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# The Vehicle Routing Problem with Profits and Consistency Constraints

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## Abstract

This paper models and solves a new transportation problem of practical importance; the Consistent Vehicle Routing Problem with Profits. There are two sets of customers, the frequent customers that are mandatory to service and the non-frequent potential customers with known and estimated profits respectively, both having known demands and service requirements over a planning horizon of multiple days. The objective is to determine the vehicle routes that maximize the net profit, while satisfying vehicle capacity, route duration and consistency constraints. A new mathematical model is proposed that captures the profit collecting nature, as well as other features of the problem. For addressing this computationally challenging problem, an Adaptive Tabu Search has been developed, utilizing both short- and long-term memory structures to guide the search process. The proposed metaheuristic algorithm is evaluated on existing, as well as newly generated benchmark problem instances. Our computational experiments demonstrate the effectiveness of our algorithm, as it matches the optimal solutions obtained for small-scale instances and performs well on large-scale instances. Lastly, the trade-off between the acquired profits and consistent customer service is examined and various managerial insights are derived.

*Keywords:* Metaheuristics, consistency, Vehicle Routing Problems with profits, customer-oriented transportation services

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## 1. Introduction

Customer services offered by numerous companies require that employees, such as drivers, sales representatives, technicians and medical personnel, visit customers on a regular basis. Consistent patterns with respect to the time of visit and the visiting service provider can enhance brand loyalty and customer satisfaction by allowing the company's employees to develop relations and create bonds with the customers. Consistency in customer service can act as an order winner, being a key characteristic of high quality service and, hence, of high customer satisfaction. As a consequence, in recent years, more and more companies have focused on and invested in customer relationship management, resulting in a heightened interest in providing consistent customer-oriented services. What complicates matters further is how to design vehicle routes for serving a mix of regular and on the spot customers, while ensuring that consistent service is provided to the regular customers. To that end, this paper addresses a combined orienteering and multi-period vehicle routing problem that aims to maximize the overall profits when a mixed set of customers is served with consistent service constraints.

The literature in recent years has flourished with customer-oriented routing problems. A seminal work in the field is that of Groër et al. (2009) introducing the so-called Consistent Vehicle Routing Problem (ConVRP). The ConVRP involves designing minimum cost routes to service a set of frequent and non-frequent customers with known demands over multiple days via a homogeneous fleet of depot-returning capacitated vehicles, while satisfying vehicle capacity, route duration and consistency constraints. The single-vehicle version of the problem, the Consistent Traveling Salesman Problem (ConTSP), has been recently introduced, and focuses on the time consistency constraints (Subramanyam and Gounaris, 2016). It is also worth mentioning that variants of the problem that allow the introduction of waiting times at the depot or customer locations are presented in the works of Kovacs et al. (2014b) and Subramanyam and Gounaris (2017) for the ConVRP and the ConTSP, respectively. Different types of consistency have been defined and discussed in the literature, i.e. *time consistency*, *person consistency* and *delivered quantity consistency*, along with a number of practical applications, such as courier services (Groër et al., 2009), in-home care and nursing services, the transportation of handicapped (Feillet et al., 2014) and elderly people (Braekers and Kovacs, 2016), pharmaceutical distribution (Campelo et al., 2018), home meal delivery (Hewitt et al., 2015), aircraft fleet schedul-

ing (Ioachim et al., 1999) as well as cleaning services (Tarantilis et al., 2012b). A detailed literature review on VRPs in which consistency is important can be found in Kovacs et al. (2014a).

As far as person consistency is concerned, it is desirable by both employees and customers as it brings about several advantages. Employees can familiarize themselves with customers’ regions and feel more competent in handling unexpected situations, such as an unpredicted detour due to a blocked road or congestion. Additionally, they become familiar with customer needs and requirements and are able to provide a better customized service, creating bonds with the customers (Janssens et al., 2015; Rodríguez-Martín et al., 2018). In a similar manner, person consistency helps customers to become more comfortable with the visiting employee, especially in home-care services when physical contact is required, and may also help in applying security or administrative procedures (Feillet et al., 2014; Spliet and Dekker, 2016). Time consistency is important as well, as it facilitates the visited customers’ planning and organising (Maya Duque et al., 2015). Consistency in the delivered quantity is also desirable, especially in the context of vendor-managed inventory systems, as reduced variations in the delivery quantity facilitate customers’ warehouse management operations (Coelho et al., 2012).

Different ConVRP variants adopt different types of constraints. Harder consistency constraints impose driver and arrival time consistency at the same time (Groër et al., 2009), whereas softer versions of consistency constraints are concerned with time consistency alone (arrival times belonging to the same time-class) (Feillet et al., 2014) or with the objective of limiting the number of drivers/personnel that visit a customer (Kovacs et al., 2015a; Luo et al., 2015; Braekers and Kovacs, 2016). Furthermore, in a number of papers consistency is not addressed in the form of constraints but is included in the objective function, either in an aggregated form (Kovacs et al., 2015a; Sungur et al., 2010) or as multiple objective functions (Kovacs et al., 2015b; Lian et al., 2016). In this paper, we follow the same rationale as Groër et al. (2009), imposing driver and arrival time consistency constraints at the same time. All the aforementioned existing ConVRP variants make the assumption that the company’s resources are adequate to cover all customer requirements. However, this is not the case in a number of applications.

There are real-life cases in which the company’s employees have to regularly and consistently visit customers with long-term relations, i.e. existing customers, to advertise their new products or receive feedback on current products. On the other hand, in an effort to attract new customers and ex-

pand the existing customer base, potential customers, usually located close to the existing ones, need to be added to the current routes (Tricoire et al., 2010). However, resources are limited and often inadequate to accommodate both existing and potential customers; thus it is critical to determine the subset of the potential customers that will be included in the routing plans.

A similar setting is also faced by small package shipping companies. Commercial customers need to be visited multiple times during the planning horizon, while residential customers need to be visited only once on an ad hoc basis (Groër et al., 2009). While demand (requested quantity and service frequency) is quite stable and repetitive for commercial customers, this is not the case for residential customers and, as a result, demand can fluctuate considerably. For example, an unpredictably high volume of requests may be received and, due to vehicle capacity and working hours restrictions, it may not be possible to fulfill all customer requests. Thus, managers need to make decisions on a tactical and operational level as to which customers are going to be serviced. The aforementioned problems pose the same challenge; designing a set of profitable routes with the aim of visiting a set of mandatory customers consistently and, at the same time, deciding which additional customers will be serviced.

Whenever the available resources are insufficient to service all customers, the problem can be modeled as a Vehicle Routing Problem with profits (VRP with profits) (Aksen and Aras, 2006; Archetti et al., 2014b). Given a set of potential customers with known profits, the aim is to determine the subset of customers to be serviced and to construct vehicle routes that optimize the given objective function and satisfy all operational constraints. There are two conflicting objectives; the first is to maximize the total acquired profit, while the second is to minimize the total traveling cost. As a result, three alternative objective functions appear in the literature: a) the maximization of the total acquired profit while constraining the total traveled distance; called Team Orienteering Problem, shortened to TOP, b) the maximization of the net profit, i.e. the difference between the total profit and the total traveling cost; called Capacitated Profitable Tour Problem, shortened to CPTP and c) the minimization of the total traveled distance ensuring that a minimum total profit is gained; called Prize Collecting Vehicle Routing Problem, shortened to PCVRP (Tarantilis et al., 2013).

VRPs with profits have been studied to a great extent and can be used to model a wide variety of applications including home fuel delivery, tourist trip design, mobile crowdsourcing, athlete recruitment, routing of technicians,

aerial reconnaissance, under-way replenishment, fisheries patrol, collection of used products and freight transportation (Feillet et al., 2005; Archetti et al., 2014b; Gunawan et al., 2016). A number of papers present variants that involve additional features such as capacity constraints (Archetti et al., 2009), time windows (Tricoire et al., 2010), multiple depots (Stenger et al., 2013), split deliveries (Archetti et al., 2014a), incomplete service (Archetti et al., 2013), non-linear cost functions (Stenger et al., 2013) and constraints appearing in tourist trip design such as multiple time windows and budget constraints (Souffriau et al., 2013; Kotiloglu et al., 2017). It is also worth highlighting the work of Gendreau et al. (1998), the so-called Undirected Selective Traveling Salesman Problem. The objective is to construct a single maximal profit Hamiltonian cycle, while constraints of visiting compulsory and optional vertices are imposed. Detailed surveys regarding node routing problems with profits can be found in Feillet et al. (2005), Vansteenwegen et al. (2011), Archetti et al. (2014b) and Gunawan et al. (2016).

The contribution of this paper is fourfold. First, a new problem is introduced and modeled, the so-called Consistent Vehicle Problem with Profits (ConVRP with Profits). This problem aims at routing a regular set of customers in a consistent manner and at the same time attempts to decide from a set of customers that appear on the spot which non-regular customers to service additionally and include in the routing plans. The objective is to maximize the net acquired profits ( $profit - cost$ ). Second, a new three-index formulation is proposed. Compared to existing four-index formulations proposed recently for the ConVRP, our mathematical model is more compact, as it uses less variables. Third, a novel Adaptive Tabu Search algorithm has been designed and developed with relatively few user-defined parameters. The proposed metaheuristic uses a multi-start mechanism to generate a number of diversified initial solutions to allow a more thorough exploration of the search space. A key element is the use of both short- and long-term memory structures to guide the local search process. The short-term memory avoids revisiting the same solutions, while the long-term memory exploits and reuses the “good” solution characteristics in an efficient way. Fourth, an extensive set of computational experiments is reported. Our solution approach seems to perform well compared to state-of-the-art approaches for the ConVRP and obtains new improved heuristic upper bounds. Furthermore, new benchmark problem datasets are generated. Initially small-scale instances are solved to optimality and on return these solutions are used to evaluate the effectiveness of the proposed algorithm. In most cases the opti-

mal solutions were matched (worst case performance is 3.28%). Lastly, the trade-off between the acquired profits and consistent customer service is examined, taking into consideration both variable and fixed costs, i.e. routing cost and fleet size, and various managerial implications are discussed.

The remainder of the paper is organized as follows. Section 2 provides the necessary notation and proposes a formal mathematical formulation for the ConVRP with Profits. Section 3 presents the proposed solution method and describes in detail all algorithmic components and mechanisms. Computational experiments on existing and newly generated small, medium and large-scale instances are presented, discussed and analyzed in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Problem Formulation

The ConVRP with Profits can be defined on a complete undirected graph  $G = (N, E)$ , where  $N = \{0, 1, 2, \dots, n\}$  is the node set and  $E = \{(i, j) : i, j \in N, i \neq j\}$  is the edge set. The depot is located at node 0 and the set of customers is denoted by  $N_c = N \setminus \{0\}$ . A non-negative cost  $c_{ij}$  is associated with each edge  $(i, j) \in E$ , while the corresponding travel cost matrix  $[c_{ij}]$  is symmetric, i.e.  $c_{ij} = c_{ji}$ , and satisfies the triangle inequality. A homogeneous fleet of  $K$  depot-returning vehicles with maximum carrying capacity  $Q$  that operate for no more than  $T$  time units is available. Furthermore, there is a set of periods  $P = \{1, \dots, h\}$ , i.e. the planning horizon, with specific service requirements  $r_{ip}$  for each customer  $i \in N_c$  on period  $p \in P$ , i.e.  $r_{ip} = 1$  if customer  $i$  requires service on period  $p$  and 0 otherwise. Additionally, each customer  $i \in N_c$  has a predefined profit  $g_{ip}$ , a service time  $s_{ip}$  and a non-zero demand  $q_{ip}$ , ( $0 < q_{ip} \leq Q$ ). For notational convenience,  $E_p$  denotes a reduced set of edges,  $E_p = \{(i, j) \in E : r_{ip}r_{jp} = 1\}$  and  $V_p$  a reduced set of customers,  $V_p = \{i \in N_c : r_{ip} = 1\}$  for each period  $p \in P$ .

The customer set  $N_c$  can be divided into two disjoint subsets based on the customers' service requirements. The set of frequent customers  $N_f$  contains all the customers that require at least two visits during the planning horizon, while the set of non-frequent customers  $N_{nf}$  contains the remaining customers that require service only once during the planning horizon. Due to route duration and vehicle capacity restrictions, it is not possible to service all customers but it is obligatory to service all frequent customers  $i \in N_f$ . Thus, the set  $N_{nf}$  is split into two disjoint subsets; the set of serviced customers  $N_s$  and the set of non-serviced customers  $N_u$ . The objective is to determine

the subset of customers that will be serviced, along with the corresponding visiting sequences such that the difference of the total collected profit minus the total traveling cost is maximized, satisfying the following constraints:

- each vehicle route starts and ends at the depot
- the accumulated quantity carried by each vehicle  $k \in K$  does not exceed the total carrying capacity  $Q$
- each vehicle route lasts at most  $T$  time units
- each frequent customer  $i \in N_f$  must be visited only once (or alternatively each profit  $g_{ip}$  of customer  $i \in N_f$  is collected only once) by the same vehicle  $k \in K$  on each period  $p \in P$  they require service
- each non-frequent customer  $i \in N_{nf}$  can be visited at most once (or alternatively each profit  $g_{ip}$  of customer  $i \in N_s$  is collected at most once) by one vehicle  $k \in K$  on the period  $p \in P$  they require service
- the maximum difference between the earliest and latest vehicle arrival times to a frequent customer  $i \in N_f$  over the planning horizon does not exceed  $L$  time units

In this paper, a new three-index formulation is introduced. In particular, three groups of variables associated with the customer visiting sequence, the customers' assignment to vehicle routes and the vehicle arrival time to serviced customers are utilized. Let binary variables  $x_{ijp}$  count the number of times edge  $(i, j) \in E$  is traversed on period  $p$ , binary variables  $y_{ik}$  indicate if customer  $i$  is serviced by vehicle  $k$  and continuous variables  $a_{ip}$  depict the arrival time to customer  $i$  on period  $p$  in the optimal solution. Given the above representation, the ConVRP with Profits can be mathematically depicted as follows:

$$\max \sum_{i \in N_c} \sum_{k \in K} \sum_{p \in P} y_{ik} g_{ip} - \left( \sum_{p \in P} \sum_{(i,j) \in E} x_{ijp} c_{ij} + \sum_{p \in P} \sum_{k \in K} \sum_{i \in N_c} y_{ik} s_{ip} \right) \quad (1)$$

Subject to

$$\sum_{k \in K} y_{ik} = r_{ip} \quad \forall p \in P, i \in N_f \quad (2)$$



$$\sum_{k \in K} y_{ik} \leq r_{ip} \quad \forall p \in P, i \in N_{nf} \quad (3)$$

$$\sum_{j \in N_c} x_{0jp} = \sum_{i \in N_c} x_{i0p} = K \quad \forall p \in P \quad (4)$$

$$\sum_{j \in N} x_{ijp} = \sum_{j \in N} x_{jip} = \sum_{k \in K} y_{ik} \quad \forall i \in N (i \neq j), p \in P \quad (5)$$

$$\sum_{i \in V_p} q_{ip} y_{ik} \leq Q \quad \forall k \in K, p \in P \quad (6)$$

$$1 - x_{ijp} - x_{jip} \geq y_{ik} - y_{jk} \quad \forall (i, j) \in V_p \times V_p : i \neq j, k \in K, p \in P \quad (7)$$

$$a_{0p} = 0 \quad \forall p \in P \quad (8)$$

$$a_{ip} + x_{ijp}(s_{ip} + c_{ij}) - (1 - x_{ijp})T \leq a_{jp} \quad \forall (i, j) \in V_c \times V_c : i \neq j, p \in P \quad (9)$$

$$a_{ip} + r_{ip}(s_{ip} + c_{i0}) \leq r_{ip}T \quad \forall i \in V_c, p \in P \quad (10)$$

$$a_{ip} - a_{ip'} \leq L \quad \forall i \in V_p \cap V_{p'}, p \in P, p' \in P : p \neq p' \quad (11)$$

$$y_{0k} = 1 \quad \forall k \in K \quad (12)$$

$$y_{ik}, x_{ijp} \in \{0, 1\}, a_{ip} \geq 0 \quad \forall i, j \in N, k \in K \quad (13)$$

$$r_{ip}c_{0i} \leq a_{ip} \leq T - s_{ip} - c_{i0} \quad \forall i \in N_f, p \in P \quad (14)$$

The objective function (1) maximizes the net acquired profit, i.e. the difference between the total collected profit and the total traveling cost. Constraints (2) and (3) dictate that frequent customers must be visited on each period they require service while the non-frequent customers must be visited at most once on the period they require service. Constraints (4) impose that  $K$  vehicles leave and return to the depot in every period. Constraints (5) ensure route connectivity. Constraints (6) are capacity restrictions for each vehicle. Constraints (7) show that each frequent customer must be serviced by the same vehicle. Constraints (8),(9) and (10) calculate the arrival times to the depot and to each serviced customer on each period. They constitute the Miller-Tucker-Zemlin (MTZ) constraints for time duration and, thus, function also as subtour elimination constraints (Kara et al., 2004). Constraints (11) ensure that the maximum difference between the earliest and latest arrival time to each frequent customer does not exceed a predefined threshold  $L$ . Constraints (12) impose that all vehicle routes start from the depot on every period of the planning horizon. Finally, the last sets impose binary conditions to  $x$  and  $y$  variables as well as lower and upper bounds for the continuous  $a$ -variables.

### 3. Solution Method

#### 3.1. Basic Concept & Solution Framework

The ConVRP with Profits can be seen as a single-objective problem, consisting of two conflicting components. The first component seeks to maximize the total collected profit, whereas the second component seeks to minimize the total traveling distance, while providing consistent customer service. These components are linked with different decisions. The profit maximization is linked with determining the subset of frequent customers that will be assigned to and consistently visited by a vehicle, along with the subset of potential non-frequent customers that will be serviced. The distance minimization refers to determining the visiting sequence of the serviced frequent and non-frequent customers, satisfying all operational constraints. On the basis of the above, the proposed Adaptive Tabu Search (ATS) metaheuristic algorithm takes advantage of different search landscapes and seeks to explore the solution space on the basis of the total collected profit and the total traveling distance.

From the algorithmic point of view, the proposed ATS can be seen as a multi-start trajectory local search approach that utilizes short- and long-term memories and multiple neighborhood moves to allow a more thorough exploration of the search space. It uses a greedy randomized constructive heuristic to generate a number of diversified initial solutions, which are further improved by the ATS algorithm. A key element of the ATS solution framework is the use of short- and long-term memory structures. The short-term memory, being a key component of the ATS algorithm, prevents cycling and revisiting the same solutions during the local search, while the long-term memory records the “good” solution characteristics obtained during the execution of the ATS algorithm. This information is then used to construct initial solutions with “elite” characteristics. Furthermore, multiple neighborhood moves that operate on different search landscapes are utilized and explore the solution space in terms of the acquired profit and the traveling cost.

Algorithm 1 presents the pseudocode of the proposed ATS scheme. At first, a greedy randomized constructive heuristic (Line 4) is utilized to generate initial feasible solutions, which are then improved by the ATS algorithm (Line 5) and the best obtained solution is updated accordingly (Lines 6-8). At this point, the adaptive memory component is triggered (Lines 9-16). A feasible solution is constructed using the long-term memory (“elite moves”)

recorded during the ATS execution (Line 10). In the next step, after emptying the adaptive memory (Line 11), ATS is triggered again to improve the given solution (Line 12). The procedure is repeated until a number of  $z_{in}$  iterations is completed (Line 9). The ATS solution framework terminates after  $\Theta$  iterations (Line 3) (termination criterion) and the best encountered solution  $s^*$  is returned. Input parameters,  $z_t, u_t, z_{in}$  and  $u_{mem}$  control the termination condition for the ATS (number of iterations without observing any improvement), the tabu tenure (tabu list size), the number of adaptive memory component iterations and the adaptive memory size, respectively.

---

**Algorithm 1:** ATS solution framework

---

**Input:**  $z_t, u_t, z_{in}, u_{mem}$

```

1  $s, s^* \leftarrow \emptyset$ 
2  $m^* \leftarrow \emptyset$  //Generate empty elite moves set
3 for  $\theta \leftarrow 1$  to  $\Theta$  do
4    $s \leftarrow \text{Constructive Heuristic}(\theta)$ 
5    $s' \leftarrow \text{ATS}(s, z_t, u_t, u_{mem}, m^*)$ 
6   if  $f(s') > f(s^*)$  then
7      $s^* \leftarrow s'$ 
8   end if
9   for  $i \leftarrow 1$  to  $z_{in}$  do
10     $s \leftarrow \text{Constructive Heuristic}(m^*)$ 
11     $m^* \leftarrow \emptyset$  //Empty elite moves set
12     $s' \leftarrow \text{ATS}(s, z_t, u_t, u_{mem}, m^*)$ 
13    if  $f(s') > f(s^*)$  then
14       $s^* \leftarrow s'$ 
15    end if
16  end for
17 end for
Output:  $s^*$ 

```

---

### 3.2. Constructive Heuristic

A randomized constructive heuristic is proposed in this paper to create initial feasible solutions. Algorithm 2 provides an overview of the proposed constructive heuristic. The generated solutions contain all the required routing information, i.e. the list of daily routes and the corresponding customers'

visiting sequence. The initial feasible solutions are constructed in two phases. In the first phase, the frequent customers are sorted in descending order on the basis of their service requirements and their profit (Line 2). If two customers have the same service requirements (tie-breaker) they are sorted in descending order of their profits. Then, for each frequent customer the  $\theta$  least-cost feasible vehicle assignments are considered and the customer is assigned to one of these randomly (Line 3-9). The  $\theta$  least-cost vehicles are selected using a greedy function that takes into account the traveling cost for all periods the customer under consideration requires service. After assigning the frequent customer to a specific vehicle, for each of the periods of service requirement, all feasible insertion positions are considered and the customer is routed to the one that causes the minimal traveling cost increase. After routing all frequent customers, the non-frequent customers are sorted in descending order based on their profit (Line 10). Thus, non-frequent customers with higher profits are favored with the aim of maximizing the total collected profit. For each non-frequent customer all feasible insertion positions are considered and the one with the least cost is selected (Line 11-18). The constructive heuristic terminates after attempting to route all non-frequent customers.

During the adaptive memory component execution, an initial feasible solution is constructed in a deterministic way, using the information stored in the long-term memory (Line 12 in Algorithm 1). Specifically, the stored “elite moves” are considered consecutively and the corresponding frequent customers are routed in the least-cost positions in the appropriate vehicles. If it is infeasible to assign a frequent customer to the corresponding memory-defined vehicle, then this customer is later considered for insertion with the remaining frequent customers. The remaining unrouted frequent customers are sorted in the same way as discussed in the randomized constructive heuristic above and are placed in the least-cost feasible insertion positions, following a greedy rationale. After routing all frequent customers, the non-frequent ones are selected and included in the solution in the same way as described in the randomized constructive heuristic scheme.

It is worth noting that there might be cases where adding a non-frequent customer decreases the objective value, i.e. the net profit. There are two categories of non-frequent customers that decrease the objective value; the ones with an estimated profit equal to zero and the ones whose estimated profit is greater than zero, but when including them in the initial solution the required traveling cost is greater than their estimated profit. As expected,

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**Algorithm 2:** Randomized constructive heuristic scheme

---

**Input:**  $\theta$

```
1  $s \leftarrow \emptyset$  //Generate empty solution
2 Sort Frequent Customers()
3 for  $i \leftarrow 1$  to  $N_f$  do
4   for  $k \leftarrow 1$  to  $K$  do
5     Find  $\theta$  Feasible Least-Cost Vehicles() // Find  $\theta$  suitable
      vehicles
6   end for
7    $v \leftarrow$  Select Random() // Choose a vehicle randomly
8    $s \leftarrow$  Insert Customer( $v$ ) // Route customer to this vehicle
9 end for
10 Sort Non-Frequent Customers()
11 for  $i \leftarrow 1$  to  $N_{nf}$  do
12   for  $k \leftarrow 1$  to  $K$  do
13      $v \leftarrow$  Find Feasible Least-Cost Vehicle()
14   end for
15   if( $v$ ) then // If a suitable vehicle is found
16      $s \leftarrow$  Insert Customer( $v$ ) // Route customer to this vehicle
17   end if
18 end for
Output:  $s$ 
```

---

the proposed constructive heuristics do not allow the inclusion of the former category in the initial solution. However, they take into account the latter category, as this can be altered during the ATS execution. Due to the fact that during the solution construction phase the routing plans are in a preliminary stage, the traveling cost is likely to be improved as the ATS improvement phase progresses. This would make the non-frequent customers belonging in the second category profitable and, therefore, they should be included in the routing plans. For this reason, our constructive heuristics take into consideration the aforementioned non-frequent customers.

### 3.3. Neighborhood moves

In the proposed implementation of ATS two different groups of moves are utilized: the ones that affect the total collected profit and the ones that affect the total traveling distance. The moves belonging to the first group,

namely 1-1 Replace, 2-1 Replace and 1-2 Replace, change the decisions made regarding the subset of serviced non-frequent customers (Figure 1). The 1-1 Replace, 2-1 Replace and 1-2 Replace moves proposed by Tarantilis et al. (2013) are utilized. Figure 1 shows the moves concerning the selection of non-frequent customers. The 1-1 Replace seeks to swap a non-frequent customer  $i$  from a vehicle  $v_a$  with a non-serviced non-frequent customer  $j$ . All possible insertion positions for  $j$  are examined prior to and after the removal of  $i$  from  $v_a$ . If a feasible insertion position for  $j$  is found - without removing customer  $i$  - then a single customer Insertion is performed instead in the least-cost feasible position. The complexity per iteration of 1-1 Replace is  $O(n^3)$ . In Figure 1, non-frequent customer 1 is no longer serviced while non-frequent customer 7 is included in the solution. In a similar manner, the 2-1 Replace attempts to remove two serviced non-frequent customers from a vehicle  $v_a$  - regardless their position - and insert a non-serviced one. All possible insertion positions are examined after the removal of both serviced customers; as a result the complexity per iteration of 2-1 Replace is  $O(n^4/2)$ . In our example, non-frequent customers 1 and 5 are removed from the vehicle and non-frequent customer 7 is included. Finally, 1-2 Replace attempts to remove one serviced non-frequent customer and to insert two non-serviced ones. As previously, all feasible insertion positions are examined for the pair of non-serviced non-frequent customers. The complexity per iteration of 1-2 Replace is  $O(n^5/2)$ . It is noteworthy that the evaluation of each solution within the neighborhood structures described above has a complexity of  $O(1)$ , i.e. to perform all the necessary feasibility checks and calculate the cost components. As shown in Figure 1, non-frequent customer 2 is no longer serviced while customers 8 and 9 are included for service.

For the routing counterpart of the ConVRP with Profits, the neighborhood moves considered in the proposed implementation are traditional  $O(n^2)$  edge-exchange local moves, namely intra- and inter-route 2-Opt, 1-0 Relocate and 1-1 Exchange (Tarantilis et al., 2012b), along with the ChangeVehicle move (Figure 2). The ChangeVehicle move seeks to alter the assignment of frequent customer  $i$  from vehicle  $v_a$  to vehicle  $v_b$ . Specifically, for each of the remaining vehicles and for each day of required service, all possible insertion positions are examined and the least-cost ones are taken into account for the calculation of the move cost. The complexity per iteration of ChangeVehicle is  $O(n^2|P|)$ . As shown in Figure 2, the vehicle assignment of frequent customer 5 changes for all days of the planning horizon. In particular, customer 5 is serviced by Vehicle 1, before applying the ChangeVehicle move, while

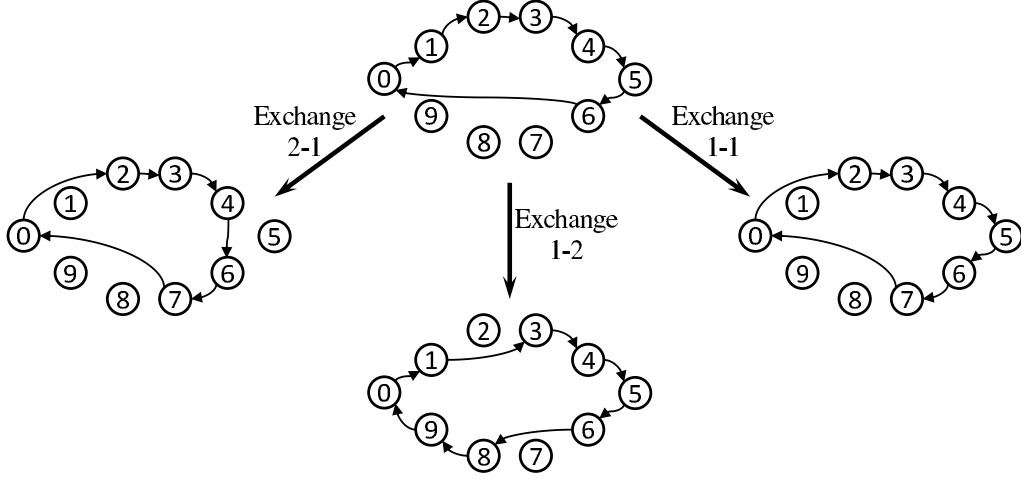


Figure 1: Moves affecting the selection of non-frequent customers

afterwards Vehicle 2 visits customer 5 on each day of service requirement. It should be noted that day 2, in which customer 5 does not require service, remains unchanged.

From the implementation viewpoint, the key advantage of all aforementioned neighborhood moves is their simplicity and flexibility as well as their ease of implementation. For their evaluation a lexicographic search is followed. Emphasis is given on direct feasibility gains to accelerate the evaluation process, i.e. to avoid unnecessary feasibility checks and early pruning of the search tree. For example, we do not examine relocation moves for a customer that cannot fit in terms of capacity to a particular vehicle. Furthermore, we only focus on feasible solutions and we do not take into account infeasible neighbors. As far as the neighborhood exploration is concerned, our approach does not incorporate any spatiotemporal decomposition schemes or heuristic restriction procedures to accelerate the neighborhood search. The reader is referred to the works of Irnich et al. (2006) and Zachariadis & Kiranoudis (2010) for techniques to accelerate neighborhood exploration.

#### 3.4. Adaptive Tabu Search

The ATS algorithm is used to further improve the solutions generated via the constructive heuristic. Algorithm 3 provides an overview of the proposed ATS. ATS seeks to explore the solution space by moving at each iteration from a solution  $s$  to the best admissible solution  $s'$  in a subset  $\Phi_w(s)$  of a

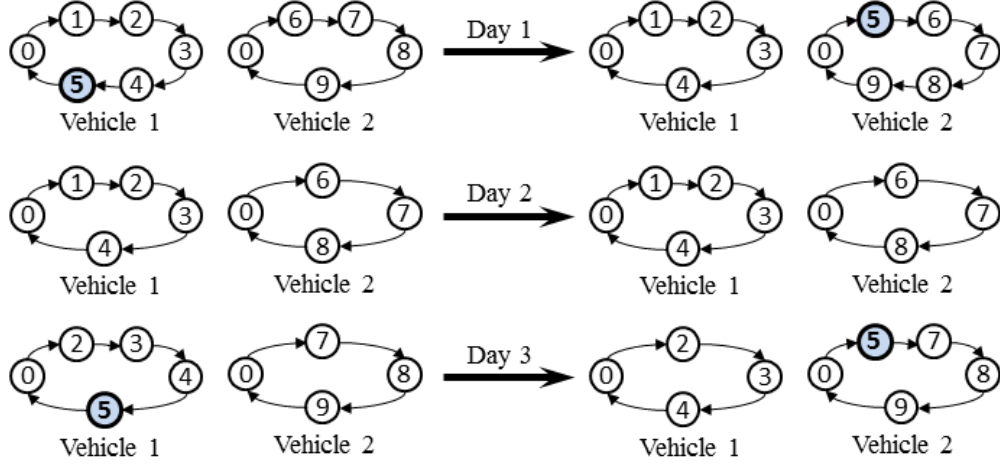


Figure 2: ChangeVehicle move - affecting the assignment of frequent customers to vehicles

neighborhood move  $w$ . The oscillation among all moves is purely random with equal selection probability and a best-accept strategy is followed. During the search, deteriorating solutions are accepted in order to escape from local optima, and the search history is used (short-term memory) to avoid cycling. This memory keeps track of the most recently encountered solution attributes; its size is called tabu tenure ( $u_t$ ) (Lines 4-7).

During the ATS execution, the best improving moves (“elite moves”) concerning the frequent customer assignment to vehicles are recorded in the adaptive memory (long-term memory), whose size is denoted as  $u_{mem}$  (Line 8). The overall procedure terminates when a maximum number of iterations  $z_t$  without any further improvement is reached (Line 3) and the best encountered local optimum  $s_t$  is returned.

### 3.5. Long-term memory structure

The long-term memory structure aims to exploit and reuse the “good” solution characteristics obtained during the ATS execution, by storing the corresponding “elite moves”. This information can be beneficial in the construction of successive initial solutions. In particular, the moves encountered during the execution of the ATS improvement method that fulfill certain selection criteria are stored in the long-term memory and considered as “good” solution characteristics (or alternatively “elite moves”). In our case, the best improving feasible ChangeVehicle moves are selected and recorded in the



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**Algorithm 3:** ATS algorithm

---

**Input:**  $u_{mem}, z_t, u_t, m^*, s$   
//Set of neighborhood moves,  $w = 1, 2, \dots, w_{max}$ //  
1  $m_t \leftarrow \emptyset$  //Generate an empty tabu moves set  
2  $s_t \leftarrow s$   
3 **for**  $z \leftarrow 1$  to  $z_t$  **do**  
4    $w \leftarrow \text{Select Random}()$   
5    $\Phi_w(s) \leftarrow \text{Build Allowed Set}(s, w)$  //Neighborhood evaluation  
6    $s \leftarrow \arg \max_{s' \in \Phi_w(s)} \{f(s')\}$   
7    $m_t \leftarrow \text{Update Tabu List}(s, w, u_t)$   
8    $m^* \leftarrow \text{Update Adaptive Memory}(s, w, u_{mem})$   
9   **if**  $f(s) > f(s_t)$  **then**  
10      $z \leftarrow 0, s_t \leftarrow s$   
11   **else**  
12      $z \leftarrow z+1$   
13   **end if**  
14 **end for**  
**Output:**  $s_t$

---

long-term memory structure during the ATS execution. The reason for storing only these moves is that the frequent customer assignment to vehicles is a crucial decision as it directly affects the routing of frequent customers, as well as the selection and routing of the non-frequent ones. Thus, it is important to exploit the acquired information regarding the frequent customer assignment to vehicles. It is noteworthy that the long-term memory is initialized at the beginning of each ATS trigger and is subsequently updated during its execution.

The pseudocode that depicts the adaptive memory component is presented in Algorithm 4. Let  $u_{mem}$  denote the size of the adaptive memory and  $dc(s, w)$  denote the traveling cost improvement when applying move  $w$  to solution  $s$ . Each stored move consists of a customer ID  $i$  and a vehicle ID  $v$ , accompanied with the corresponding traveling cost improvement  $c'$ . If the adaptive memory is empty, then the move under consideration is stored (Lines 1-2), otherwise the stored “elite moves” are sorted in descending order according to their traveling cost improvement (Lines 6-8), after searching the adaptive memory component (Line 5). If an encountered improving move contains a frequent customer already included in another stored “elite move”,

then the “elite move” with the higher cost improvement will be the one finally recorded (Lines 9-11). It is worth highlighting that since all vehicles are identical, possible symmetry issues might occur in the adaptive memory. However, to overcome these, in the daily routing plans the vehicles are sorted according to the ID of the first routed frequent customer. Thus, it is meaningful to assign a frequent customer to a specific vehicle and record this information in the adaptive memory.

---

**Algorithm 4:** Update adaptive memory

---

**Input:**  $s, w, u_{mem}$

```

1 if ( $m^* = \emptyset$ ) then
2    $m^* \leftarrow \text{Add}(w, dc(s, w))$ 
3 else
4   for  $l \leftarrow 1$  to  $u_{mem}$  do
5      $i, v, c' \leftarrow \text{getStoredMove}(m^*)$ 
6     if ( $w(customer) \neq i$  and  $dc(s, w) > c'$ ) then
7        $m^* \leftarrow \text{Add}(w, dc(s, w))$ 
8     end if
9     if ( $w(customer) = i$  and  $w(vehicle) \neq v$  and  $dc(s, w) > c'$ ) then
10       $m^* \leftarrow \text{Replace Existing Move}(w, dc(s, w))$ 
11    end if
12  end for
13 end if
Output:  $m^*$ 

```

---

## 4. Computational Results

### 4.1. Benchmark Datasets

For the evaluation of the proposed solution method, various computational experiments were conducted using existing and new benchmark datasets. In particular, the ConVRP benchmark instances of Groër et al. (2009) were utilized to test the efficiency and robustness of the proposed ATS algorithm. In addition, to further test the performance of our solution approach, we generated 13 new small-scale instances for the ConVRP with Profits and solved them to optimality. Lastly, 36 new medium- and large-scale ConVRP with Profits instances were created for parameter tuning, algorithmic component testing and managerial insights analysis.

The existing ConVRP benchmark instances originate from the traditional VRP instances of Christofides and Eilon (1969), containing 50 to 199 customers. A 5-day planning horizon with a non-fixed fleet size is assumed, along with a low percentage of non-frequent customers, ranging up to 5%. Vehicle capacity and route duration constraints are taken into consideration. To follow our algorithm’s rationale, a profit was generated for each customer, utilizing the formula presented below, and a fleet size was determined on the basis of the number of vehicles reported in Tarantilis et al. (2012b). The fleet size specified in the aforementioned paper ensures that all customers can be serviced with the available resources; in this case the maximum profit is obtained and the ConVRP with Profits is reduced to the typical ConVRP. As no constraints are imposed on the maximum arrival time difference limit  $L$  of each benchmark instance, this was set according to the results obtained in Groër et al. (2009) to ensure consistency and a fair comparison with the existing solution methods.

In a similar manner, to create the ConVRP with Profits benchmark instances, the existing small, medium and large-scale ConVRP instances of Groër et al. (2009) were adopted and modified accordingly. A profit  $g_i$  was defined for each customer  $i$ , based on the formula:  $b_i \frac{\sum_{j \in N_c} c_{ij}}{n} l$ , where  $\frac{\sum_{j \in N_c} c_{ij}}{n}$  is the average distance of all customers belonging in the same instance,  $l$  is a random number uniformly distributed in the interval  $[0, 1]$  and  $b_i$  is a number that links the acquired profit to the transportation cost as in Chbichib et al. (2011). In a realistic logistical model, the transportation cost as a percentage of sales turnover is 5%-10% while the gross profit margin may be between 20% and 50% of the sales turnover (Rushton et al., 2010). Thus, the total profit is 3 to 5 times the transportation cost. Following this distribution, customer  $i$  can be assigned a value  $b_i$  that links the corresponding acquired profit, which is proportional to their demand, to the transportation cost. To this end, the customer  $i$  with the highest demand  $q_i$  is assigned the value 5, i.e.  $b_i = 5$  and the customer  $j$  with the lowest demand  $q_j$  is assigned the value 3, i.e.  $b_j = 3$ . All other customers with demands ranging between the minimum and maximum demands are assigned a corresponding value  $b$  according to the ranking of their demand. In this way, the total acquired profit is linked to the total transportation cost. The random number  $l$  represents the probability of gaining a potential non-frequent customer and acquiring the corresponding profit, as in Tricoire et al. (2010). For this rea-

son, for each frequent customer  $i$ ,  $l$  is equal to 1 and for each non-frequent customer  $j$ ,  $l \in [0, 1]$ .

The small ConVRP instances of Groër et al. (2009) contain 10 to 12 customers, a 3-day planning horizon as well as vehicle capacity and route duration constraints. A fleet size was determined experimentally and the capacity and route duration limits were modified accordingly, if necessary. Moreover, a specific profit was generated for each customer, as described above. Furthermore, in a number of instances, some frequent customers were randomly chosen to become non-frequent, as the original number of non-frequent customers was not sufficient. Finally, the small-scale instances set for ConVRP with Profits was enriched with some larger instances containing up to 18 customers, following the same rules.

Adopting the same process, we constructed the medium and large-scale ConVRP with Profits benchmark instances. A profit was generated for each customer; a fixed fleet size was specified and vehicle capacity and route duration limits were adapted accordingly, along with the non-frequent customers' percentages. Subsequently, the large-scale benchmark datasets for the ConVRP with Profits are divided into three sets based on their percentage of non-frequent customers. In the first set 15% of customers are non-frequent (set 1), in the second set 25% of customers are non-frequent (set 2) while in the third set 50% of the customers are non-frequent (set 3).

All experimental results reported in the following sections consider fixed parameter settings over a single simulation run; our metaheuristic algorithm was implemented in C++ and all computational experiments were performed on a 3.30 GHz Intel Core i5 PC over a single thread. It is worth highlighting that insignificant differences were observed over multiple runs.

#### 4.2. *Parameter Settings & Experimental Analysis*

The proposed ATS solution method uses four user-defined parameters, i.e. the tabu list size  $u_t$ , the maximum number of ATS iterations without observing any improvement  $z_t$ , the size of the adaptive memory  $u_{mem}$  and the iterations of the adaptive memory component  $z_{in}$ . Assuming reasonable value ranges, well performing parameter settings can be determined with modest effort, since most of these parameters are relatively insensitive to the characteristics of the problems considered.

In what follows, the effect and behavior of the novel algorithmic component and its associated parameters are experimentally examined. A rich scientific literature is devoted to Tabu Search algorithm (Tarantilis et al.,

2012b; Nikolopoulou et al., 2017; Kotiloglu et al., 2017); thus standard settings in the VRP literature are used for the ATS algorithm in all executions with  $u_t = 30$  and  $z_t = 3000$ , providing a good balance between effectiveness and computational time consumption. Additionally, as far as the multi-start procedure parameter  $\Theta$  is concerned, a small positive number ranging from 5 to 20 was used throughout the computational experiments. This number is related to the instance size and was calculated using the following formula:  $\frac{n}{10}$ , where  $n$  is the number of customers of the instance, as larger instances require more computational effort to obtain high quality solutions.

Regarding the adaptive memory component, two structural parameters are incorporated to determine the search strategy: the adaptive memory size  $u_{mem}$  and the number of maximum iterations  $z_{in}$ . Due to the fact that this is a novel algorithmic component, various combinations for both parameters were experimentally tested, in an attempt to find the one that provides a good compromise between effectiveness and computational effort. In particular, value ranges from 0 to 50 and from 5 to 15 were examined for  $u_{mem}$  and  $z_{in}$ , respectively. Figure 3 shows the results obtained for a total of 33 combinations with respect to the average objective value obtained over 6 of the 36 generated medium and large-scale ConVRP with Profits instances (Figure 3(a)) and the CPU time consumption in seconds (Figure 3(b)). The parameter tuning experiments were conducted on six randomly chosen instances to avoid overfitting. Specifically, the results presented in the following figure concern instances P3 and P4 from set 1, P13 and P15 from set 2 and P21 and P24 from set 3.

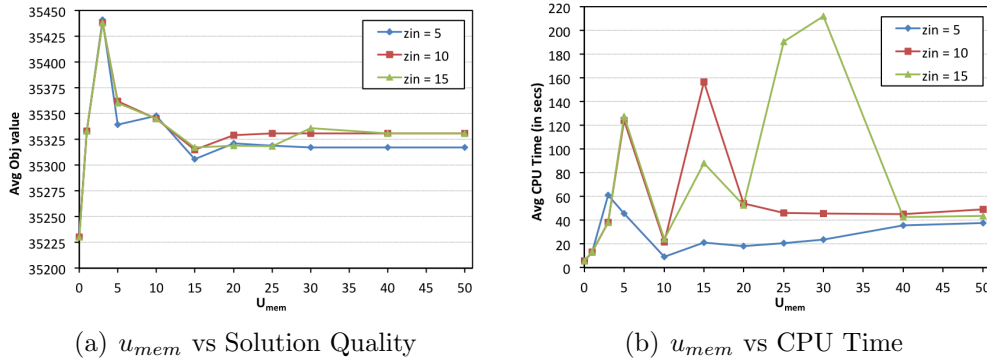


Figure 3: Performance profile of the adaptive memory component

As shown in Figure 3, the adaptive memory component seems to have a

clear contribution to the algorithm’s overall performance. This observation will be further discussed in section 4.4.2. Moreover, we observed that the adaptive memory component performance seems to remain on the same levels after a certain adaptive memory size. This is expected as there is a limit to the number of “elite moves” encountered during the local search. However, from the computational effort viewpoint the trade-off and the expense of computational time seem to increase. On this basis, an adaptive memory size of 10 moves ( $u_{mem} = 10$ ) and a number of maximum iterations equal to five ( $z_{in} = 5$ ) seem to provide a good compromise between solution quality and computational effort. These parameter settings have been used in all the experiments described in the following sections.

#### 4.3. Results for ConVRP Benchmark Instances

This section discusses the results obtained from our algorithm on the ConVRP instances and compares them to the state-of-the-art. The ConVRP instances of Groër et al. (2009) were modified, as described in section 4.1, to follow our algorithm’s rationale and the algorithmic parameter settings described in section 4.2 were utilized. Tables 1 and 2 show the corresponding results and the computational times in seconds (where available), respectively. The proposed ATS algorithm was used on the existing ConVRP instances and was compared to all available metaheuristics, i.e. the Record-To-Record travel algorithm for the ConVRP (ConRTR) (Groër et al., 2009), the Master And Daily Scheduler (MADS) (Sungur et al., 2010), the template-based Tabu Search (TTS) (Tarantilis et al., 2012b), the template-based Adaptive Large Neighborhood Search (TALNS) (Kovacs et al., 2014b), the Large Neighborhood Search (LNS) (Kovacs et al., 2015a) and the Decomposition, Repair and Distance Reduction approach (DRDR) (Luo et al., 2015). For each algorithm and each instance, the total travel time (TT) and the maximum arrival time difference ( $L_{max}$ ) are reported.  $TT_m$  is the best result obtained in five runs for the TTS and the TALNS and in 10 runs for the LNS and the DRDR. It is noteworthy that as far as the DRDR approach is concerned, the authors report the solutions with the smallest maximum arrival time difference, instead of the ones with the smallest traveling cost. For this reason, the solutions presented in this table are denoted as TT and not  $TT_m$ . Additionally, both MADS and DRDR do not bound  $L_{max}$ . Therefore, only the instances with a maximum arrival time difference smaller or equal to those found by Groër et al. (2009) are reported to ensure fairness and consistency. The best solutions reported in the literature in terms of the

Table 1: Comparative analysis on ConVRP instances

#	Best		ConRTR		MADS		TTS		TALNS		LNS		DRDR		ATS		
	TT	$L_{max}$	TT	$L_{max}$	TT	$L_{max}$	TT	$L_{max}$	TT <sub>m</sub>	$L_{max}$	TT <sub>m</sub>	$L_{max}$	TT	$L_{max}$	TT	$L_{max}$	%Gap
P1	2124.21	20.18	2282.14	24.38	2281.05	24.27	2210.56	21.99	2124.21	23.72	<b>2124.21</b>	20.18	2150.57	23.84	2135.70	23.63	0.54
P2	3540.80	25.41	3872.86	34.26	3954.99	30.00	3622.71	27.75	3600.41	31.86	<b>3540.80</b>	25.41	3653.62	25.22	3559.43	34.11	0.53
P3	3280.47	21.22	3628.22	22.87	-	-	3451.1	21.92	3326.12	22.21	<b>3280.47</b>	21.22	3492.05	22.63	3355.89	20.94	2.30
P4	4473.31	19.85	4952.91	27.53	-	-	4572.00	25.15	4556.33	24.19	<b>4473.31</b>	19.85	4711.01	20.12	4560.09	26.05	1.94
P5	5632.22	17.69	6416.77	26.93	-	-	5732.62	19.99	5664.06	22.69	<b>5632.22</b>	17.69	5802.72	15.48	5650.02	26.72	0.32
P6	4051.48	63.29	4084.24	63.47	-	-	4096.87	55.38	<b>4051.48</b>	63.29	4070.72	40.39	4283.40	51.61	4053.96	55.69	0.06
P7	6673.61	43.72	7126.07	83.96	7062.52	60.88	6752.36	63.28	6770.49	76.62	<b>6673.61</b>	43.72	7065.47	56.74	6675.69	75.73	0.03
P8	7126.29	50.27	7456.19	73.04	7461.98	67.49	7279.39	62.01	7129.79	65.97	<b>7126.29</b>	50.27	7571.57	49.82	7167.81	68.08	0.58
P9	10370.60	104.22	11033.54	106.43	10872.44	74.87	10585.10	84.76	10381.90	88.85	10390.70	59.07	10974.20	61.55	<b>10370.60</b>	104.22	0.00
P10	12955.10	55.19	13916.8	60.17	13646.84	59.48	13120.40	57.17	13102.70	57.95	<b>12955.10</b>	55.19	13992.70	50.16	13019.90	59.83	0.50
P11	4471.22	13.91	4753.89	16.10	-	-	4721.09	15.68	4485.37	15.33	<b>4471.22</b>	13.91	4643.78	14.63	4590.83	15.31	2.68
P12	3497.93	16.50	3861.35	17.58	3938.11	16.82	3607.88	16.91	<b>3497.93</b>	16.50	3521.88	13.63	3617.33	11.24	3505.77	16.94	0.22
Avg	5683.10	37.62	6115.42	46.93	7031.13	47.69	5812.67	39.33	5724.23	42.43	5688.38	31.71	5996.54	33.59	5720.47	43.94	0.81

total traveling time are presented (Best), along with the corresponding %Gap obtained by our metaheuristic algorithm compared to the best reported solutions. The average results over all instances are given in the last row of the table. The best-known solutions are indicated in bold.

Table 2 presents the corresponding reported computational times (in seconds). As different machines and different numbers of simulation runs were utilized to obtain the results reported in literature, in order to provide an efficiency indication of the different solution approaches, we followed the procedure discussed in Bräysy and Gendreau (2005) and Tarantilis et al. (2012a). The bottom section of the table describes the machine used, the number of runs, the relative speed of the machine, and the normalized average computational time. The relative speed of each machine is derived with respect to an Intel Xeon 2.8GHz, using the PassMark® CPU marks ([https://www.cpubenchmark.net/CPU\\_mega\\_page.html](https://www.cpubenchmark.net/CPU_mega_page.html)). To that end, the normalized computational time is calculated from the relative speed multiplied by the mean computational time (in seconds) and the number of runs. It is worth mentioning that this procedure provides an efficiency indication for each solution approach; however, it cannot be used as a basis for direct comparisons.

Combining the values reported in Tables 1 and 2, Figure 4 illustrates the two-dimensional space with respect to the average travel time and the average normalized computational time (in seconds) of each approach, following the rationale suggested by Bräysy and Gendreau (2005). The closer a point is to the lower left corner (low traveling cost requiring the least computational effort), the better is the associated solution approach. It is noteworthy that ConRTR and MADS are not plotted in Figure 4, as there are no computational times reported for ConRTR and MADS cannot be directly compared

Table 2: Computational times in seconds of different algorithms for ConVRP

#	TTS	TALNS	LNS	DRDR	ATS
P1	80.00	5.45	15.13	8.00	10.20
P2	93.00	14.69	18.82	6.00	10.05
P3	369.00	25.58	40.22	29.00	20.34
P4	388.00	84.31	62.71	31.00	37.36
P5	550.00	122.24	87.26	38.00	43.04
P6	70.00	6.63	14.58	5.00	26.04
P7	161.00	18.33	19.68	4.00	30.19
P8	539.00	32.24	31.27	16.00	42.29
P9	947.00	97.39	50.24	23.00	40.27
P10	1052.00	146.32	78.73	31.00	84.33
P11	480.00	35.96	83.63	56.00	30.00
P12	172.00	25.60	27.41	17.00	14.25
Average	408.42	51.23	44.14	22.00	32.36
Machine	IX 2.80 <sup>1</sup>	IX 2.67 <sup>2</sup>	IX 2.67 <sup>2</sup>	IX 2.27 <sup>3</sup>	ICi5 3.30 <sup>4</sup>
Runs	5	10	10	10	1
Relative speed	1.00	3.19	3.19	3.81	4.30
Normalized time	2042.10	1634.24	1408.07	838.20	139.15

<sup>1</sup> Intel Xeon 2.80 GHz<sup>2</sup> Intel Xeon X5550 2.67 GHz<sup>3</sup> Intel Xeon 2.27 GHz<sup>4</sup> Intel Core i5 3.30 GHz

to the other metaheuristics, as explained above.

As shown in Table 1, the ATS algorithm performed well, compared to the state-of-the-art, and obtained a new heuristic lower bound for instance P9. As far as the computational time is concerned, the ATS framework required a very competitive computational time, proving its efficiency. As illustrated in Figure 4, our metaheuristic is non-dominated, providing a very good combination of solution quality and speed.

From the algorithmic viewpoint, the majority of the existing metaheuristic algorithms addressing the ConVRP adopt the concept of template routes. A template is a set of predetermined artificial routes, containing only the frequent customers, that are used as a guide to design the actual daily schedules. According to the literature and the obtained results, the methods utilizing the rationale of template routes as a focal algorithmic component do not perform as well as the ones that construct and handle the daily routes accounting for all customers (Kovacs et al., 2015a). However, Groër et al. (2009) highlight that in optimal ConVRP solutions the frequent customers are routed in the same order over the planning horizon, following the idea of a template. Investigating this statement further, Gounaris (personal communication, October 5, 2017) reports that, in the optimal ConTSP solutions they have obtained, 75% of frequent customer pairs appear in the same order



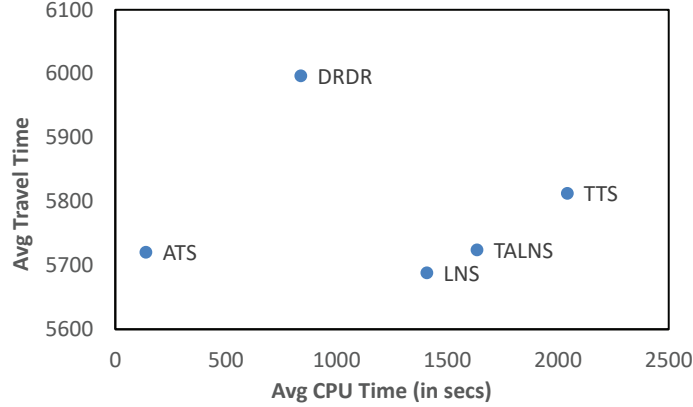


Figure 4: Efficiency analysis - ConVRP metaheuristics

in all common periods and, in only two optimal solutions out of more than 400, all pairs of customers satisfied the precedence rule. Our findings are in line with the findings reported by Gounaris, as none of our solutions satisfies the precedence rule fully. However, it is worth highlighting that in our ConVRP solutions, 95% of frequent customer pairs appear in the same order in all common periods. Taking all the above into consideration, we believe that a solution framework tackling a Vehicle Routing Problem with consistency constraints could utilize the idea of template routes as a means to construct an initial solution rather than a fixed decision that restricts the local search space.

#### 4.4. Results for ConVRP with Profits Benchmark Datasets

In this section we discuss all the computational results obtained on the new small- and large-scale benchmark instances for the ConVRP with Profits. In particular, the small-scale instances are solved to optimality and compared to our algorithm’s results. Moreover, we use the medium- and large-scale instances to test the efficiency of our algorithmic components as well as to examine the trade-off between the collected profits and the fixed fleet size and consistency constraints. In all the aforementioned experiments, the focus is on the offered service level, i.e. the maximum arrival time difference ( $L_{max}$ ) and the percentage of unvisited non-frequent customers (%UNFC). Both metrics are of high importance and lead to increased customer satisfaction. Low  $L_{max}$  values result in less diverse and “more consistent” daily

Table 3: Comparative analysis on ConVRP with Profits small-scale instances

#	CPLEX						ATS				
	NP	$L_{max}$	%UNFC	Root Node	CT	# Nodes	NP	$L_{max}$	%UNFC	%Gap	% $L_{gap}$
P1	357.31	4.67	50.00	369.17	1.97	2529	357.31	4.67	50.00	0.00	0.00
P2	289.04	4.63	0.00	293.78	2.36	1914	279.56	3.93	0.00	3.28	-15.12
P3	375.12	2.50	0.00	398.80	8.36	5433	375.12	2.50	0.00	0.00	0.00
P4	413.11	3.98	0.00	422.87	11.23	5592	413.11	3.98	0.00	0.00	0.00
P5	391.69	4.17	0.00	406.60	12.37	5393	389.50	4.40	0.00	0.56	5.52
P6	440.98	3.93	0.00	477.30	222.11	76280	440.87	3.93	0.00	0.02	0.00
P7	303.77	4.25	50.00	308.25	0.38	358	303.77	4.25	50.00	0.00	0.00
P8	431.56	3.34	0.00	446.91	14.65	4791	431.56	3.34	0.00	0.00	0.00
P9	377.63	4.21	0.00	396.25	51.20	25481	369.40	4.61	0.00	2.18	9.50
P10	305.08	4.48	67.00	310.92	2.43	1628	298.87	4.15	67.00*	2.04	-7.37
P11	588.61	4.99	0.00	605.15	5759.34	1374780	581.21	2.03	0.00	1.26	-59.32
P12	715.63	3.48	75.00	735.35	340.38	104901	715.63	3.48	75.00	0.00	0.00
P13	698.92	4.66	75.00	735.46	1038.31	244180	698.92	4.66	75.00	0.00	0.00
Average	437.57	4.10	24.38	454.37	574.24	142558.50	434.99	3.84	24.38	0.72	-5.14

schedules, while lower percentages of unvisited non-frequent customers contribute not only to customer satisfaction but also to higher acquired profits.

#### 4.4.1. Small-scale instances

To further test the performance of our algorithm, we used the small-scale benchmark instances and the corresponding optimal solutions were obtained by solving exactly the proposed mathematical model, using a commercial solver (ILOG CPLEX 12.7). Table 3 presents the optimal solutions and the results obtained by the ATS framework. The objective function value (NP),  $L_{max}$ , the percentage of unvisited non-frequent customers, the root node, the number of nodes being examined during the CPLEX execution and the corresponding computational time in seconds (CT) are reported for each optimal solution. The obtained objective function value, the percentage of unvisited non-frequent customers and  $L_{max}$  are reported for our metaheuristic algorithm. The gap in the objective function value and the maximum arrival time difference obtained by our metaheuristic compared to the optimal solutions, i.e. %Gap and % $L_{gap}$  respectively, are calculated. In all cases, ATS took less than a second to find the presented solutions.

As shown in Table 3, our metaheuristic managed to obtain high quality solutions. Specifically, it found seven optimal solutions, while its average deviation from the optimal solutions was 0.72% (worst case performance is 3.28%). As far as consistency is concerned, the proposed solution method produced “more consistent” solutions, improving the average  $L_{max}$  by 5.14%. However, this result should be treated with caution as it occurs not only due

to our metaheuristic’s effectiveness to improve consistency but also as a trade-off between the net acquired profits and the service consistency constraints. It is worth noting that ATS identified the optimal set of non-frequent customers in 12 out of 13 instances. The only case that ATS did not identify the optimal set of non-frequent customers to be serviced was P10 (noted by asterisk), where the solution obtained by ATS included the same percentage of non-frequent customers as the optimal solution, however there was a difference in the subset of serviced non-frequent customers (one non-frequent customer was different). All the aforementioned results demonstrate the effectiveness of the proposed framework.

#### 4.4.2. Medium and Large-scale instances

As shown in the previous section, the computational time for solving optimally instances of realistic size is excessive. To this end, larger instances were solved via the proposed metaheuristic algorithm. Several computational experiments were performed on these instances in order to test the performance of the introduced algorithmic mechanisms and the effect of the fixed fleet size to the overall profits.

Initially, the cooperative and synergistic effect between the long-term memory structure, i.e. the adaptive memory, and the multi-start mechanism, is examined. For this purpose, the proposed ATS framework is compared to Tabu Search (TS), a well-known, robust and effective metaheuristic. As far as the TS implementation is concerned, a number of 30,000 iterations without observing any further improvement and a tabu list size equal to 30 were used. The corresponding computational results are summarized in Table 4. For each algorithm and each instance, the obtained objective function value,  $L_{max}$ , the computational time in seconds and the percentage of non-frequent customers not included in the final solution are reported. Additionally, we have calculated the %Gap between the two metaheuristic algorithms. It is worth highlighting that both ATS and TS use the same multi-start mechanism to ensure fairness in their comparison.

All in all, comparable results are obtained by both solution frameworks, indicating the efficiency and the effectiveness of the proposed ATS metaheuristic. In particular, ATS found slightly better solutions in terms of the objective function value than the TS algorithm on average, obtaining improvements of up to 1.16%. The reason behind this improvement is that the ATS solution framework managed to include more non-frequent customers in the final solution by 1.1% on average than the TS algorithm, with the highest

Table 4: Results on ConVRP with Profits medium and large-scale instances

#	ATS				TS				%Gap
	NP	$L_{max}$	CT	%UNFC	NP	$L_{max}$	CT	%UNFC	
P1	15352.90	128.79	0.97	25.00	15354.80	132.18	6.47	25.00	-0.01
P2	27915.10	106.63	4.48	27.00	27915.10	106.63	29.91	27.00	0.00
P3	35400.00	135.90	2.33	7.00	35253.70	160.28	21.01	13.00	0.41
P4	55536.00	127.15	14.62	9.00	55536.00	127.15	123.47	9.00	0.00
P5	73648.30	128.00	253.39	3.00	73480.10	124.52	317.14	7.00	0.23
P6	15938.50	248.73	0.70	14.00	15938.50	248.73	6.12	14.00	0.00
P7	26596.10	217.04	2.78	18.00	26596.10	217.04	25.63	18.00	0.00
P8	32442.40	284.04	4.30	0.00	32067.60	273.27	28.96	7.00	1.16
P9	49065.30	258.91	14.76	0.00	49065.30	258.91	88.33	0.00	0.00
P10	67020.00	262.75	162.92	0.00	66765.00	272.77	199.80	3.00	0.38
P11	69669.70	139.98	5.67	6.00	70077.50	237.38	49.84	0.00	-0.59
P12	36145.10	125.74	6.19	0.00	36412.60	123.81	49.38	0.00	-0.74
Average (set 1)	42060.78	180.31	39.43	9.08	42038.53	190.22	78.84	10.25	0.07
P13	13699.80	149.11	4.33	15.00	13572.40	163.61	25.99	15.00	0.93
P14	25865.20	104.64	0.58	5.00	25865.20	104.64	0.55	5.00	0.00
P15	31186.40	209.80	13.53	8.00	31130.10	185.22	33.62	12.00	0.18
P16	50824.60	140.72	240.96	16.00	50610.80	145.60	263.06	14.00	0.42
P17	66436.80	161.01	625.07	10.00	66279.00	128.75	489.71	6.00	0.24
P18	14208.60	225.13	4.83	17.00	14053.40	222.87	7.79	8.00	1.09
P19	23893.00	179.04	4.20	21.00	23706.40	183.81	36.55	16.00	0.78
P20	29118.60	293.90	6.08	16.00	29118.60	293.90	58.99	16.00	0.00
P21	43315.20	259.94	14.02	0.00	42867.90	271.79	69.21	3.00	1.03
P22	59187.10	247.00	440.18	20.00	59109.50	273.97	316.28	22.00	0.13
P23	64016.90	209.80	2.90	0.00	64016.90	209.80	2.73	0.00	0.00
P24	32948.60	126.92	5.66	8.00	32893.10	134.77	78.66	8.00	0.17
Average (set 2)	37891.73	192.25	113.53	11.33	37768.61	193.23	115.26	10.42	0.41
P25	9926.88	140.97	1.83	20.00	10009.40	153.36	47.33	12.00	-0.83
P26	17699.00	139.93	21.59	40.00	17696.90	139.93	204.79	39.00	0.01
P27	21893.20	184.24	110.35	30.00	21794.10	199.28	448.58	28.00	0.45
P28	35915.20	158.87	1451.27	22.00	35762.40	160.58	2836.18	22.00	0.43
P29	46441.50	156.79	3650.98	16.00	46377.90	181.21	5360.62	18.00	0.14
P30	10299.10	174.88	18.81	0.00	10214.00	162.33	147.68	4.00	0.83
P31	16521.30	170.51	484.56	24.00	16482.80	169.39	236.80	21.00	0.23
P32	20943.40	285.36	44.19	32.00	20943.40	285.36	151.90	32.00	0.00
P33	30606.80	203.47	370.68	35.00	30606.80	203.47	1156.24	35.00	0.00
P34	42175.30	223.77	1048.58	29.00	42095.70	223.77	3015.00	32.00	0.19
P35	44489.10	218.17	106.07	5.00	44290.60	232.25	780.42	5.00	0.45
P36	22939.70	193.84	374.04	22.00	22902.70	185.69	352.55	18.00	0.16
Average (set 3)	26654.21	187.57	640.25	22.92	26598.06	191.39	1228.17	22.17	0.17
Average	35535.57	186.71	264.40	14.44	35468.4	191.61	474.09	14.28	0.22

difference being 7%. Furthermore the ATS solutions were “more consistent” than the TS solutions by 2.62%. Specifically, the highest difference concerning  $L_{max}$  (5.5%) was observed in the problem set with the lowest percentage of non-frequent customers (set 1). This is due to the fact that large numbers of frequent customers result in less diverse daily schedules and “more consistent” routes. Thus, a strategy of assigning effectively the frequent customers

to vehicles can lead to consistent solutions. Finally, ATS was much faster than the TS algorithm, requiring almost 80% less time to obtain its solutions. The above indicates that there is a clear cooperative and synergistic effect between the adaptive memory component and the multi-start mechanism. Although this is more or less expected as two different methods are combined to improve the current best solution, it is worth highlighting that their combination not only helped the search to acquire better solutions, but also expedited the overall search process in terms of computational times.

Another interesting point for investigation is the required fleet size to visit all customers and maximize the collected profit. One of the key characteristics of ConVRP with Profits is the fact that the available resources, i.e. the number of vehicles, are not adequate to serve/visit all customers. Therefore, from a managerial point of view it would be useful for the decision-makers to be aware of the trade-off between the potential acquired profit and the cost of the required vehicles. To this end, computational experiments were conducted to determine the net collected profit provided the available fleet size was sufficient to visit all customers. For all problem instances the number of available vehicles was increased until the obtained solution included all customers. The corresponding results are summarized in Table 5. The results for the original fleet size are reported under the ATS label, while the results for the increased fleet size are reported under the ATS-fi label. For each instance, we present the corresponding objective function value,  $L_{max}$ , the computational time in seconds and the number of vehicles required for the obtained solution ( $|K|$ ). The %Gap between the two objective function values and the %Gap( $|K|$ ) between the required numbers of vehicles were calculated. It is worth noting that some instances are not included in the table as our algorithm managed to include all customers utilizing the original fleet size.

Clearly, the net acquired profit increases with larger fleet size. Specifically, there is an average improvement of 1.35% in terms of the objective function value. This improvement seems to increase as the number of non-frequent customers increases, ranging up to 5.23%. This is expected as the more non-frequent customers included in the final solution, the higher the net collected profit will be. Additionally, the solutions utilizing more vehicles were “more consistent” by 8.3% and the required computational time was decreased by 84%. These differences are due to the higher number of available vehicles, increasing the degrees of freedom and enabling the ATS metaheuristic algorithm to route all customers more easily and efficiently.

Table 5: Results on increasing fixed fleet size

#	ATS				ATS-f				%Gap	%Gap( K )
	NP	$L_{max}$	CT	K	NP	$L_{max}$	CT	K		
P1	15352.90	128.79	0.97	3	15373.10	134.60	1.89	4	-0.13	33
P2	27915.10	106.63	4.48	6	28367.40	92.18	67.32	7	-1.62	17
P3	35400.00	135.90	2.33	4	35506.90	174.48	53.87	5	-0.3	25
P4	55536.00	127.15	14.62	8	55797.30	110.98	110.78	9	-0.47	13
P5	73648.30	128.00	253.39	11	73702.50	125.71	95.95	12	-0.07	9
P6	15938.50	248.73	0.70	3	16074.70	244.40	33.17	3	-0.85	0
P7	26596.10	217.04	2.78	5	26958.50	217.04	5.04	6	-1.36	20
P11	69669.70	139.98	5.67	4	70580.80	210.35	139.15	5	-1.31	25
Average (set 1)	40007.08	154.03	35.62	5.50	40295.15	163.72	63.40	6.38	-0.77	17.75
P13	13699.80	149.11	4.33	3	13760.60	137.93	2.12	4	-0.44	33
P14	25865.20	104.64	0.58	6	26390.30	85.59	6.49	7	-2.03	17
P15	31186.40	209.80	13.53	4	31307.70	126.00	17.60	5	-0.39	25
P16	50824.60	140.72	240.96	7	50961.80	117.24	170.11	9	-0.27	29
P17	66436.80	161.01	625.07	9	66681.50	134.13	91.40	10	-0.37	11
P18	14208.60	225.13	4.83	3	14316.70	213.47	2.08	3	-0.76	0
P19	23893.00	179.04	4.20	6	24093.00	176.32	8.53	7	-0.84	17
P20	29118.60	293.90	6.08	5	29623.40	275.62	97.91	6	-1.73	20
P22	59187.10	247.00	440.18	9	60032.70	222.91	52.51	10	-1.43	11
P24	32948.60	126.92	5.66	5	33288.10	91.04	7.50	6	-1.03	20
Average (set 2)	34736.87	183.73	134.54	5.70	35045.58	158.03	45.63	6.70	-0.93	18.30
P25	9926.88	140.97	1.83	2	10076.60	123.97	10.62	3	-1.51	50
P26	17699.00	139.93	21.59	4	18625.50	86.79	45.88	6	-5.23	50
P27	21893.20	184.24	110.35	3	22606.60	144.57	63.42	4	-3.26	33
P28	35915.20	158.87	1451.27	5	36401.00	131.59	40.65	6	-1.35	20
P29	46441.50	156.79	3650.98	7	47069.90	123.77	164.68	9	-1.35	29
P31	16521.30	170.51	484.56	5	16686.10	176.57	59.76	6	-1.00	20
P32	20943.40	285.36	44.19	3	21616.20	249.58	5.15	4	-3.21	33
P33	30606.80	203.47	370.68	6	31599.70	192.38	13.11	7	-3.24	17
P34	42175.30	223.77	1048.58	7	42740.30	221.83	21.17	8	-1.34	14
P35	44489.10	218.17	106.07	3	44687.00	214.32	75.08	4	-0.44	33
P36	22939.70	193.84	374.04	3	23380.70	162.62	26.19	4	-1.92	33
Average (set 3)	28141.03	188.72	696.74	4.36	28680.87	166.18	47.79	5.55	-2.17	30.18
Average	33688.85	177.43	320.50	5.14	34079.54	162.69	51.35	6.17	-1.35	22.65

This observation is consistent with the findings of Kovacs et al. (2015a), reporting that time consistency can be improved with a modest increase in the average fleet size. However, this comes with the extra cost of acquiring more vehicles to enlarge the existing fleet and serve all customers. In this case, in order to increase the net profit by 1.35%, an increase of the fleet size by 22.65% is required, utilizing one extra vehicle on average. Our results indicate that more vehicles are needed as the number of non-frequent customers increases. This is due to the fact that in ConVRP with Profits the available vehicles are sufficient to visit all the frequent customers but only a subset of the non-frequent ones. Therefore, the larger the number of non-frequent customers, the more vehicles are required. However, there should be extra

Table 6: Results on decreasing fixed fleet size

#	ATS					ATS-fd					%Gap	
	NP	$L_{max}$	CT	K	%UNFC	NP	$L_{max}$	CT	K	%UNFC		
P25	9926.88	140.97	1.83	2	20.00	9539.77	249.67	0.11	1	64.00	3.90	50.00
P26	17699.00	139.93	21.59	4	40.00	17621.10	154.67	22.71	3	47.00	0.44	25.00
P27	21893.20	184.24	110.35	3	30.00	20766.10	207.89	59.06	2	60.00	5.15	33.33
P28	35915.20	158.87	1451.27	5	22.00	34122.30	187.70	339.53	4	55.00	4.99	20.00
P29	46441.50	156.79	3650.98	7	16.00	43272.00	159.39	369.52	6	66.00	6.82	14.29
P30	10299.10	174.88	18.81	3	0.00	9852.11	195.40	3.89	2	75.00	4.34	33.33
P31	16521.30	170.51	484.56	5	24.00	15576.70	173.19	24.75	4	50.00	5.72	20.00
P32	20943.40	285.36	44.19	3	32.00	20761.00	182.72	10.78	2	38.00	0.87	33.33
P33	30606.80	203.47	370.68	6	35.00	30228.90	209.82	24.3	5	43.00	1.24	16.67
P34	42175.30	223.77	1048.58	7	29.00	39909.20	248.79	499.79	6	62.00	5.37	14.29
P35	44489.10	218.17	106.07	3	5.00	42969.60	271.88	1.93	2	37.00	3.41	33.33
P36	22939.70	193.84	374.04	3	22.00	21122.50	206.95	0.77	2	68.00	7.92	33.33
Average	26654.21	187.57	640.25	4.25	22.92	25478.44	204.00	113.10	3.25	55.42	4.18	27.24

consideration taken when making decisions regarding the fleet size with high fixed costs occurring.

Apart from investigating the effect of increasing the available fleet size, we also explored the impact of decreasing the fleet size to the net acquired profits. The instances of set 3 were utilized, as they are the ones with the largest number of non-frequent customers, and are thus more likely to obtain feasible solutions, when reducing the available fleet size. Table 6 shows the corresponding computational results. The results for the original fleet size are reported under the ATS label, while the results for the increased fleet size are reported under the ATS-fd label. For each instance, we present the corresponding objective function value,  $L_{max}$ , the computational time in seconds, the number of vehicles required for the obtained solution and the percentage of non-frequent customers not included in the final solution. The %Gap between the two objective function values and the %Gap(|K|) between the required numbers of vehicles were calculated.

As expected, the net acquired profits decrease with a smaller fleet size. Specifically, there is an average decrease of 4.18% as far as the objective value is concerned. This is due to the fact that less non-frequent customers are included in the final solution. There is a clear indication that the more non-frequent customers not included in the routing plans, the lower the net collected profit will be. Moreover, the solutions utilizing less vehicles were “less consistent” by 8.76% and the required computational time was decreased by 82%. The former observation is a result of the lower number of available vehicles, making the routing of frequent customers more difficult. The latter is due to the fact that the feasible solution search space regarding

the non-frequent customers' inclusion to the routing plans was constrained, accelerating the evaluation process of the local search and leading to lower computational times. Overall, our results indicate that reducing the available fleet size by 27.24% leads to a decrease of the net acquired profits by 4.18%. This further supports the argument that additional consideration should be exercised when decisions are made regarding the fleet size, as this could have a great impact on profitability.

#### 4.5. Cost of arrival time consistency

In this section, we discuss the price of service consistency. In particular, we examine the effect of decreasing the maximum arrival time difference on the obtained objective function value, the collected profit, the traveling cost and the number of unvisited non-frequent customers. To this end, several computational experiments were conducted, determining the value of  $L$ . For each instance, the value of  $L$  was specified as a percentage of the  $L_{max}$  obtained when  $L$  was not fixed. It is noteworthy that in our case the fleet size is fixed and no waiting time is allowed either at the depot or at the customers' locations, thus, tightening the arrival time difference constraints cannot be overcome by increasing the number of vehicles or by shifting the vehicles' departure times as in Kovacs et al. (2014b). This resulted in a number of unsolved instances as the proposed metaheuristic algorithm could not find a solution satisfying the given consistency constraints, using the available vehicles. For this reason, only the instances that the ATS solution framework managed to find a suitable solution for are presented.

Figures 5, 6, 7 and 8 summarize the obtained computational results. Figure 5 depicts the effect of decreasing  $L$  on the average objective function value on all problem sets (Figure 5(a)) and on each set respectively (Figure 5(b), 5(c) and 5(d)). As expected, constraining the value of  $L$  leads to reduced net profits. As shown in Figure 5, decreasing  $L$  by 60% results in an average decrease of 0.81% in the obtained objective value (ranging from 0.6% up to 0.89%, depending on the number of non-frequent customers). In other words, an improvement in service consistency of 60% costs 0.81% in the net acquired profits.

Figure 6 shows the impact of decreasing  $L$  on the average collected profits on all problem sets (Figure 6(a)) and on each set respectively (Figure 6(b), 6(c) and 6(d)). Following the same trend as the objective value, constraining  $L$  by 60% leads to an average decrease of 0.31% in the total collected profits (ranging from 0.02% up to 0.52%, depending on the number of non-frequent



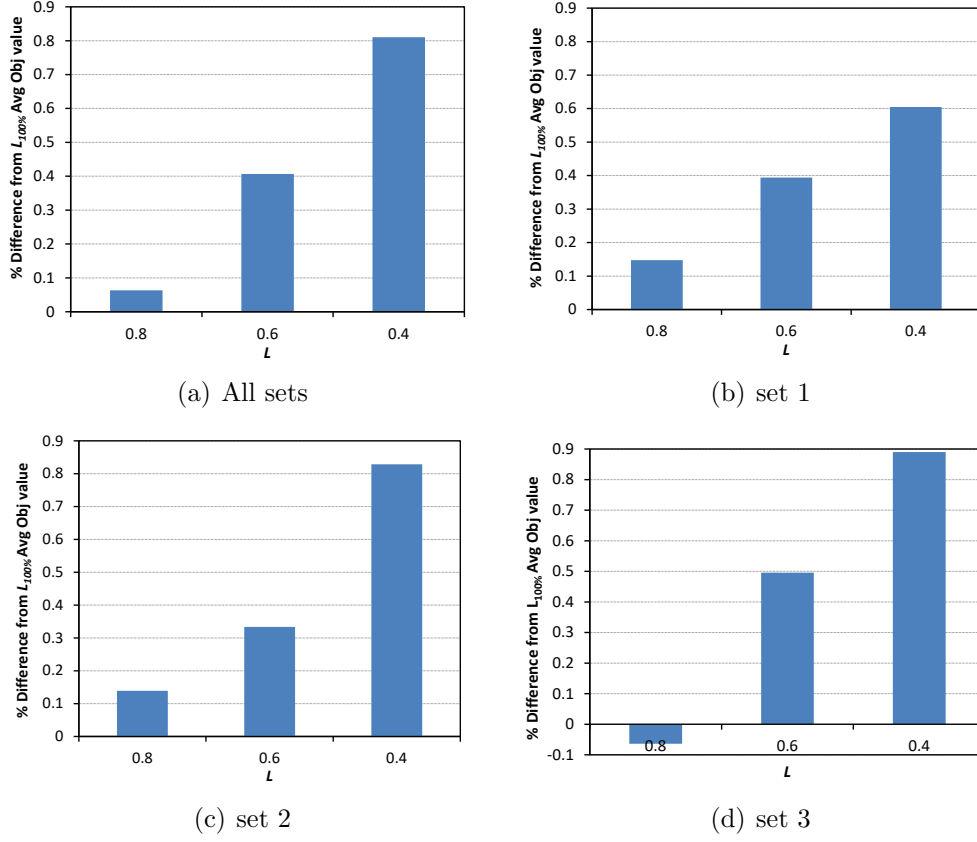


Figure 5: Effect of  $L$  value on the net profits

customers). This means that improving service consistency by 60% costs 0.31% in the total collected profits.

Figure 7 illustrates the effect of improving service consistency on the average traveling costs on all problem sets (Figure 7(a)) and on each set respectively (Figure 7(b), 7(c) and 7(d)). Clearly, constraining the value of  $L$  results in increased traveling costs. Specifically, our results indicate that decreasing  $L$  by 60% leads to an average increase of 6.18% in the required traveling cost (ranging from 3.77% up to 6.97%, depending on the dataset). In other words, an improvement in service consistency of 60% is followed by an increase of 6.18% in the traveling costs.

These findings are in line with the literature. Subramanyam and Gounaris (2016) report that an average cost increase of 1.31% occurs in order to provide consistent service in the optimal ConTSP solutions. Kovacs et al. (2015a)

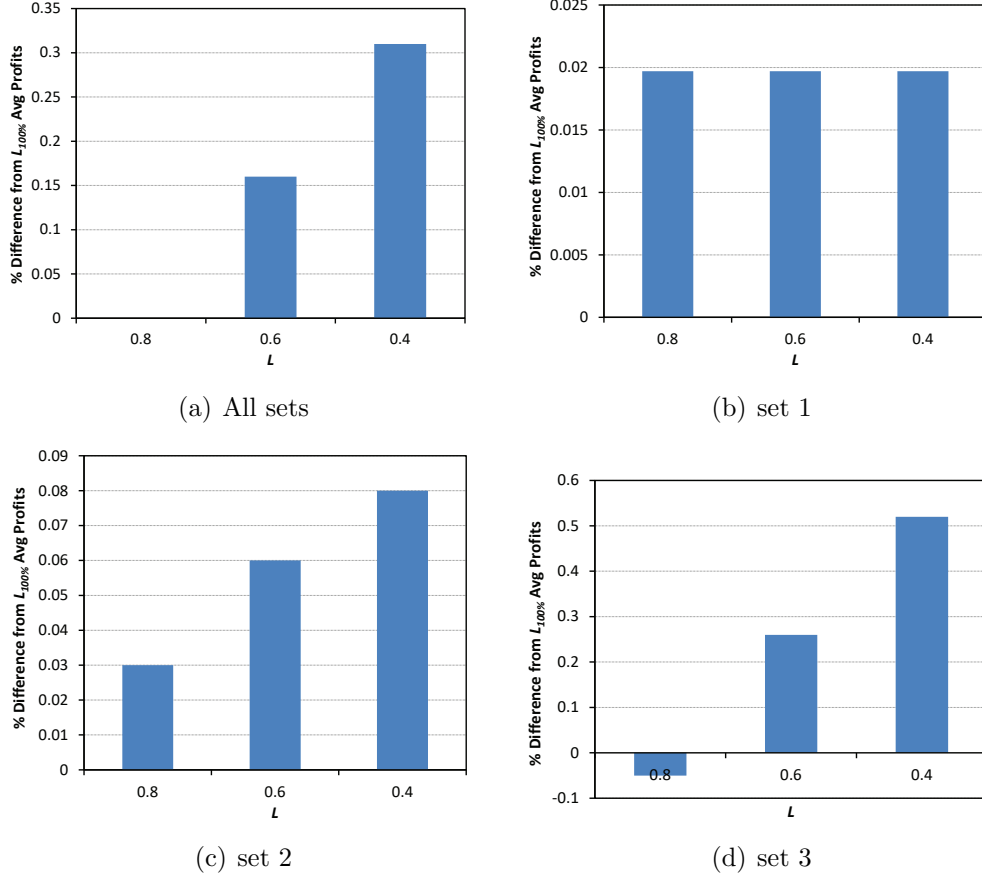


Figure 6: Effect of  $L$  value on the total profits

study the generalized version of the ConVRP and discuss that a large decrease in  $L$  can be achieved with a modest increase in the total traveling cost. Additionally, Kovacs et al. (2015b) examined the arrival time consistency cost in their multi-objective version of the ConVRP and showed that in the case of restricting driver consistency to one driver per customer, reducing  $L$  by 70% causes an average increase in the traveling distance of 2.43%. In contrast, Kovacs et al. (2014b) demonstrate that in the ConVRP a 60% reduction on  $L$  can lead to a cost increase by up to 186.15%. This is due to the fact that in ConVRP all customers need to be visited and the number of vehicles is unlimited. Therefore, in order to cope with the tightening of the arrival time constraints more vehicles are required, resulting in a substantial

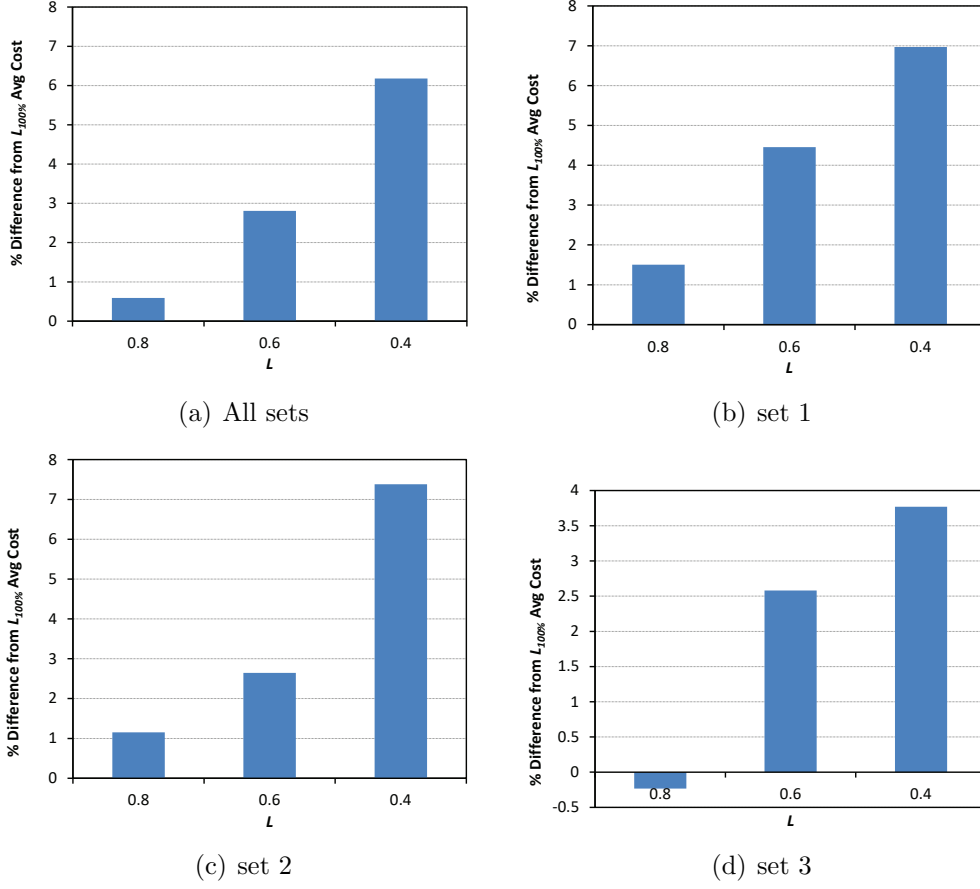


Figure 7: Effect of  $L$  value on the traveling cost

increase in the total traveling cost. Kovacs et al. (2014b) suggest that this issue can be resolved by allowing a shift in the vehicles' arrival times. In our case this problem can be avoided by altering the subset of non-frequent customers to be visited.

Figure 8 presents the effect of decreasing  $L$  on the average percentage of the unvisited non-frequent customers on all problem sets (Figure 8(a)) and on each set respectively (Figure 8(b), 8(c) and 8(d)). The figures show that there is a decrease in the number of visited non-frequent customers as time consistency improves. In particular, 2% less customers are visited on average when improving service consistency by 60% (ranging from 1% up to 4%, depending on the dataset). However, there seems to be a fluctuation in

the obtained results. As illustrated in Figure 8(c), in set 2 when decreasing  $L$  by 40% there is an increase in the number of visited non-frequent customers of 1%. Following the same trend, in set 3 when decreasing  $L$  by 20% there is an increase in the number of visited non-frequent customers of 1%. These results contradict our earlier findings which show a decrease in the objective value. This is due to the fact that different subsets of non-frequent customers were chosen to be included in the final solutions. Thus, increasing service consistency led to the inclusion of non-frequent customers with lower profits, resulting in a decrease of the final objective value as far as set 2 is concerned. The results concerning set 3 are different, demonstrating a reverse effect of increasing the net profits and total profits and reducing the total traveling cost, when constraining the value of  $L$  by 20%. We believe that this is a result of suboptimal solutions obtained for this particular set. Another explanation can be that adding more customers may help satisfy the consistency constraint, due to the fact that the vehicle departure time is always anchored at point 0. Overall, our findings indicate that decision-makers should carefully consider the service consistency level that will be offered as this can have an impact on profitability and may lead to a considerable decrease in the percentage of visited customers, imposing a negative effect on customer satisfaction.

## 5. Conclusions

Nowadays, companies focus on and invest in customer relationship management in an attempt to enhance customer satisfaction and brand loyalty by forming bonds with their customers. Along these lines, a new problem is introduced in this paper, the ConVRP with Profits. It is a customer-oriented routing problem that aims at maximizing a company's net profits, while providing customer service in a consistent manner. In particular, given a set of mandatory frequent and potential ad hoc customers with known profits, demands and service requirements over a planning horizon of multiple days, the goal is to design vehicle routes that maximize the difference between the total profits and the overall traveled distance, utilizing all available resources.

A novel Adaptive Tabu Search (ATS) algorithm has been developed with few user-defined parameters to address this complex transportation problem. The proposed solution approach starts by generating a random initial solution, which is improved by the ATS algorithm. The "elite moves" encountered during the local search execution are stored in the long-term

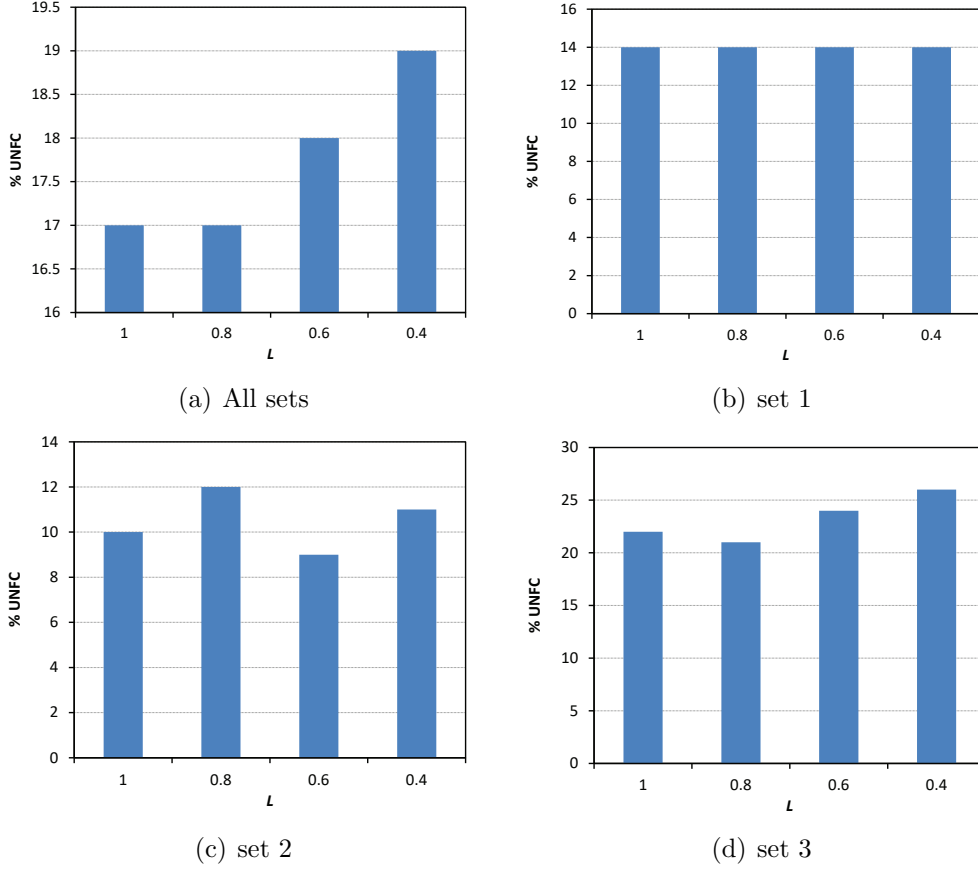


Figure 8: Effect of  $L$  value on the %UNFC

memory component and are then used as a basis to construct solutions with “good characteristics”. For the performance evaluation of the proposed meta-heuristic algorithm, existing and newly generated benchmark instances were utilized. In particular, our solution approach was tested on the ConVRP benchmark dataset, performing well compared to the state-of-the-art and obtaining new improved heuristic upper bounds. Furthermore new small benchmark instances were constructed and solved to optimality. The average gap from the optimal solutions was 0.72%. Finally, as far as the newly generated medium and large-scale instances are concerned, the proposed solution approach was compared to TS, producing slightly better results and requiring 80% less computational time. All the aforementioned computational experiments highlight the efficiency and effectiveness of the proposed

ATS framework.

Our computational study indicates that in order to increase the net acquired profits by 1.35% on average, an increase of the fleet size by 22.65% is required. Additionally, when reducing the available fleet size by 27.24% the net acquired profits are decreased by 4.18% on average. Thus, decision-makers are advised to carefully determine the company's fleet size, as it can have an impact on profitability. Furthermore, the price of the arrival consistency was examined. Our findings suggest, in accordance with the literature, that an improvement in the service consistency of 60% leads to a decrease of 0.81% in the net acquired profits, an average decrease of 0.31% in the total collected profits and an average increase of 6.18% in the total traveling costs, while decreasing the percentage of visited non-frequent customers by 2%. Therefore, careful consideration is required when defining the offered level of service consistency as this may have a negative impact on both net profits and customer satisfaction. One research direction worth pursuing would be to examine the multi-objective version of the ConVRP with Profits, including collected profit, routing cost, arrival time consistency and driver consistency as independent objectives of the problem.

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