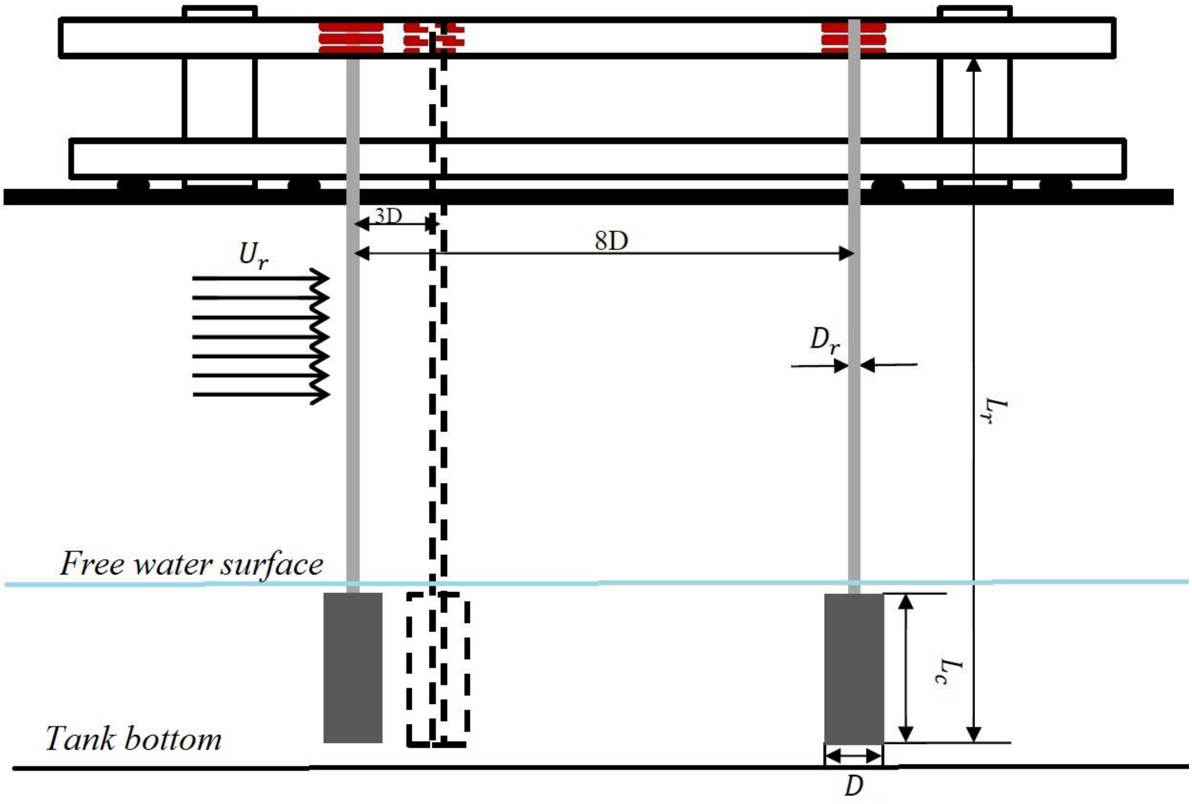

Figure 2 schematic view of the experiment rig

The aspect ratio was deemed high enough to consider the sample cylinders as slender so that the acquired data can be used as a guideline for offshore structures that mostly fall into this category. The lower end of the Cylinders were located 25 mm above the tank bottom and the upper ends located 50 mm below the water surface so no separation occurred at either end. This assured a 2D flow so no end-plate was required. The mass of each cylinder was 20kg which led to a mass ratio (m^*) of 2.36 for each cylinder.

The motion of each cylinder was tracked using a Qualisys optical tracking system in all six DOFs at sampling rate of 137 kHz. This system was constantly tracking the four infrared reflector balls that were attached to each cylinder. All reflectors were mounted on a collar and fixed to the supporting rod very close to the water surface. Five Qualisys optical tracking cameras were arranged to track a minimum of three reflectors on each cylinder at any given time. The experiment setup is shown in Figure3.

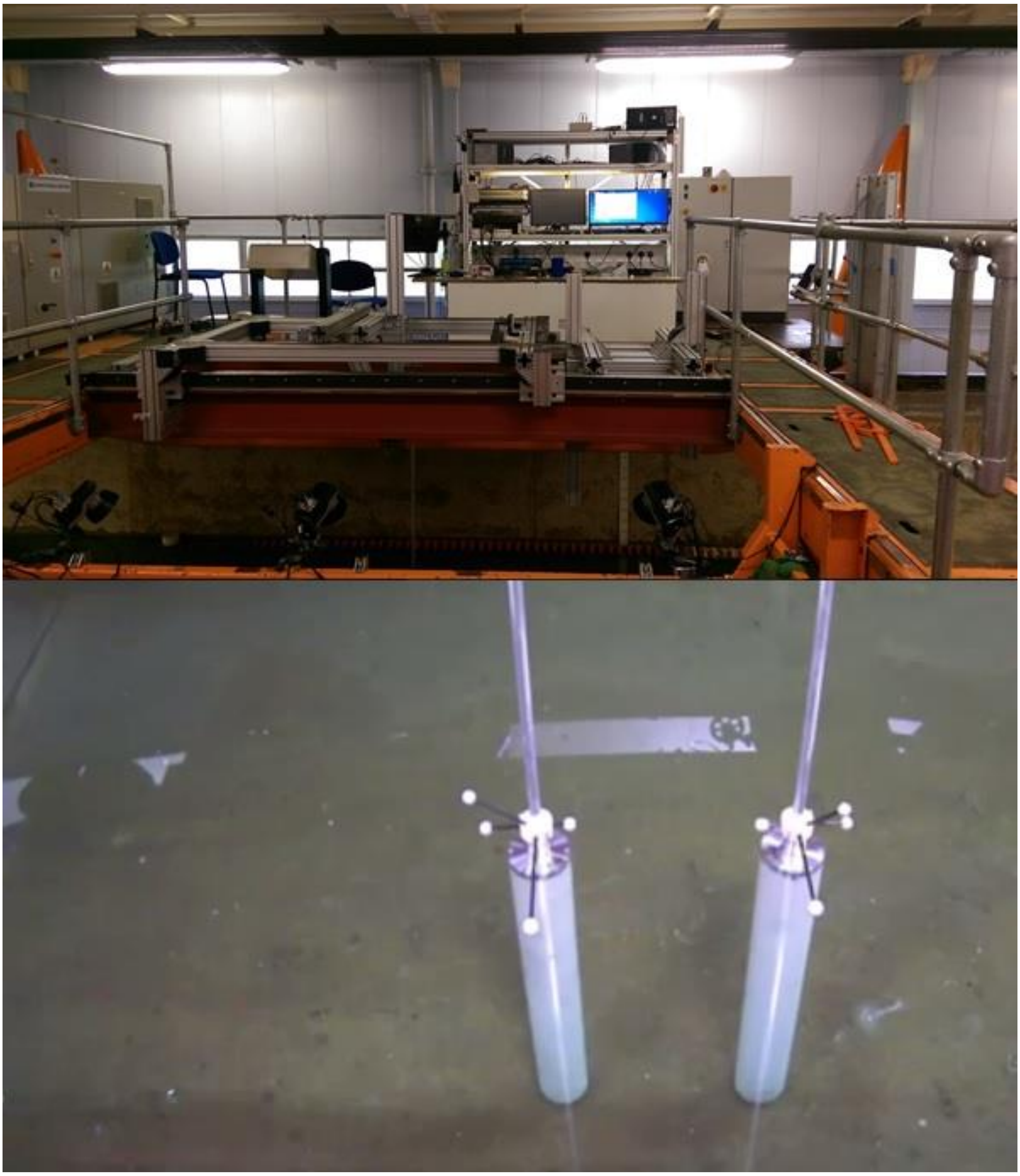

Figure 3 Experiment setup at Kelvin Hydrodynamic Laboratory

After placing the cylinders at the targeted spacing the rig was secured so that the gap was maintained during the test. For each run the carriage was started at rest, accelerated up to the targeted speed, maintained the speed for the maximum available time, and then stopped. The speed of the carriage ranged between 0.08 to $0.885\ m/s$ which was monitored by a built-in speedometer on the carriage and a separate speedometer up to three decimal points; running the test at different velocities allowed to observe how reduced velocity (Gabbai and

Benaroya, 2005) effects the response of two cylinders. Targeted spacings for the first set of experiments were 20, 15, 10, 8, 5, 4 and 3.5 cylinder diameters. Due to design limitation two cylinders would have collided if the gap between two cylinders had become any smaller.

3. Single cylinder

Both cylinders were mounted on the rig individually and underwent free decay tests in transverse and in-line directions in water and in air; this indicated the natural frequency and critical damping ratio of the system. Each test was repeated three times to assure the accuracy of the results. Table 1 shows the results obtained from these tests.

Table 1.Natural frequency and damping ratio of the two cylinders in air and water

	Water				Air			
	Cross flow		In-line		Cross flow		In-line	
Property / Cylinde	f_n (Hz)	ξ	f_n (Hz)	ξ	f_n (Hz)	ξ	f_n (Hz)	ξ
Leading cylinder	0.3669	0.0155	0.3763	0.0111	0.5052	0.0077	0.5058	0.0070
Leeward cylinder	0.3649	0.0146	0.3665	0.0113	0.5058	0.0083	0.5061	0.0063

A single cylinder VIV test was carried out on the upstream cylinder. Figure 4 shows the standard deviation of the cylinder response amplitude against reduced velocity, $U_r = U/f_nD$. The results were compared with similar works on VIV response of a single cylinder with two DOFs in order to validate the experiment apparatus.

The results from present work shows the typical VIV behaviour of a single cylinder, however some key features are different from previous studies. A smaller value of maximum amplitude in the experiments by Srinil et al. (2013) or Blevins and Coughran (2009) is due to higher damping in these two experiments. Damping ratio is 0.05 for Srinil et al. (2013) and 0.02 for Blevins and Coughran (2009) experiments. Also, the difference in mass ratio resulted into different lock-in range width. It is established in the literature that structures with lower mass ratio have wider lock-in ranges. T these experiments, mass ratio was 1.75 and 2.8 respectively.

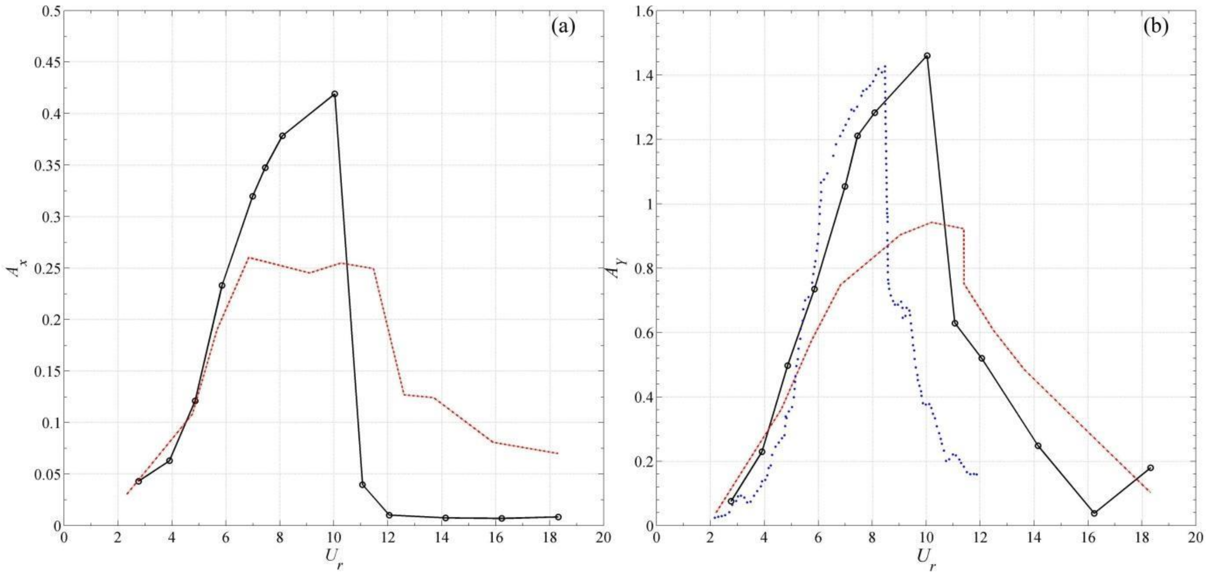

Figure 4. Amplitude of vibration obtained from single cylinder test (━●━) and (Srinil et al.)(‒ ‒) and (Blevins and Coughran)(· · ·) in (a) stream-wise and (b) cross-flow

The initial lock-in range in the stream-wise direction (caused by the in-line oscillation frequency coinciding with the system natural frequency) was not captured in the isolated cylinder test due to the high initial reduced velocity. Due to the low mass-damping ($m^*\xi$) parameter three branches of excitation can be observed based on Govardhan and Williamson (2000) study. The initial branch approximately starts at U_r=2 to 6 during which transverse response amplitude is limited to 1D; afterwards, upper branch can be observed from U_r=8 to 10 while response is at its maximum, and finally lower branch starting at U_r=11 after a sudden drop of the oscillation amplitude.

4. Two identical cylinders in tandem

In the next stage of the test campaign, the trailing cylinder was placed in the wake of the first cylinder, initially at the centre to centre distance of 20*D*. A set of tests was carried out over the same range of velocities used for the single cylinder. After each set of tests the trailing cylinder was moved closer to the upstream cylinder and the spacing was systematically reduced until the cylinders collided during the test at *L/D* = 3 and a reduced velocity of 10 and the test was terminated. This occurred when the upstream cylinder was at its maximum oscillation. The root mean square (RMS) of the response amplitude for various spacings versus reduced velocity is shown in figure 5.

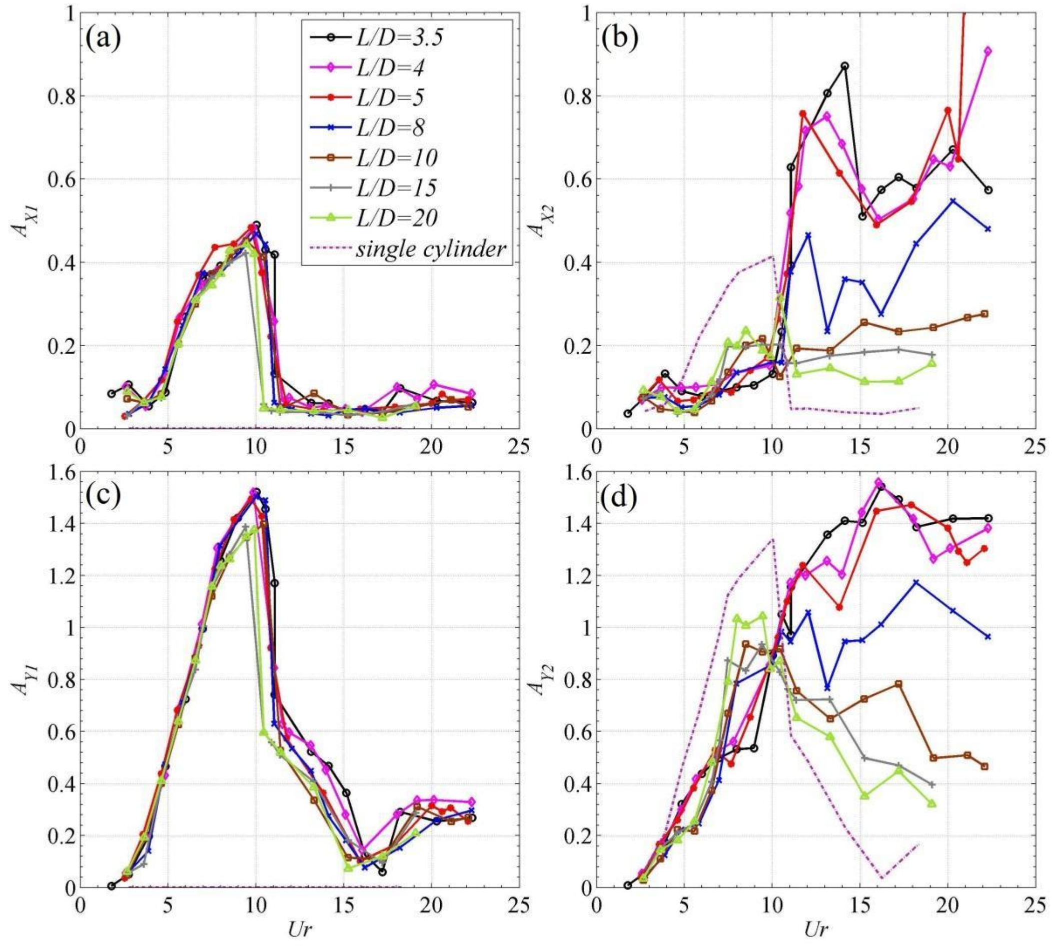

Figure 5. RMS of response amplitude versus reduced velocity. (a)Leading cylinder's stream-wise motion, (c)Leading cylinder's cross-flow motion, (b)Trailing cylinder's stream-wise motion, (d)Trailing cylinder's cross-flow motion

The initial lock-in range in the stream-wise direction can be observed in the leading cylinder at spacings of 3.5*D* and 4*D*; as the spacing grew larger it can be seen again at spacings of 10*D* and 20*D*. The in-line lock-in range of the trailing cylinder shifted to higher reduced velocities as the two bodies are placed closer together. As Xu and Zhou (2004) observed in their study, when two cylinders are placed at close proximity the shear layers do not have enough space to roll up behind the upstream cylinder. However, when the Reynolds number increases, separation happens earlier and the shear layers have enough time to roll into vortices. This may explain why lock-in range is shifting to higher U_r in smaller gaps. Papaioannou et al. (2008) also claimed that when the upstream cylinder started to shed vortices, it acted similar to an isolated cylinder. It could be concluded that when the leading cylinder is undergoing lock-in range, the wake behind it is fully developed due to high $U_r$thus spacing has no effect on the lock-in range of the leading cylinder.

The response of the leading cylinder in cross-flow is greater than that in the stream-wise direction, similar to a single cylinder. The trailing cylinder response was strongly affected by the leading cylinder and the spacing had significant influence on its response. The behaviour of the trailing cylinder should be interpreted considering two important facts. First, the velocity of flow passing the trailing cylinder is less than the free stream value due to the upstream cylinder's shielding effect. Second, vortices washed from the leading cylinder buffet the trailing body. It appears that at small spacings the trailing cylinder has the same maximum amplitude as that of the leading cylinder. Nonetheless, the maximum oscillation amplitude occurs at a higher reduced velocity due to the shielding effect. As the trailing cylinder is shifted further downstream, the vortices lose a fraction of their energy travelling downstream so the buffeting force on the trailing cylinder reduces significantly. It can be observed that the response amplitude reduces as the gap grows, which may be assumed to be due to this reduction. The response graph of larger spacings ($L/D > 5$) suggests that the force exerted on the trailing cylinder by upstream vortices has a larger impact on the cylinder in comparison to its own wake forces. Increase in spacing means reduction in vortices buffeting force due to viscous resistance which explains the reduction of amplitude response at moderate spacings ($L/D = 8, 10$). However, the wake force becomes dominant at very large spacings ($L/D = 15, 20$) where there is enough for time the flow velocity to recover. As the flow velocity in the gap increases, due to reduction of shielding effect, lift and drag rise significantly and compensates the reduction in buffeting vortices energy and response amplitude rises again as the gap grows larger.

The in-line response graph of the trailing cylinder shows four peaks regardless of the spacing. The first peak can be observed at the same U_r as that of the leading cylinder. The second peak happens between reduced velocities of 8 to 10 where the first cylinder is undergoing its largest oscillations. This peak becomes dominant as the gap grows larger which indicates that cylinder's response is evolving towards that of a single cylinder. The third peak happens between $U_r = 10$ to 15 which is the most dominant for small and moderate spacings and appears to be the lock-in range induced by vortices generated from the cylinder itself. The width of this lock-in range has an inverse relationship with the gap size and disappears as the gap grows larger. The last peak happens at a reduced velocity of approximately 20; this pick occurs at this reduced velocity for all spacings and appears to receive no influence from changing the spacing until very large spacings at which the influence of buffeting vortices is dominated by the wake force.

The cross-flow response of the trailing cylinder presents three distinct peaks. Considering of the reduced velocity at which each peak occurs as well as the effect of spacing could lead to a conclusion about the excitation mechanism of each lock-in. The first peak happens at U_r =5 to 10 which is during upstream cylinder's lock-in range. The change in spacing does not have an effect on the reduced velocity at which the first peak occurs, however the magnitude of this peak changes with spacing, showing a direct relation with the size of the gap. As observed in figure 5 the peak is at its highest value at spacing of 20*D.*

The second peak in transverse direction corresponds with the second pick in stream-wise direction at similar reduced velocities, and the variation in magnitude of these peaks due to spacing change is similar as well, which suggests that lock-in phenomenon in stream-wise direction is affecting the cross flow response. This is contrary to the case of a single cylinder. This peak slowly disappears as the spacing increases further. The third peak, which is the most dominant and exclusive to cross flow response of the trailing cylinder, occurs at very high reduced velocities. Nevertheless, the peak loses its dominance from spacing of 8*D* and larger, and transforms to something that is similar to sudden increase at very high velocities for an isolated cylinder.

Looking at the frequency of the motion will help to further explore the excitation mechanism corresponding to each peak. The frequency content of transverse oscillation showed only one dominant frequency in all cases. Figure 6 compares the transverse oscillation frequencies of both cylinders normalised to the natural frequency in water. It is evident that the trailing cylinder was oscillating at a lower frequency in comparison with its upstream counterpart, specifically during leading cylinder lock-in range, which is due to lower flow velocity in the gap. However, results are in contrast to observations of Xu and Zhou (2004) and Kitagawa and Ohta (2008) in their experiments on two identical **fixed** cylinders in tandem, which showed that both cylinders had the same Strouhal number regardless of *Re*. Moreover, the first cylinder's shedding frequency graph display a significantly larger jump compare to its trailing counterpart (figure 6) which can be a sign of change in its wake regime but more investigation is required.

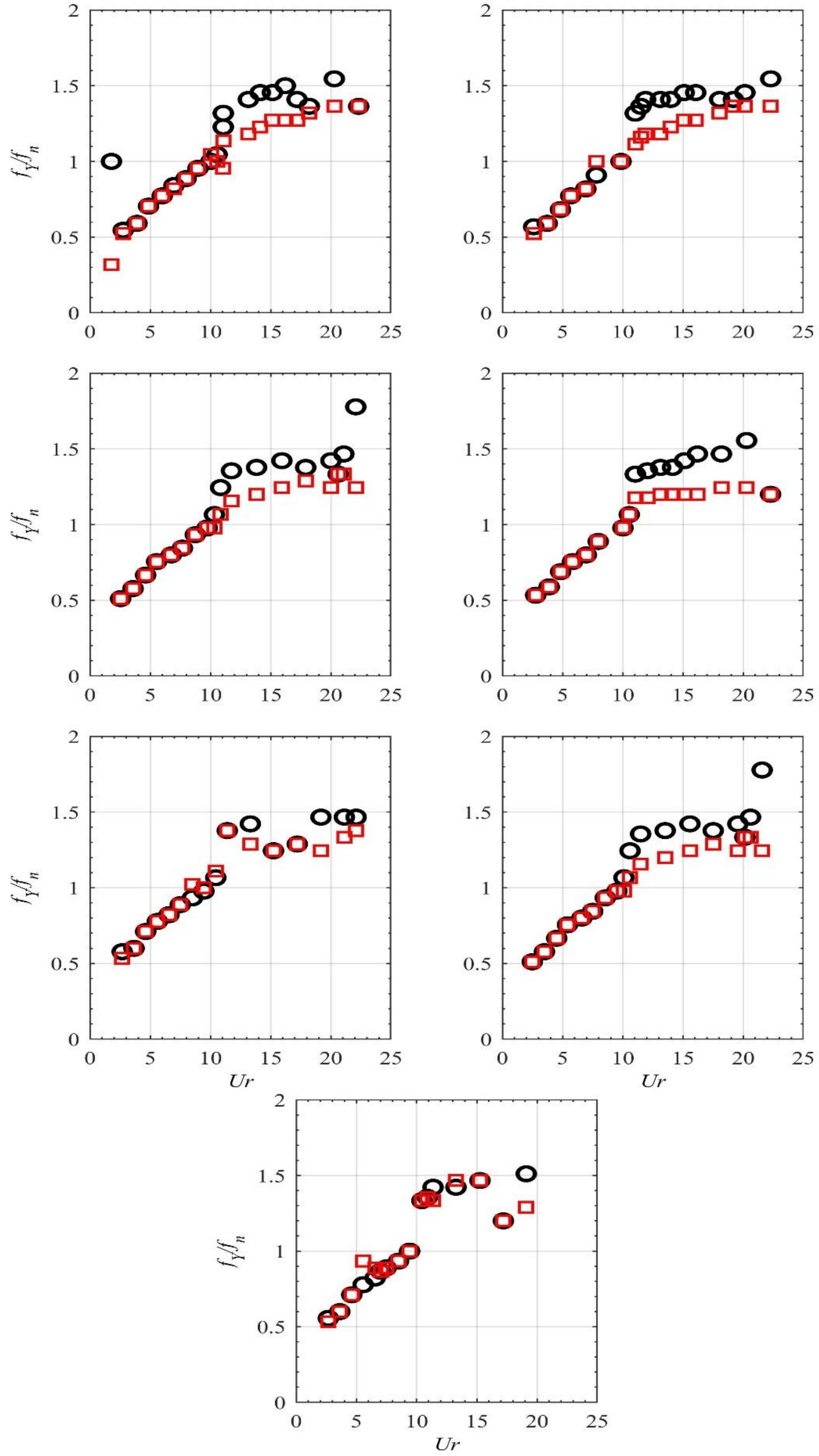

Figure 6 Comparison between frequency response of leading (○) and trailing (□) cylinders in cross-flow direction at various spacings. L/D = (a) 3.5, (b) 4, (c) 5, (d) 8, (e) 10, (f) 15, (g) 20.

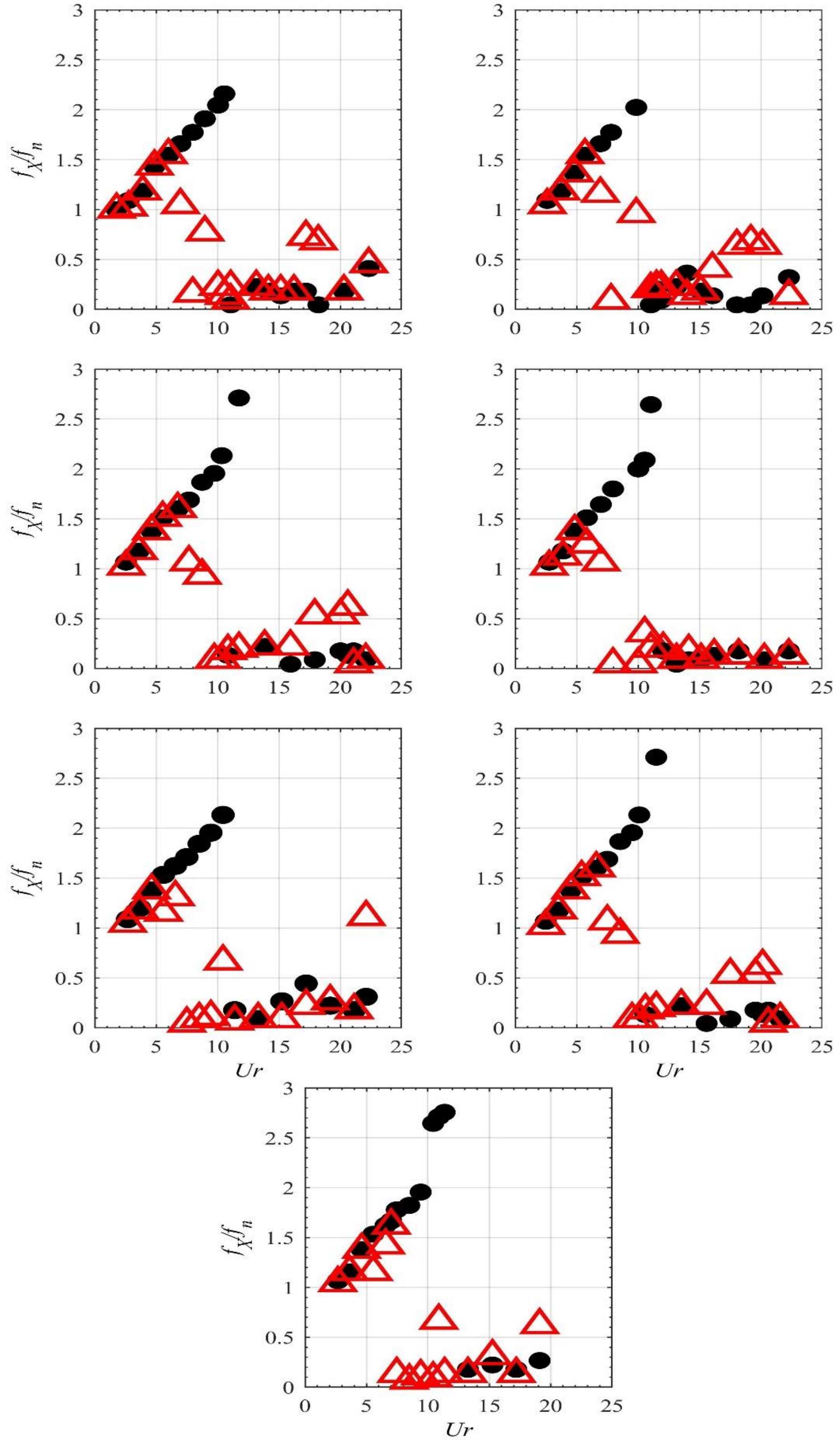

Figure 7 Comparison between frequency response of leading (●) and trailing (△) cylinder in stream-wise direction at various spacings. L/D= (a) 3.5, (b) 4, (c) 5, (d) 8, (e) 10, (f) 15, (g) 20.

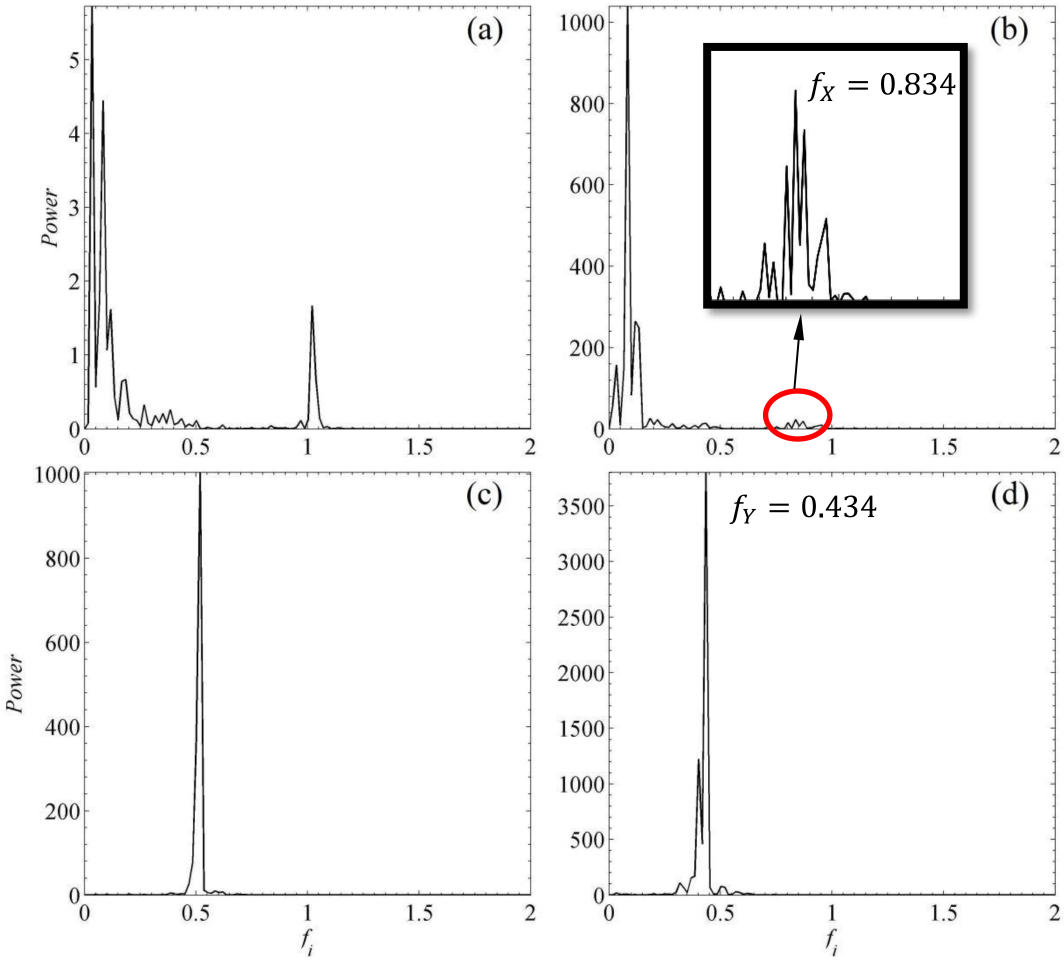

Figure 8. Power spectrum sample of a test with U_r =12 and $L/D = 4$. (a) Leading cylinder's stream-wise response frequency, (b) trailing cylinder's stream-wise response frequency, (c) leading cylinder's cross-flow response frequency, (d) trailing cylinder's cross-flow response frequency

The in-line oscillation frequency for either cylinder does not follow the behaviour of an isolated cylinder (figure 7) (Jauvtis and Williamson, 2004; Srinil et al., 2013). Very low frequencies are dominant (figure 8) at relatively high reduced velocities while for reduced velocities of the initial branch, the stream-wise frequency is twice the transverse frequency as observed by Bearman (1984). The domination of low frequency in the power spectrum of stream-wise response was observed by Huang and Sworn (2011) as well.

By considering the frequency spectrum of both cylinders in cross-flow and stream-wise directions, it is observed that the stream-wise responses of both cylinders consist of different motions with different frequencies. Figure 8 shows an example of such a test where the gap is 4*D* and the reduced velocity is approximately 12. It is observed that although the dominant frequency in stream-wise direction is not twice that observed in the cross-flow direction, a distinct peak still can be found at the frequency equal to twice the cross-flow one. The time history of this test is also shown in figure 9. It is clear that the response of both cylinders in the stream-wise direction consist of two frequency components, one very low frequency with relatively high amplitude and one high frequency with smaller oscillation amplitude, especially for the trailing cylinder. The high frequency motion is due to impact of the upstream vortices on the trailing cylinder (WIV). Two cylinders are not in phase so the vortices arrive to the trailing cylinder at random positions on its trajectory; therefor, they may amplify or damp the amplitude response which overall add a secondary motion to the time history. This motion can be seen in the oscillation trajectory of the trailing cylinder. If the amplitude of each cylinder in cross-flow was drawn against its amplitude in stream-wise direction, it yields a trajectory of its oscillation. Figure 10 shows trajectories for different reduced velocities where $L/D = 4$.

It is evident that the trailing cylinder trajectory is different from the observations of Dahl & Hover during their experiment on a single cylinder. The leading cylinder trajectories are similar to those seen in their observations at all reduced velocities and resemble a figure of eight, hence it is called figure of eight (*fo8*) in literature. However, the trailing cylinder trajectory maintains fo8 up to around $U_r = 10$; beyond that value, a secondary motion appears in the stream-wise direction which disrupts the cylinder's motion. One could identify and distinguish these movements from low frequency motions observed in figure 9.

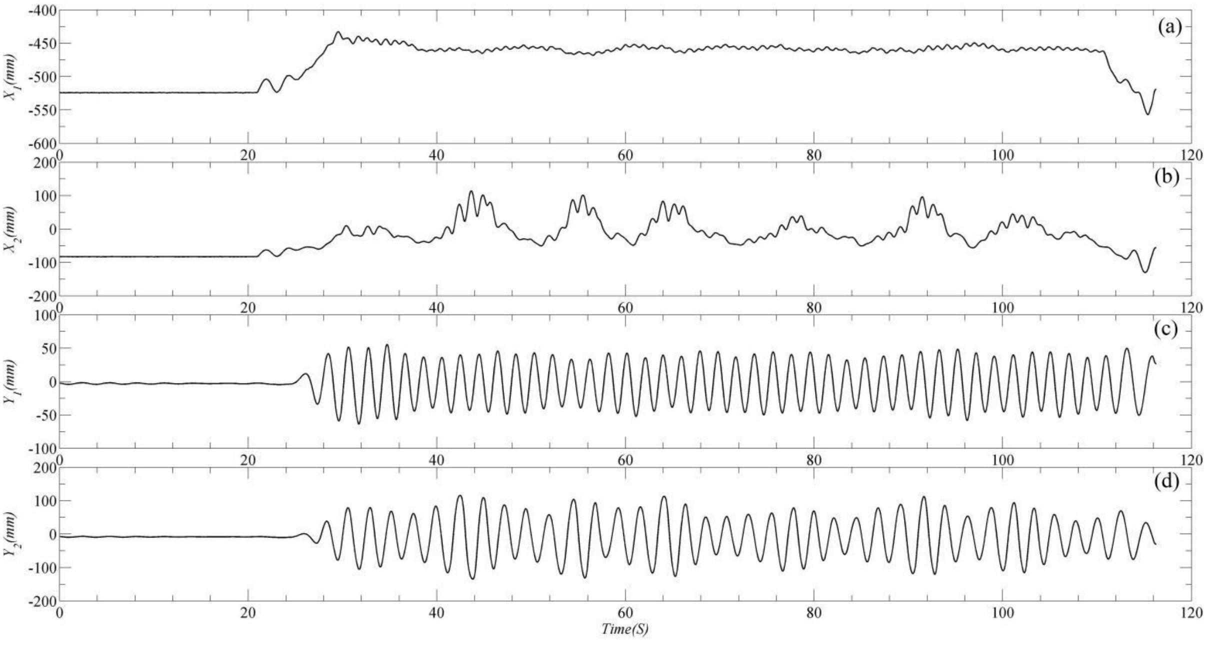

Figure 9 Time history of a test with U_r =12 and $L/D = 4$. (a) Leading cylinder's stream-wise motion, (b) trailing cylinder's stream-wise motion, (c) leading cylinder's cross flow motion, (d) trailing cylinder's cross-flow motion.

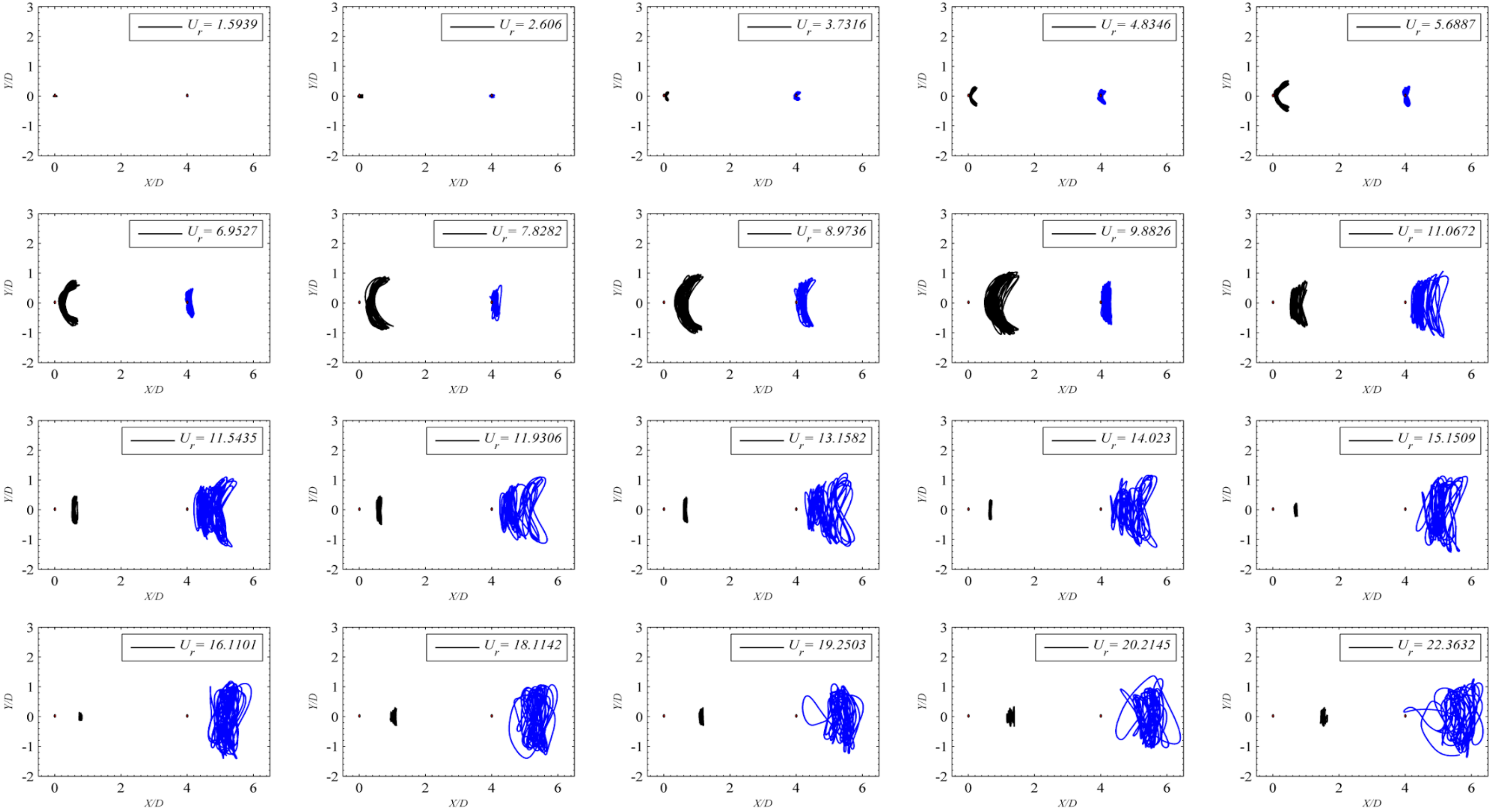

Figure 10: Oscillation trajectories for $L/D = 4$; leading cylinder at (0,0) and trailing cylinder at (0,4)

Such observations can confirm the hypothesis about multiple excitation forces involved in the trailing cylinder's response specifically in the stream-wise direction. Power spectrums extracted from the tests reveal that multiple peaks appear in the stream-wise spectrum of the trailing cylinder after $U_r = 11$. This is approximately the speed after which leading cylinder exits lock-in range and generates relatively weaker vortices. From this reduced velocity onward, vortices detached from the upstream cylinder cannot excite the structure in the wake anymore; nevertheless, they interrupt its VIV motion (note the difference between trajectory of the single cylinder from Dahl et al. (2006) and that of the trailing cylinder in this study). Observations during the experiment suggest that since the two cylinders are not in phase, the vortices from the upstream cylinder reach the trailing cylinder at different position on its trajectory. Consequently, the cylinder’s response to colliding vortices adds to its conventional VIV response. Depending on where the collision happens on the cylinder’s trajectory the final stream-wise motion time history appears to be random. The trailing cylinder stream-wise motion in figures 9 and 10 shows how the time history could be made up of different frequencies and the cylinder’s response to vortices could be combined with the cylinder VIV motion. The upstream vortices do not have a significant effect on the trailing cylinder in cross-flow direction due to its strong motion in this direction. On the other hand, since the motion response amplitude is very small in stream-wise direction, vortices can significantly affect the trailing cylinder’s motion in this direction.

In order to reach a better understanding of this phenomenon an attempt was made to separate the VIV response from the other frequency components and their respective oscillation amplitude in the drag direction using a Fourier transform. It is well-established that in VIV, the drag excitation frequency is always twice that of lift. Since the shedding frequency in the cross-flow direction always has a strong motion and a dominant frequency, it is possible to double this value and assume that it is the approximate stream-wise VIV oscillation frequency. The methodology used was to perform an FFT on the stream-wise response time history from each cylinder and chose the dominant frequency in a bandwidth around the value that was expected to be the shedding frequency. This value should be twice the cross-flow frequency based on observations from isolated cylinder studies. The size of such a bandwidth was chosen to be $0.3345 Hz$ (which is 10 point of FFT sampling on each side of the targeted frequency) to accommodate for any possible discrepancy in exact values of oscillation frequency. Therefore, there will be two matrices; one is containing frequency components of VIV motion and the other is including frequency components of the motion induced by interaction between two cylinders. Then, by performing inverse FFT on each matrix two separate time history corresponding to these two motions can be obtained. These two new time histories could be used to calculate RMS of oscillation amplitude.

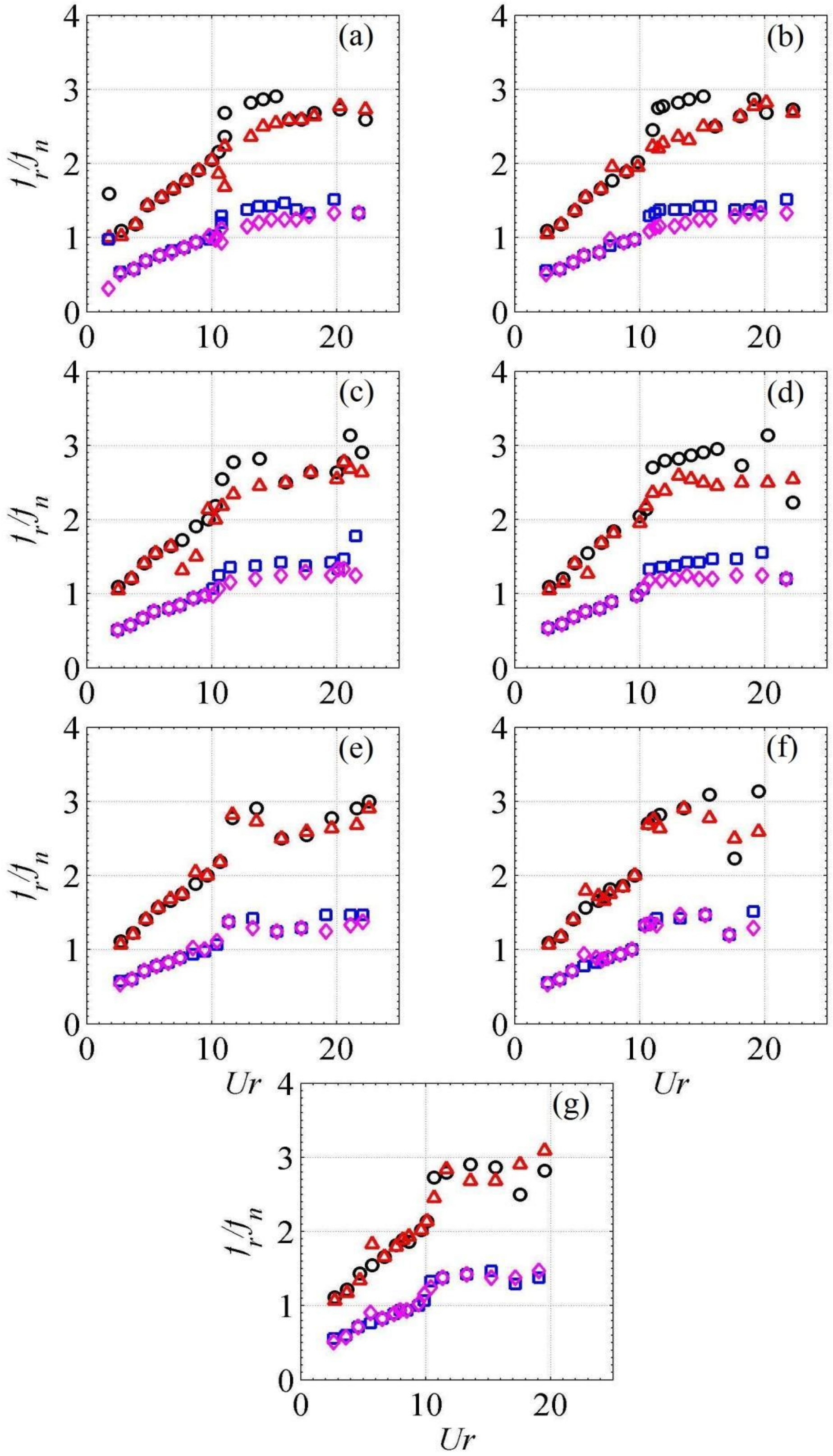

Figure 11 Comparison between frequency response of leading (○) and trailing (△) cylinder in stream-wise direction, leading (□) and trailing (◇) cylinder in cross-flow direction at various spacings. L/D = (a) 3.5, (b) 4, (c) 5, (d) 8, (e) 10, (f) 15, (g) 20.

Figure 11 shows the motion frequency of the VIV response of both cylinders in drag and lift directions obtained using this process. It is evident that the frequency of two cylinders in stream-wise direction is almost identical except for a small interval between reduced velocities of 10 to 16, the period during which trailing cylinder goes through a lock-in-like response by being excited in the stream-wise direction; however the authors cannot propose an explanation for such behaviour at the moment. Nevertheless, it is clear that two cylinders have a significant effect on each other in the stream-wise direction compared to cross flow. The difference between the frequencies of the two cylinders in this direction reduces as the gap grows larger and it is clear that two cylinders have almost identical response frequencies at large spacings.

The different nature of the two motion responses is clearer if the RMS values of two time histories, obtained by inverse FFT, are compared. Figure 12 shows VIV and wake induced vibrations in both directions. Comparison of the stream-wise amplitude responses (figure 12 a and b) reveals that motion induced by chaotic wake of the leading cylinder makes up a larger part of the trailing cylinder's oscillation. On the contrary, vibration of the trailing cylinder in cross-flow (figure 12 c, d) is mostly induced by vortices generated by itself.

5. Conclusion

An experimental investigation was conducted on a pair of identical cylinders in tandem at subcritical *Re*. The two rigid cylinders were flexibly mounted on a rig and tested at various spacings.

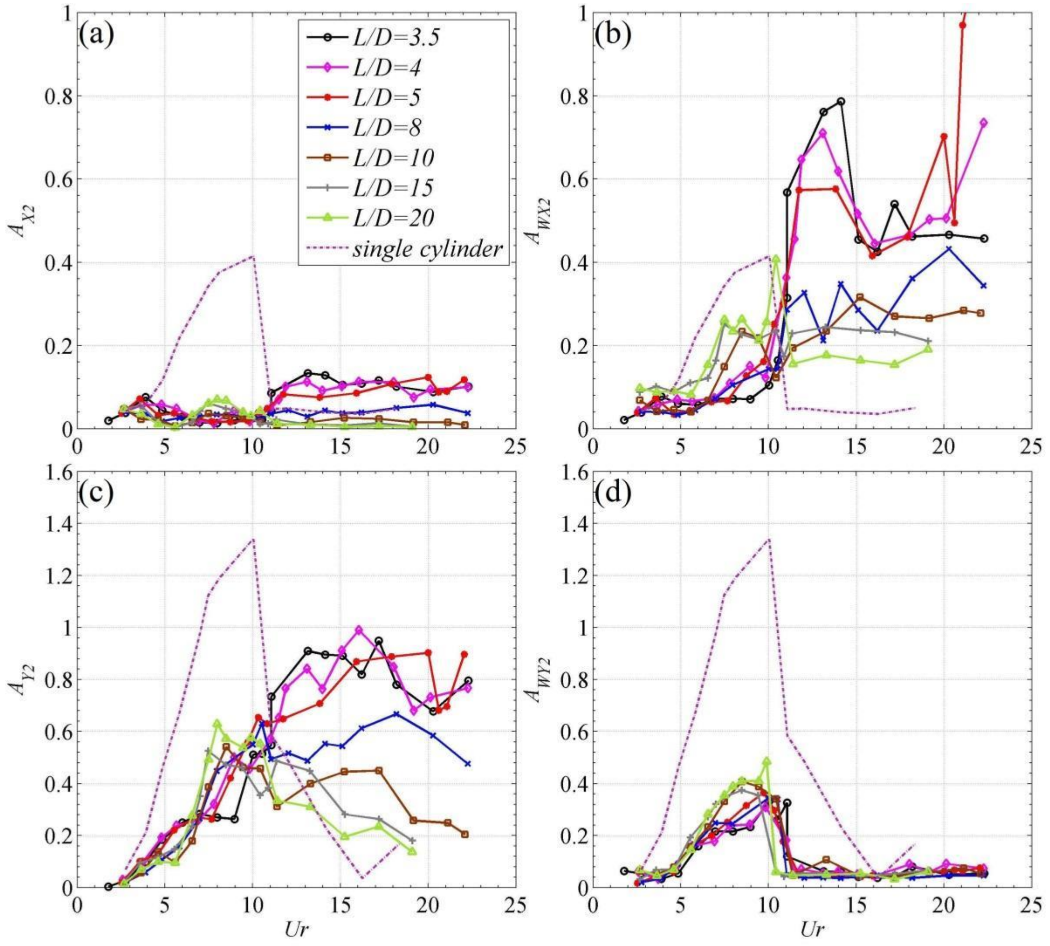

Figure 12: Response amplitude of all spacings in first experiment set-up against reduced velocity.(a)Trailing cylinder's stream-wise VIV response amplitude,(b) trailing cylinder's stream-wise wake induced response amplitude, (c) trailing cylinder's cross-flow VIV response amplitude, (d)trailing cylinder's cross-flow wake induced response amplitude

The motion response of the leading cylinder showed an only a small change with variation of spacing. In contrast, the oscillation of the trailing cylinder was dramatically affected by the wake of the leading cylinder although the maximum amplitude did not exceed that of its upstream counterpart.

- The trailing cylinder underwent a resonance-like response as the flow velocity increased and did not calm down up to the highest reduced velocity for small and moderate spacings.
- On the other hand, As the gap increased ($L/D > 10$), the response of the trailing cylinder became more similar to that of an isolated cylinder. which is in agreement with those papers
- The amplitude of oscillation decreased with increase in the gap until $L/D = 10$ and then increased to become more similar to response of an isolated cylinder. This change in trend suggest a change in the dominant excitation mechanism and the flow regime.

- It was observed that the response of the trailing cylinder has multiple peaks; based on their frequencies they could be divided into 1) peaks excited by vortices coming from the upstream cylinder (WIV) and 2) peaks excited by the coinciding of the trailing cylinder's natural frequency and vortices generated by the trailing cylinder (VIV).
- By separating the motion induced by each mechanism, it was shown that WIV has more influence on stream-wise motion which was not apparent in experiments where cylinder was oscillating only in cross-flow direction.
- Although the trailing cylinder was shielded from the free flow by the leading cylinder, its oscillation frequency was the same as the leading cylinder up to a reduced velocity of around 11; it is concluded that the first cylinder dictated the oscillation frequency of the system. Nevertheless, after the lock-in range, the upstream wake was not powerful enough to keep the trailing cylinder excited; the trailing cylinder then continued to oscillate with a lower frequency than the leading cylinder presumably due to a lower flow velocity in the gap. This observation is in contrast with results obtained from two **fixed** cylinders in tandem where both cylinders oscillate at the same frequency.
- At high velocities, the motion of the trailing cylinder was mainly induced by its VIV and impinging vortices only disturbed its harmonic motion. Since the two cylinders were not oscillating in phase, interaction between trailing cylinder and vortices from the upstream cylinder did not follow a regular pattern; thus, the buffeting vortices randomly impacted the amplitude of oscillation depending on where in the oscillation trajectory they collided. Due to this irregularity, the trailing cylinder oscillation trajectory appeared chaotic. Using frequency analysis the amplitude response regarding each excitation mechanism was extracted and drawn.
- The trailing cylinder's oscillation frequency in transverse direction was not affected by variations of spacing and was solely the result of interaction between fluid and cylinder itself. This allowed to separate motions due to VIV and WIV.

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