Interplane cross-saturation in multiphase machines

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Abstract: The use of electrical machines in electric vehicles and high-power drives frequently requires multiphase machines and multiphase inverters. While appropriate mathematical models under the linear magnetic conditions are readily available for multiphase machines, the same cannot be said for the models of the saturated multiphase machines. This paper examines the saturation in an asymmetrical six-phase induction machine under different supply conditions and addresses the applicability of the existing saturated three-phase machine models for representation of saturated multiphase machines. Specifically, the mutual coupling between different sequence planes in the vector space decomposed model under saturated conditions is analyzed. The paper relies on analytical considerations, finite element analysis and experimental results. It is shown that the saturation of the main flux path is influenced by the current components in the orthogonal (non-fundamental) sequence plane. This implies the need to develop new multiphase machine models which take this effect into account.

1 Introduction

During the last fifteen years or so a rapid pace of development has taken place in the area of multiphase (more than three phases) machines and drives. Such machines are suitable for numerous niche applications, due to the advantages offered by the existence of more than three phases (e.g. locomotive traction, electric ship propulsion), very high power industrial applications, electric and hybrid electric vehicles, more-electric aircraft concept, remote off-shore wind energy generation) [1–3].

The control strategies for multiphase drive applications require a good knowledge of the machine parameters to ensure a high quality of the dynamic and steady-state drive performance [4]. The performances of a machine controller, which depend on knowledge of the machine’s magnetic properties, can be worsened by the phenomenon of magnetic saturation. Proper understanding and modeling of the saturation phenomenon plays a key role in determining the flux weakening capability and better control performance of multiphase drives, since the precise estimation of controlled quantities (e.g. machine currents) and the control algorithms are all based on multiphase modeling. If such control systems can operate properly in the presence of magnetic saturation, a smaller machine may be used for the same purpose [5].

One of the standard assumptions of the general theory of electrical machines is that the main flux saturation can be neglected. This however proves to be inadequate in many operating regimes of three-phase and it is even not possible to study by simulation certain transients under this assumption (e.g. self-excitation of a three-phase stand-alone induction generator). It is for this reason that, over the years, a large research effort has been put into development of modified three-phase machine models that can account for the main flux saturation phenomenon in an accurate way. Nowadays, dozens, numerous improved models are available for both three-phase induction and synchronous machines that enable appropriate representation of the saturation within the circuit equations used to describe the machine. In general, three common approaches related to the main flux saturation modeling in three-phase machines can be identified: modeling in phase coordinates [6], dq model approach [7–12] and voltage-behind-reactance (VBR) approach [13–15]. In many ways, this research topic is now closed as far as the three-phase machinery is concerned.

Since multiphase machines are still not as common in industry as their three-phase counterparts, a huge effort has been made recently to improve multiphase machine parameter estimation techniques [16, 17]. While appropriate mathematical models under the linear magnetic conditions are readily available for multiphase machines [18, 19], the same cannot be said for the models of the saturated multiphase machines. A relatively few works have dealt with this topic [20–23] and there appears to be still a large scope for improvement.

By vector space decomposition (VSD) approach, the original phase-domain model of a multiphase machine can be decomposed into several equivalent circuits that represent the decoupled vector subspaces (planes): the fundamental (dq) plane, identical to that of a three-phase machine, one or multiple orthogonal (xy) planes and one or two zero-sequence components [2]. The advantages of the VSD model regarding the component decoupling become questionable if saturation and mutual leakage between stator windings is considered. The analysis of mutual coupling between the dq and xy planes carried out in [21, 23] assumes a synchronously rotating xy magnetomotive force (mmf) which contributes to air-gap flux and thus to the saturation of the main flux path. On the other hand, xy current components at fundamental frequency generate a subsynchronous rotating mmf which results in a flux density confined to leakage flux paths, due to the rotor cage reaction. Fundamental frequency currents in the xy plane are certain to occur in all post-fault scenarios that exploit fault tolerance [24], in all schemes that suggest power sharing control of the machine with multiple three-phase windings [25–28], as well as in the regenerative testing methods recently developed for multiphase machines [29, 30]. It is unknown if coupling between different orthogonal planes occurs under saturated conditions in such cases. Therefore, it is debatable whether the existing saturated dq machine models can be used to adequately take the magnetic saturation in multiphase machines into account when fundamental-frequency xy current components are present.

For the machine control purposes, it is common to take only the saturation of the main flux path into account. The leakage inductances are not affected by magnetic saturation, which is reasonable except in fault and overload conditions [9]. At low flux values, the inductances remain constant, but as the flux increases the machine starts to saturate and the inductances decrease. This is important when the machine is designed to be slightly saturated in the rated operating point in order to maximize the torque production [31, 32].
In this paper, the influence of fundamental frequency x-y plane quantities on the saturation of the main flux path in an asymmetrical six-phase induction machine (6PHIM), with a 30° electrical shift between the two three-phase windings, will be investigated. It will be examined whether the saturation of the main flux path has an effect on the decomposition between the dq and xy planes. The influence of the xy plane on saturation will be investigated analytically, through Finite Element Analysis (FEA), and experimentally. According to the research presented in the following sections, it is concluded that the main flux path occurs mostly due to the torque-producing (dq) plane, but an influence of the orthogonal (xy) plane exists. This mutual influence between subspaces is termed “interplanar cross-saturation”. According to the results obtained from the upcoming analyses, it is not possible to adequately include the saturation effect by considering only the currents in the dq plane, since the effect of the xy plane needs to be included as well.

This paper is organized as follows. The existing linear VSD model and a proposed approach for inclusion of magnetic saturation are described in the second section. An intuitive qualitative approach to the analysis of interplane cross-saturation will be presented in the third section. Results obtained using FEA will be given in the fourth section, whereas the experimental verification is given in the fifth section. The discussion of the results is given in the sixth section, and the conclusions are presented in the final section.

2 Theoretical background

In electrical machine theory, the following assumptions are frequently made when considering saturation phenomena [33]:

- the total flux linkages of each coil are the sum of the leakage and mutual flux components,
- the magnetic circuit saturation depends on the total air gap flux linkages,
- the leakage flux paths are not subject to saturation (except in transients and overload conditions), and
- hysteresis and eddy current effects (iron losses) are neglected.

Three main approaches to multiphase machine modeling exist:

- the phase-variable, multiple dq (for multiphase machines with multiple, three-phase windings) and VSD model. The phase-domain has the advantage of directly representing physical quantities, which simplifies the modeling of the machine with the power system network and allows more accurate representation of internal machine phenomena. The negative aspect of the phase-variable model is that it is difficult to interface this model with the external components or power electronics circuits modeled in the phase domain. Therefore, the voltage-behind-reactance approach was recently proposed as an alternative solution [21, 22]. The widely used VSD model is based on transforming the phase-domain variables of a multiphase machine into a fundamental (torque-producing) plane, one or more orthogonal (non-torque-producing) planes and one or two zero-sequence subspaces. The fundamental and non-fundamental subspaces are completely decoupled, which provides valuable benefits in terms of machine analysis and control [35, 36]. The VSD model equivalent circuit of a multiphase machine is identical to that of a three-phase machine, making the existing control techniques directly applicable to multiphase machines [34]. This approach can adequately describe the machine in both transient and steady-state operating conditions, both for sinusoidal and non-sinusoidal supply.

- Decoupling between subspaces facilitates modeling and control (and the model is questionable under saturated conditions. Only the coupling between the dq and xy components will be studied, as the zero-sequence components can always be avoided by simply isolating the neutral points. The 6PHIM is commonly operated with separated neutral points, as this reduces the system dimensionality and thus simplifies the control algorithm [34]. The unsaturated VSD model of a 6PHIM will be presented, followed by an assumed extension to a model involving saturation.

2.1 Unsaturated VSD model

The voltage equations of a 6PHIM in the VSD domain are given as [37]:

\[
\begin{align*}
\dot{u}_{d} &= R_{a}i_{d} + \frac{d\psi_{r}}{dt} - \omega_{e}\psi_{q} + \frac{L_{s}}{L_{r}}\psi_{d} + \frac{1}{L_{r}}\psi_{dq} \\
\dot{u}_{q} &= R_{a}i_{q} + \frac{d\psi_{r}}{dt} - \omega_{e}\psi_{d} + \frac{L_{s}}{L_{r}}\psi_{q} + \frac{1}{L_{r}}\psi_{dq} \\
\dot{\psi}_{r} &= J_{e} + \frac{1}{L_{r}}u_{d} - \frac{1}{L_{r}}u_{q} \\
\dot{\phi}_{r} &= J_{m} + L_{m}\dot{\psi}_{r} + L_{m}\dot{\psi}_{pq} \\
\end{align*}
\]

(1)

where \(\psi_{d}, \psi_{q}\) is the rotor electrical angular speed, \(\omega_{e}\) (rad/s) is the arbitrary angular speed of the rotating reference frame, and:

\[
\xi_{a,b} = [\xi_{a,x} \xi_{b,x} \xi_{a,y} \xi_{b,y} \xi_{a+z} \xi_{b-z}]^{T}
\]

(2)

where \(I_{0,0}\) is an identity matrix of the sixth order and \(\xi\) stands for an arbitrary electrical quantity (voltage, current or flux linkage).

The stator flux linkages are given in space vector form as (analogous expressions hold for rotor flux linkages):

\[
\begin{align*}
\vec{\psi}_{dq} &= (L_{m} + L_{d})\vec{\psi}_{dq} + L_{m}\vec{\psi}_{dpq} \\
\vec{\psi}_{yx} &= \vec{\psi}_{dx} - \vec{\psi}_{dy} \\
\vec{\psi}_{dx} &= \vec{\psi}_{dx} - \vec{\psi}_{dy} \\
\end{align*}
\]

(3a)

(3b)

(3c)

(3d)

where \(L_{m}\) is the magnetizing inductance and \(L_{d}\) is the stator leakage inductance. Note that there is no mutual influence between the quantities of different subspaces. With no saturation involved, all inductances in (3) are constant. The remaining equations needed to complete the model are the torque equation:

\[
T_{r} = p\xi_{dx} (\vec{\psi}_{dq} - \vec{\psi}_{dpq})
\]

(4)

and the electromechanical motion equation:

\[
T_{e} - T_{L} = J\frac{d\omega}{dt} + k_{f}\Omega
\]

(5)

where \(p\) is the pole pair number, \(T_{e}\) is the load torque, \(J\) is the moment of inertia, \(\Omega\) is the mechanical angular rotor speed, and \(k_{f}\) is the friction coefficient. The given equations are obtained when applying the power invariant decoupling transformation matrix [37]. Note that the given model is simplified as mutual leakage inductance is neglected in flux equations (3). According to [38], mutual leakage terms occur in dq and zero-sequence flux equations. This effect is not essential for the analysis in this paper, so it will be discarded for the sake of simplicity.

2.2 Hypothesis - saturation modeling

It is already known from [11, 39] that coupling between windings in spatial quadrature (cross-saturation) exists in saturated smooth air-gap machines. By analogy with this phenomenon, it is of interest to determine how the main flux saturation affects the mutual coupling between the dq and xy planes, that are decoupled under unsaturated conditions. This research is necessary in order to investigate if the cross-coupling effect exists between different VSD subspaces. If it is proven that the multiphase machine main flux saturation can be modeled solely in the fundamental (dq) plane, all existing conclusions
regarding the modeling of saturated three-phase machines would
apply to multiphase machines as well. It will therefore be assumed
that saturation occurs solely under the influence of fundamental (dq)
planar components and that no-torque producing subspaces do not
contribute to saturation. In other words, it will be considered that
the decoupling between the orthogonal subspaces is still valid in
saturated conditions. According to this assumption, saturation inclu-
sion in the model requires addition of the following equation to the
unsaturated VSD model (1)–(5):
\[ L_m = f(l_m), \quad l_m = \sqrt{(l_{ds} + l_{dp})^2 + (l_{qs} + l_{qp})^2}, \] (6)
where \(l_m\) is the magnetizing current of the machine. Note that the
decoupling of subsystems is not affected by this modification, as
already stated. It is the goal of this paper to confirm or rebut this
assumption.

It should be noted that the machine model (1)–(6) is given here in a
generic form. Its subsequent formulation in terms of state-space vari-
bles would lead to the introduction of the dynamic cross-saturation
in the \(dq\) equations in accordance with the selected state-space vari-
able set, in the same manner as for a three-phase machine [7–11].

Importantly however, if (6) is sufficient to model the saturation effect\footnote{3} then all the three-phase \(dq\) models become directly appli-
cable to multiphase machines, as \(xy\) equations of the model (1)–(3)\footnote{26} remain fully decoupled from the \(dq\) equations.

\section{Analytical approach}

It is of interest to determine whether the \(xy\) currents affect the reluc-
tance of iron parts of the main flux path and, if so, under which
conditions. For this purpose, an appropriate magnetic equivalent
circuit of the machine is developed and examined. By solving the cir-
cuit equations under different conditions, the influence of \(xy\) current
components on the saturation of the main flux path can be studied.

\subsection{Magnetic equivalent circuit}

Only the stator magnetic circuit will be modeled. A part of the cir-
cuit spanning an arbitrary slot is shown in Fig. 1. All dimensions
displayed in Fig. 1 are defined in Table 3 in the Appendix. The
model spans one pole pair, i.e. \(Q_{pp}\) slots. A similar concept is
proposed in [40] for calculating the core reluctance of an induction
circuit equations under different conditions, the influence of
saturated conditions. According to this assumption, saturation inclu-
sion in the model requires addition of the following equation to the
unsaturated VSD model (1)–(5):
\[ F_{xy,i} = \left(R_{th} + R_{ts,i} + R_{ts,i+1} + R_{th,i+1} \right) \Phi_{bs,i}, \] (7)
where \(i \in \{1, \ldots, Q_{pp}\}\), and \(\Phi_{bs,i}\) is the self flux corresponding to the \(i\)-th slot, i.e. the flux generated solely by the slot mmf \(F_{xy,i}\). This
flux component is designated by a dashed line in Fig. 1. Note that \(\Phi_{bs,i}\) is confined to the leakage flux path, which is in accordance with the
fact that the \(xy\) currents produce only leakage flux [17]. As stated in assumption (ii), the stator slot bridge reluctance \(R_{th}\) is
considered constant and equal for each slot. Stator yoke and tooth
reluctances depend on the corresponding total flux densities, which
are defined as:
\[ B_{y,i} = \frac{\Phi_{bs,i} + \Phi_{yadq,i}}{b_{ys,i}}, \] (8a)
\[ B_{ts,i} = \frac{\Phi_{bs,i} + \Phi_{yadq,i} - \Phi_{bs,i+1}}{w_{ts,i}}, \] (8b)
where \(l_a\) is the axial length of the machine (Table 3 in the Appendix), \(B_{yadq,i}\) is the main flux through one slot pitch, and \(\Phi_{yadq,i}\) is
the yoke flux obtained by integrating the main flux density over
the perimeter of the machine. In this model, the yoke flux corre-
sponding to the portion of the yoke above the \(i\)-th slot is calculated
approximately as:
\[ \Phi_{yadq,i} = \sum_{n=1}^{i} \Phi_{yadq,n}, \] (9)
The yoke, tooth, and slot bridge reluctances are given as:
\[ R_{y,i} = \frac{B_{y,i}}{\mu_{y,i} B_{y,i} b_{y,i}}, \] (10a)
\[ R_{ts,i} = \frac{B_{ts,i}}{\mu_{ts,i} (B_{ts,i}) w_{ts,i}}, \] (10b)
\[ R_{th,i} = \frac{w_{th,i}}{\mu_{th,i}}, \] (10c)
where \(\mu_{y,i}\) and \(\mu_{ts,i}\) are the yoke and tooth iron permeability,
respectively, and \(\mu_{th,i}\) is the permeability of free space. Note that the prior two are dependent on the corresponding
flux densities. The dependence \(\mu(B)\) is obtained from the saturation

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{Part of the developed stator magnetic circuit surrounding one slot}
\end{figure}
characteristic of a commercial laminated steel and expressed as a piecewise linear function.

The unknown quantities in (7) are the fluxes \( \Phi_{x,i} \), \( \Phi_{x,i-1} \) and \( \Phi_{y,i} \). Values of \( \Phi_{x,0} \) and \( \Phi_{y,0} \) are obtained directly from the given air-gap flux density, according to assumption (i), and therefore represent input quantities. In order to obtain a square system with a unique solution, (7) needs to be formulated for each of the \( Q_{pp} \) slots under one pole pair, thereby constituting a system of \( Q_{pp} \) nonlinear algebraic equations. By noting that \( \Phi_{x,i} = \Phi_{x,i-1} \) and \( \Phi_{y,i} = \Phi_{y,i-1} \), the number of variables reduces to \( Q_{pp} \) as well and a square system of nonlinear algebraic equations is obtained. In the following section, the model of the analyzed 6PHIM will be synthesized and solved for different combinations of \( xy \) mmf and main flux density.

### 3.2 Calculation results

The calculations are performed using the data of the actual machine given in Table 3 in the Appendix. The main flux density distribution in the air-gap is given as:

\[
B_{dq}(\theta) = B_0 \cos \theta, 
\]

where \( B_0 \) is the magnitude of the fundamental air-gap flux density and \( \theta \) is the electrical angle, with \( \theta = 0 \) corresponding to the middle of the first tooth (\( ts,1 \)) of the developed magnetic circuit model. The mmf corresponding to each slot to the currents of the top and bottom layer and the number of conductors per layer (\( ts,2 \)) are obtained directly from assigning appropriate currents to each phase according to (37):

\[
i_{a1,xy} = I_a \cdot \cos \varphi_{a,xy} \\
i_{b1,xy} = I_b \cdot \cos \varphi_{b,xy} - 4\pi/3 \\
i_{c1,xy} = I_c \cdot \cos \varphi_{c,xy} - 2\pi/3 \\
i_{a2,xy} = I_a \cdot \cos \varphi_{a,xy} - \pi/6 \\
i_{b2,xy} = I_b \cdot \cos \varphi_{b,xy} - \pi/6 \\
i_{c2,xy} = I_c \cdot \cos \varphi_{c,xy} - 3\pi/2 
\]

The current magnitude \( I_a \) will be held constant, whereas the phase angle \( \varphi_{a,xy} \) will be varied in order to change the position of the \( xy \) mmf. This angle will be referred to as the "\( xy \) phase shift".

The mmf distribution corresponding to the \( xy \) subspace is achieved by assigning appropriate currents to each phase according to (37):

\[
i_{a1,xy} = I_a \cdot \cos \varphi_{a,xy} \\
i_{b1,xy} = I_b \cdot \cos \varphi_{b,xy} - 4\pi/3 \\
i_{c1,xy} = I_c \cdot \cos \varphi_{c,xy} - 2\pi/3 \\
i_{a2,xy} = I_a \cdot \cos \varphi_{a,xy} - \pi/6 \\
i_{b2,xy} = I_b \cdot \cos \varphi_{b,xy} - \pi/6 \\
i_{c2,xy} = I_c \cdot \cos \varphi_{c,xy} - 3\pi/2 
\]

The unknown components, \( \varphi_{a,xy} \) and \( \varphi_{b,xy} \), are obtained directly from the given air-gap flux density, according to assumption (i), and therefore represent input quantities. In order to obtain a square system with a unique solution, (7) needs to be formulated for each of the \( Q_{pp} \) slots under one pole pair, thereby constituting a system of \( Q_{pp} \) nonlinear algebraic equations. By noting that \( \Phi_{x,i} = \Phi_{x,i-1} \) and \( \Phi_{y,i} = \Phi_{y,i-1} \), the number of variables reduces to \( Q_{pp} \) as well and a square system of nonlinear algebraic equations is obtained. In the following section, the model of the analyzed 6PHIM will be synthesized and solved for different combinations of \( xy \) mmf and main flux density.

### Fig. 2

Main flux density and \( xy \) mmf fifth harmonic distributions under one pole pair for different phase shifts.

\[
U_{ydq} = \int_0^{Q_{y,i}} H_{ydq}(\theta) \, \phi_{y,i} \, d\theta, 
\]

where \( H_{ydq} \) is the dq yoke field intensity attributed to the main flux and \( \phi_{y,i} \) is the radius of the yoke centerline. Note that only the \( dq \) flux component is used in the calculation, but the influence of the \( xy \) current component on the reluctance is the most pronounced in those parts of the magnetic circuit that are already saturated by the main (\( dq \)) field component.

In order to quantify the saturation of the main flux path, the magnetic voltage across the stator yoke is calculated as:

\[
U_{ydq} = \int_0^{Q_{y,i}} H_{ydq}(\theta) \, \phi_{y,i} \, d\theta, 
\]

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The resulting data in Fig. 4 clearly indicate that the \( xy \) current component has an influence on the saturation of the main flux path. Note that this influence is the most pronounced when the magnetic

\[
I_{y,5} = 5 \, \text{A} 
\]

where \( j \) and \( k \) denote the elements of corresponding arrays defined in (13). The relative magnetic voltage values are displayed in Fig. 4.

The following conclusions can be derived from the given diagrams:

- The \( xy \) current component has a substantial effect on yoke saturation only when the main flux density is sufficiently high, in the sense that the magnetic circuit is previously saturated by the \( dq \) flux component.
- The magnetic voltage can either increase or decrease due to the \( xy \) current component, depending on the \( xy \) phase shift.

Note that this influence is the most pronounced when the magnetic...
The results displayed in Fig. 4 correspond to scenarios 1 and 2 (analytical model). The diagrams for the unsaturated and saturated cases are displayed in Fig. 5. Note that, under saturated conditions ($B_\delta = 0.9$ T), the field intensity obtained when the $dq$ and $xy$ current components act together differs significantly from the value obtained when only the $dq$ current component is present, which indicates the presence of interplane cross-saturation. When the magnetic circuit is unsaturated ($B_\delta = 0.4$ T), the influence of the $xy$ current component is practically negligible. This confirms the conclusions of the analysis in section 3, as the influence of the $xy$ component on the field distribution in the stator yoke is obviously much more pronounced when the magnetic circuit is saturated by the main flux.

In order to determine the influence of the $xy$ current components on main flux path saturation, the yoke magnetic voltages in scenarios 1 and 3 need to be compared. Only the yoke magnetic voltage caused by the main flux is of interest. Therefore, the fundamental spatial component of the yoke flux density is obtained for each scenario, and the magnetic voltage is determined as:

$$U_y = \int B_y d\theta$$

where $B_y$ denotes the fundamental spatial component of the yoke flux density. The magnetic material permeability is calculated as:

$$\mu(\theta) = \frac{B_y(\theta)}{H_y(\theta)}$$

The yoke field intensity distribution is obtained in each case. The diagrams for the unsaturated and saturated cases are displayed in Fig. 5. Note that, under saturated conditions ($B_\delta = 0.9$ T), the field intensity obtained when the $dq$ and $xy$ current components act together differs significantly from the value obtained when only the $dq$ current component is present, which indicates the presence of interplane cross-saturation. When the magnetic circuit is unsaturated ($B_\delta = 0.4$ T), the influence of the $xy$ current component is practically negligible. This confirms the conclusions of the analysis in section 3, as the influence of the $xy$ component on the field distribution in the stator yoke is obviously much more pronounced when the magnetic circuit is saturated by the main flux.

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The yoke field intensity distribution is obtained in each case. The diagrams for the unsaturated and saturated cases are displayed in Fig. 5. Note that, under saturated conditions ($B_\delta = 0.9$ T), the field intensity obtained when the $dq$ and $xy$ current components act together differs significantly from the value obtained when only the $dq$ current component is present, which indicates the presence of interplane cross-saturation. When the magnetic circuit is unsaturated ($B_\delta = 0.4$ T), the influence of the $xy$ current component is practically negligible. This confirms the conclusions of the analysis in section 3, as the influence of the $xy$ component on the field distribution in the stator yoke is obviously much more pronounced when the magnetic circuit is saturated by the main flux.

In order to determine the influence of the $xy$ current components on main flux path saturation, the yoke magnetic voltages in scenarios 1 and 3 need to be compared. Only the yoke magnetic voltage caused by the main flux is of interest. Therefore, the fundamental spatial component of the yoke flux density is obtained for each scenario, and the magnetic voltage is determined as:

$$U_y = \int B_y d\theta$$

where $B_y$ denotes the fundamental spatial component of the yoke flux density. The magnetic material permeability is calculated as:

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yoke magnetic voltage values are calculated by dividing the values obtained from (16) in scenarios 1 and 3 and given in Fig. 6. These results are very similar to those obtained from the analytical magnetic circuit model (Fig. 4). Of course, an exact match cannot be expected, as the air-gap flux density in the FEA model changes with the addition of the $xy$ current component, and the magnetic circuit model itself is of limited accuracy. For instance, the leakage flux generated by $dq$ currents was neglected in the magnetic circuit model. However, this flux is very pronounced in the FEA model at 1.2 T, as the magnetic circuit is highly saturated at such a high air-gap flux density, hence the required $dq$ current is several times larger than the rated value. Nevertheless, the FEA confirms the conclusions derived in section 3. The influence of the $xy$ component is significant if the magnetic circuit is already saturated due to the main flux. The level of saturation, i.e. the magnetic voltage, can decrease or increase depending on the position of the $xy$ mmf wave (phase shift $\varphi_{xy}$). The results obtained from FEA confirm the presence of interplane cross-saturation indicated by the results of the analytical model.

5 Experimental verification
The influence of $xy$ current components on the main flux saturation will be studied by observing the currents of the 6PHIM. For this purpose, measurements are performed in three operating modes characterized by the applied voltage components:

1) $dq$ voltage supply,
2) $xy$ voltage supply, and
3) $dq + xy$ voltage supply.

![Fig. 6](image_url)

**Fig. 6** Relative stator yoke magnetic voltage as a function of the $xy$ phase shift for different values of the main flux density (FEA)

![Fig. 7](image_url)

**Fig. 7** Experimental setup: 1-6PHIM, 2-three-phase inverter boards, 3-DC bus, 4-variac, 5-DC bus voltage measurement, 6-microcontroller, 7-auxiliary motor, 8-current probes, 9-four-channel oscilloscope, 10-voltage probe (PWM1 signal), 11-voltage probe (air-gap voltage), 12-two-channel oscilloscope
The tests are performed for different levels of saturation. The saturation of the main flux density, i.e. the magnetic permeability, is varied by changing the amplitude of the dq component of supply voltage.

The experimental setup is displayed in Fig. 7. The 6PHIM is supplied from two three-phase inverters connected to a common DC bus and controlled from a 32-bit digital signal controller with 6 PWM channels. According to [34], such a configuration is applicable when the neutral points of the two three-phase windings are separated. The DC voltage is obtained from a three-phase diode bridge rectifier supplied from a variable autotransformer. The PWM outputs are controlled in such a way that the initial phase angle corresponding to the first channel is always equal to zero, whereas the phase angles corresponding to other outputs are assigned so that the required voltage components (dq or xy) are obtained. The four-channel oscilloscope is used for the measurement of four phase currents – two in one three-phase winding, and two in the other. As the neutral point of each three-phase winding is isolated, the remaining two currents are easily calculated. The two-channel oscilloscope is used for: a) measurement of the voltage signal on the first PWM channel (PWM1) further on), which is used for time-synchronization of the current waveforms obtained in different operating modes, and b) measurement of the induced voltage of a single-turn coil placed under one pole of the 6PHIM (approximately proportional to the air-gap flux).

The auxiliary motor is a four-pole induction motor used for running the 6PHIM at approximately no-load speed in operating mode 2. In operating modes 1 and 3, the auxiliary motor is disconnected from the supply and the 6PHIM is operated in no-load conditions.

The tests are conducted for two values of DC bus voltage – 

\[ U_{dc} = 300 \text{ V} \]  

\[ U_{dc} = 600 \text{ V} \]  

Operation with \( U_{dc} = 300 \text{ V} \) will be referred to as the “unsaturated case”, whereas operation with \( U_{dc} = 600 \text{ V} \) represents the “saturated case”. The fundamental voltage component corresponds to the rated frequency of 50 Hz (see Table 3 in the Appendix). The fundamental of \( xy \) voltage is maintained equal at both DC voltage levels by setting the appropriate values of the modulation index, so that approximately equal \( xy \) currents are obtained in both cases. The values of the modulation indices and the corresponding rms values of the supply voltage fundamental for each component and DC voltage level are given in Table 1.

The fundamental component of \( dq \) voltage was set to the same value in modes 1 and 3, in order to obtain an approximately equal air-gap flux density in these two cases. Note that the sum of the modulation indices corresponding to the \( dq \) and \( xy \) component may not exceed 1, otherwise overmodulation would occur in operating mode 3 (pure sinusoidal PWM is used, without zero-sequence injection).

Obviously, the phase voltage could have been decreased by reducing the modulation index without lowering the DC bus voltage. However, this would lead to a reduction of the fundamental harmonic of current, while the ripple would remain unchanged, thereby reducing the measurement accuracy. This is a significant matter, as the oscilloscopes provide only 8-bit vertical resolution.

The oscilloscope screenshots of phase current waveforms corresponding to all three operating modes are shown in Figs. 8 and 9. The current measurement resolution rate is set to 10 mA/A. The motor was operated at no-load in modes 1 and 3, and rotated at approximately no-load speed by means of the auxiliary motor in operating mode 2.

It was necessary to rotate the machine under \( xy \) supply in order to achieve the same rotor cage reaction to \( xy \) current components as in mode 3. Note that the currents in mode 1 are highly unbalanced, even though the supply voltages form a balanced six-phase system.

This is the consequence of the winding asymmetry, i.e. the different winding distribution of the first and second three-phase winding (see Fig. 2). Therefore, an \( xy \) current component is present even under balanced supply. This does not represent a problem though, as the influence of the additional \( xy \) component corresponding to mode 2 can be observed regardless of the inherent \( xy \) components in mode 1.

The displayed waveforms indicate that the currents corresponding to 300 Vdc are sinusoidal in all three operating modes with no notable distortion. On the other hand, the currents corresponding to 600 Vdc exhibit a certain amount of distortion, especially in operating mode 3.

In order to better visualize the influence of the \( xy \) currents on the saturation of the magnetic circuit, the following waveforms are overlapped in Fig. 10, representing:

- the sum of currents in operating modes 1 and 2, and
- the current in operating mode 3.

All waveforms were synchronized in time with respect to the fundamental harmonic of the measured PWM1 signal. The PWM1 signal was recorded on a separate two-channel oscilloscope. In order to obtain the current and PWM1 measurements at the same instant, a single-shot external trigger was applied to both oscilloscopes.

<table>
<thead>
<tr>
<th>Table 1 Supply voltage information</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC bus voltage</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>300 V</td>
</tr>
<tr>
<td>300 V</td>
</tr>
<tr>
<td>600 V</td>
</tr>
<tr>
<td>600 V</td>
</tr>
</tbody>
</table>

Operating mode 3

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8
If the decoupling assumption were correct, the waveforms obtained by superposition of currents in mode 1 and mode 2 should be nearly identical to those obtained in mode 3. According to Fig. 10a, this is true for waveforms obtained for the unsaturated case (300 Vdc). On the other hand, the waveforms obtained in the saturated case (600 Vdc) differ noticeably (Fig. 10b). This indicates that the dq and xy subspaces are not decoupled when the magnetic circuit is saturated, i.e. that interplane cross-saturation is present.

The second channel of the two-channel oscilloscope was used for measuring the emf induced in a test coil placed under one pole of the stator. This emf can be considered approximately proportional to the air-gap flux. However, a certain amount of tooth-tip and zig-zag leakage is inevitably present in the flux linkage of the test coil.

For purely exemplary purposes, the recorded emf waveform corresponding to 600 Vdc, operating mode 1, is shown in Fig. 11. The magnitude of the fundamental of air-gap flux density is obtained as:

$$B_1 = \frac{\mu E_1}{\sqrt{2} D_{a1} f_c}$$  \hspace{1cm} (18)

where $E_1$ is the rms value of the test coil emf fundamental. All other quantities from (18) are defined in Table 3 in the Appendix.

The obtained values of fundamental air-gap flux density in operating modes 1 and 3, in both the unsaturated and saturated case, are given in Table 2. These values are very close to those selected in the analytical approach and FEA, see (13). It is important to note that the flux densities in mode 1 and 3 differ very slightly, which is most likely the consequence of increased leakage flux due to $xy$ current components in mode 3. The air-gap flux density under rated operating conditions (rated load and 180 V per phase) was determined to be 0.78 T. By observing the results of Table 2 and considering that the magnetic circuit is moderately saturated under rated operating conditions, it follows that saturation is negligible at 300 Vdc, whereas it is highly pronounced at 600 Vdc.

The phase current waveforms are not sufficient to determine the influence of the $xy$ current components on the saturation of the main flux path. Therefore, an additional analysis of $dq$ current components is necessary. The time-varying amplitude of the space vector of the

### Table 2: Test coil fundamental emf and air-gap flux density values

<table>
<thead>
<tr>
<th>DC bus voltage (operating mode)</th>
<th>Emf fundamental</th>
<th>Air-gap flux density</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 V (1)</td>
<td>0.658 V</td>
<td>0.41 T</td>
</tr>
<tr>
<td>300 V (3)</td>
<td>0.676 V</td>
<td>0.42 T</td>
</tr>
<tr>
<td>600 V (1)</td>
<td>1.453 V</td>
<td>0.90 T</td>
</tr>
<tr>
<td>600 V (3)</td>
<td>1.479 V</td>
<td>0.92 T</td>
</tr>
</tbody>
</table>


The influence of interplane cross-saturation is present in the $xy$ plane as well. This can be observed from Fig. 13, where the waveforms of current $i_x$ in the saturated and unsaturated cases are displayed. A comparison is made between the current corresponding to operating mode 3 and the sum of currents corresponding to modes 1 and 2. In the unsaturated case (Fig. 13a), the two waveforms are nearly identical. In the saturated case (Fig. 13b) there is a significant increase in the current magnitude. The Fourier analysis of the waveforms reveals that the fundamental (50 Hz) component is the most affected, with a relative increase of nearly 40%. Higher order harmonics are also inflicted by saturation, but are still much lower than the fundamental.

![Test coil induced emf waveform (operating mode 1, 600 Vdc)](image)

**Fig. 11:** Test coil induced emf waveform (operating mode 1, 600 Vdc)

6 Discussion

Results obtained from the magnetic circuit model, FEA analysis and experiment confirm the presence of mutual coupling between the $dq$ and $xy$ subspaces under saturated conditions. This implies a requirement for an improved multiphase machine model which includes this phenomenon, termed interplane cross-saturation. A summary of the obtained results is in order:

1. The $dq$ and $xy$ subspaces are decoupled under unsaturated conditions (see Figs. 12a and 13a);
2. The addition of $xy$ current components under saturated conditions increases the magnetizing ($dq$) current component (see Fig. 12b);
3. Saturation of the magnetic circuit, i.e. the increase of magnetizing ($dq$) current increases the $xy$ current component (see Fig. 13b);
4. The $xy$ current component does not affect the air-gap flux density, regardless of the saturation level (see Table 2).

These observations can be used as a starting point to formulate a model that can adequately deal with the observed saturation effects. The intention is to retain the basic model formulation similar to (1)-(6) and to accommodate the findings of this paper through modifications of the flux linkage equations (3). Since any such new

**Fig. 12:** Comparison of magnetizing current waveforms in operating mode 3 and the sum of currents in modes 1 and 2

- **a** Unsatuated case (300 Vdc)
- **b** Saturated case (600 Vdc)

\[
i_{dq} = |i_{ds} + j_{qs}|
\]
shown to be incorrect. The results of all analyses indicate that interplane cross-saturation is present and needs to be taken into account for control and modeling purposes. The results obtained in this paper reveal a need to develop a new multiphase machine model or modify the existing models in order to include interplane cross-saturation.

Guidelines for obtaining such a model were given in this paper. The exact formulation and verification of the model will be the focus of future research.

8 References

9 Appendix

The rated data and dimensions of the analyzed 6PHIM are given in Table 3. The 6PHIM was obtained by rewinding an existing three-phase machine. Due to a limited number of stator slots and the requirement to keep the same number of poles in order to avoid excessive yoke saturation, the winding was executed with 1.5 slots per pole and phase. The given rated power corresponds to that of the original three-phase machine, which is a reasonable assumption considering that the cross-section of the conductors was retained. A thermal test would need to be conducted in order to determine the rated power of the 6PHIM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation (Unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power (estimated)</td>
<td>( P_\text{r} ) (W)</td>
<td>4000</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>( f_\text{r} ) (Hz)</td>
<td>50</td>
</tr>
<tr>
<td>Rated current</td>
<td>( I_\text{r} ) (A)</td>
<td>5.2</td>
</tr>
<tr>
<td>Rated voltage (per phase)</td>
<td>( U_\text{r,ph} ) (V)</td>
<td>180</td>
</tr>
<tr>
<td>No. of poles</td>
<td>2p</td>
<td>4</td>
</tr>
<tr>
<td>No. of stator slots</td>
<td>( Q_\text{s} ) (( \varphi ))</td>
<td>36</td>
</tr>
<tr>
<td>No. of rotor slots</td>
<td>( Q_\text{r} ) (( \varphi ))</td>
<td>28</td>
</tr>
<tr>
<td>No. of turns/phases</td>
<td>( N_\text{ph} )</td>
<td>264</td>
</tr>
<tr>
<td>No. of conductors/p( \varphi )</td>
<td>( s_\varphi )</td>
<td>44</td>
</tr>
<tr>
<td>Conductor diameter</td>
<td>( a ) (mm)</td>
<td>1.0</td>
</tr>
<tr>
<td>Outer stator diameter</td>
<td>( D_\text{os} ) (mm)</td>
<td>184</td>
</tr>
<tr>
<td>Inner stator diameter</td>
<td>( D_\text{is} ) (mm)</td>
<td>116</td>
</tr>
<tr>
<td>Air gap length</td>
<td>( \delta ) (mm)</td>
<td>0.5</td>
</tr>
<tr>
<td>Stack length</td>
<td>( l_\text{s} ) (mm)</td>
<td>125</td>
</tr>
<tr>
<td>Stator slot height</td>
<td>( h_\text{ss} ) (mm)</td>
<td>16</td>
</tr>
<tr>
<td>Stator slot width</td>
<td>( w_\text{ss} ) (mm)</td>
<td>6.2</td>
</tr>
<tr>
<td>Stator slot opening height</td>
<td>( h_\text{bs} ) (mm)</td>
<td>1.2</td>
</tr>
<tr>
<td>Stator slot opening width</td>
<td>( w_\text{bs} ) (mm)</td>
<td>1.8</td>
</tr>
<tr>
<td>Stator tooth width</td>
<td>( w_\text{ts} ) (mm)</td>
<td>5.4</td>
</tr>
<tr>
<td>Stator yoke height</td>
<td>( h_\text{ty} ) (mm)</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3 Machine data